WAVES: INTERNAL TIDES

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Introduction

Oceanic internal tides are internal waves with tidal periodicities. They are ubiquitous throughout the ocean, although generally more pronounced near large bathymetric features such as mid-ocean ridges and continental slopes. The internal vertical displacements associated with these waves can be extraordinarily large. Near some shelf breaks where the surface tides are strong, internal displacements (e.g., of an isothermal surface) can exceed 200 meters. Displacements of 10 meters in the open ocean are not uncommon. The associated current velocities are usually comparable to or larger than the currents of the surface tide. On continental shelves internal tides can occasionally generate packets of internal solitons, which are detectable in remote sensing imagery. Other common nonlinear features are generation of higher harmonics (e.g., 6-hr waves) and wave breaking. Internal tides are known to be an important energy source for mixing of shelf waters. Recent research suggests that they may also be a significant energy source for deep-ocean mixing.

Internal tides were first recognized in the early part of the 20th century, yet as late as the 1950s arguments were still being waged over what causes them. Their wavelengths, generally shorter than 200 km, are poorly matched to the planetary-scale astronomical tidal potential, so the generation mechanism for surface tides appears inapplicable. Various theories invoking hypothetical resonances at inertial latitudes (where tidal and Coriolis frequencies are equal) were put forth, but they are not compelling, not least because the inertial latitude for the dominant tide M_2 is in the far polar latitudes (74.5°). The now accepted explanation for internal tides is that they are generated by the interaction of the barotropic surface tide with bottom topography. As the tide sweeps stratified water over topographic features, it disrupts normal (equilibrium) isopycnal layers, setting up pressure gradients than induce secondary internal motions at the same frequency as the tide. (More esoteric mechanisms sometimes play a role—for example, internal tides can be generated when the surface tide impinges upon an intense mesoscale eddy.)

Since internal tides are a special kind of internal wave, much of our knowledge of internal waves is immediately applicable. For example, an internal tide always displays current shear—that is, the associated horizontal current velocities change with depth—whereas the surface tide's horizontal current is independent of depth. And like other internal waves, an internal tide displays the seemingly odd property that its group velocity is perpendicular to its phase velocity. The fundamental properties of internal tides, including whether or not they even exist, are controlled by the relative magnitudes of three basic frequencies: the tidal frequency ω, the local Brunt-Väisälä or buoyancy frequency N, and the local Coriolis frequency f. Depending on which of these frequencies is highest and which lowest, internal tides may propagate freely away from their generation point, they may be reflected in some manner, or they may be evanescent. For mid-latitude semidiurnal tides, typically $f < \omega < N$, a regime allowing free propagation.

Given that the generation and propagation of internal tides depend strongly on the stratification, it is not surprising that most observations have found internal tides to be highly variable, often with pronounced seasonal variations. Sometimes they appear only during spring tides (when solar and lunar tides are at maximum). In many locations they appear randomly intermittent, evident for several days and then disappearing for reasons unknown. Some observations, primarily from the open ocean, have revealed a component that does remain temporally coherent with the astronomical potential (see below), but the dominant characteristic of internal tides in most regions is one of incoherence.

Modes and Beams

Two complementary dynamical frameworks are used for analyzing internal tides: decomposition into vertical modes and propagation along characteristics. Generally, the latter description is more useful near generation points, and the modal description more useful elsewhere, but in any particular situation one or the other approach may be advantageous.

Both approaches require knowledge of the stratification, which is usually parameterized by the buoyancy frequency N. This is the frequency with which a vertically displaced fluid element would oscillate because of restoring buoyancy forces. It is given by $N = \sqrt{(g\rho^{-1}\,\partial\rho/\partial z)}$, where g is the acceleration of gravity and ρ is the average potential density, a function of position and depth. The Coriolis frequency $f = 2\Omega \sin\phi$, where ϕ is the latitude and Ω is the Earth's sidereal rotation frequency ($7.2921 \times 10^{-5}\ \mathrm{s}^{-1}$).

