

# APPLICATION OF ANFIS TO PHASE ESTIMATION FOR MULTIPLE PHASE SHIFT KEYING

**JEFFREY T. DRAKE and NADIPURAM R. PRASAD**

*New Mexico State University*

*Department of Electrical and Computer Engineering*

*Las Cruces, NM USA 88003*

*jeff.drake@gsfc.nasa.gov, rprasad@nmsu.edu*

## ABSTRACT

The paper discusses a novel use of Adaptive Neuro-Fuzzy Inference Systems (ANFIS) for estimating phase in Multiple Phase Shift Keying (M-PSK) modulation. A brief overview of communications phase estimation is provided. The modeling of both general **open-loop**, and **closed-loop** phase estimation schemes for M-PSK symbols with unknown structure are discussed. Preliminary performance results from simulation of the above schemes are presented.

**KEYWORDS:** ANFIS, phase estimation, phase synchronization, coherent communication, neural-fuzzy inferencing, M-PSK

## INTRODUCTION

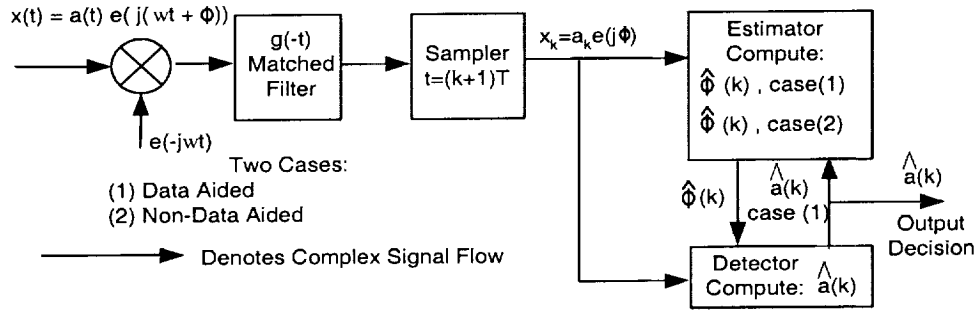
Synchronization (estimation) is a critical function in any modern coherent digital communications system. In synchronous digital transmissions the information is conveyed by uniformly spaced pulses, and the received signal is completely known except for the data symbols and a group of variables referred to as *reference parameters*. Though it is the ultimate task of the receiver to generate an accurate replica of the symbol sequence with no regard to the reference parameters, this is only possible by exploiting knowledge of these parameters. Coherent demodulation is used with pass-band digital communications. In coherent communications the baseband data signal is derived making use of a local reference with the same frequency and phase as the incoming carrier. Carrier or phase synchronization is the function of aligning the phase and frequency of the receiver oscillator with that of the transmitter oscillator when the information is modulated onto the carrier.

The coherent receiver structure that forms the basis for our work is shown in Figure 1. Referring to Figure 1, for carrier phase estimation we represent the received signal by the sufficient statistic, namely,

$$x(k) = a_k e^{j\phi} + n(t) \quad (1)$$

Here,  $a_k$  is the complex valued information symbol,  $\phi$  is the unknown carrier phase, and  $n(t)$  is complex additive white Gaussian noise (AWGN). It is shown by Van Trees in [4] that  $\{x(k)\}$  forms a set of sufficient statistics for estimating the phase  $\phi$ .

Historically the approaches to synchronization structure can be divided into two categories, which we denote as ad-hoc structures and derived or analytical structures.



**Figure 1.** Coherent receiver structure

Arguably, the most commonly used analytical method for phase estimation is that of maximum likelihood (ML). The ML estimator has several important theoretical advantages (“best” w.r.t. chosen criterion, can attain lower bound). The ML estimate is given by the *likelihood equation*

$$\left. \frac{\partial \ln p_{\mathbf{x}|\phi}(\mathbf{X}|\phi)}{\partial \phi} \right|_{\phi=\hat{\phi}_{ML}(\mathbf{x})} = 0 \quad (2)$$

Unfortunately, in most practical cases where digital modulation is present, derived structure criterion leads to highly non-linear systems, which in general cannot be solved for the optimum solution and only implicit solutions are arrived at. To find explicit solutions approximations must be made, which leads to results that are valid only for ranges of the parameters and are in essence sub-optimal to the true ML or maximum *a posteriori probability* (MAP) estimates.

