Final Report for
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Jet Mixing Enhancement by Feedback Control

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1 Summary

The objective of this work has been to produce methodologies for high speed jet noise reduction based on natural mechanisms and enhanced feedback control to affect frequencies and structures in a prescribed manner. In this effort the two-point hot wire measurements obtained in the Langley jet facility by Ukeiley were used in conjuction with linear stochastic estimation (LSE) [1] to implement the LSE component of the complementary technique (v. Bonnet, Cole, Delville, Glauser and Ukeiley 1994, Experiments in Fluids). This method combines the Proper Orthogonal Decomposition (POD) [4] and LSE to provide **an experimental low dimensional time dependent description** of the flow field. From such a description it should be possible to identify short time high strain rate events in the jet which contribute to the noise (v. J.M. Seiner 1997, AIAA Fluid Dynamics Conference in Snowmass Village, CO). The main task completed for this effort is summarized below:

LSE experiments were performed at the downstream locations where the two point hot wire measurements have been obtained by Ukeiley. These experiments involved sampling simultaneously hot wire signals from a relatively course spatial grid in r and θ. From this simultaneous data, coupled with the two-point measurements of Ukeiley via the LSE components of the complementary technique, an experimental low dimensional description of the jet at 4, 5, 6, 7 and 8 diameters downstream was obtained for Mach numbers of 0.3 and 0.6. We first present an overview of the theory involved. We finish up with a statement of the work performed and finally provide charts from a 1999 APS talk which summarizes the results.

2 Introduction

The complementary technique consists of projecting the POD eigenfunctions onto an estimated velocity field obtained from application of LSE as described by Cole et al [2] to obtain estimated random coefficients. These estimated random coefficients are then used in conjunction with the first couple of POD eigenfunctions to reconstruct the estimated random field, i.e., a low dimensional description. Bonnet et al 1994 presented a qualitative comparison between, the first POD mode representation of the estimated random velocity field, and that obtained utilizing the original **measured** field, and found that the two are remarkably similar, in both the axisymmetric jet and two dimensional mixing layer. In order to quantitatively assess the technique, the root mean square (RMS) velocities were computed from the estimated and original velocity fields and comparisons made. In both flows the RMS velocities captured using the first POD mode of the estimated field are very close to those obtained from the first POD mode of the unestimated original field. These results showed that the complementary technique, which combines LSE and POD, allows one to obtain time dependent information from the POD while greatly reducing the amount of instantaneous data required. Hence, it is not necessary to measure the instantaneous velocity field at all points in space *simultaneously* to obtain the phase of the structures, but only at a few select spatial positions. Hence this type of an approach can be used to obtain experimental low dimensional descriptions of the jet. The work of Bonnet et al 1994 applied the estimation in the radial direction only. In this study the method was extended to include the azimuthal direction. A brief review of the POD and LSE are included below followed by an overview of the complementary technique.

2.1 POD Theory

The POD was proposed by Lumley in 1967 as a mathematically unbiased technique for examining coherent structures in turbulent flows. He proposed that the coherent structure is the structure which has the largest mean square projection on the velocity field. In the following equations $\vec{x}$ denotes r and $d\vec{x}$ denotes rdr for the jet. If $\vec{\phi}(\vec{x},t)$ is taken to be the candidate structure and then projected onto the velocity vector field, $\vec{u}(\vec{x},t)$, in the following manner,

$$\overline{|\vec{u}\cdot\vec{\phi}|^2} = \overline{|\alpha|^2} \tag{1}$$

a resulting structure which maximizes energy can be chosen. Equation 1 is assumed to be normalized by the modulus of $\vec{\phi}(\vec{x},t)$ since it is the degree of projection not the amplitude that is of interest in this study. Through the use of variational calculus this projection process can also be written as,

$$\int_D R_{ij}(\vec{x},\vec{x}',t,t')\phi_j^{(n)}(\vec{x}',t')d\vec{x}'dt' = \lambda^{(n)}\phi_i^{(n)}(\vec{x},t) \tag{2}$$

where the kernel of this integral eigenvalue problem is the cross-correlation tensor, $R_{ij}(\vec{x},\vec{x}',t,t')$. Since R_{ij} is a symmetric function the solutions can be discussed in terms of the Hilbert-Schmidt theory(v. Lumley[4]). This implies that there is not one, but an infinite number of orthogonal solutions(eigenmodes) which can be normalized such that,

$$\int \phi_i^{(n)}(\vec{x},t)\phi_i^{(m)}(\vec{x},t)d\vec{x} = \delta_{nm}. \tag{3}$$

The original random vector field can be reconstructed in terms of $\phi_i^{(n)}$ as follows:

$$u_i(\vec{x},t) = \sum_{n=1}^{\infty} a_n(t)\phi_i^{(n)}(\vec{x},t) \tag{4}$$

where the random coefficients $a_n(t)$ can be calculated from:

$$a_n(t) = \int u_i(\vec{x},t)\phi_i^{(n)}(\vec{x},t)d\vec{x}. \tag{5}$$

The contribution from each eigenmode to the turbulent stresses can be determined as follows:

$$\overline{u_i u_j} = \sum_n \lambda^{(n)}\phi_i^{(n)}(\vec{x},t)\phi_j^{(n)*}(\vec{x},t). \tag{6}$$

In the jet study the spectral tensor may be defined by the following equation,

$$S_{ij}(r,r',f,m,\overline{z}) = \int R_{ij}(r,r',\tau,\theta,\overline{z})e^{-i(2\pi f)}e^{i(m\theta)}d\tau d\theta, \tag{7}$$

where f denotes frequency, m denotes azimuthal mode number, τ is the separation in time, θ the separation in the azimuthal direction and $\overline{z}$ represent streamwise locations where the correlation tensors were measured (in the Ukeiley data, 4,5,6,7,8 diameters downstream for Mach numbers of 0.3, 0.6 and 0.85). Equation 2 now becomes

$$\int S_{ij}(r,r',f,m,\overline{z})\psi_j^{(n)}(r',f,m,\overline{z})r'dr' = \lambda^{(n)}(f,m)\psi_i^{(n)}(r,f,m,\overline{z}), \tag{8}$$

where the ψ's are frequency and azimuthal mode number dependent eigenfunctions and $\lambda^{(n)}(f,m)$ now represents the eigenspectra.

The numerical approximation, detailed by Glauser et al[3], simply consists of replacing the integral in equation 8 by an appropriate quadrature rule (in this study a trapezoidal rule). $S_{ij}(r,r',f,m,\overline{z})$ is then obtained from experimental measurements of Ukeiley for the various downstream locations and utilized in equation 8 to obtain the eigenvalues and eigenfunctions. These eigenfunctions and eigenvalues will then be used in the complementary technique as described below.

2.2 Stochastic Estimation Theory

In the following equations x denotes r in the jet. Only estimates in r are shown here. It is straight forward to extend the analysis to include additional directions as shown by Cole et al (see Cole et al APS 1997 and www.clarkson.edu/g̃lauser) in their application to the time dependent PIV (for this study it will be extended to include the θ direction). In general, the conditional average is defined as

$$\overline{g(u)|E} = expected\ \ value\ \ of\ \ g(u) \tag{9}$$

provided that the event E, the detector of coherent structure, occurs. The properties of these coherent structures are not known beforehand, therefore it is difficult to select reliable unambiguous and unbiased detector events. Adrian [1] suggested choosing

$$g(u) = u(x') \tag{10}$$

and

$$E = e \leq u(x) < e + de. \tag{11}$$

This confines the velocity vector to a small interval between e and $e + de$ where e is any arbitrary vector. This can be expressed as

$$\tilde{u}(x') = \overline{u(x')|u(x)} \tag{12}$$

which can be approximated, using a Taylor series expansion about $u(x) = 0$, as

$$\tilde{u}_i(x') = A_{ij}(x')u_j(x) + B_{ijk}(x')u_j(x)u_k(x) + C_{ijkl}(x')u_j(x)u_k(x)u_l(x) + \cdots \tag{13}$$