Modes. The governing dynamical equations for internal tides, under linear, hydrostatic, inviscid, Boussinesq and flat-bottom assumptions, may be solved by separation of variables. The equation for the vertical displacement leads to an eigenvector problem with eigenvalue α_n^2:

$$\frac{\partial^2}{\partial z^2}G(z) + \alpha_n^2 N^2(z)\, G(z) = 0.$$

The eigenvectors $G_n(z)$ provide a complete, orthogonal basis for the internal vertical displacements. The corresponding equations for horizontal dependence yield expressions for the horizontal wavenumber k, phase velocity $c_p = \omega/k$, and group velocity $c_g = d\omega/dk$ in terms of α_n:

$$\begin{aligned} k^2 &= \alpha_n^2(\omega^2 - f^2) \\ c_p^2 &= \omega^2/[\alpha_n^2(\omega^2 - f^2)] \\ c_g &= (c_p\alpha_n^2)^{-1} \end{aligned}$$

If N is taken constant, then G_n can be found analytically: $G_n(z) = a\sin(n\pi z/D)$ for an ocean depth D. If N is taken more representative of deep-ocean conditions, with a peak at the pycnocline, then the oscillations in G_n are shifted upward (Figure 1). Notice that the displacements are small (or zero) at the top and bottom of the water column, and that each G_n has $n-1$ crossings of the origin. Horizontal velocity modes are given by $\partial G_n/\partial z$, and they have n crossings (and hence nonzero shear for all modes). Most observations of internal tides (excepting those very near the generation point) are adequately described by a superposition of a few low-order modes.

In the deep ocean typical phase speeds c_p are of order 3 $\mathrm{m\,s^{-1}}$ for n

$= 1$. Corresponding wavelengths $\lambda = 2\pi/k$ are between 100 and 200 km. Higher order modes have speeds and wavelengths given roughly by c_p/n and λ/n, respectively. On continental shelves both speeds and wavelengths may be an order of magnitude smaller. These values are for semidiurnal tides; wavelengths of diurnal tides are approximately twice as large.

From the above expressions for k and c_p it is apparent that internal tides cannot freely propagate unless $\omega > f$. They are evanescent polewards of the critical latitudes where $\omega = f$. Diurnal internal tides are therefore confined to the small band between latitudes $\pm 30°$. (In fact, unambiguous observations of diurnal internal tides are fairly rare, but this is partly due to extremely weak barotropic forcing and higher background noise levels.)

Beams. A complementary approach to modal analyses stems from, for example, the hydrodynamic equation for the two-dimensional stream function, which is hyperbolic (in spatial coordinates) and may therefore be solved by the method of characteristics. The group velocity, and hence the energy propagation, follow these characteristics, which are along lines of slope

$$c = \tan\theta = \pm\left(\frac{\omega^2 - f^2}{N^2 - \omega^2}\right)^{1/2} .$$

That is, the group velocity is confined to the direction θ relative to horizontal, independent of wavenumber. From a given internal-tide generation point, energy will thus propagate along beams at the angle θ. (Such beams comprise a large number of modes, with modal cancellations occurring outside the allowed beam.) A numerical example of beam-like propagation from a shelf break is shown in Figure 2.

Generation of internal tides is apparently especially efficient when the seafloor slopes at precisely the critical value c. Barotropic flow is then coincident with the motion plane for free internal waves, resulting in near-resonant conditions in which even smallish surface tides can generate internal tides. With nominal values of $N \sim 50\,\mathrm{cpd}$, $\omega \sim 2\,\mathrm{cpd}$, $f \sim 0.6\,\mathrm{cpd}$, then θ is $2°$. Continental slopes commonly exceed this, so c would be attained near the shelf break, as depicted in Figure 2.

When an internal wave is reflected from the ocean bottom (or ocean surface), energy propagation is still confined to the angle θ, which makes for a curious variation on the usual laws of reflection. If the wave is incident upon bathymetry that is steeper than θ (supercritical case), energy is reflected backwards into deeper water. If the bathymetry is subcritical, energy is reflected forward toward shallower water. See Figure 3. Ocean observations of this behavior are not easy to obtain, since mooring instruments must be precisely placed (depending on the ambient N); but measurements of internal tides in the Bay of Biscay have not only observed the downward energy propagation from the generation point, but also the subsequent reflection from the ocean bottom. In the Bay of Biscay, as in most places, N diminishes with depth, so θ grows larger and the beams become steeper as they approach the bottom (Fig. 3 is drawn for constant N.)

