INTEGRATION OF THE RESPONSE SURFACE METHODOLOGY WITH THE COMPROMISE DECISION SUPPORT PROBLEM IN DEVELOPING A GENERAL ROBUST DESIGN PROCEDURE

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ABSTRACT

In this paper we introduce a *comprehensive* and *rigorous* robust design procedure to overcome some limitations of the current approaches. A *comprehensive* approach is general enough to model the two major types of robust design applications, namely,

- robust design associated with the minimization of the deviation of performance caused by the deviation of noise factors (uncontrollable parameters), AND
- robust design due to the minimization of the deviation of performance caused by the deviation of control factors (design variables).

We achieve *mathematical rigor* by using, as a foundation, principles from the design of experiments and optimization. Specifically, we integrate the Response Surface Method (RSM) with the compromise Decision Support Problem (DSP). Our approach is especially useful for design problems where there are no closed-form solutions and system performance is computationally expensive to evaluate. The design of a solar powered irrigation system is used as an example. Our focus in this paper is on illustrating our approach rather than on the results *per se.*

I OUR FRAME OF REFERENCE

The fundamental principle in robust design is to improve the quality of a product by minimizing the effects of variation without eliminating these causes. There are two broad categories, of robust design based on the source of variation:

Type I Robust Design - minimizing performance deviation caused by deviation of noise factors (uncontrollable parameters).

Type II Robust Design - minimizing performance deviation caused by control factor deviation (design variables).

The Taguchi method was developed for Type I applications (Phadke, 1989). This is basically a two-part orthogonal array for experimental design using the signal-to-noise-ratio as an optimization criterion. Although applications of the Taguchi method have yielded useful results, this method has also been criticized by statisticians (Box, 1988, Nair, 1992, Palatino, Tsui, 1992). Tsui, 1992, argues that many of Taguchi's statistical methods, e.g., orthogonal arrays, linear graphs and accumulation analysis, are not statistically efficient and proposes to overcome these limitations using alternative experimental formats, design criteria, analysis techniques, graphical tools and optimization strategies. Box, 1988, points out that there are various mathematical difficulties/ requirements associated with the use of signal-to-noise-ratio. Alternatively, Welch, et al., 1990, propose combining control and noise factors into a single array thus modeling the response rather than expected loss, and approximating a prediction model for loss based on the fitted-response model, this approach is further developed by Shoemaker, et al., 1991. The proposed modifications to the Taguchi method, however, involve a *single* performance metric. In our opinion, as there are multiple objectives to be satisfied in design it follows that there must be multiple aspects to quality. This opinion is reinforced by some preliminary insight we gained using quality engineering techniques to reduce the number of the trajectory simulations of a LifeSat space vehicle (Mistree, et al., 1993b). Further, we investigated the benefits and limitations of using the Taguchi method in the early stages of design by applying it to concurrent concept selection and system synthesis of a solar powered irrigation system (Chen, et al., 1994). We assert that quality loss depends on several quality characteristics and each of these may have different degrees of importance. Related to this view, Otto and Antonsson, 1991, argue the necessity of incorporating constraints in robust design.

On another front, nonlinear programming methods can be used for both Type I and II applications. Ramakrishnana and Rao, 1991, formulate the robust design problem as a nonlinear optimization problem with Taguchi's loss function as the objective. They emphasize the numerical evaluation of total quality loss but fall short in addressing the nature of variations. Sundaresan et al., 1993, incorporate a Sensitivity Index (SI) in the optimization procedure to determine a "robust optimum". The SI is defined as the root mean square value of the difference between the value of the function for the worst case and the target value. In this case, it is difficult to determine the weighting factors for target performance and variance in the objective function as designers may be uncertain about the ranges for the actual performance and its variance. When using nonlinear programming methods, Sundaresan et al. use concepts from the design of experiments to approximate the variation in performance at each iteration. However, this is computationally inefficient for large, analysis-intensive and computationally-intensive problems.
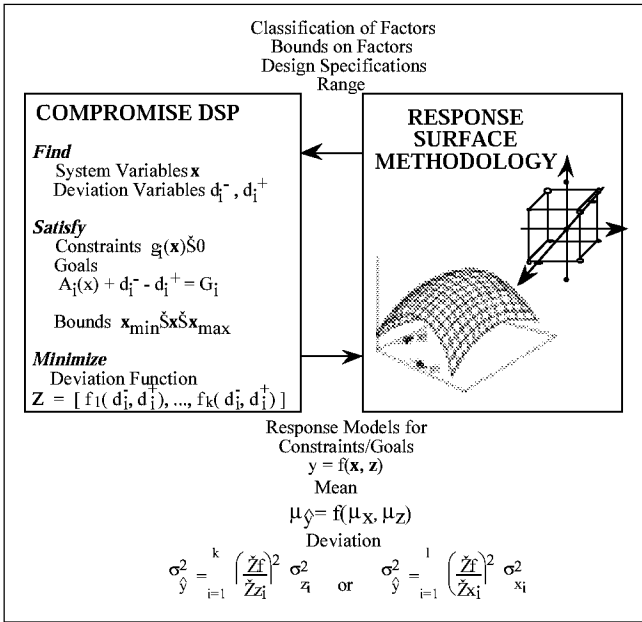

Classification of Factors
Bounds on Factors
Design Specifications
Range

COMPROMISE DSP

Find
  System Variables x
  Deviation Variables d_i^-, d_i^+

Satisfy
  Constraints $g_i(x) \check{S} 0$
  Goals
  $A_i(x) + d_i^- - d_i^+ = G_i$

