FINITE ELEMENT MODELING OF MAGNETICALLY-DAMPED CONVECTION DURING SOLIDIFICATION

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ABSTRACT

A fully 3-D, transient finite element model is developed to represent the magnetic damping effects on complex fluid flow, heat transfer and electromagnetic field distributions in a Sn-35.5%Pb melt undergoing unidrectional solidification. The model is developed based on our in-house finite element code for the fluid flow, heat transfer and electromagnetic field calculations. The numerical model is tested against numerical and experimental results for water as reported in literature. Various numerical simulations are carried out for the melt convection and temperature distribution with and without the presence of a transverse magnetic field. Numerical results show that magnetic damping can be effectively applied to stabilize melt flow, reduce turbulence and flow levels in the melt and over a certain threshold value a higher magnetic field resulted in a greater reduction in velocity. Also, for the study of melt flow instability, a long enough running time is needed to ensure the final fluid flow recirculation pattern. Moreover, numerical results suggest that there seems to exist a threshold value of applied magnetic field, above which magnetic damping becomes possible and below which the convection in the melt is actually enhanced.

1. INTRODUCTION

The imposition of a DC magnetic field on a moving, electroconducting fluid, such as molten metal or semiconductor melt, produces an opposing Lorantz force that results in a reduction of flow velocity in the liquid. This effect has been explored in both the semiconductor and metals industries to control natural convection and/or surface tension induced Marangoni convection as well as thermally induced flow instability during solidification processing of these melts [1-6]. A mathematical model can be of great use in developing both fundamental understanding of the fluid-magnetic field interactions in these magnetically-assisted processing systems and basic guidelines for process design, control and optimization.

This paper presents a transient 3-D mathematical model for magnetically damped fluid flow and heat transfer in a solidifying Sn-Pb melt undergoing unidirectional solidification and some preliminary results concerning the flow instability and magnetic damping effects to suppress the flow instability in the solidifying melt. The model development is based on the finite element solution of the Navier-Stokes equations, energy equations and equations for induced electric field. The model is tested against reported results from a water model and is then applied to study the fluid flow and thermal distribution in the Sn-Pb melt with and without a magnetic field.

2. MATHEMATICAL FORMULATION

Let us consider a simplified model for a undirectional solidification process as shown in Figure 1 below. In this model, the solid-liquid phase change associated with solidification is neglected. As such, the top and bottom surfaces are assumed flat and are at fixed temperatures, with the top surface at a lower temperature and adiabatic along the side wall. This represents the case where a crystal pulled from the top. The gravity points downward, which interacts with the temperature field to generate fluid flow, when a sufficiently large temperature gradient exists in the melt. A DC magnetic field is imposed along the x-direction and interacts with the melt flow to produce the magnetic damping effects to reduce the melt flow. The mathematical equations governing the fluid flow, temperature distribution and electromagnetic field distributions in the melt, along with appropriate boundary conditions, are written in the dimensionless form as follows [2, 6],

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + Pr\nabla^2 \mathbf{u} - RaPrT\mathbf{e}_z + Ha^2 Pr(-\nabla\Phi + \mathbf{u} \times \mathbf{B}) \times \mathbf{B} \tag{2}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla^2 T \tag{3}$$

$$\nabla^2 \Phi = \nabla \cdot (\mathbf{B} \times \mathbf{u}) = \mathbf{B} \cdot (\nabla \times \mathbf{u}) \tag{4}$$

The boundary conditions for the problem represent the physical constraints needed to define the numerical model, and are given by the following equations,

$$\mathbf{n} \cdot \nabla T = 0, \quad \mathbf{u} = 0, \quad \mathbf{n} \cdot \nabla \Phi = 0; \qquad r=0.5 \tag{5}$$

$$T = 0, \quad \mathbf{u} = 0, \quad \mathbf{n} \cdot \nabla \Phi = 0; \qquad z=1.0 \tag{6}$$

$T = 1, \qquad \mathbf{u} = 0, \qquad \mathbf{n}\cdot\nabla\Phi = 0; \qquad z=0 \qquad (7)$

