ABSTRACT

Impact of Eccentricity on East-West Stationkeeping for the GPS Class of Orbits

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INTRODUCTION

There exists a strong relationship between eccentricity and the potential for a repeating groundtrack orbit to exhibit chaotic motion. This is true at all values of eccentricity, but, perhaps most dramatic, is that it is true even for orbits that are nearly circular.[1] These complex motions can have a significant impact on the east-west stationkeeping process for maintaining the repeating groundtrack property of a commensurate orbit. Ely and Howell[3] have shown that traditional stationkeeping (SK) methods are unable to maintain a repeating groundtrack in the presence of complex dynamics, such as with chaotic motion. They developed an alternate SK method that is able to maintain a repeating groundtrack for eccentric, commensurate orbits. The focus of the current study is to investigate orbits with characteristics that are similar to GPS satellites except with modestly larger eccentricities. It will be shown that at eccentricities larger than ~ .01 the chaotic regions become significant, and the need arises for a robust stationkeeping approach, such as developed in [3]. Furthermore, the investigation will reveal that the influence of luni-solar perturbations contributes to the growth of eccentricity, thus increasing the probability of encountering chaotic motion during a typical satellite lifetime.

ANALYSIS

A Poincaré section is a useful tool for analyzing the relationship between eccentricity and the chaotic regions of phase space produced by an eccentric, commensurate orbit. Figure 1 displays two plots, the upper one is a Poincaré section of orbits with initial values of $(a, e, i, \Omega, \omega, M) = (\text{various periods near 12 hrs}, 55°, .01, 0°, 65°, -47°)$; the lower plot is similar except the initial eccentricity has been increased to .02. The dynamical model used to produce the trajectories is long period with secular oblateness effects and critical tesseral harmonics up to 4^{th} order and degree. Note the presence of chaotic regions in both plots, and the marked increase in the complexity and extent of the regions in the plot with e = .02 as compared to e = .01. The implication of this result is the traditional east-west stationkeeping method is much more likely to become unstable (that is, the action rate changes sign during a stationkeeping cycle) for e = .02. To assess this possibility, it is instructive to analyze the equations of motion associated with a satellite that is controlling its stroboscopic mean node to remain within a specified deadband region (i.e., east-west SK),

$$\dot{I} = \sum_{q=0} m h_{lmpq}^{\text{Tess}}(a^*, e^*, i^*) \sin[m(\lambda - \lambda_{lm})] + \sum_{q \neq 0} m h_{lmpq}^{\text{Tess}}(a^*, e^*, i^*) \sin[m(\lambda - \lambda_{lm}) - q\omega(t)], \tag{1a}$$

$$\dot{\lambda} = -\frac{3}{s_o^2 a^{*2}} I, \tag{1b}$$

$$\dot{\omega} \equiv \dot{\omega}_{\sec} = \frac{\partial H_o}{\partial I_2^*} = \frac{3}{2} \frac{J_2 n^*}{\left(1 - e^{*2}\right)^2} \left(\frac{R_e}{a^*}\right)^2 \left(2 - \frac{5}{2} \sin^2 i^*\right) \tag{1c}$$

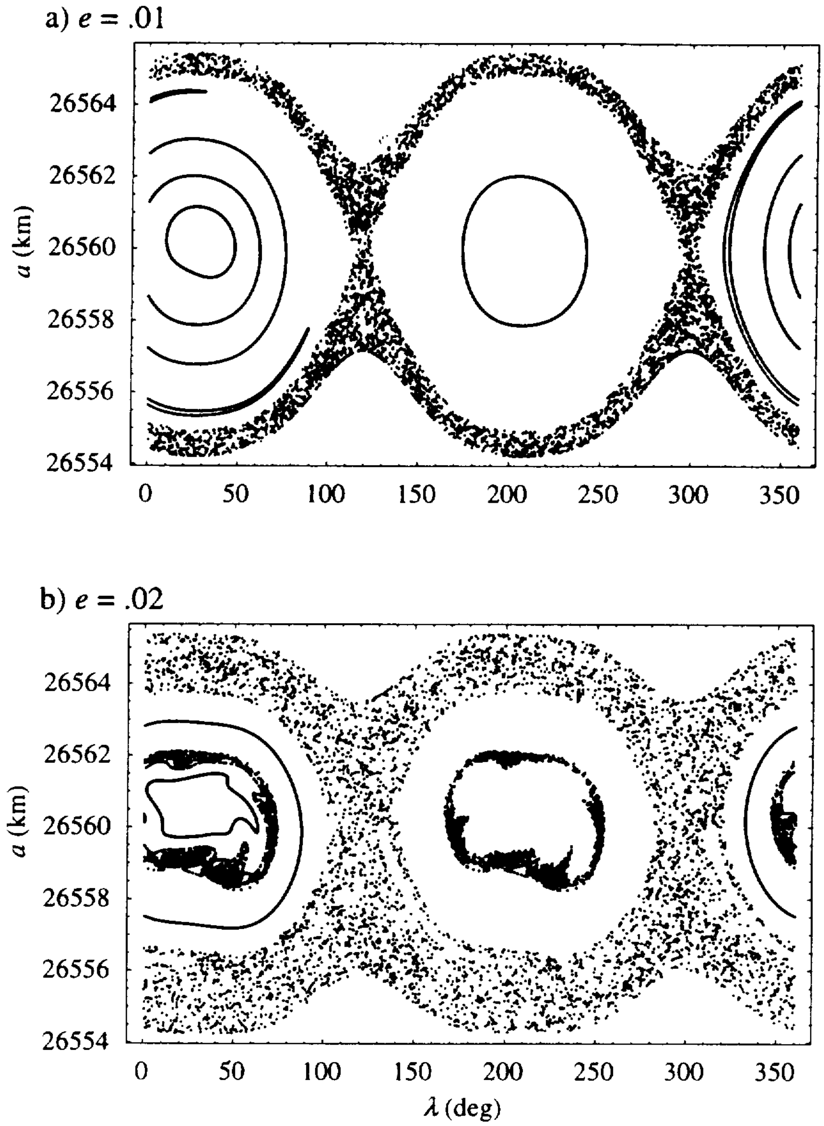

Figure 1: Poincaré section of GPS class of orbits for different eccentricities (ω= 65°)