  Bounds $x_{min} \check{S} x \check{S} x_{max}$

Minimize
  Deviation Function
  $Z = [f_1(d_i^-, d_i^+), ..., f_k(d_i^-, d_i^+)]$

RESPONSE SURFACE METHODOLOGY

Response Models for
Constraints/Goals
$y = f(x, z)$
Mean
$\mu_{\hat{y}} = f(\mu_x, \mu_z)$
Deviation
$$\sigma_{\hat{y}}^2 = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial z_i}\right)^2 \sigma_{z_i}^2 \quad \text{or} \quad \sigma_{\hat{y}}^2 = \sum_{i=1}^{l} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2$$

FIGURE 1. INTEGRATION OF RESPONSE SURFACE METHODOLOGY WITH THE COMPROMISE DSP

Our focus in this paper is to introduce a *comprehensive* and *rigorous* robust design procedure to overcome some of these limitations. By *comprehensive*, we mean that our approach is general enough to model the two major types of robust design applications, namely, robust design to minimize the deviation of noise factors (uncontrollable parameters), AND robust design to minimize deviation of control factors (design variables). Thus, our approach is general enough to be used for both Type I and II robust design. We achieve *mathematical rigor* by using principles from the design of experiments and optimization. Specifically, we integrate the Response Surface Method (RSM) with the compromise Decision Support Problem (DSP), Figure 1. The

Response Surface Method (RSM) is a collection of statistical techniques which support the design of experiments and fitting a response model (Khuri and Cornell, 1987). The compromise DSP is a multiobjective decision model (Mistree, et al., 1993a) which enables a designer to determine values of design variables which satisfy a set of constraints to achieve a set of goals. The objective is to minimize the deviations of different goals from target values using lexicographic minimization (Ignizio, 1985).

We build approximating functions using the response surface model to relate each response to all important control- and noise–factors. Then we solve the problem using the compromise DSP and these approximations rather than solving the problem directly using a computationally expensive analysis model. In this way, the variance of performance can be predicted instead of running multiple experiments at each iteration of the solution. The compromise DSP provides a generic approach to achieve robust design by enabling a designer to find values of control factors to achieve a performance which is as close as possible to the targets and to minimize variations around the targets, subject to engineering constraints. Using the compromise DSP, it is possible to address individually the issues of maximizing the intensity of the signal on target and minimizing variation. These become separate goals in the multiobjective compromise DSP. This approach helps a designer focus on individual contributions to mean and variation and to identify parameters which affect the attainment of specific goals.

In the following, we discuss the two major types of robust design. We then show the integration of Response Surface Method and the compromise DSP into a general procedure for robust design. The design of a solar powered irrigation system is used as an example. Our focus in this paper is on explaining the approach rather than on the results *per se.*

II A DESCRIPTION OF OUR APPROACH

Two Types of Robust Design

The concepts behind the two major types of robust design application are illustrated in Figure 2. On the left-hand side of Figure 2, we use a P-diagram (Phadke, 1987) to represent different types of parameters in robust design, their relationships with the whole system, and thus the differences in source of variation in response for Type I and Type II applications. *Control factors (x)* are parameters which can be specified freely by a designer; *noise factors (z)* are parameters that are not under a designer's control; and the *signal factor (M)* is the intended value for the *response (y)* of a product/process. In Type I applications, the variance of the response is caused by variations in the noise factor, z. Type II is different from Type I in that its input does not include a noise factor. The variation in performance is caused solely by variations in control factors or design variables in the region $\pm \Delta x$.

On the right hand side of Figure 2 we present a schematic of the different concepts behind the two types robust design. Taguchi's robust design method is a Type I method and is highlighted in the upper right block of Figure 2. In the Taguchi method, a designer adjusts control factors, *x,* to dampen the variations caused by the noise factor, z. The two curves represent

the performance variation as a function of noise factor when x is at two different levels, x = a and x = b. If the design objective is to achieve a performance as close as possible to the target, M, the designs at both levels are acceptable because their means are the target M. However, introducing robustness, when x = a, the performance varies significantly with the deviation of noise factor, z, however when x = b, the performance deviates much less. Therefore, x = b is a better than x = a as a design solution which dampens the effect of the noise factors.
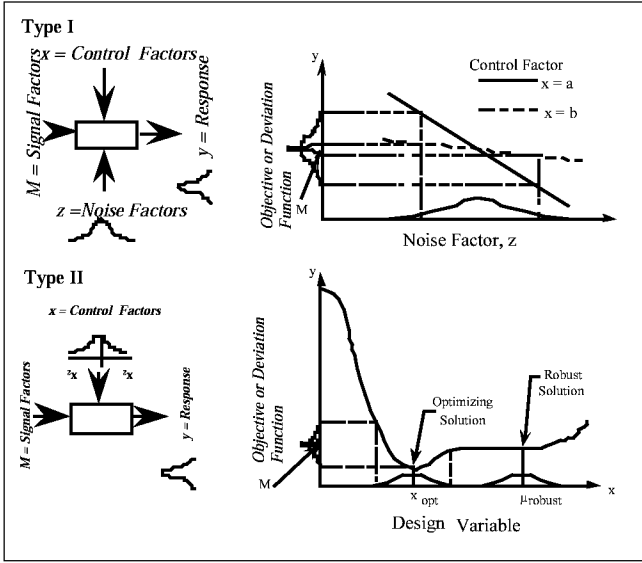

FIGURE 2. A COMPARISON OF TWO TYPES
OF ROBUST DESIGN

The concept behind Type II robust design is represented in the lower right block of Figure 2. For purposes of illustration, assume that performance is a function of only one variable, x. Generally, in this type of robust design, to reduce the variation of response caused by the deviations of design variables, instead of seeking the optimum value, a designer is interested in the flat part of a curve near the performance target. If the objective is to move the performance function towards target M and if a robust design is not sought then obviously the point x = a is chosen. However, for a robust design, x = b is a better choice. This is because if design variables vary within the region $\pm\Delta x$ of their means, the resulting variation of response of the design at x = b is much smaller than that at x = a, while the means of the response at two designs are close.