In the above equations, **u** is the velocity, p the pressure, T the temperature, $\mathbf{g}^*$ gravity vector, Φ the electric potential, **B** the applied magnetic field vector, t the time, and **n** the outward normal. Also the following scales have been used in nondimensionalization, $L=d$, $U=\alpha/L$, $t_0=d^2/\alpha$, $P = \rho\alpha^2/d^2$ and $\Phi^*=\alpha B$, where ρ is the density, α the thermal diffusivity and B the applied magnetic field strength. The temperature is nondimensionalized as $T = (\theta - T_m)/(T_h - T_m)$ where θ is the dimensional temperature, T_h the melt inlet temperature and T_m the melting point. The nondimensionalization process also resulted in a set of nondimensional system parameters, that is, Rayleigh number, $Ra = \beta g(T_h - T_m)L^3/\nu\alpha$, Prandtl number, $Pr = \nu/\alpha$, and Hartmann number, $Ha = BL\sqrt{\sigma_m/\mu_m}$, with g being the earth gravity, β the thermal expansion, ν the kinematic viscosity, D the diffusion coefficient of the species, V_m the growth velocity, σ_m the electrical conductivity and μ_m the molecular viscosity.

Following the standard Galerkin finite element procedure, the governing equations along with the boundary conditions are cast in the following matrix form. The above matrix equations are solved using the successive substitution method and the time derivatives are approximated using the implicit finite difference scheme. Both variable time steps with automatic error tracking and time step automation and fixed time steps may be applied. The majority of the results presented in the report are calculated using fixed time steps.

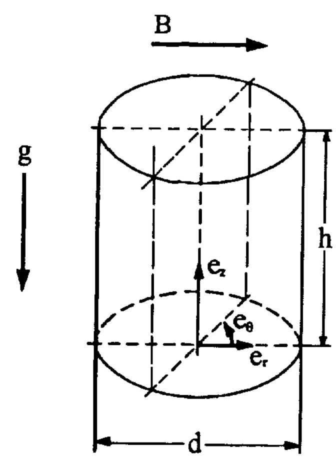


Figure 1. Schematic representation of the 3-D model for magnetic damping studies.

3. NUMERICAL RESULTS AND DISCUSSION

The above equations are solved using an in-house finite element code developed at Washington State University. The code has been tested against available analytical solutions and other numerical codes available for various 2-D and 3-D fluid flow and thermal calculations. Simulations have been carried out for magnetically damped flows in Sn-35.5%Pb melts for various operating conditions and 8-node brick elements were used in calculations. The parameters for calculations are given in [6]. To test the mesh dependency of our calculations, numerical experiments were carried out using different mesh sizes or different numbers of elements. Basically, the finite element meshes were refined until the results no longer showed any significant mesh dependency. Unless stated otherwise, all calculations for magnetic damping assumed that the magnetic field is applied in the x-direction (see Figure 1).

All the computations are transient and start with a temperature pulse or test function. The system becomes stable if the pulse effect decays and no fluid motion prevails after a sufficiently long time. Otherwise, the internal convection will evolve to a steady state with a defined fluid flow pattern. Before it was used to obtain the results for the Sn-35.5% Pb melt flows, the computer model was tested against experimental measurements of flow instability of water in a cylinder heated from below for various aspect ratios. Figure 2 shows the comparison between the computed results and the results from a finite difference model and corresponding water model reported by Muller's group [4,5]. Apparently, the results computed from the present 3-D finite element model compare very well with the existing work.