Values for the estimation coefficients, $A_{ij}(x')$, $B_{ijk}(x')$, $C_{ijkl}(x')$ are selected such that the mean square error is minimized, ie.,

$$e_i = \overline{[\tilde{u}_i(x') - \overline{u_i(x')|u(x)}]^2} \tag{14}$$

for $i = 1, 2, 3$. This minimization requires that

$$\frac{\partial e_i}{\partial A_{ij}(x')} = \frac{\partial e_i}{\partial B_{ijk}(x')} = \frac{\partial e_i}{\partial C_{ijkl}(x')} = 0 \tag{15}$$

which produces the following equation

$$\begin{aligned} &\overline{u_j(x)u_k(x)}A_{ik}(x') + \overline{u_j(x)u_k(x)u_l(x)}B_{ikl}(x') \\ &+\overline{u_j(x)u_k(x)u_l(x)u_m(x)}C_{iklm}(x') = \overline{u_j(x)u_i(x')}. \end{aligned} \tag{16}$$

2.2.1 Linear Stochastic Estimation

Tung and Adrian [5] have shown that linear stochastic estimation produces reasonable qualitative estimates and little is to be gained by using second order or higher. By applying linear stochastic estimation only the first term on the right hand side of equation 13 is retained. As a result, equation 13 becomes

$$\tilde{u}_i(x') = A_{ij}(x')u_j(x) \tag{17}$$

This also reduces equation 16 to:

$$\overline{u_j(x)u_k(x)}A_{ik}(x') = \overline{u_j(x)u_i(x')} \tag{18}$$

where $\overline{u_j(x)u_k(x)}$ is the Reynolds stress tensor and $\overline{u_j(x)u_i(x')}$ is the two-point correlation tensor.

Linear stochastic estimation is a useful tool for the identification of coherent structures in the axisymmetric jet mixing layer as has been demonstrated by Cole et al [2]. After applying this technique to u,v data, the matrices that are a direct result of the expansion of equation 13 for a two wire estimate are:

First System:

$$\begin{bmatrix} \overline{u^2_{ref_1}} & \overline{u_{ref_1}v_{ref_1}} & \overline{u_{ref_1}u_{ref_2}} & \overline{u_{ref_1}v_{ref_2}} \\ \overline{v_{ref_1}u_{ref_1}} & \overline{v^2_{ref_1}} & \overline{v_{ref_1}u_{ref_2}} & \overline{v_{ref_1}v_{ref_2}} \\ \overline{u_{ref_2}u_{ref_1}} & \overline{u_{ref_2}v_{ref_1}} & \overline{u^2_{ref_2}} & \overline{u_{ref_2}v_{ref_2}} \\ \overline{v_{ref_2}u_{ref_1}} & \overline{v_{ref_2}v_{ref_1}} & \overline{v_{ref_2}u_{ref_2}} & \overline{v^2_{ref_2}} \end{bmatrix} \begin{bmatrix} A^{ref_1}_{11w} \\ A^{ref_1}_{12w} \\ A^{ref_2}_{11w} \\ A^{ref_2}_{12w} \end{bmatrix} = \begin{bmatrix} \overline{u_{ref_1}u_w} \\ \overline{u_{ref_1}v_w} \\ \overline{u_{ref_2}u_w} \\ \overline{u_{ref_2}v_w} \end{bmatrix} \tag{19}$$

Second System:

$$\begin{bmatrix} \overline{u^2_{ref_1}} & \overline{u_{ref_1}v_{ref_1}} & \overline{u_{ref_1}u_{ref_2}} & \overline{u_{ref_1}v_{ref_2}} \\ \overline{v_{ref_1}u_{ref_1}} & \overline{v^2_{ref_1}} & \overline{v_{ref_1}u_{ref_2}} & \overline{v_{ref_1}v_{ref_2}} \\ \overline{u_{ref_2}u_{ref_1}} & \overline{u_{ref_2}v_{ref_1}} & \overline{u^2_{ref_2}} & \overline{u_{ref_2}v_{ref_2}} \\ \overline{v_{ref_2}u_{ref_1}} & \overline{v_{ref_2}v_{ref_1}} & \overline{v_{ref_2}u_{ref_2}} & \overline{v^2_{ref_2}} \end{bmatrix} \begin{bmatrix} A^{ref_1}_{21w} \\ A^{ref_1}_{22w} \\ A^{ref_2}_{21w} \\ A^{ref_2}_{22w} \end{bmatrix} = \begin{bmatrix} \overline{v_{ref_1}u_w} \\ \overline{v_{ref_1}v_w} \\ \overline{v_{ref_2}u_w} \\ \overline{v_{ref_2}v_w} \end{bmatrix} \tag{20}$$

where ref_1 and ref_2 refer to reference wires 1 and 2 respectively, and w refers to the wire number. It should be noted, that for these systems of equations, only the two-point space-time correlation data is utilized. These systems **are not** a function of the condition being investigated. The estimated velocity components for the two wire reference case can then be found from the expansion of equation 17 ie.,

$$\tilde{u}_w = A^{ref_1}_{11w}uc_{ref_1} + A^{ref_1}_{12w}vc_{ref_1} + A^{ref_2}_{11w}uc_{ref_2} + A^{ref_2}_{12w}vc_{ref_2} \tag{21}$$

and

$$\tilde{v}_w = A^{ref_1}_{21w}uc_{ref_1} + A^{ref_1}_{22w}vc_{ref_1} + A^{ref_2}_{21w}uc_{ref_2} + A^{ref_2}_{22w}vc_{ref_2}. \tag{22}$$

It is in these estimated velocity equations that the condition selected plays a role (i.e., through uc_{ref_1}, uc_{ref_2}, vc_{ref_1} and vc_{ref_2}). A single wire estimate is obtained by merely setting all terms containing $ref_2 = 0$. Without much trouble this system can easily be expanded to include addition wires distributed in the θ direction as done in this study.

2.3 Complementary Technique

Mathematically the stochastic estimates of the random coefficients are calculated for a given location downstream from:

$$a^{est}_n(f,m) = \int \hat{u}^{est}_i(r,f,m)\psi^{(n)*}_i(r,f,m)rdr \tag{23}$$

where $\hat{u}^{est}_i(x,f,m)$ is either a single or multipoint linear stochastic estimate of the random field and $\psi^{(n)*}_i(r,f,m)$ is obtained from the original POD eigenvalue problem. The estimated streamwise or radial velocity can be reproduced in Fourier space by

$$\hat{u}^{est}_i(x,f,m) = \sum_{n=1}^{\infty} a^{est}_n(f,m)\psi^{(n)}_i(r,f,m) \tag{24}$$

and then inverse transformed to obtain $u^{est}_i(r,\theta,t)$.

In this study we have been concentrating on the first POD mode representation $u^{(1)est}_i(r,\theta,t)$, the low dimensional discription.

3 Work Performed

3.1 Experimental Low Dimensional Description via LSE

We ran experiments at the downstream locations where the two point hot wire measurements were obtained by Ukeiley and applied the LSE. These experiments involved sampling simultaneously hot wire signals from a relatively course spatial grid in r and θ. From this simultaneous data, coupled with the two-point measurements of Ukeiley via the LSE, an experimental low dimensional description of the jet at 4, 5, 6, 7 and 8 diameters downstream was obtained for Mach numbers of 0.3 and 0.6. An APS talk is attached which summarizes the work. Also reference the web site www.clarkson.edu/ etrl/taylorja/CompressibleJet/index.html for more details and animations of the LSE time series.

References

[1] Adrian, R.J. (1975) "On the Role of Conditional Averages in Turbulence Theory". In *Proc. 4th Biennal Symp. on Turbulence in Liquids.*

[2] Cole, D.R., Glauser, M.N. and Guezennec, Y.G., (1992), "An Application of Stochastic Estimation to the Jet Mixing Layer". *Physics of Fluids A.*,**4**(1).

[3] Glauser, M.N., Leib, S.J. and George, W.K. (1987) "Coherent Structures in the Axisymmetric Jet Mixing Layer". *Turbulent Shear Flows 5*, Springer Verlag.