Although the concepts behind the two major types of robust design are different, robust design is always concerned with aligning the peak of the bell shaped response distribution with the targeted quality, (bring the mean to the target) and making the bell shaped curve thinner (reduce the variance). This makes it possible to develop a general procedure for robust design in which these objectives are modeled as separate goals in a compromise DSP.

Developing a General Robust Design Procedure

As mentioned in Section I, to overcome the limitations of Taguchi's loss model approach, we use the Response Surface Method, RSM, starting with the response-model approach (Welch, et al., 1990: Shoemaker, et al., 1991). The major elements of the response-model approach are:
- Combining control and noise factors in a single array instead of using Taguchi's inner- and outer-array approach,
- modeling the response itself rather than expected loss,
- approximating a prediction model for loss based on the fitted-response model.

The RSM, is used to support the design of experiments and fitting a response model. Instead of using Taguchi's orthogonal array as the combined array for experiments, other fractional factorial designs or central composite designs can be chosen. In general, the response-model postulates a single, formal model of the type

$$\hat{y} = f(\mathbf{x}, \mathbf{z}), \qquad (1.1)$$

where $\hat{y}$ is the estimated response scalar and $\mathbf{x}$ and $\mathbf{z}$ are the settings of the control and noise variables. The surface model is usually modeled with a second order equation, from this, it is possible to estimate response mean and variability. For Type I applications in which the deviations of noise factors are the sources of variation:

Mean of the response $\qquad \mu_{\hat{y}} = f(X, \mu_z) \qquad (1.2)$

Variance of the response $\quad \sigma_{\hat{y}}^2 = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial z_i}\right)^2 \sigma_{zi}^2 \qquad (1.3)$

where μ is the mean value, k is the number of noise factors in the response model and σ_{zi} is the standard deviation of each noise factor. In Eqn. (1.3), it is assumed that the noise variables are independent. In Type II robust design, i.e., when the deviations of control factors are the source of variation, Eqns. (1.2) and (1.3) are modified:

Mean of the response $\qquad \mu_{\hat{y}} = f(x) \qquad (1.4)$

Variance of the response $\quad \sigma_{\hat{y}}^2 = \sum_{i=1}^{1} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2. \qquad (1.5)$

Using the response-model approach, robust design for either type of application can be achieved by bringing the mean given in Eqn (1.2) or (1.4) to the target and minimizing the variance, Eqn. (1.3) or (1.5). Because the classical RSM is restricted to unconstrained searching for a local optimum of a single response (or the variance of that response) a compromise Decision Support Problem, is introduced to handle multiple quality aspects and engineering constraints.

A typical structure for a compromise DSP is presented in Figure 1 and described in detail elsewhere (e.g., Mistree, et al., 1993a). The compromise DSP assists the designer in modeling and handling multiple trade-offs simultaneously. In the compromise DSP, each goal, A_i, has two associated deviation variables d_i^- and d_i^+ which indicate the extent of the deviation from the target G_i. To effect a solution, on the basis of preference, goals may be rank-ordered into priority levels using the lexicographic minimum. (Ignizio 1985).

In Figure 1, the integration of the compromise DSP and RSM are illustrated. When formulating the compromise DSP, different parameters are classified as system variables, system parameters (constant variables) or system performances (constraints/goals). All the decisions in classifying parameters are used to help building response surface models. Using the quality engineering terminology, response surface models are developed for responses (system performances) as functions of control factors (system variables) and noise factors (system parameters with deviation) over the region of interest (defined by the bounds of design variables). Based on a response surface model, the mean and variance of a response can be derived as functions of control factors according to the sources of variation (either the variation of noise factors or control factors themselves). The explicit forms of these equations are passed to the compromise DSP where they are either used as constraints or goals. Instead of using a single criterion and minimizing the signal-to-noise ratio, an important aspect of this work is to permit a designer to independently bring the mean on target and minimize the variation around this target. This is accomplished by modeling these objectives separately.

We believe this is a comprehensive robust design procedure. If a surface model is established over the space formed by all the parameters involved, the same response surface model can be used for either type of compromise DSP. It is not difficult to imagine that these two formulations can be combined to achieve a robust design where there are deviations of both control and noise factors.

We use a worst case scenario instead of *expected values* of system performance for constraints to study variations of constraints caused by the deviations of controllable or uncontrollable parameters. Using the *expected values* of system performance for constraints might permit the constraints to be violated, especially when the constraints are close to being active at the solution point. Using the worst case for the constraints, instead of using $E[g_j(x,z)] \leq 0$,

$$E\,[g_j(x,\,z)] + \sum_{i=1}^{k} \left| \frac{\partial g_j}{\partial z_i} \Delta z_i \right| \leq 0, \text{ or } E\,[g_j(x)] + \sum_{i=1}^{l} \left| \frac{\partial g_j}{\partial x_i} \Delta x_i \right| \leq 0 \quad (2.1)$$

where Δz_i is the deviation of a noise parameter and Δx_i, the deviation of a sensitive design variable (control factor); k and l are the numbers of noise parameters and sensitive design variables, respectively. Next, we use the design of a solar-powered irrigation system to demonstrate our approach.