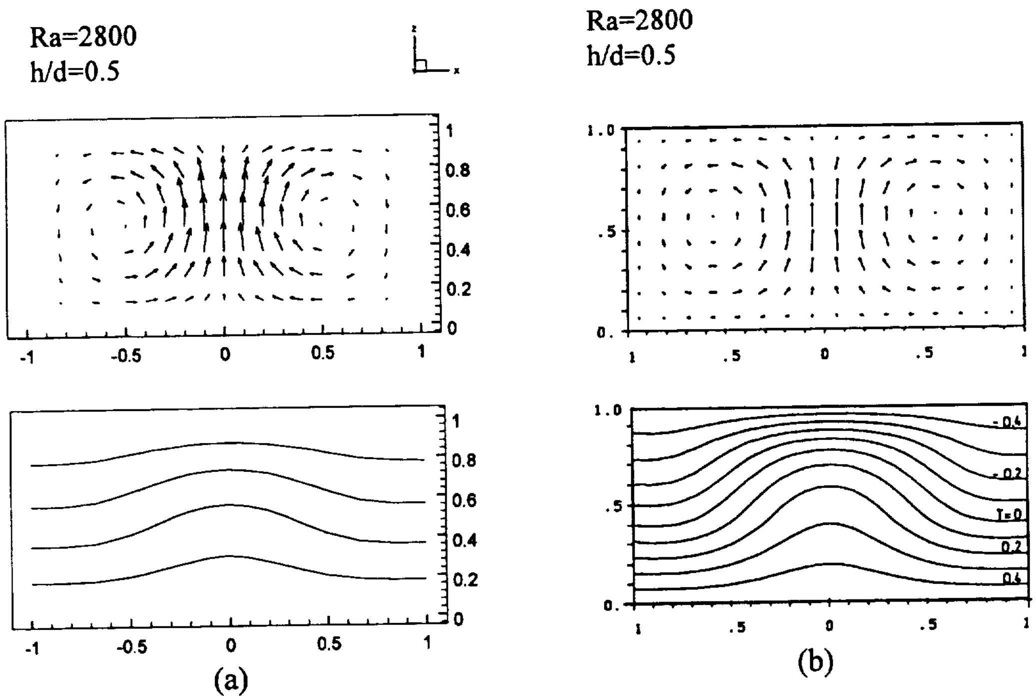


Figure 2. Comparison of our 3-D finite element model predictions for the velocity and temperature distributions (a) with those (b) reported in literature [4] for natural convection of water (Pr=6.7) in a cylindrical container heated from below.

Figure 3 illustrates the evolution of the temperature and fluid flow fields in the Sn-35.5% Pb melt in the cylindrical cavity of h/d=0.5. For simplicity, only the 2-D views (plane views) are presented. Clearly, both the fluid flow and temperature fields evolve starting from some arbitrary temperature pulse and eventually arrive at a steady state after 2000 time steps (corresponding to the real time of 3.05 seconds and dimensionless t=2.0). At the final steady state, a symmetric pattern is attained with the flow moving downward at the center and upward from the side wall. It should be stressed that for such an unstable case a final stage of the flow pattern is strongly affected by the initial temperature pulse to induce the flow. When an appropriate temperature pulse is used, a flow pattern opposite to what is shown is also possible. This is also consistent with water model measurements [4,5].

Figure 4 shows the transient development of the fluid flow and temperature distributions in the Sn-35.5% Pb melt under the same condition as in Figure 3 but with an applied magnetic field (B=0.05 T). Comparison of Figures 3 and 4 clearly indicates the magnetic damping effects. In particular, when the system reaches the steady state, the fluid flow level is damped substantially and the temperature profile is basically flat. Note also that the steady state is reached within about 800 time steps when a magnetic field is applied.