[4] Lumley, J.L. (1967) "The Structure of Inhomogeneous Turbulent Flows". *Atm. Turb. and Radio Wave Prop.*. Yaglom and Tatarsky eds. Nauka, Moscow.

[5] Tung, T.C. and Adrian, R.J. (1980) "Higher-Order Estimates of Conditional Eddies in Isotropic Turbulence", *Phys. Fluids* **23**,. 1469.

Linear Stochastic Estimation in a Compressible Axisymmetric Mixing Layer

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SUB-SONIC JET
NOISE REDUCTION

*** Requires ***

An Improved Understanding of the
Turbulent Jet Mixing Layer

Introduction - Some Previous Studies

Arndt, R., George, W.K.

What's the relationship between Coherent Structures and Jet Noise? (1972)

Arndt, R., and Long, D. - University of Minnesota

George, W.K. and Glauser, M.N.- SUNY at Buffalo/Clarkson University

x/D: 3
Re # ~ 100,000
Ma # ~ 0.0
13 single component hot-wire or 4 x-wire probes

George, W.K. and Citriniti, J.H. - SUNY at Buffalo

x/D: 3
Re # ~ 80,000
Ma # ~ 0.0
138 single component hot-wire probes

Ukeiley, L.S. and Seiner, J.M. - NASA Langley/University of Mississippi

x/D: 4, 5, 6, 7, 8, 9, 10, 11, 12
Re # ~ 300,000; 600,000; 1,000,000
Ma # ~ 0.3, 0.6, 0.85
12 X-Wire probes

Current Study - Clarkson University/University of Mississippi

x/D: 4, 4.5, 5, 6, 7, 8
Re # ~ 300,000; 600,000
Ma # ~ 0.3, 0.6
15 X-Wire probes

<u>Linear Stochastic Estimation</u> - Brief Review

Adrian, R.J. (1977)

Proposed the use of "Stochastic Estimation" as a means of using conditional information on a course grid of points to estimate, or infer, the behavior of the flow on a finer grid.

Cole, D.R., Glauser,M.N., and Guezennec,Y.G. (1992)

Proposed using the instantaneous fluctuating velocity on a course grid as the input variable, or condition, in the estimation as a means of reducing experimental complexity.

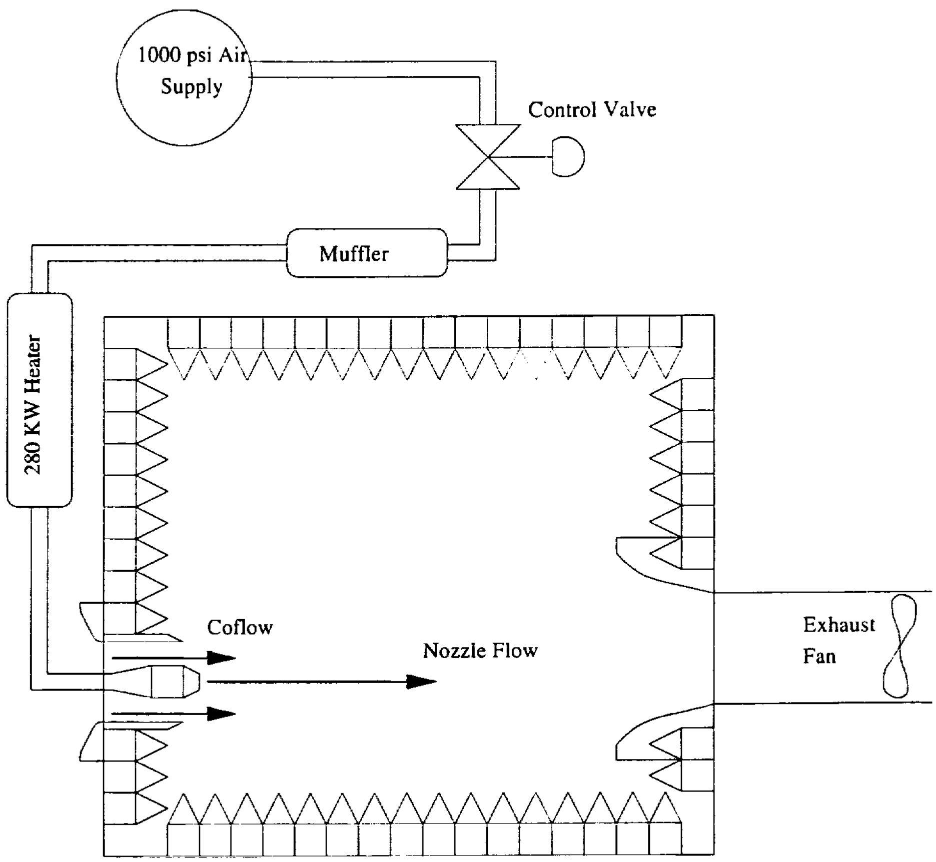
Bonnet, J.P., Cole, D.R., Deville, J., Glauser, M.N., and Ukeiley, L.S. (1994)

Proposed the "Complementary Technique" as a combination of the LSE and POD.

1. Obtain two-point correlation tensor
2. Solve the POD for the eigenfunctions/eigenvalues
3. Use the LSE to obtain an estimate of the instantaneous flow field
4. Project the eigenfunctions onto the flow field to generate an experimental low-dimensional description of the flow

Experimental Description - Facility

Small Anechoic Jet Facility (SAJF)



Features:

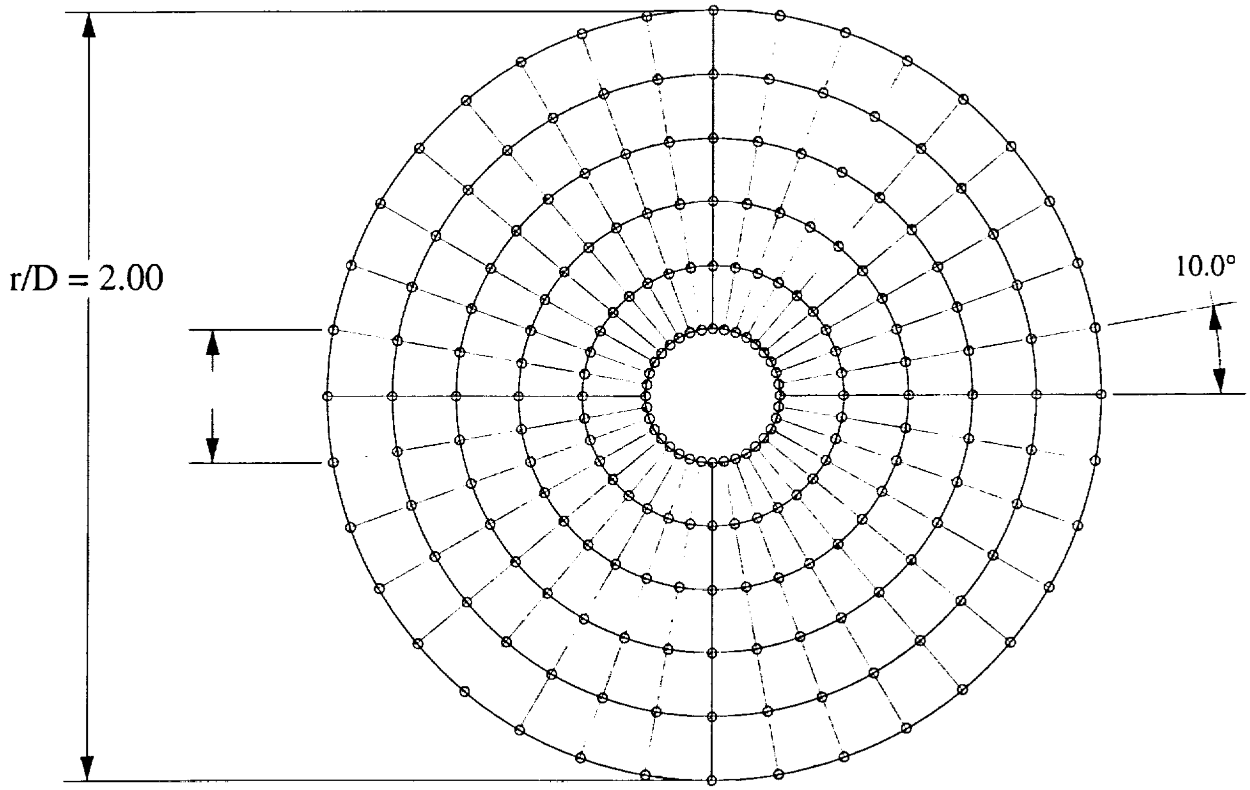
Continuous Operation at 2 lbm/sec (0.9 kg/sec)