III A SOLAR POWERED IRRIGATION SYSTEM DESIGN EXAMPLE

Previously, we have used the solar powered irrigation system design as an example to show the application of Taguchi techniques to concurrent subsystem embodiment and system synthesis (Chen, et al., 1994). We used the inner- and outer-arrays for experiments and the "signal-to-noise ratio" as our measurement and found that there are many limitations of the Taguchi's approach, Section 1. It was difficult to use this method when making tradeoffs among multiple quality characteristics and incorporating constraints and factor interactions. In this paper, using the same problem, we show how our newly proposed method can be used to overcome these difficulties and extend the

robust design applications. The layout of the solar-powered irrigation system is shown in Figure 3. It is assumed that parabolic trough N-S tracking is used for solar collection and water is the working fluid. The aim in the preliminary system design is then to determine:

- maximum operating press., Rankine Cycle, X_A (MPA)
- maximum operating temp., Rankine Cycle, X_B (°K)
- maximum temp. drop in the solar collectors, X_C (°K)
- working fluid flow rate, X_D (kg/s)

The design must meet the specifications:

- pumped load (power output) must be as close as possible to 20 kW
- overall efficiency must be maximized as close as possible to 20%
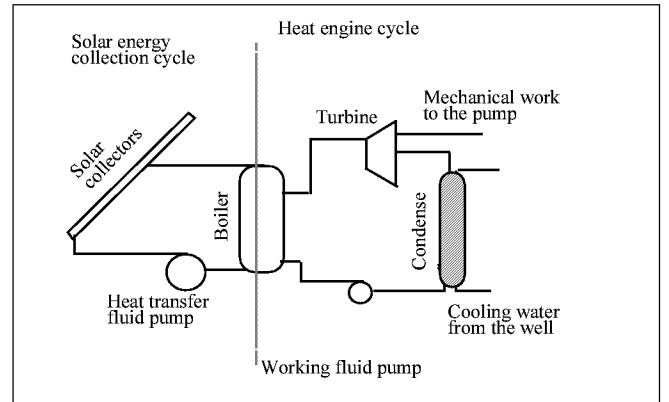- economic benefits must be maximimzed as near as possible to $150,000.

FIGURE 3. SYSTEM CONFIGURATION

The design also must satisfy thermal system requirements, i.e., the maximum temperature in the cycle has to be smaller than the stagnation temperature at all times (The stagnation temperature cannot be less than the maximum temperature corresponding to the maximum pressure). In this work, we combine thermodynamic property prediction software (Shamsundar, 1989) with an economic analysis routine (Bascaran, 1990) for our computer simulation module. Simulation inputs include definitions for components, system parameters involved in system synthesis and the operating environment. System performances, e.g., cycle efficiency, power output, total efficiency and economic benefits are generated. An important aspect of thermal system design is to reduce the variance in system performance caused by the deviation of an uncontrollable operating environment (noise factors). In our case, the noise factors are ambient temperature and level of insolation. Type I robust design can be applied. Another important application of robust design is to reduce the variance of performance caused by deviations in control factors considering that there may be adjustments to these factors in the later stages of design. In other words, it is necessary to find a flat region of design space rather than the optimum. We demonstrate how these two types of robust design can be formulated and solved.

The first step is to classify the parameters as control factors, noise factors, held-constant factors or responses. The ranges of these parameters and their relationship to the whole solar powered irrigation system are shown in Figure 4. There are four controllable design parameters, representing decisions on thermal cycle design. Two noise factors (uncontrollable parameters) are associated with the unstable operating environment. The held-constant factors are the constant parameters and the four responses are system performances which can either be considered as goals or constraints. After classifying parameters, it is necessary to fit a surface model for each of the four responses over the space formed by control factors, X_A, X_B, X_C, and X_D, and noise factors, Z_a and Z_B.

We use the fitted response surface model and a derived function of variance of the response as input to the compromise DSP instead of using an expensive computer simulation at each solution iteration. Using this approach, there is always a tradeoff between the number of experiments used and the accuracy of the estimated model. Studies show that the response-model approach relies on the adequacy of simple models over a small parameter range. A common strategy is to use sequential experimentation, i.e., perform low order screening experiments first over the whole design space to identify appropriate regions of interest, and then build a higher order response model over reduced regions, probably also using a reduced number of factors. Here we assume the reduced parameter ranges are X_A [2.0-3.0] MPA, X_B [450-520] $^{\circ}$K, X_C [550-650] $^{\circ}$K, and X_D [0.01-0.05] kg/s. The ranges for the noise factors Z_a and Z_b are as before.

Noise Factors
Ambient temperature Z_a [293–303] K
Level of insolation Z_b [800–1000] W/m^2

Control Factors
Maximum pressure
X_A [0.3–3.0] MPA
Maximum temperature
X_B [450–520] K
Max. collector temperature
X_C [520–800] K
Working fluid flowrate
X_D [0.01–0.2] kg/s

Solar Powered Irrigation System

Response
Power output
Overall efficiency
Yearly savings
Stagnation temperature

Held-Constant Factors
Solar collector concept
Working fluid concept
Turbine efficiency
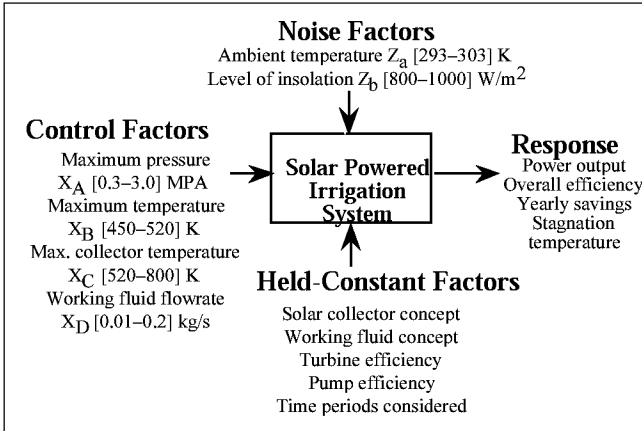Pump efficiency
Time periods considered