With such reflection properties, internal waves incident on a subcritical sloping bottom will be focused into the shallows (as in Fig. 3a), with energy density correspondingly intensified. The same mechanism tends to trap internal-wave energy within steep (supercritical) canyons, where the canyon sides reflect energy ever deeper, focusing it toward the canyon floor. If the floor is subcritical, then energy is futher focused toward the canyon head. Intense internal-tide currents and large kinetic energy densities have indeed been observed in canyons, and especially near canyon heads.

In the presence of internal viscosity or other dissipative mechanisms, internal tidal beams widen. Decay scales are more rapid for higher order modes, so beams tend to disintegrate rapidly into the few low-order modes that are most commonly observed.

Observations

Internal tides have been observed with a great multitude of instruments and techniques, both *in situ* and remote. Two distinctly different examples are given here which serve to highlight a number of characteristic features of deep-sea internal tides.

Moored current meters. Because of their widespread deployments, these provide probably the most common method for observing, or at least detecting, internal tides. Sufficient vertical sampling is required for decoupling the internal modes from the surface tide (and unfortunately sufficient sampling is not so common). Figure 4 is an example of adequate vertical sampling; it shows tidal current estimates extracted from moored meters near 110°W on the Pacific equator. Estimates are given for each of ten months, at ten depths throughout the water column. The current ellipses are fairly uniform below 1000 m; these depths are dominated by the stable, depth-independent currents of the surface tide. In contrast, large temporal variation, and occasionally much larger amplitudes, are evident in the shallower estimates; in these depths, where the buoyancy frequency (and its change) is maximum, the tidal signal is dominated by the internal tide. Modal analysis reveals that the internal tide is essentially random, isotropic, and without a dominant mode for these 10 months.

Such observations are characteristic of *in situ* observations of internal tides. However, in a few locations in the deep ocean, a component of the internal tide has been observed that is not so variable and that maintains phase-lock with the astronomical tide. The famous MODE experiment in the western Atlantic found that approximately 50% of the internal tide variance was temporally coherent with the astronomical tide. Such observations imply a nearly constant ocean stratification, at least to the extent that it determines generation and propagation properties.

Satellite altimetry. Recently satellite altimetry has been shown capable of providing a near-global view of the coherent component of internal tides. It does this by detecting the very small surface displacements associated with internal tides. These are given roughly by the tide's internal displacements scaled by $\Delta\rho/\rho$, the fractional difference in water density, typically of order 0.2%, thus implying surface displacements of a few cm for internal displacement of tens of meters. Altimetry detects such small waves as modulations (with wavelenths 100–200 km for internal mode 1) of the surface tide as estimated along satellite tracks. Because tides can be estimated from altimeter data only by gathering multi-year time series of elevations at a particular site, only the component of the internal tide that maintains phase-lock with the surface tide is capable of being detected.

Figure 5 gives an example of the first detection of such waves, near the Hawaiian Ridge. The waves are roughly 5 cm amplitude near the ridge and decay slowly with distance. Phase estimates (not shown) reveal clearly that the waves are propagating away from the ridge. Evidently they are created by the barotropic tide striking the ridge (at nearly right angle from the north) and generating an internal tide that propagates both northwards and southwards. The picture reveals three important aspects of deep-ocean internal tides: (1) that in some locations they maintain temporal coherence over several years, thus allowing altimetry to measure them, (2) that they maintain spatial coherence over a wide area, and (3) that they are capable of propagating hundreds to thousands of km before being dissipated. All three aspects contrast sharply with the usual picture of incoherence obtained from *in situ* observations.

Waves similar to those in Figure 5 have been detected in many regions throughout the global ocean. However, altimetry is incapable of detecting internal tides in a region where they are temporally incoherent. Such is apparently the case, for example, off the northwest European shelf, a region known for some of the largest internal tides in the world, but where the coherent signals in altimeter data are extremely weak. Internal tide studies with satellite altimetry are relatively new, and further work should reveal new facets from a global perspective.