Electric Heat up to 1000 °F (538 °C)

Exit Turbulence Intensities ≈ 0.15%

Coflow ≈ 2% of main flow

Experimental Description - Two-Rake Experiment



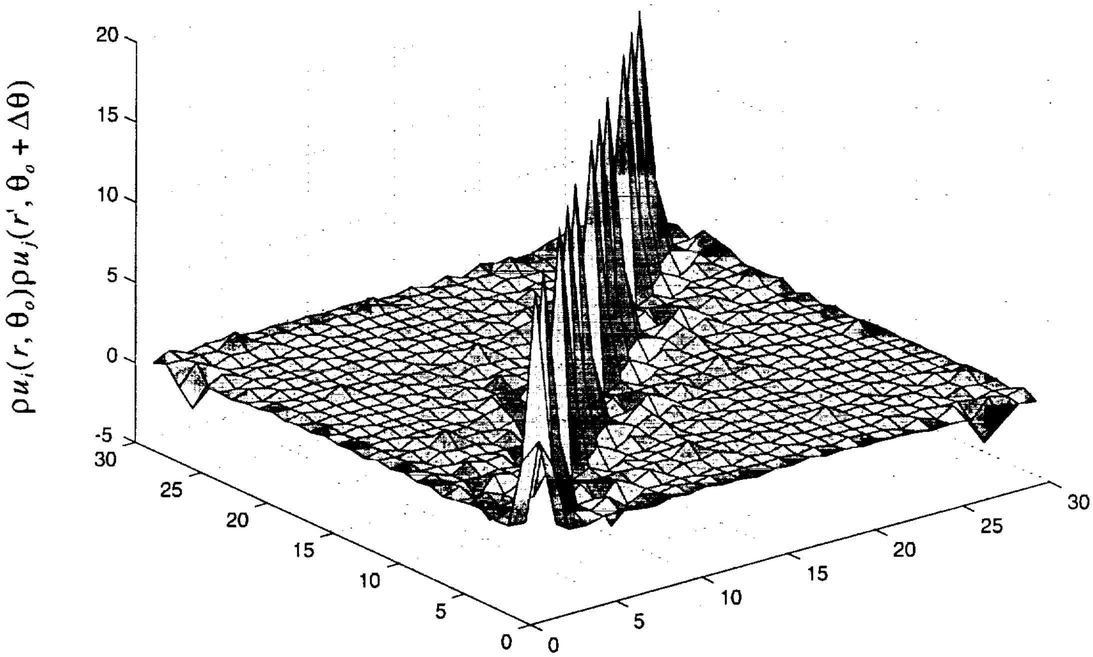
Experimental Parameters:

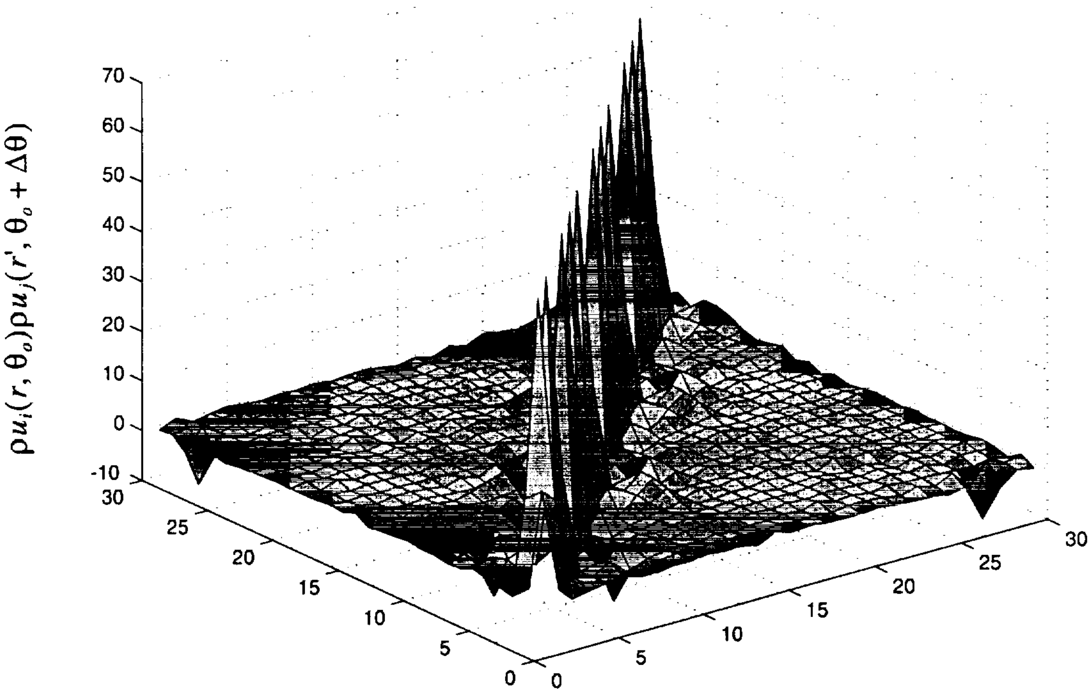
- 2 Rakes of Probes
- 12 Auspex X-Wire probes
- 19 Azimuthal Locations (0°:10°:180°)
- 6 Radial Locations (r/D: 0.175, 0.339, 0.504, 0.668, 0.833, 0.999)
- 3 Mach Numbers (0.3, 0.6, and 0.85)

Data Acquisition Parameters:

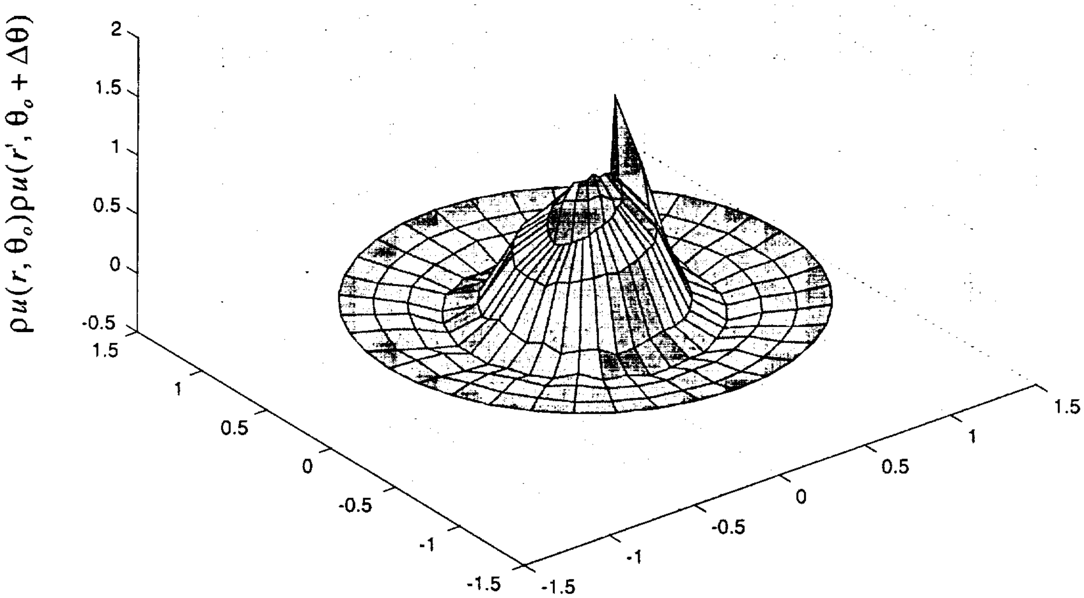
- 65 kHz Sampling Frequency
- 31.5 kHz Low-Pass Filter Cut-Off Frequency
- 16 bit A/D Conversion
- 256 blocks with 2048 samples/block