FIGURE 4. CLASSIFICATION OF PARAMETERS

From low-order screening experiments, we also notice that noise factor Z_b and its associated interactions have no effect on the power and economic benefits while control factor X_D and associated interactions do not influence the behavior of efficiency and stagnation temperature. Thus Central Composite Design (CCD) with 43 experiments for 5 factors can be used to construct quadratic response surfaces for each response (Montgomery, 1991). The CCD is composed of a 2^5 (32) factorial design, 10 "star" points and 1 center point. The suggested value for α, a parameter varied to meet conditions of equal precision of

estimation in all directions, is 2.378. Based on the results of the experiment, the least squares method from regression analysis is used to fit a quadratic surface model:

$$f(x_1,...x_n) = \beta_0 + \beta_1 x_1 + ... + \beta_n x_n + \gamma_1 x_1^2 + ... + \gamma_n x_n^2$$
$$+ \beta_{12} x_1 x_2 + ... + \beta_{n,n-1} x_{n-1} x_n$$

A summary of the response surface models for the four system performances is presented in Table 1. Variables X_A, X_B, X_C, X_D, Z_a and Z_b used in the equations are all normalized from -1 to 1. One benefit of using normalized variables is that the total response variance can be calculated easily when there are several sources of variance. Based on the coefficients of the response model, a designer can see clearly the significance (contributions) of different terms (linear, quadratic and interaction terms). The functions in Table 1 are the reduced models with some trivial effects ignored, e.g., those items with very small coefficients. If, after normalization, the contribution of an item is less than 1%, it is considered trivial. The sequence of items is arranged in descending order of significance.

The normalized quadratic surface model can also help to identify potential control factors for reducing the noise effects by examining the control-noise interactions. For example, to reduce the variance of power caused by the noise factors Z_a, the potential adjustment factors are X_A, X_B and X_D. Further, using X_D is most effective because the coefficient of $X_D Z_a$ is greater. Based on the response models, we can formulate compromise DSPs for Type I and Type II robust design applications.

TABLE 1. RESPONSE MODELS FOR PERFORMANCE

	Response Model
Power (objective) kW	= 24.947 + 16.011X_D + 1.306X_B + 0.820$X_B X_D$ - 0.785Z_a - 0.497$X_D Z_a$ + 0.228$X_A X_B$ + 0.212X_A - 0.153X_B^2 + 0.130$X_A X_D$ - 0.110X_A^2 - 0.034$X_B Z_a$ + 0.002$X_A Z_a$ (3.1)
Efficiency (objective)	= 0.18507 + 0.01041X_A + 0.00380Z_b - 0.00366Z_a - 0.00350X_C - 0.00157X_B (3.2)
Savings (objective) $	= 174695.73 + 112114.69X_D + 9133.80X_B + 5733.05$X_B X_D$ - 5487.76Z_a - 3478.84$X_D Z_a$ + 1586.48$X_A X_B$ + 1486.84X_A - 1067.42X_B^2 + 916.26$X_A X_D$ - 768.90X_A^2 - 242.88$X_B Z_a$ + 152.40$X_A Z_a$ (3.3)
Tstag K (constraint)	= 1873 - 5Z_a - 175 Z_b (3.4)

Compromise DSP Type I Reducing the variance of system performance caused by the deviation of uncontrollable operating environment is associated with the Type I robust design application, Table 2. Using the compromise DSP, the problem becomes how to choose X_A, X_B, X_C and X_D to reduce the influence from Z_a and Z_b while keeping the performance as close as possible to the targets.

To handle multiple objectives simultaneously, each objective is modeled as a goal. Tradeoffs are implemented by minimizing the total deviation function. As we choose to address separately the issues of bringing the mean to target and minimizing the deviation, for three system performance targets, there will be six goals. f_1- f_6 in the deviation function represent different priority levels, assigned to the six goals. Initially, all goals are at the same priority level and assigned equal weights. The deviation function

becomes:

$$Z = 0.167(d_1^- + d_1^+) + 0.167(d_2^- + d_2^+) + 0.167(d_3^-) +$$

$$0.167(d_4^- + d_4^+) + 0.167(d_5^-) + 0.167(d_6^- + d_6^+) \qquad (3.12)$$

Further information about the formulation and solution of compromise DSPs is available in Mistree, et al, 1993b.