Implications for Energetics and Mixing

Internal tides can be an important energy source for vertical mixing, especially in coastal waters where they help maintain nutrient fluxes from deep water to euphotic zones on the shelf. A good example is the Scotian Shelf off Nova Scotia where internal tides are responsible for a strip of enhanced concentrations of nutrients and biomass along the shelf break. During each tidal cycle one or two strong (50 meter) internal solitons are generated near the shelf edge, moving shoreward but dissipating rapidly (possibly within 10 km). Estimated energy fluxes of 500 $\mathrm{W\,m^{-1}}$ appear more than adequate to maintain observed nutrient supply to the mixed layer. Similar mixing mechanisms have been observed in the Celtic Sea and elsewhere.

In the open ocean it seems reasonable that internal tides dissipate by transferring energy into the internal wave continuum or by directly generating pelagic turbulence, but the dominant mechanisms associated energies are unclear. Nonlinearity is a common feature of internal tides (e.g. occurrences of higher harmonics), so "diffusion" into the continuum is possible via nonlinear (resonant triad) interactions. It is possible that bottom scattering of low-mode tides into higher modes may play a role. Wave reflections off sloping seabeds tend to intensify kinetic energy densities and may lead to shear instabilities and wave breaking. The traditional view is that both the internal-wave continuum and the pelagic turbulence and mixing are maintained by non-tidal mechanisms such as wind generation; whether internal tides play a major or minor role in this is unknown. At a minimum, improved quantitative estimates are needed for the global internal tide energy budget.

The internal tide energy budget also bears on a long-standing geophysical problem: finding the energy sink for the global surface tide. If the generation/dissipation rate for internal tides is sufficiently large, then internal tide generation conceivably supplements the the traditional sink of bottom friction in shallow seas. Dissipation rates for the surface tide are well determined at 3.7 terawatts (2.5 TW for the principal tide M_2). How much of this is accounted for by conversion into internal tides is not well determined; published estimates range from less than 100 GW to more than 1 TW. There is fairly wide agreement that generation of internal tides at continental slopes provides a fairly small energy sink. Both models and measurements suggest that typical energy fluxes at shelf breaks are of order 100 $\mathrm{W\,m^{-1}}$, leading to a global total of order 15 GW (0.015 TW). This is perhaps an underestimate because it may not fully account for shelf canyons and other three-dimensional features, but the order of magnitude seems reliable. Internal tide generation by deep-ocean topography, however, may be far more important. Recent research based on global tide models as well as on empirical estimates of tidal dissipation deduced from satellite altimetry suggests that generation of internal tides by deep sea ridges and seamounts could account for 1 TW of tidal power. Refining such estimates, and understanding the role that internal tides play in generation of the background internal-wave continuum, in vertical mixing, and in maintenance of the abyssal stratification, are some of the outstanding issues of current research.

Further reading

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FIGURE CAPTIONS

Figure 1: Baroclinic displacement modes 1, 3, and 5, computed for a bouyancy-frequency profile from the deep ocean.

Figure 2: Theoretical internal wave beam from a shelf edge. (a) Vertical displacements following beam at constant slope $\tan\theta$; (b) phase contours (in degrees) of the vertical displacements relative to the surface tide. Notice how phase propagation is at right angles to the beam; i.e. the phase velocity is perpendicular to group velocity (and to the direction of energy propagation). From Prinsenberg and Rattray (*Deep Sea Res.,* 22: 251–263, 1975).

Figure 3: Successive reflections of an internal wave along (a) a subcritical seafloor and (b) a supercritical seafloor. Arrows denote direction of energy propagation.

Figure 4: M_2 tidal current ellipses by month at each of ten depths obtained from moored current meters near 0°, 110°W. Scale bar for velocity is at upper right. Each ellipse indicates how the direction and magnitude of the

horizontal current velocity evolves over one tidal cycle. All ellipses are polarized clockwise except those marked with a plus sign. From Weisberg et al. (*J. Geophys. Res.*, 92: 3821–3826, 1987).