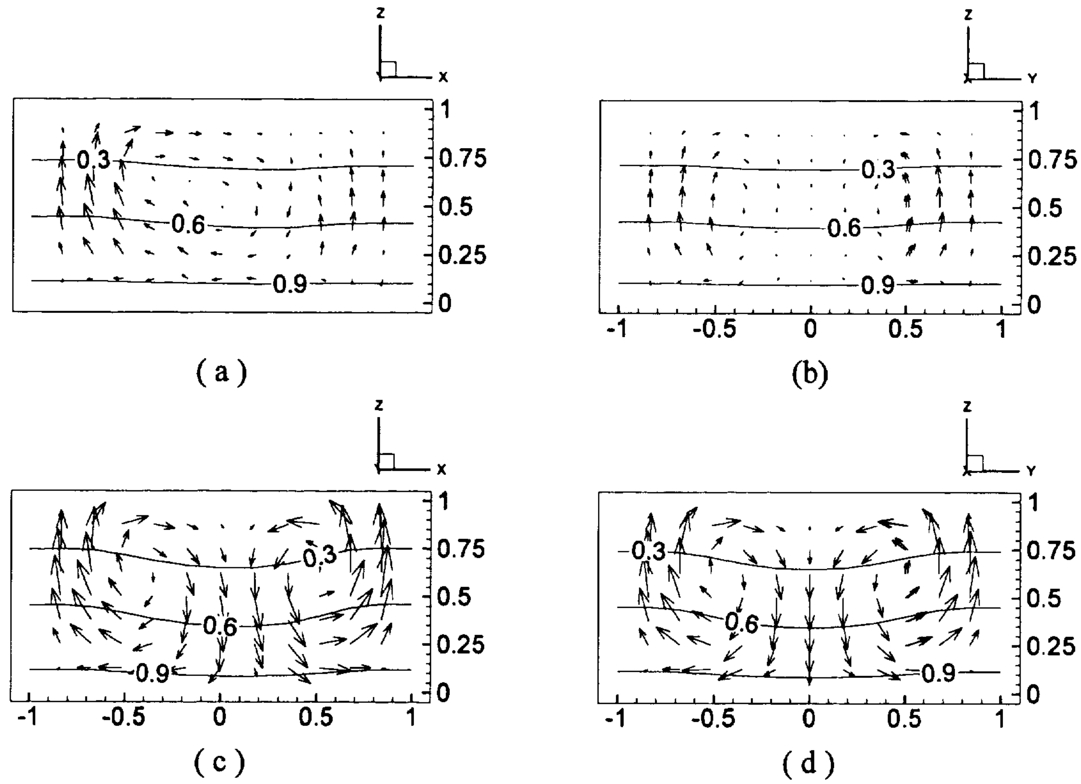
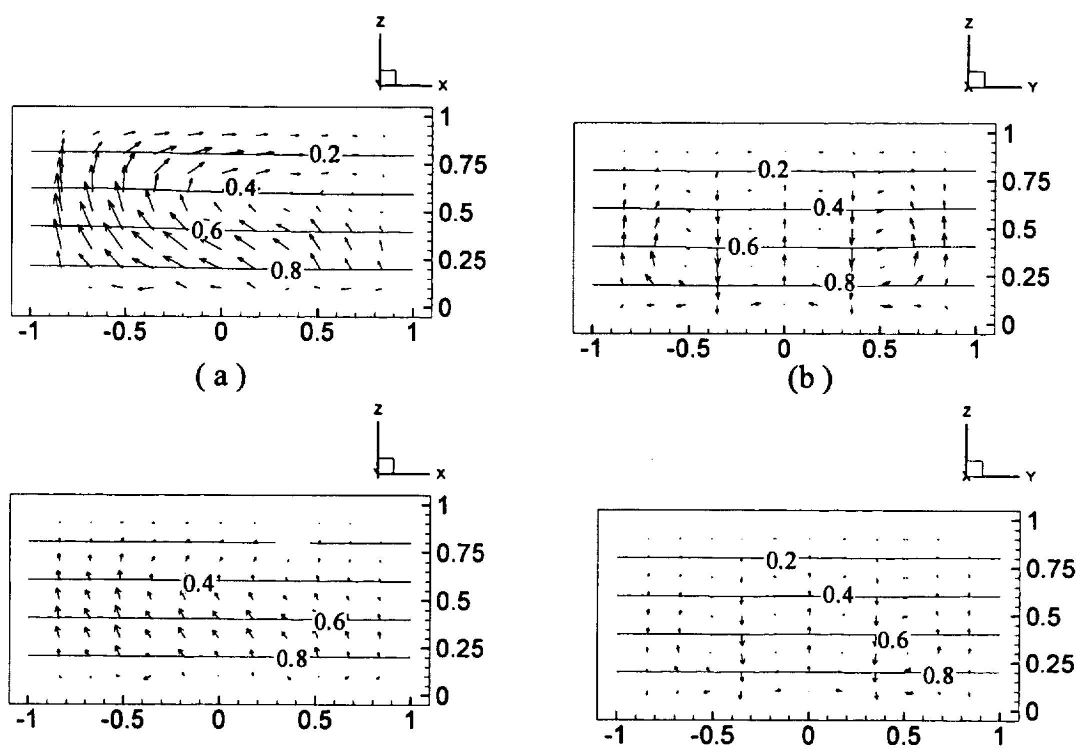