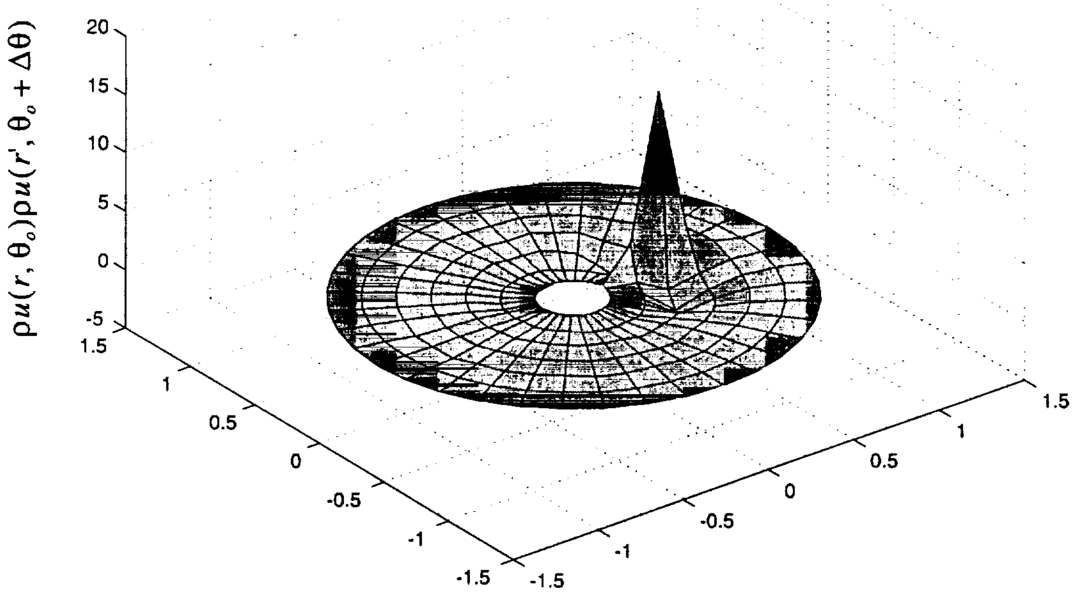
Reynold's Stress Tensor: Mach 0.3 & Mach 0.6, x/D = 4



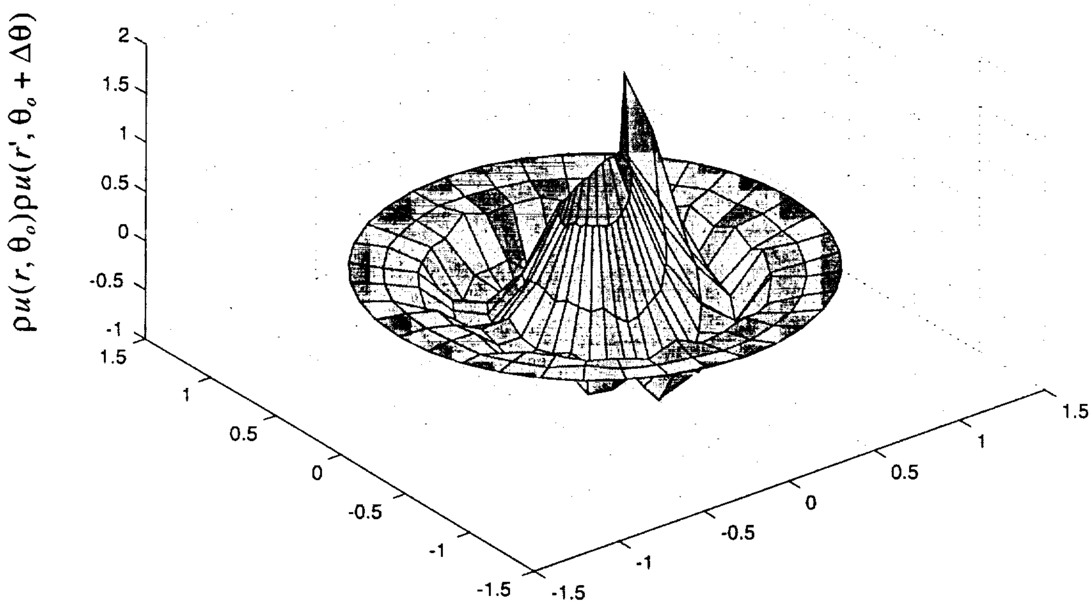


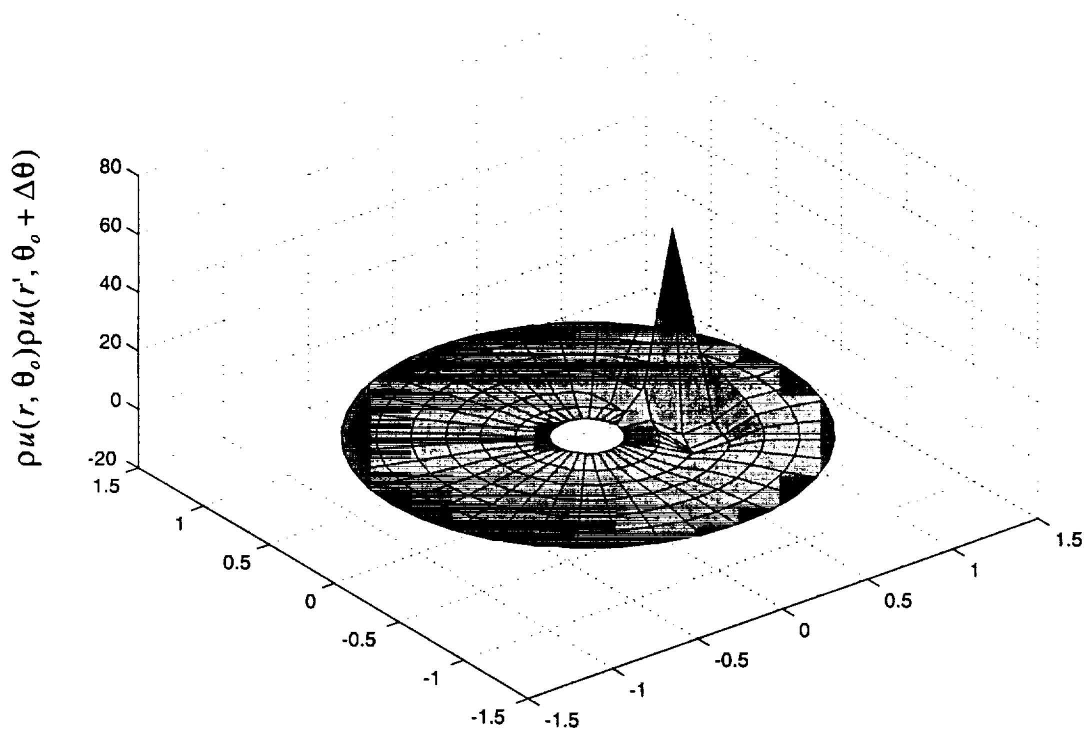
Two Point Correlations: Mach = 0.3, x/D = 4