TABLE 2. THE COMPROMISE DSP TYPE I

Given
- Response model of Power, Overall Efficiency, Savings and Stagnation Temperature as functions of X_A, X_B, X_C, X_D, Z_a and Z_b.
- The mean and variance of noise factors, $\mu_{Za} = 0$, $\mu_{Zb} = 0$, $\sigma_{Za} = 1/3$, $\sigma_{Za} = 1/3$ (all normalized).
- Target for Power and its variance, $T_{Powmean} = 20$ kW (the nominal the better), $T_{Powvar} = 0$.
- Target for Efficiency and its variance, $T_{Effmean} = 20\%$ (the bigger the better), $T_{Effvar} = 0$.
- Target for Savings and its variance, $T_{Savmean} = \$150000$ (the bigger the better), $T_{Savvar} = 2.60E6(\$^2)$ (equivalent to $1612 standard deviation)

Find
- The values of control factors
  X_A, Normalized cycle maximum pressure —
  X_B, Normalized cycle maximum temperature —
  X_C, Normalized collector maximum temperature —
  X_D, Normalized working fluid flow rate —
- The values of deviation variables d_i^-, d_i^+ (i =1, 6) –

Subject to
- The constraint:
  The stagnation temperature cannot be less than the maximum temperature in cycle
  $E[Tstag(X_A, X_B, X_C, X_D, Z_a, Z_b)] - |\partial Tstag/\partial Z_a|*3\sigma_{Za} - |\partial Tstag/\partial Z_b|*3\sigma_{Zb} \geq X_B*520 \qquad (3.5)$
  where E represents the statistical *expected value* of a function
- The goals:
  Achieve the mean of power as close as possible to the target
  $Pow(X_A, X_B, X_C, X_D, \mu_{Za}, \mu_{Zb})/T_{Powmean} - d_1^+ + d_1^- = 1, \qquad (3.6)$
  Minimize the variance of power
  $(\partial Pow/\partial Z_a)^2 + (\partial Pow/\partial Z_b)^2 - d_2^+ + d_2^- = 0, \qquad (3.7)$
  Maximize mean of overall efficiency to the target
  $Eff(X_A, X_B, X_C, X_D, \mu_{Za}, \mu_{Zb})/T_{Effmean} - d_3^+ + d_3^- = 1, \qquad (3.8)$
  Minimize the variance of overall efficiency
  $(\partial Eff/\partial Z_a)^2 + (\partial Eff/\partial Z_b)^2 - d_4^+ + d_4^- = 0, \qquad (3.9)$
  Maximize mean of savings to the target
  $Sav(X_A, X_B, X_C, X_D, \mu_{Za}, \mu_{Zb})/T_{Savmean} - d_5^+ + d_5^- = 1, \qquad (3.10)$
  Minimize the variance of savings
  $[(\partial Sav/\partial Z_a)^2 + (\partial Sav/\partial Z_b)^2]/T_{Savvar} - d_6^+ + d_6^- = 0, \qquad (3.11)$
- Bounds on the design variables:
  $-1 \leq X_A \leq 1 \qquad -1 \leq X_B \leq 1$
  $-1 \leq X_C \leq 1 \qquad -1 \leq X_D \leq 1$
- $d_i^+ \cdot d_i^- = 0$, with d_i^+, $d_i^- \geq 0$

Objective
  Minimize the total deviation function
  $Z = [f_1(d_1^- + d_1^+), \ f_2(d_2^- + d_2^+), \ f_3(d_3^-), \ f_4(d_4^- + d_4^+), f_5(d_5^-), \ f_6(d_6^- + d_6^+)]$

  (3.12)

Eqns. (3.5-3.11) can be expanded using the response surface models in Table 1, e.g., using Eqn. (3.1), Eqn. (3.6) becomes:

$(24.947 + 16.011*X_D + 1.306*X_B + 0.820*X_B*X_D +$

$0.228*X_A*X_B + 0.212*X_A - 0.153*X_B^2 + 0.130*X_A*X_D -$

$0.110*X_A) / T_{Powmean} - d_1^+ + d_1^- = 1 \qquad (3.14)$

Similarly Eqn. (3.7) can be expanded:

$(-0.785 - 0.497*X_D - 0.034*X_B + 0.002*X_A)^2 - d_2^+ + d_2^- = 0 \qquad (3.15)$

In the compromise DSP, to calculate the performance variance, instead of using $\sum_{i=1}^{k} \left(\frac{\partial f}{\partial z_i}\right)^2 \sigma_{z_i}^2$, only $\sum_{i=1}^{k} \left(\frac{\partial f}{\partial z_i}\right)^2$ is required. When there are several noise factors, z_i, after normalization, σ_{zi} has the same value if the standard deviation is proportional to the range of design variables. This is equal to 1/3 assuming that the deviation follows a normal distribution. With respect to the goal of minimizing variance, it is desirable that the target approach zero. However, it is not always advisable to use zero as a target because tremendous resources are often required to reduce variance to zero. In this work, we first calculate a reasonable minimum value for the variance of a response without considering any other goals and use this value as the target for the variance goal. For Power and Efficiency zero becomes the target. The target for variance of economic benefits is $2.60E+6$ (^{2}).

The compromise DSP in Table 2 is solved using the DSIDES software (Mistree, et al., 1993a) and the results are:
$X_A = 3.0MPA$, $X_B = 450K$, $X_C = 550K$ and $X_D = 0.0258(kg/s)$ At this solution point, the values of mean and variance of each performance with the total deviation function value, Z, equal to 0.02577:

$Pow_{mean} = 20.0027$ (kW) $\quad Pow_{var} = 0.0068$ (kW^2)
$Eff_{mean} = 19.45\%$ $\qquad\qquad Eff_{var} = 4.93E-06$
$Sav_{mean} = 141143$ ($) $\qquad Sav_{var} = 3.3E+06$ (^{2})

To confirm the results, we compare values from the response model with those from a complete simulation, Table 3.