The *true* ML estimator of carrier phase for M-PSK modulation obtained in terms of the received signal over the immediate past  $N$  symbols is nonimplementable [2]. It is the very fact that the true ML estimator is nonimplementable and highly non-linear that suggests possible application of soft computing methods as an alternative for phase estimator design. Neural networks and adaptive fuzzy systems have the ability to approximate the model structure using only target system sample data. Detailed insight into the target system helps set up the initial model structure, but it is not mandatory.

## ANFIS BASED PHASE ESTIMATION

This section briefly describes a hybrid platform that incorporates fuzzy/neural structures namely the, Adaptive Network Fuzzy Inference System (ANFIS). In the following sections we discuss its application to phase estimation for M-PSK in three forms; general unknown *open-loop* structure, known *open-loop* structure, and unknown *closed-loop* structure.

### Introduction to ANFIS

ANFIS refers to a class of adaptive network-based fuzzy inference system which are functionally equivalent to fuzzy inference systems [1]. Specifically the ANFIS system

of interest here is functionally equivalent to the Sugeno first-order fuzzy model. A common Sugeno rule set with two fuzzy If-Then rules is the following:

Rule 1: If  $x$  is  $A_1$  and  $y$  is  $B_1$ , Then  $f_1 = p_1x + q_1y + r_1$ ,

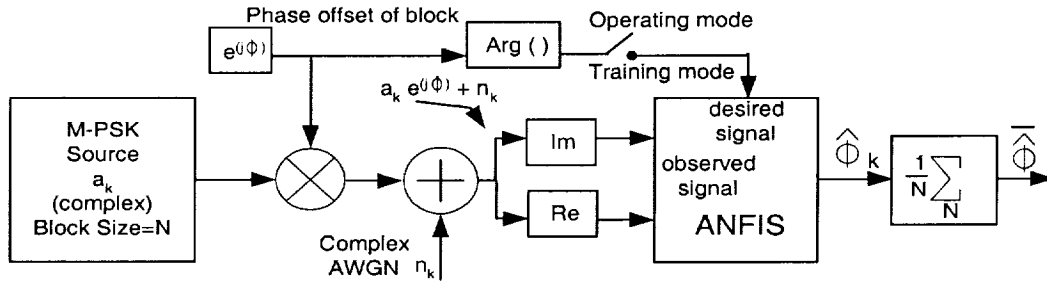
Rule 2: If  $x$  is  $A_2$  and  $y$  is  $B_2$ , Then  $f_2 = p_2x + q_2y + r_2$ ,  $f = \frac{w_1f_1 + w_2f_2}{w_1 + w_2}$

Through the use of the *Hybrid Learning Algorithm* [1], which combines gradient descent and the least-squares method, the equivalent fuzzy inference system can be rapidly trained and adapted. ANFIS is a *universal approximator* and as such is capable of approximating any real continuous function on a compact set to any degree of accuracy [1]. Thus in parameter estimation where the given data is such that it associates measurable system variables with an internal system parameter, a functional mapping may be constructed by ANFIS that approximates the process of estimation of the internal system parameter.

### ANFIS M-PSK Open-Loop Phase Estimation - Unknown Structure

We employ ANFIS to develop a model for an open-loop estimator of phase using no *a-priori* knowledge of the signal and limited intuition from experts. The ANFIS system is trained with the observed signal and the target signal which consist of; phase rotated packets of length  $N$  containing M-PSK symbols plus AWGN, and the target signal uniformly distributed random phase from  $[-\frac{\pi}{M}, \frac{\pi}{M})$ , respectively.