Figure 5: Mean elevations at the sea surface of internal tides near Hawaii, deduced from altimeter measurements of the Topex/Poseidon satellite. Positive values (north of the trackline) indicate that the internal tide's surface elevation is in phase with the barotropic tide's elevation. Scale bar for elevations at upper left. Background shading corresponds to bathymetry, with darker shading denoting shallower depths and the main axis of the Hawaiian Ridge. Only internal tides that are coherent with the surface tide over the entire measurement period (here 3.5 years) can be detected in this manner.

Keywords: internal tides, internal waves, mixing, stratification, tides, tidal currents, tidal energy budget.

Cross-references: Waves–tides; Waves–internal waves; Satellite Remote Sensing–altimetry; Turbulence and Diffusion–internal tidal mixing.

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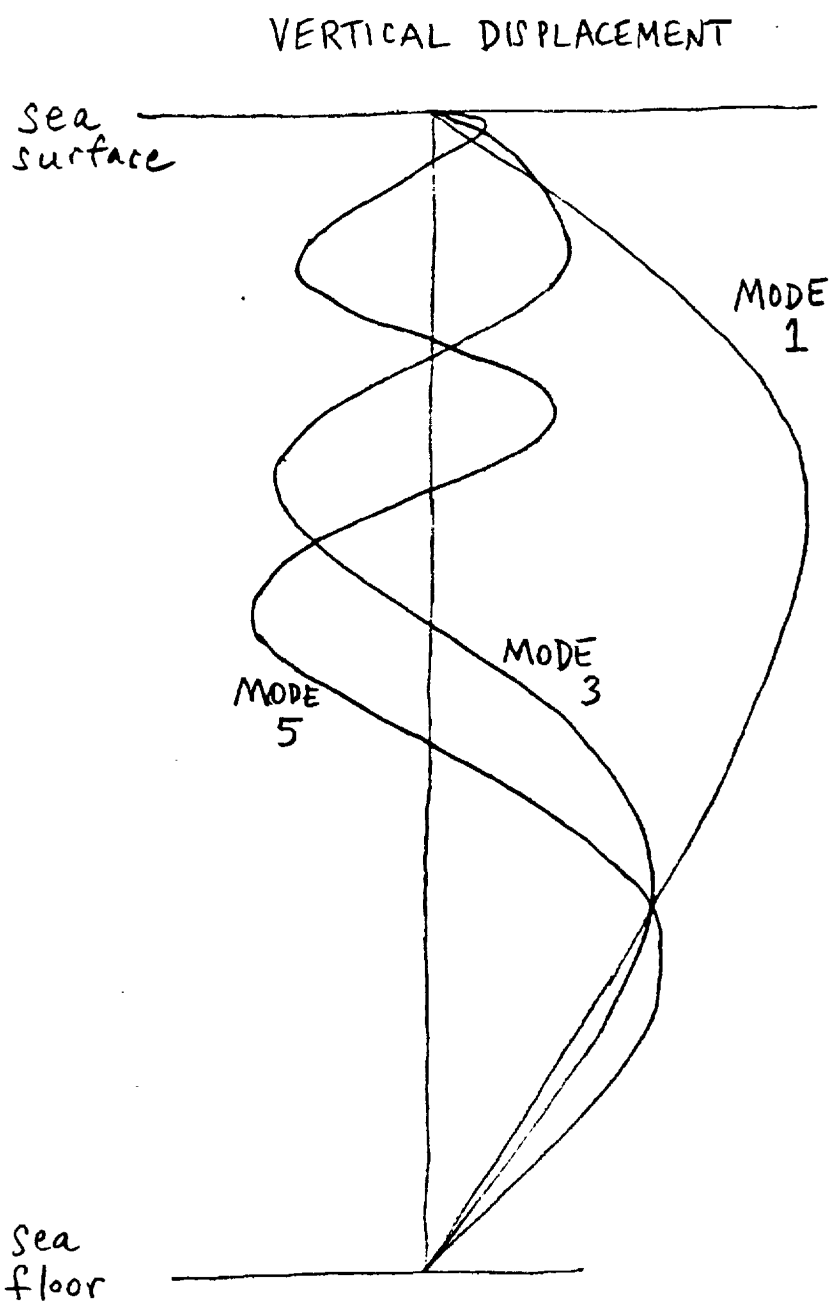
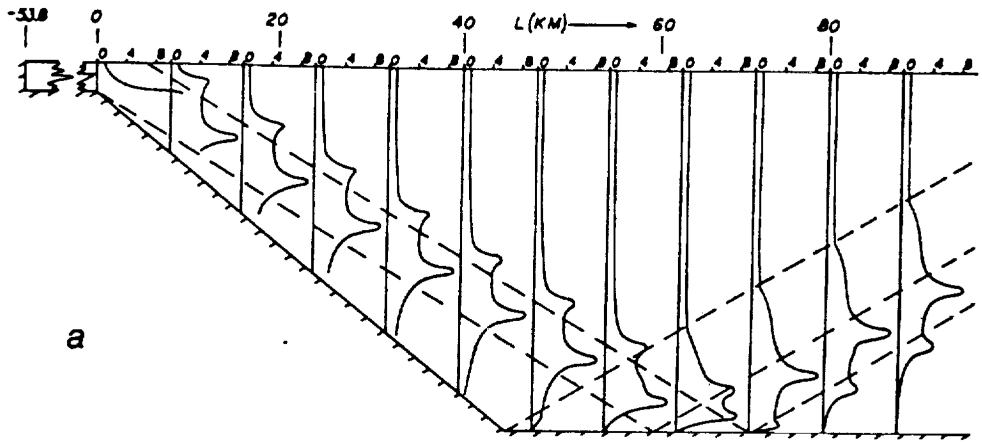