TABLE 3. CONFIRMATION OF RESULTS

	Values from the second-order response model	Values from original simulation	Error
Mean Power (kW)	20.003	19.680	1.6%
Mean Efficiency	19.45%	19.45%	0
Mean Savings ($)	141143	137795	2.4%

TABLE 4. GOAL PRIORITIES FOR MEAN AND VARIANCE

	Scenario I	Scenario II	Scenario III
Preemptive formulation of deviation function	Level 1–all goals equal weights	Level 1–Mean Level 2–Variance	Level 1–Variance Level 2–Mean

System variables			
X_A (MPA)	3.0	3.0	3.0
X_B (K)	450	450	450
X_C (K)	550	550	550
X_D (kg/s)	0.0258	0.0258	0.01
Mean on Target			
Power (Target 20 kW)	20.0027	20.0027	8.04411
Efficiency Target 20%)	19.45%	19.45%	19.45%
Savings (Target $150,000)	141143	141143	57397.9
"Reducing Variance"			
Power (Target 0)	0.0068	0.0068	9.405E-03
Eff. (Target 0)	4.93E-06	4.93E-06	4.927E-06
Savings (Target 2.6E+06)	3.3E+06	3.3E+06	460465
Dev. Function Value, Z			
level 1	0.02577	0.0286434	0.0031053
level 2		0.0222924	0.410113

In addition to the comparison at solution points, we also use other random points to check the accuracy of our predictor. This shows that the accuracy of our response model is satisfactory within the range of study. This increases our confidence in using the response model as a fast analysis module for further study.

When considering multiple aspects of quality, designers may have different preferences for whether it is more important to bring the mean on target or reduce variation. In the compromise DSP, different design scenarios can be modeled by assigning goals different priority levels. In Table 4, the results of three different preemptive formulations of deviation function are given.

It is interesting that the result for Scenario II are the same as those for Scenario I, Table 4. The means obtained are very close to their targets in Scenario I, so there is not much room for improvement in Scenario II. In Scenario III, variation has been reduced significantly, however, this is achieved by sacrificing the goals of bringing the mean on target. Studies on different goal priority levels for mean and variance can help designers obtain a clearer idea of what is possible and understand the major obstacles to achieving a robust design.

Compromise DSP Formulation II

Given that X_A, X_B, X_C and X_D are system-level design variables, adjustments are possible after the design at the component level is finalized. Type II robust design reduces the variance of performance due to deviations of control factors. Suppose that within the design parameter range, e.g., X_A [2.0-3.0MPA], X_B [450-520°K], X_C [550-650°K], and X_D [0.01-0.05kg/s], the possible deviation for each variable is constant, after normalization, say $\Delta X_A = \Delta X_B = \Delta X_C = \Delta X_D = 0.2$, then the compromise DSP for robust design in this new situation is given in Table 5. We assume that noise factors Z_a and Z_b become constants at their means $Z_a = 298K$ and $Z_b = 900W/m^2$.

The target for each variance goal is the best possible value obtained without considering the other goals. We use three different deviation functions to test solutions for different scenarios, Table 6. In Scenario I, all the goals are at the same priority level with equal weights; while in Scenario II, six goals are on the same priority level with higher weights for reducing the variance. In Scenario III all six goals have the same priority

but higher weights are assigned to bring the mean on target.

Based on the results from Table 6, we observe:

- The results for X_B, X_C, and X_D are stable under different scenarios. However, X_A jumps between its lower and upper bounds. Thus a higher cycle pressure is preferred for bringing the mean on target, and a lower cycle pressure is preferred for reducing variance.
- For Power, the goal of bringing the mean on target can always be achieved exactly, while for Efficiency, the mean on target goal must be sacrificed if reducing the variance is assigned a higher weight. For Saving, it is always difficult to bring the mean on target.
- The goals associated with reducing variance can be better achieved when they are given a higher weight. The variance in efficiency is always a small constant because it is not a function of design variables.

These observations match the mathematical structure of the response model, e.g., for the second observation, the equation of efficiency indicates that the larger X_A, the easier it is to bring it on target. However, this contradicts the notion of reducing variance of the other responses. If variance reduction is assigned a higher weight, an appropriate trade-off results.

TABLE 5. THE COMPROMISE DSP TYPE II

Given

- Response model of Power, Overall Efficiency, Savings and Stagnation Temperature as functions of X_A, X_B, X_C, X_D, Z_a and Z_b.
- $Z_a = Z_b = 0$
- $\Delta X_A = \Delta X_B = \Delta X_C = \Delta X_D = 0.2$
- Target for Power and its variance,
  $T_{Powmean} = 20$ kW (the nominal the better),
  $T_{Powvar} = (3)^2$ kW^2.
- Target for Efficiency and its variance,
  $T_{Effmean} = 20\%$ (the bigger the better), $T_{Effvar} = 0$.
- Target for Savings and its variance,
  $T_{Savmean} = \$150000$ (the bigger the better),
  $T_{Savvar} = (21107)^2$ $\$^2$

Find

- The values of control factors
  X_A, Normalized cycle maximum pressure, X_B, Normalized cycle maximum temperature, X_C, Normalized collector maximum temperature X_D, Normalized working fluid flow rate
- The values of deviation variables d_i^-, d_i^+ (i =1, 6)