Figure 3. Transient development of flow fields and temperature profiles for Sn-35.5% Pb viewed in the x-z and y-z planes: Ra=4000 (T(bootm) = 660 K), Pr=0.0281, h/d=0.5; (a) and (b) t=0.20; (c) and (d) t=0.400; (a) and (b) t=1.000; (c) and (d)



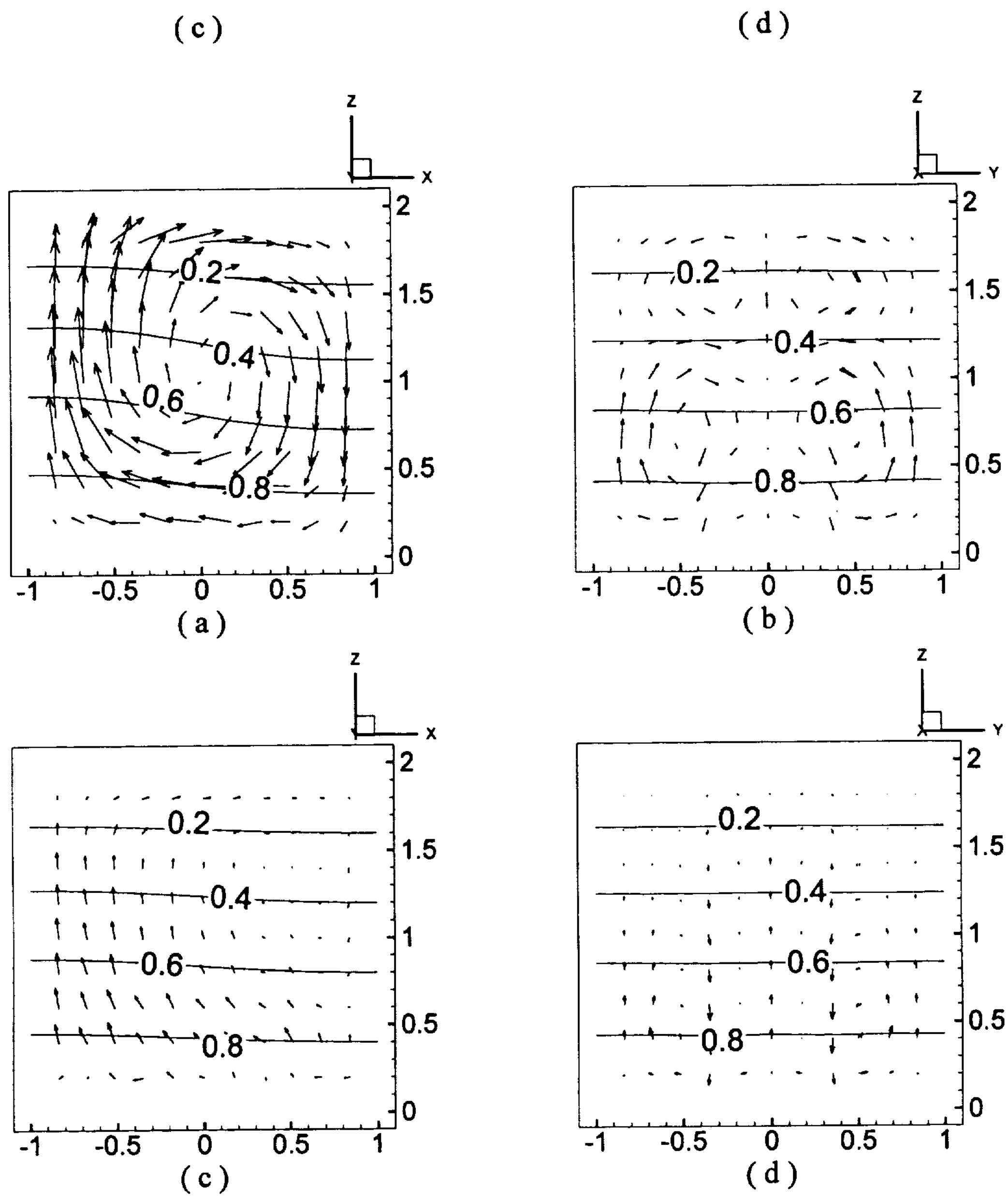


Figure 5. Fluid flow and thermal fields in the Sn-35.5%Pn melt (Pr=0.0281, Ra=10^4, h/d=1.0) without (a) and (b) and with a magnetic field B=0.05T (c) and (d). (t=0.8, Umax=0.93 for (a) and (b); t=0.8, Umax=0.42 for (c) and (d)).