Fig 1



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a

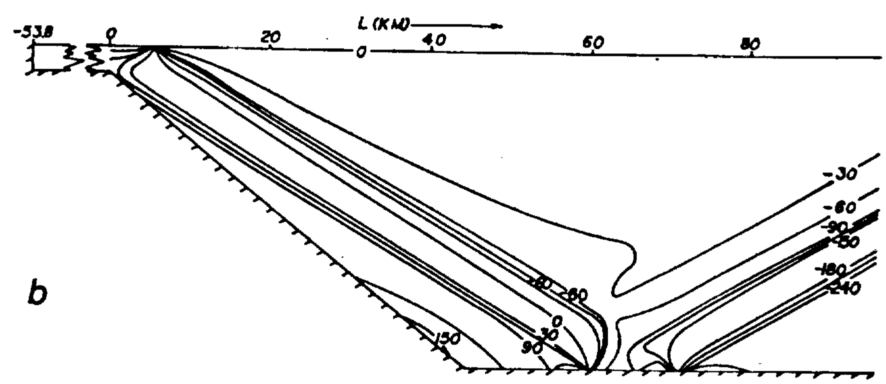

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Fig 2

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(a)

(b)

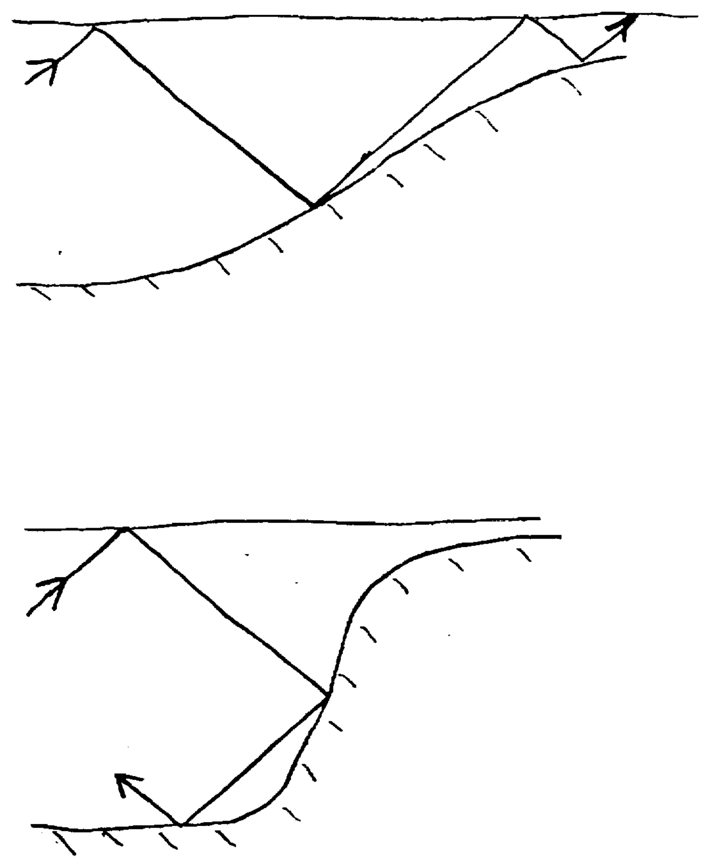

Fig 3

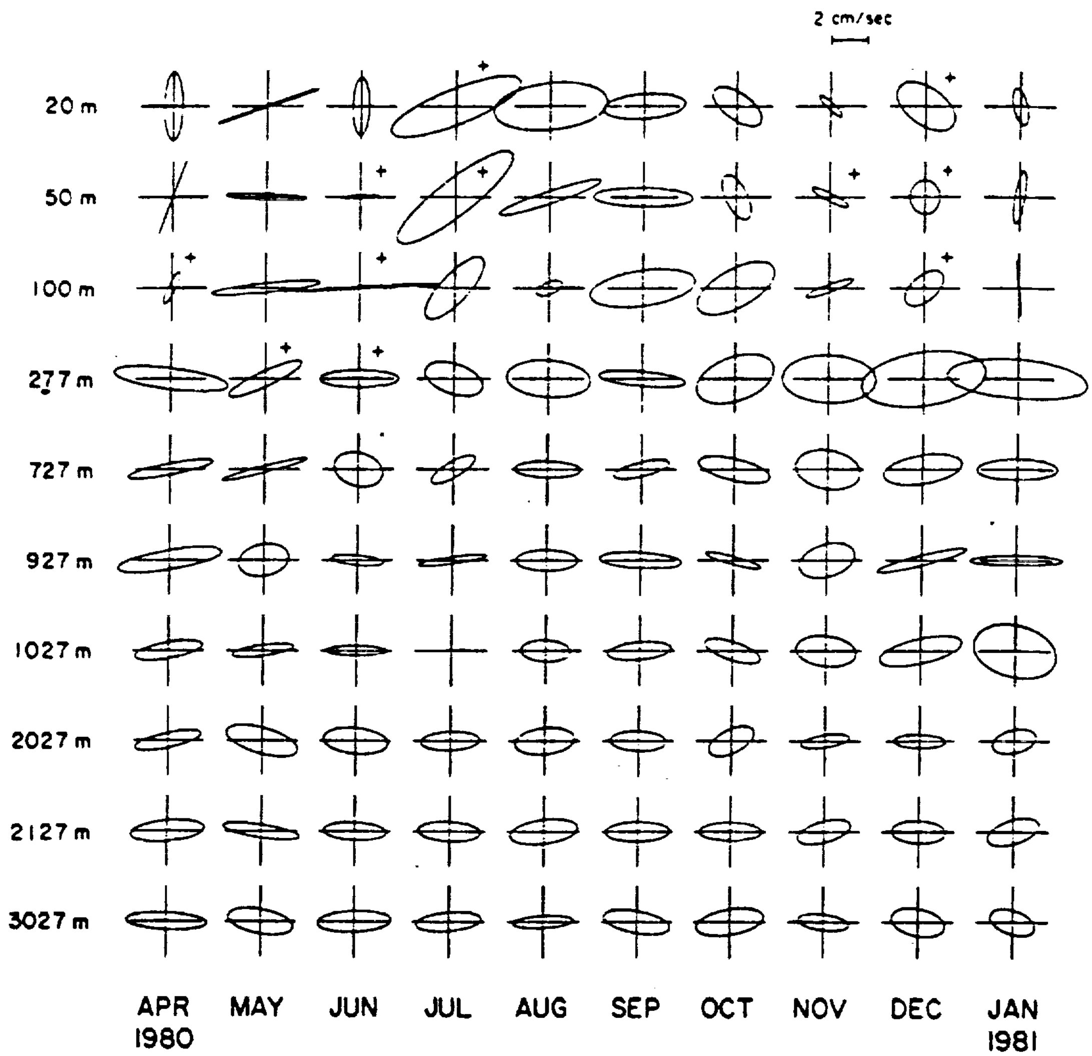

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