Subject to

- The constraints:
  The stagnation temperature cannot be less than the maximum temperature in cycle
  $E[Tstag(X_A, X_B, X_C, X_D, Z_a, Z_b)] - |\partial Tstag/\partial X_A| * \Delta X_A - |\partial Tstag/\partial X_B| * \Delta X_B - |\partial Tstag/\partial X_C| * \Delta X_C - |\partial Tstag/\partial X_D| * \Delta X_D \geq (X_B + 1 * \Delta X_B) * 500$ (3.16)

  where E represents the statistical *expected value* of a function
- The goals:
  Achieve the mean of power as close as possible to the target
  $Pow(X_A, X_B, X_C, X_D, Z_a, Z_b)/T_{Powmean} - d_1^+ + d_1^- = 1$, (3.17)
  Minimize the variance of power
  $[(\partial Pow/\partial X_A)^2 * \Delta X_A^2 + (\partial Pow/\partial X_B)^2 * \Delta X_B^2 + (\partial Pow/\partial X_C)^2 * \Delta X_C^2 + (\partial Pow/\partial X_D)^2 * \Delta X_D^2]/T_{Powvar} - d_2^+ + d_2^- = 0$, (3.18)
  Maximize mean of overall efficiency to the target
  $Eff(X_A, X_B, X_C, X_D, Z_a, Z_b)/T_{Effmean} - d_3^+ + d_3^- = 1$, (3.19)
  Minimize the variance of overall efficiency
  $(\partial Eff/\partial X_A)^2 * \Delta X_A^2 + (\partial Eff/\partial X_B)^2 * \Delta X_B^2 + (\partial Eff/\partial X_C)^2 * \Delta X_C^2 + (\partial Eff/\partial X_D)^2 * \Delta X_D^2 - d_4^+ + d_4^- = 0$, (3.20)
  Maximize mean of savings to the target
  $Sav(X_A, X_B, X_C, X_D, Z_a, Z_b)/T_{Savmean} - d_5^+ + d_5^- = 1$, (3.21)

Minimize the variance of savings
$$[(\partial Sav/\partial X_A)^2 * \Delta X_A{}^2 + (\partial Sav/\partial X_B)^2 * \Delta X_B{}^2 + (\partial Sav/\partial X_C)^2 * \Delta X_C{}^2$$
$$+ (\partial Sav/\partial X_D)^2 * \Delta X_D{}^2] / T_{Savvar} - d_6{}^+ + d_6{}^- = 0, \quad (3.22)$$

- Bounds on the design variables
$$-1 \le X_A \le 1; -1 \le X_B \le 1; -1 \le X_C \le 1; -1 \le X_D \le 1$$

- $d_i{}^+ \cdot d_i{}^- = 0$, with $d_i{}^+, d_i{}^- \ge 0$

Objective

Minimize the total deviation function
$$Z = [f_1(d_1{}^- + d_1{}^+), \ f_2(d_2{}^- + d_2{}^+), \ f_3(d_3{}^-), \ f_4(d_4{}^- + d_4{}^+), f_5(d_5{}^-), \ f_6(d_6{}^- + d_6{}^+)]$$
$$(3.23)$$

IV CLOSURE

A *comprehensive* and *rigorous* procedure is introduced to overcome some of the limitations of existing experiment-based and nonlinear programming approaches to robust design. We incorporate useful ideas from the existing approaches and to handle two different types of robust design. The integration of Response Surface Methodology with the compromise DSP appears to have several advantages:

- Engineering constraints can be introduced and there is great flexibility for studying tradeoffs among multiple design objectives.
- The interaction effects and nonlinear effects can be considered using the response surface model, e.g., quadratic surfaces. Compared to Taguchi's linear model approach, this approach yields more accurate results.
- Response surface models can serve as fast analysis modules for different types of robust design application. By introducing a parameter as a variable in the response surface model, we extend our scope of study.
- The compromise DSP addresses individually the issues of bringing the mean on target and reducing variation. This provides designers more flexibility to make decisions based on different robust design criteria.

Robust design procedures can be applied to various kinds of engineering problems. We demonstrate the application of this method to the design of a thermal system. In the future, we will apply quality concepts to improving efficiency and effectiveness in designing more complex systems. Potential applications include decomposition of complex system design into subproblems using Type I robust design, and the generation of flexible design specifications for increasing design freedom using the Type II robust design. We are in the process of developing a Robust Concept Exploration Model, RCEM, which is used to support the rapid evaluation of different design alternatives to generate competitive top-level design specifications. Improving the response surface model by narrowing the design space to the region of greatest interest is an important part of this procedure.

TABLE 6. RESULTS FOR TYPE II

	Scenario I	Scenario II	Scenario III
System variables			
X_A (MPA)			
X_B (K)	2.974	2.000	2.973
X_C (K)	450	450	450
X_D (kg/s)	550	550	550
	0.0256	0.0255	0.0256
Mean			
Power	20.0	20.0	20.0
Efficiency	20%	17.97%	20%
Savings	140085	140062	140085
Variance			
Power	9.492	9.132	9.492
Efficiency	4.92E-06	4.92E-06	4.92E-06
Savings	4.654E+08	4.477E+08	4.654E+08
Deviation Function Value Z	0.0276	0.0213	0.0253

Preemptive formulations of the deviation function in Table 6:

Scenario I: $Z = 0.167(d_1{}^- + d_1{}^+) + 0.167 (d_2{}^- + d_2{}^+) + 0.167(d_3{}^-)$
$$+ 0.167(d_4{}^- + d_4{}^+) + 0.167 (d_5{}^-) + 0.167 (d_6{}^- + d_6{}^+)$$

Scenario II: $Z = 0.1(d_1{}^- + d_1{}^+) + 0.233(d_2{}^- + d_2{}^+) + 0.1(d_3{}^-)$
$$+ 0.233(d_4{}^- + d_4{}^+) + 0.1(d_5{}^-) + 0.233(d_6{}^- + d_6{}^+)$$

Scenario III: $Z = 0.233(d_1{}^- + d_1{}^+) + 0.1(d_2{}^- + d_2{}^+) + 0.233 (d_3{}^-)$
$$+ 0.1(d_4{}^- + d_4{}^+) + 0.233 (d_5{}^-) + 0.1(d_6{}^- + d_6{}^+)$$

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