Computed results were also obtained for different aspect ratios (h/d=0.5, h/d=1 and h/d=3) and different applied magnetic field strengths. Figure 5 illustrates the results for the case of h/d=1. These results show that the higher magnetic field the more damping effect on the fluid flow and temperature distributions in the Sn-35.5% Pb melt. For all the cases studied, the damping effects are sufficiently strong that the temperature distribution in the system is basically controlled by diffusion (see Figure 5). One of the important reasons for applying the magnetic damping during solidification is to reduce the possible melt flow turbulence. The current model has been applied to gain some insight into the effectiveness of magnetic damping effects on the melt flow turbulence in the system. Computed results illustrate that for the system of h/d=1 and Ra=8 x 10^5, melt flow is in the turbulent regime. With a small magnetic field present (B=0.005 T), the flow remains turbulent. With a greater magnetic field (B=0.05T), the turbulence is substantially reduced and melt flows in the laminar region. The turbulence is further reduced when B=0.5T.

Numerical results on the effect of the applied field direction also show that the magnetic field, when applied in the transverse direction provides the most effective damping effects. For symmetric flow pattern, the x- and y-direction magnetic field seems to produce similar results. For a non-symmetric flow pattern, the damping effect is stronger when the magnetic field is applied at the plane in which flow circulation is strongest [6].