The general open-loop ANFIS simulation block diagram is shown in Figure 2. One



**Figure 2.** General open-loop ANFIS training configuration

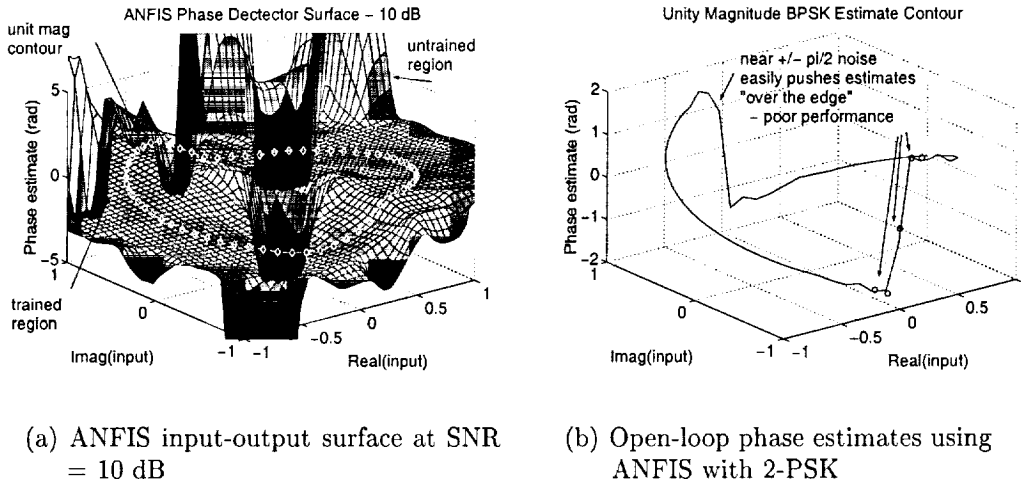
element of *a-priori* knowledge incorporated in the ANFIS open-loop estimator is the use of an averaging scheme. From an estimation theory standpoint averaging should reduce the variance of the phase estimate due to the AWGN.

Training involves using ANFIS to modify the initial parameters of the model structure to emulate the training data presented to it by modifying the membership function parameters according to the ANFIS error criterion.

Figure 3(a) illustrates the input-output surface defined by the resulting FIS for the 2-PSK system with two inputs namely, the real and imaginary parts of the received signal, trained at 10 dB. Following the *unit magnitude* contour plot we see that it traverses a wide and flat *plain* on the the surface. We have removed any *canyon* walls from the front portion of the figure to expose the *unit magnitude* contour plot. The received signal has a mean magnitude equal to one, but due to the AWGN

the individual received (observed) signals no longer have unit magnitude. Thus the ANFIS trained with noise has a larger *trained* domain, although as we can see (Figure 3(a)) *untrained* regions remain. With no noise the *trained* domain is restricted to the unit circle. It should be obvious that it is impossible to train the ANFIS for the entire domain of the received signal (given by (1)) as it is infinite.

The phase estimate derived in this manner, while accurate for estimation of the degenerate case of blocks of size  $N=1$ , is inappropriate for blocks with  $N > 1$ . This should be apparent from the periodic step discontinuities in the input-output function. Figure 3(b) illustrates that in the presence of even small amounts of noise, if the phase offset is large (near a discontinuity) noise will push some estimates (the small circles in the figure) over the discontinuity. Averaging of the estimates over this large discontinuity then results in large errors.



**Figure 3.** 2-PSK ANFIS input-output relations

### ANFIS M-PSK Open-Loop Phase Estimation - Known Structure

We now employ ANFIS in modeling a *specific*, known, open-loop phase estimation scheme for blocks of  $N$ ,  $M$ -PSK symbols. In contrast to the previous section, here we *do* have knowledge of the structure. We employ ANFIS to develop a model of the Viterbi and Viterbi (V&V) [5] open-loop estimator of phase for packets containing  $N$  symbols of  $M$ -PSK modulation. The V&V algorithm is a generalization of the familiar  $M$ -power synchronizer and may be described by writing  $x(k)$  in polar form as

$$\hat{\phi} = \frac{1}{M} \arg \left\{ \sum_{k=0}^{N-1} F[\rho(k)] e^{jM\phi(k)} \right\} \quad (3)$$

where  $F[\rho(k)]$  is an appropriately chosen function of  $\rho(k)$ .