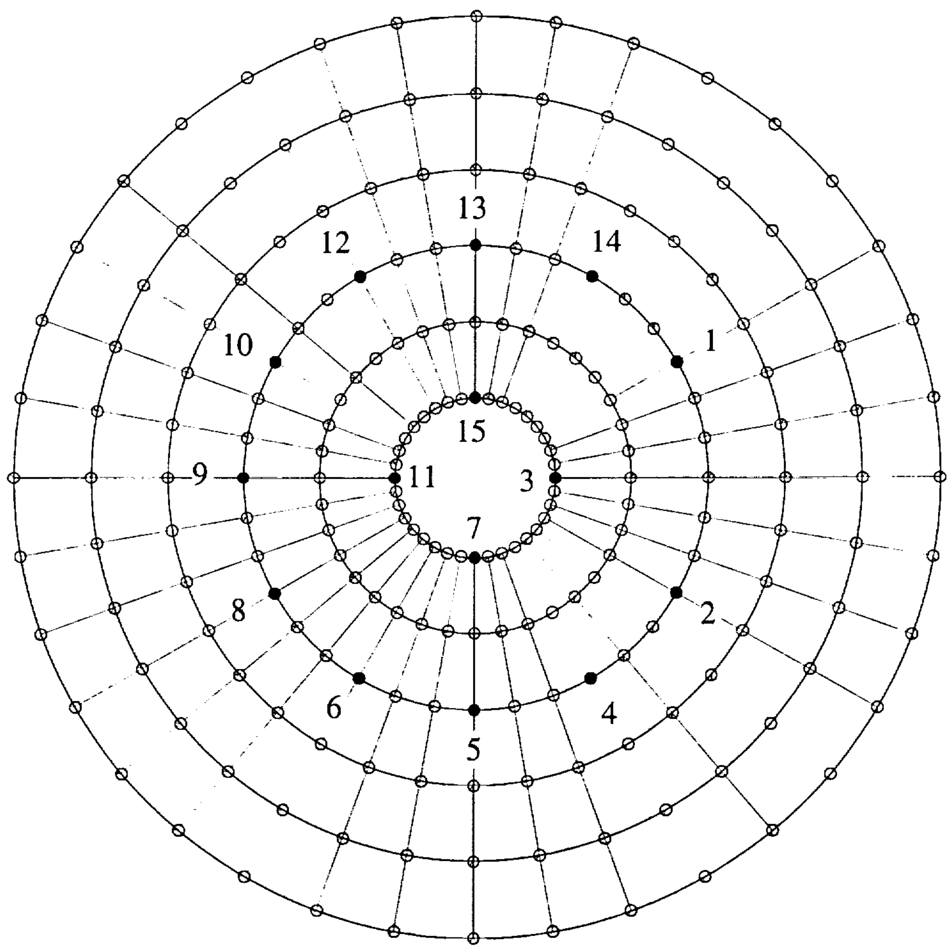


Two Point Correlations: Mach = 0.6, x/D = 4





Experimental Description - LSE Experiment



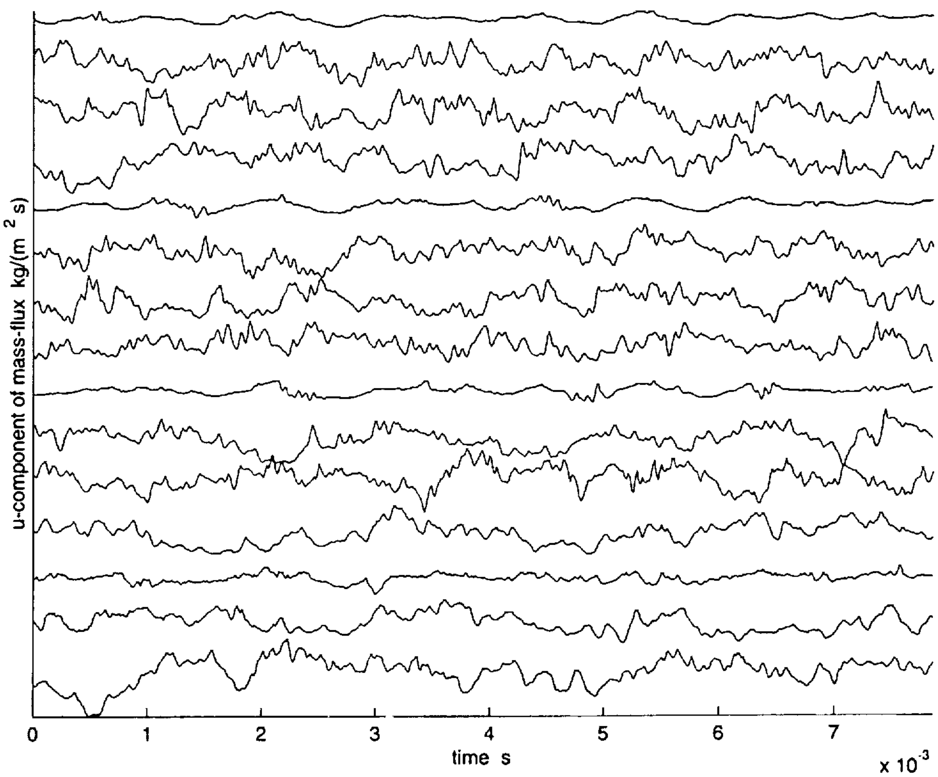
Experimental Parameters:

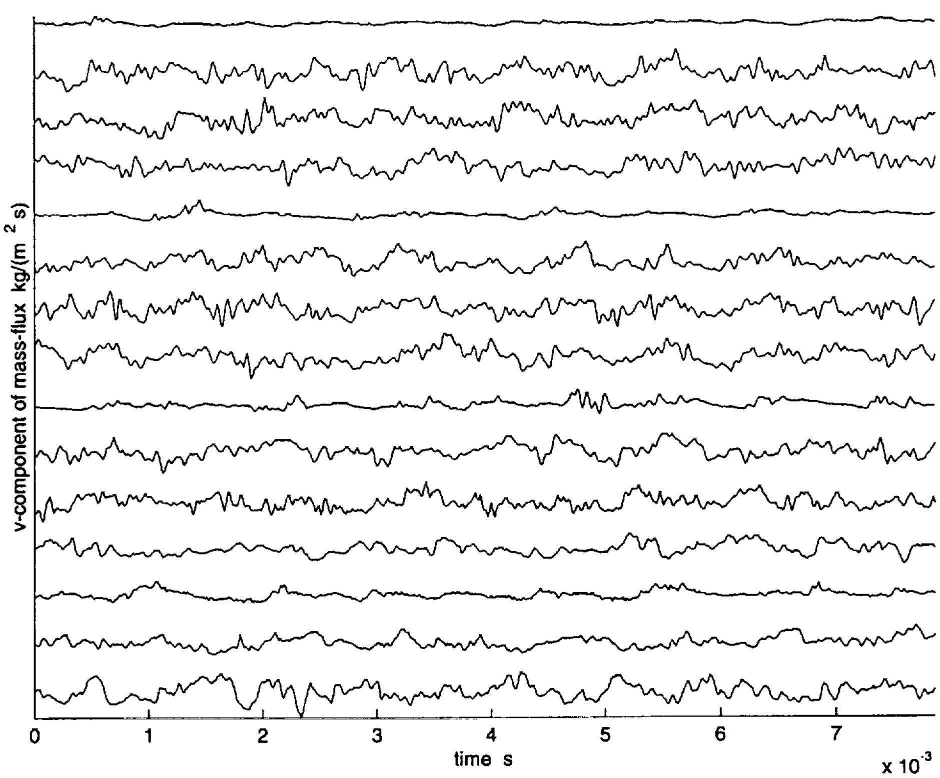
- Single Probe Holder on a Course Grid
- 15 Auspex X-Wires Probes
- 12 Azimuthal Locations (0°:30°:360°)
- 2 Radial Locations (r/D: 0.175, 0.504)

Data Acquisition Parameters:

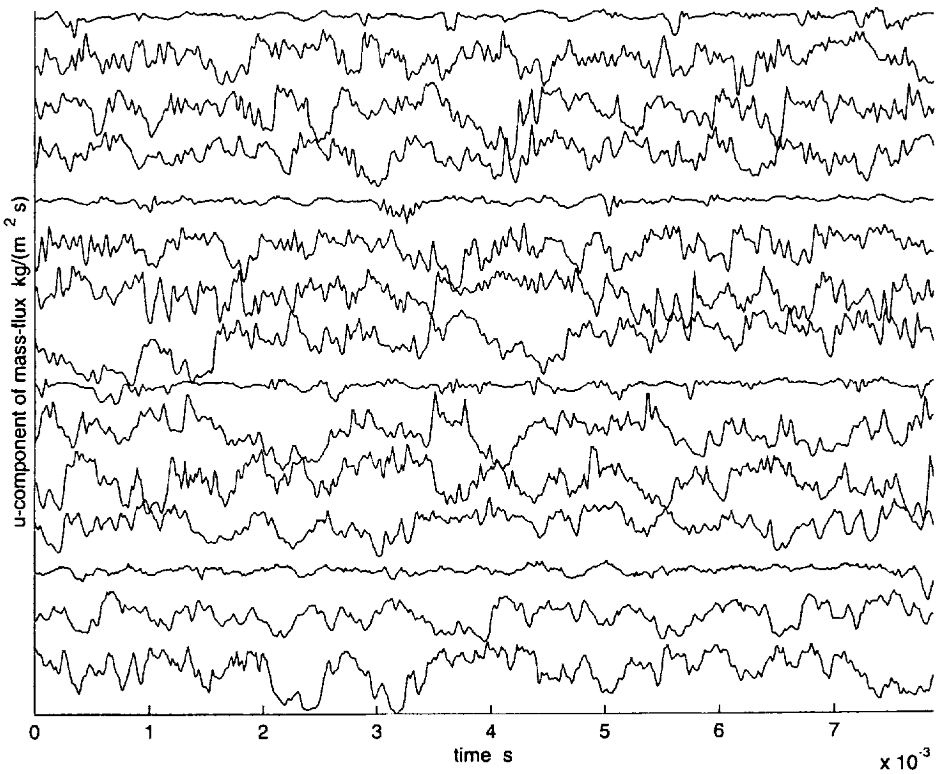
- 65 kHz Sampling Frequency
- 25 kHz Low-Pass Filter Cut-Off Frequency
- 16 bit A/D Conversion
- 100 blocks with 2048 samples/block

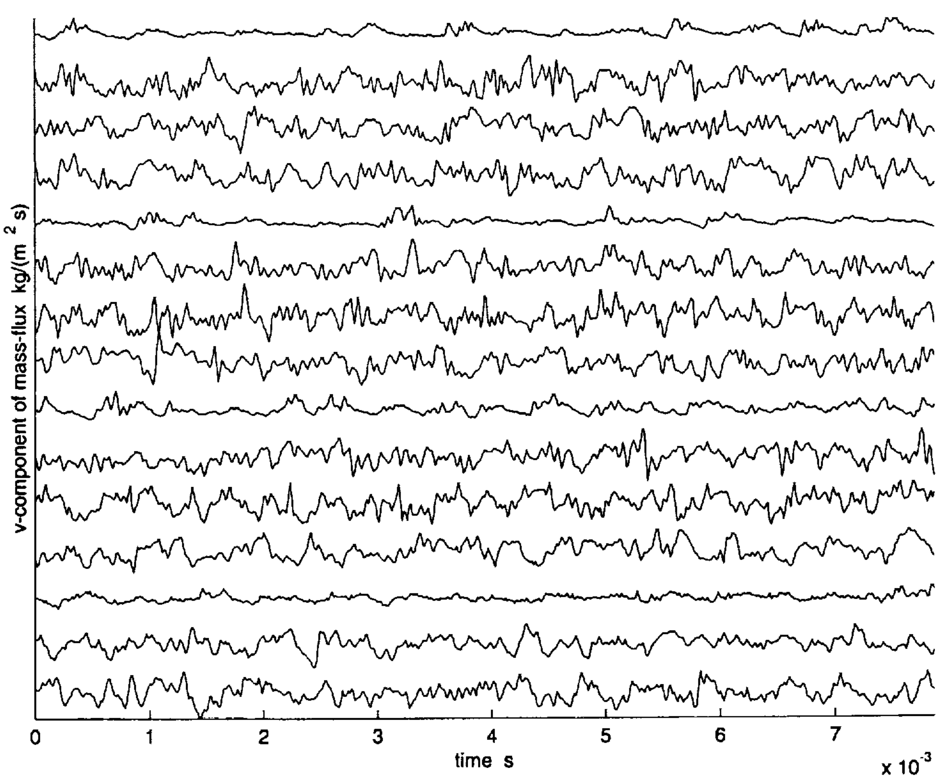
Results - Time Histories: Mach = 0.3, x/D = 4



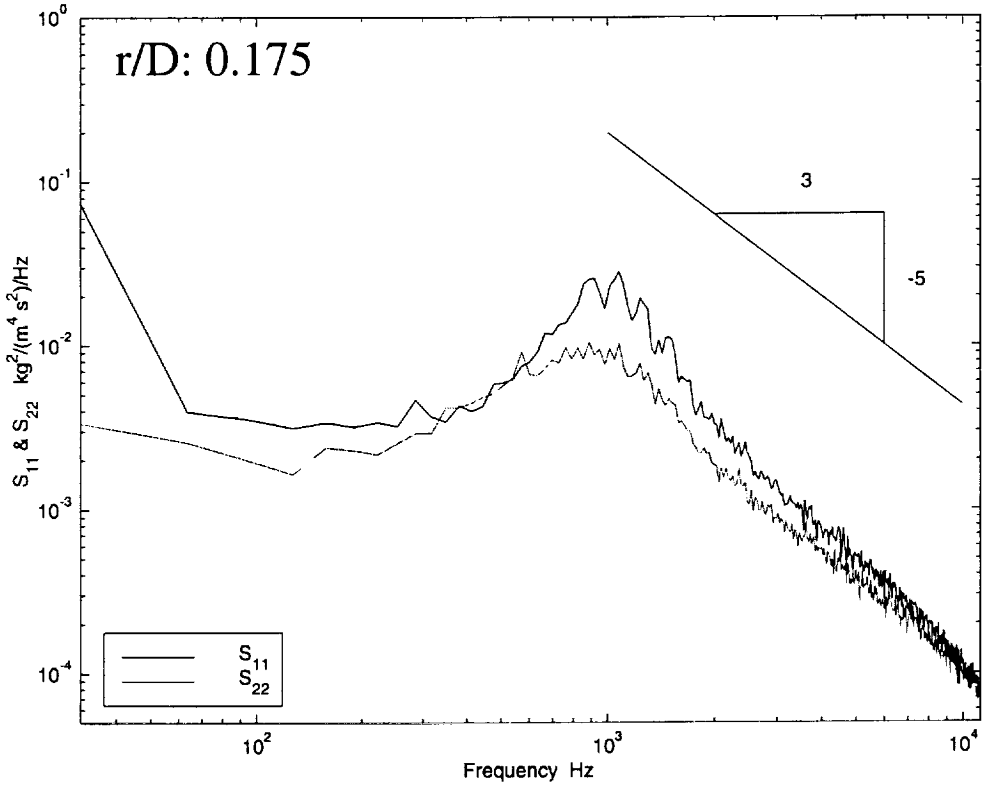


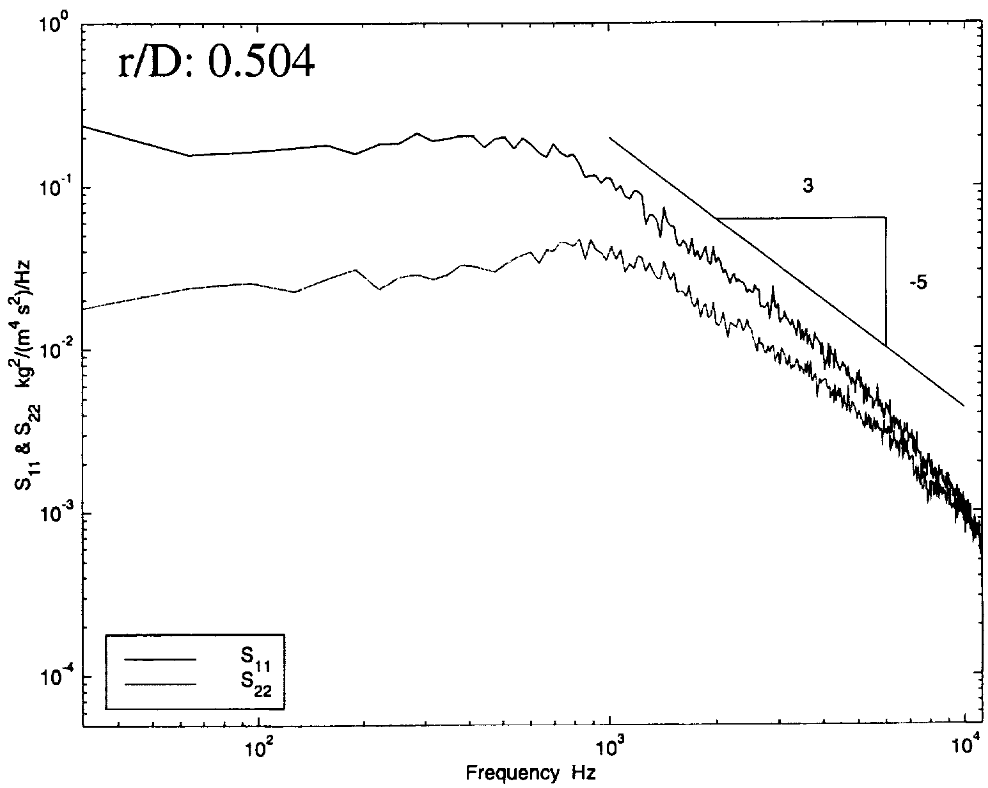
Results - Time Histories: Mach = 0.6, x/D = 4