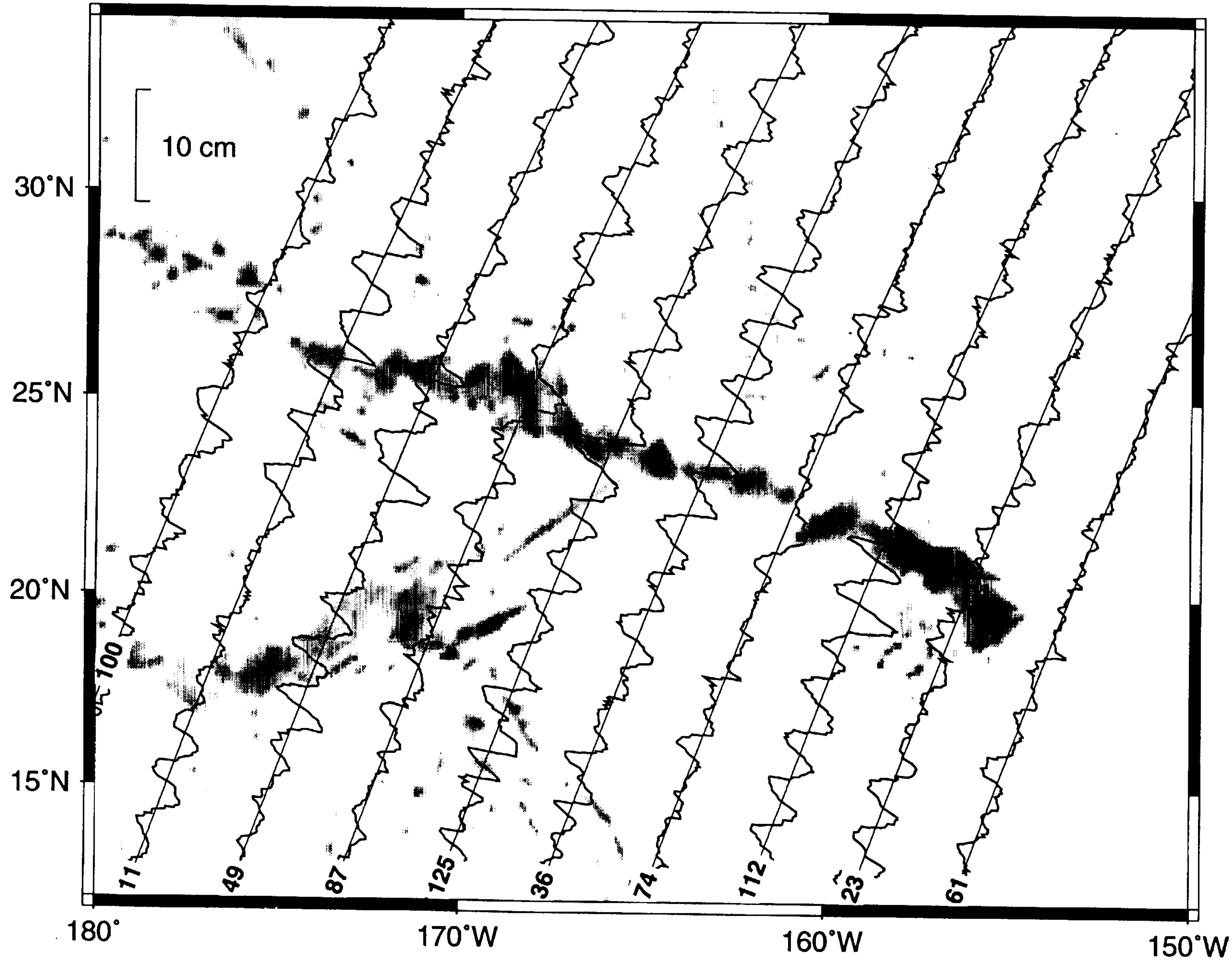
Fig 4



10 cm
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61
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Fig 5.