We train the ANFIS to model the complex, nonlinear function described above. The ANFIS system is trained with the observed and target signals. The target signal is the complex phasor  $e^{jM\phi_k}$ . Figure 4 illustrates the V&V algorithm using ANFIS to model the nonlinear function. The imaginary and real components of the signal are computed by individual ANFIS modules.

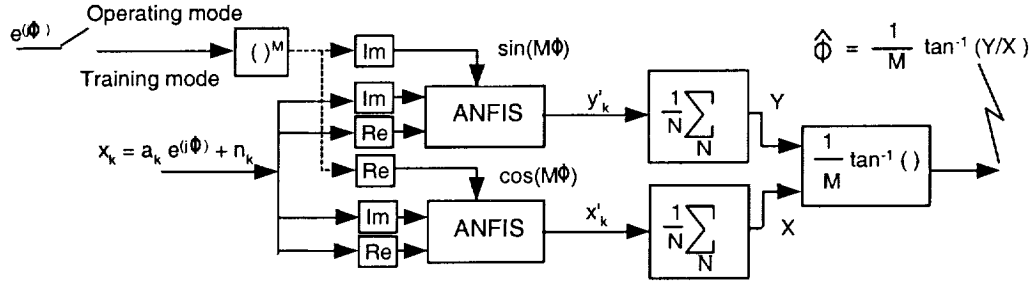


Figure 4. ANFIS based V & V phase estimator algorithm for M-PSK

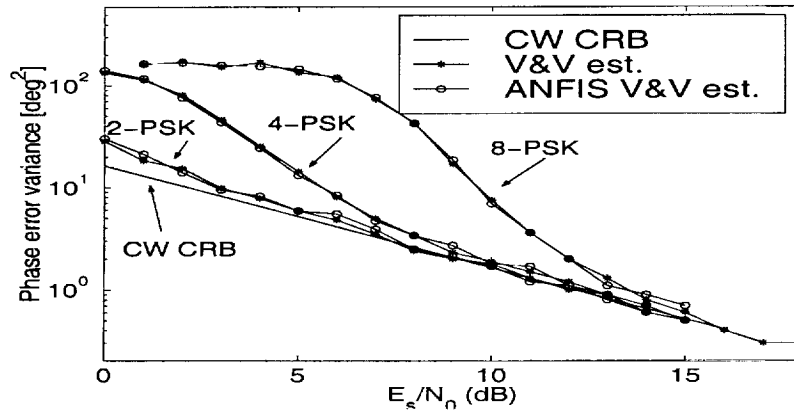


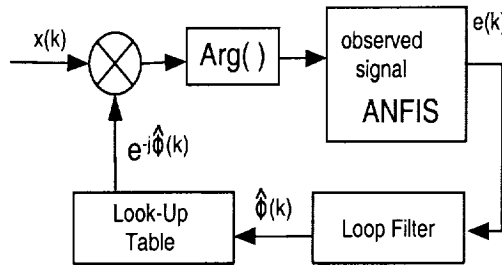
Figure 5. Phase estimate variance V&V and ANFIS V&V algorithms

### ANFIS M-PSK Closed-Loop Phase Estimation - Unknown Structure

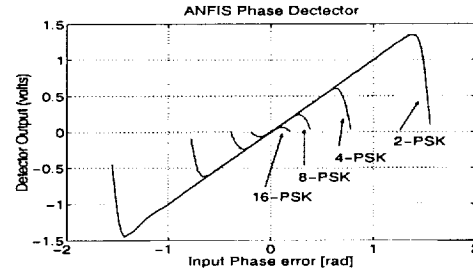
ANFIS is used to identify a *novel closed-loop* phase error detector for use with M-PSK and its performance is compared to the commonly used decision directed (DD), *modified* M-PSK Costas loop (High SNR approximate ML estimate) [3].