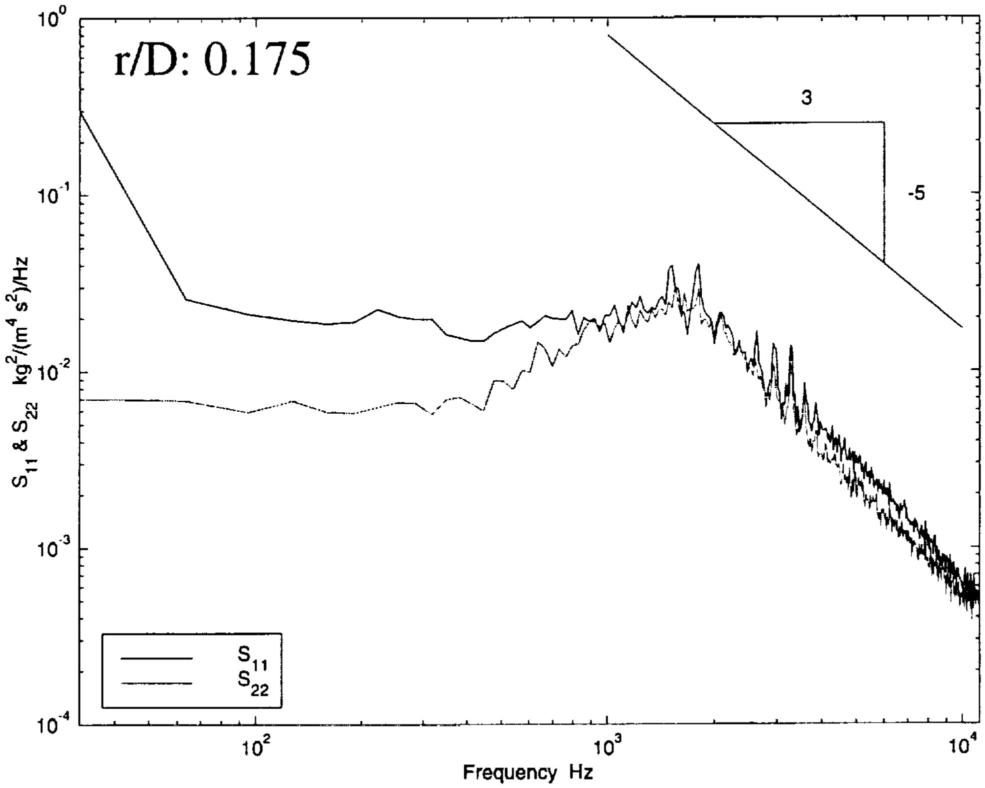


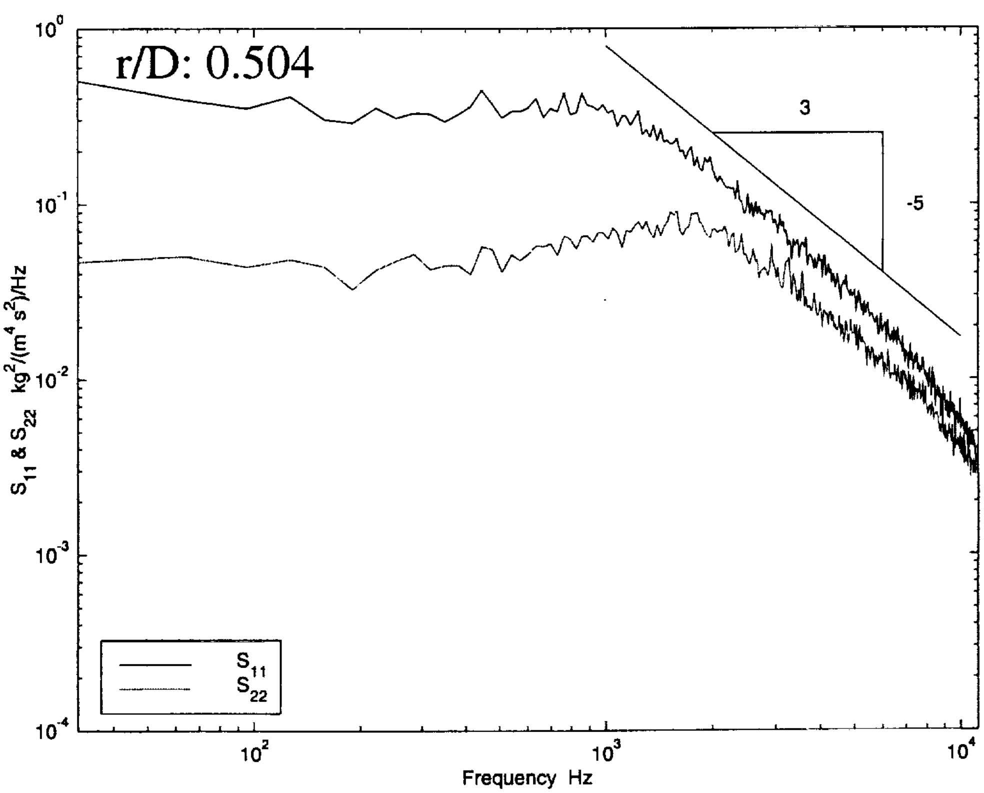
Results - Autospectra: Mach = 0.3, x/D = 4





Results - Autospectra: Mach = 0.6, x/D = 4

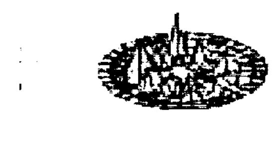




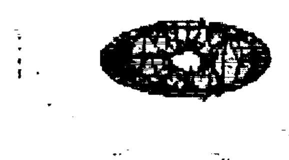
Results - Movies

Citriniti, J.H.: Instantaneous Velocity
Ma # ~0.0
Re # ~80,000
x/D: 3

Current Study: Estimated Mass-Flux
Ma # ~0.3
Re # ~300,000
x/D: 4



Current Study: Estimated Mass-Flux
Ma # ~0.6
Re # ~600,000
x/D: 4



Future Work -
Experimental Low-Dimensional Description
using the Complementary Technique

Citriniti, J.H.:
1 POD Mode; 0,3,4,5,6 Az. Modes
Ma # ~0.6
Re # ~600,000
x/D: 4

Summary

Jet mixing layer has a very similar multi-point, statistical behavior at:

Ma # ~ 0.0 Re # ~100,000
Ma # ~ 0.3 Re # ~300,000
Ma # ~ 0.6 Re # ~600,000

(i.e., Azimuthal integral length scales are similar)

Time dependent behavior also exhibits similar behavior.

(i.e., coherent in the potential region, with a higher azimuthal mode structure towards the outside of the shear layer)

Future Work

How does the low-dimensional description of the axi-symmetric shear layer change with Reynold's number? Mach number?