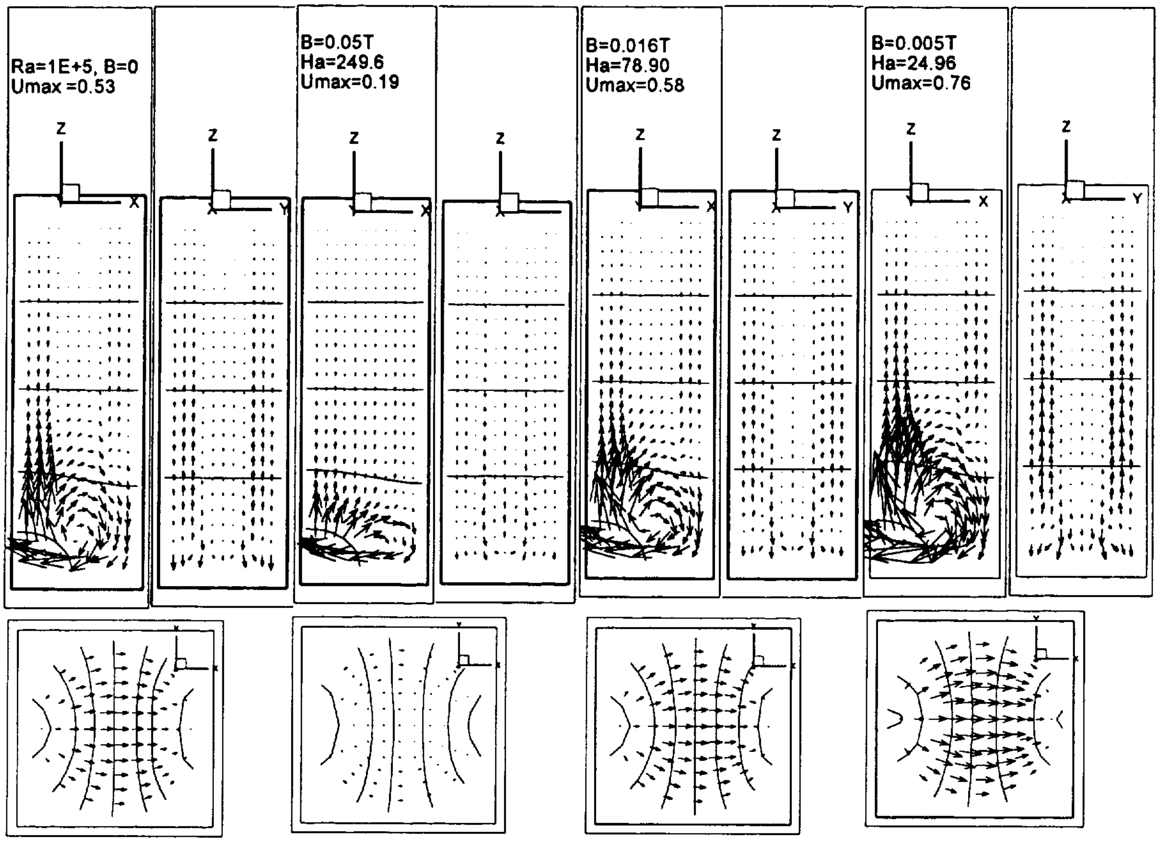


Figure 6. Effects the applied magnetic field strength on the Sn-35.5%Pn melt flow. Note that with a weak field, the convection is actually enhanced. The threshold value of the applied magnetic field to achieve damping effects seems to be slightly larger than 0.016T.

Our numerical simulations have revealed two important points that deserve a discussion. For the Sn-35.5% Pb case of h/d=0.5, it was found that calculations had to proceed for a longer time than the 600 (dimensionless) suggested in literature in order to resolve the ordered pattern [4,5]. According to Muller's numerical study of molten melts with similar properties in which numerical simulations were carried out up to 600 time steps (undimensionalized the same as this study) [5], no symmetric flow pattern would exist for this condition. Our results compare well with theirs up to the time they terminated their calculations (time step=600). However, the flow is actually still evolving and a longer run time is needed for this case to reach a steady state. For this particular case, the steady state is attained at about time step=2000 and by then a symmetric pattern appears. It is noted here that Muller's work may have used a different starting pulse because his flow pattern is just opposite to that reported here; nonetheless this would not change the conclusion that a steady state symmetric pattern may prevail if a long run time is used.

From our numerical experiments, we also found that a pure application of magnetic field does not necessarily decrease the fluid flow, as one would normally expect from simple Hartmann flow studies. On the contrary, a DC magnetic field may actually *enhance* the liquid convection in melts when a weak magnetic field is applied. A set of these results is given in Figure 6. It is clear that with B=0.005T, the flow velocity is actually increased and damping effects are not obvious until the applied magnetic field is about 0.016 T. This phenomenon seems to occur in many other cases tested. Our studies indicate that this flow augmentation seems to be dependent on the combination of the Rayleigh number of the system and the strength of the applied magnetic field. In essence, there seems to exist a magnetic field strength threshold above which the magnetic field acts to reduce the convection and below which the applied field actually enhances the convection. Our numerical experiments suggest that in general a higher threshold value is needed for a higher Rayleigh number. This is a very important issue that requires further study, considering the widespread use of magnetic damping in solidification processes.

4. CONCLUDING REMARKS

A fully 3-D, transient numerical model has been developed to represent the magnetic damping effect on complex fluid flow, heat transfer and electromagnetic field distributions in Sn-35.5%Pb melt under unidirectional solidification. The model was developed based on our in-house finite element code for the fluid flow, heat transfer and electromagnetic field calculations, which has been tested against benchmark test problems that are solved by other commercial codes as well as analytical solutions when available. The numerical model was tested against available numerical and experimental results reported for water. With the model so tested, various numerical simulations were carried out for the Sn-35.5% Pb melt. Numerical results showed that magnetic damping can be effectively applied to reduce turbulence and flow levels in the melt undergoing solidification and over a certain threshold value a higher magnetic field resulted in higher velocity reduction. For the study of the melt flow instability, a long enough simulation time has to be applied to ensure the final fluid flow circulation pattern. Moreover, numerical simulations suggest that there seems to exist a threshold value of applied magnetic field, above which magnetic damping becomes possible and below which the convection in the melt is actually enhanced.

5. ACKNOWLEDGEMENT

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