A phase error tracking system is characterized by the following principles of operation. A phase error signal as a function of the phase alignment error is computed in a functional block called a phase error detector (PD). This error signal is then used in a feedback loop to adjust the voltage controlled oscillator (VCO). If properly designed, the feedback circuit forces the error signal to zero. The VCO is then aligned with the received signal and may serve as the reference phase in the receiver. For modulated signals the PD structures derived from (2) are complex and require approximations to enable implementation.

To force the observed signal to lie in the *trained* region of the ANFIS we use *only* the phase of the observed signal, limiting the domain to  $[-\pi, \pi)$ . Having trained the ANFIS to model the PDs for M-PSK (Figure 6(b)) we use the ANFIS in a second order PLL to estimate phase for M-PSK as shown in Figure 6(a). The variance data is presented for 2-, 4-, 8- and 16-PSK in Figure 7. The variance is estimated at each SNR as the average of 1000 observations of the steady state variance.

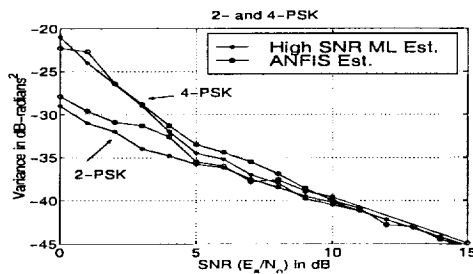


(a) ANFIS M-PSK phase error tracking system

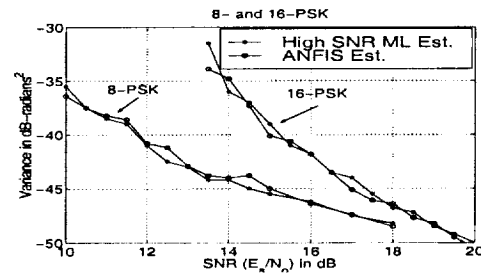


(b) ANFIS 2-,4-,8, and 16-PSK PD characteristics at SNR=20dB

**Figure 6.** ANFIS M-PSK phase error tracking system and ANFIS PD characteristics



(a) 2- and 4-PSK



(b) 8- and 16-PSK

**Figure 7.** Variance of phase error: ANFIS and *modified* Costas loop

## CONCLUSIONS

In the most general *open-loop* case, the ANFIS system does not identify a functional mapping that is adequate for phase estimation from a packet of symbols with AWGN. The ANFIS based model of the V&V phase estimation algorithm performs on par with the analytical V&V algorithm. With only 3 MFs per input ANFIS simply and accurately models the V&V non-linear function required to remove the modulation and enable averaging of estimates to reduce the effect of the AWGN.

We see that using only angle information to design *closed-loop* PDs for M-PSK using the ANFIS model free approach, results in performance equivalent to that of the approximate ML solution. It is believed improved performance can be achieved by the ANFIS solution when the complete complex signal can be processed.

## REFERENCES

1. Jang, J.-S., Sun, C.-T., and Mizutani, E. *Neuro-Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence*. Prentice Hall, Upper Saddle River, New Jersey, 1997.
2. Kam, P. Y. Maximum likelihood carrier phase recovery for linear suppressed-carrier digital data modulations. *IEEE Trans. Comm. Theory* 34, 6 (June 1986), 522-527.
3. Mengali, U. *Synchronization Techniques for Digital Receivers*. Plenum Press, New York, 1997.
4. Van Trees, H. L. *Detection, Estimation, and Modulation Theory*. John Wiley & Sons, New York, 1968.
5. Viterbi, A. J., and Viterbi, A. M. Nonlinear estimation of PSK-modulated carrier phase with application to burst digital transmission. *IEEE Trans. Info. Theory* 29, 4 (July 1983), 543-551.