Equation (1) represents a dynamical model with 3/2-DOF where the phase variables are the action $I \equiv I_1 - I_1^* = s_o(L - L^*) \approx \frac{s_o}{2}\sqrt{\mu/a^*}(a - a^*)$, the stroboscopic mean node $\lambda = 1/s_o\,(M + \omega) - (\theta_E - \Omega)$, perigee ω, and for a 12 hr orbit $s_o = 2$. A detailed development of these equations can be found in reference [3]. Proceeding in a manner similar to the classical control strategy, the node in Eq. (1a) is fixed to a specified nominal value λ_n. It is around this nominal value that the stationkeeping process controls the orbit so that it remains inside of a specified deadband region where the node is allowed to drift. Stationkeeping maneuvers are performed when the orbit reaches a boundary of the deadband. Now for this strategy, the tesseral harmonic terms ($h_{lmpq}^{\text{Tess}}(\cdot)\sin(\cdot)$ in Eq (1a)) with $q = 0$ can be considered constant, however, the terms with $q \neq 0$ are explicitly time-dependent because of their dependence on perigee, a time

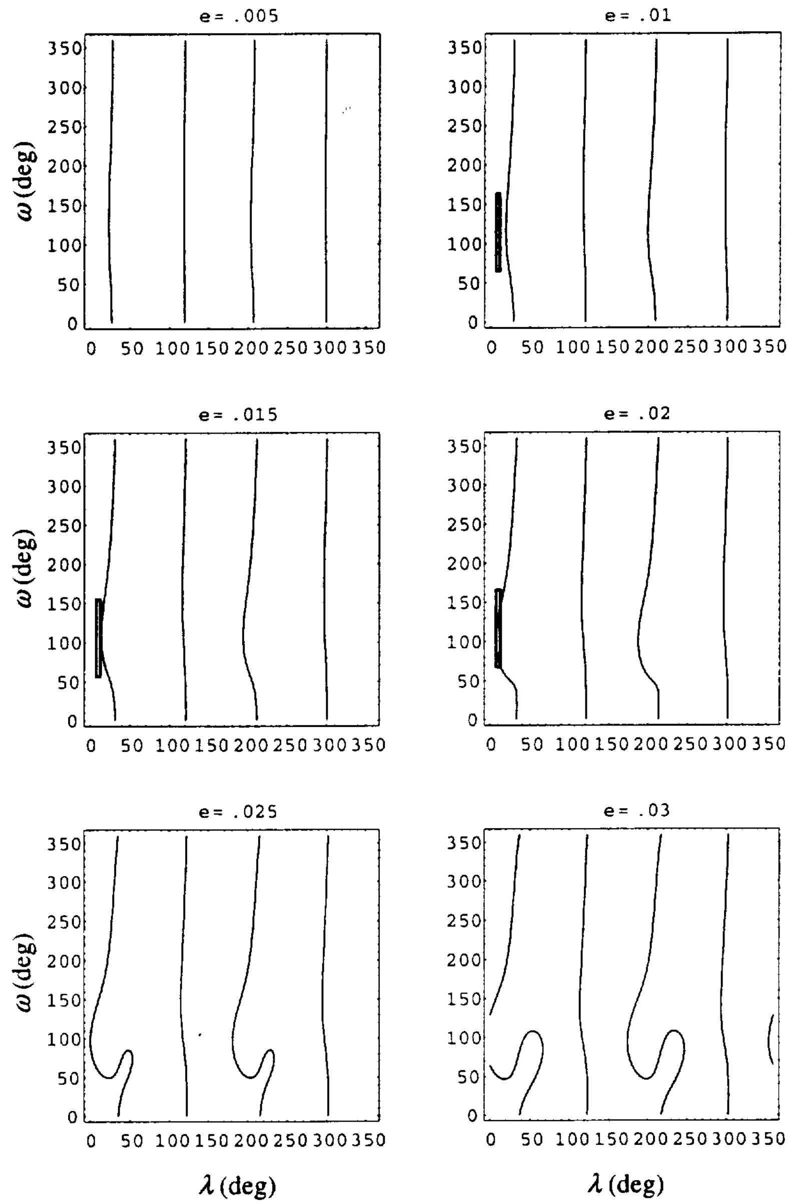

Figure 2: Action rate zeros parameterized by eccentricity

varying quantity. The time dependence invalidates two fundamental assumptions in the classical stationkeeping method: the element time histories are not symmetric around the exact resonance value, and the nodal acceleration is not guaranteed to have a constant sign within a cycle (the action rate passes through a value of zero).

To determine regions of phase space that have the potential to encounter a passage through an action rate zero set Equation (1a) equal to zero. The result yields a function of the node λ and perigee ω that is parameterized by eccentricity when the semi-major axis is set to the exact resonance value for 12 hr orbits and the inclination is set to the value associated with GPS orbits of 55°. Figure 2 depicts plots of action rate zeros as functions of perigee versus node for selected

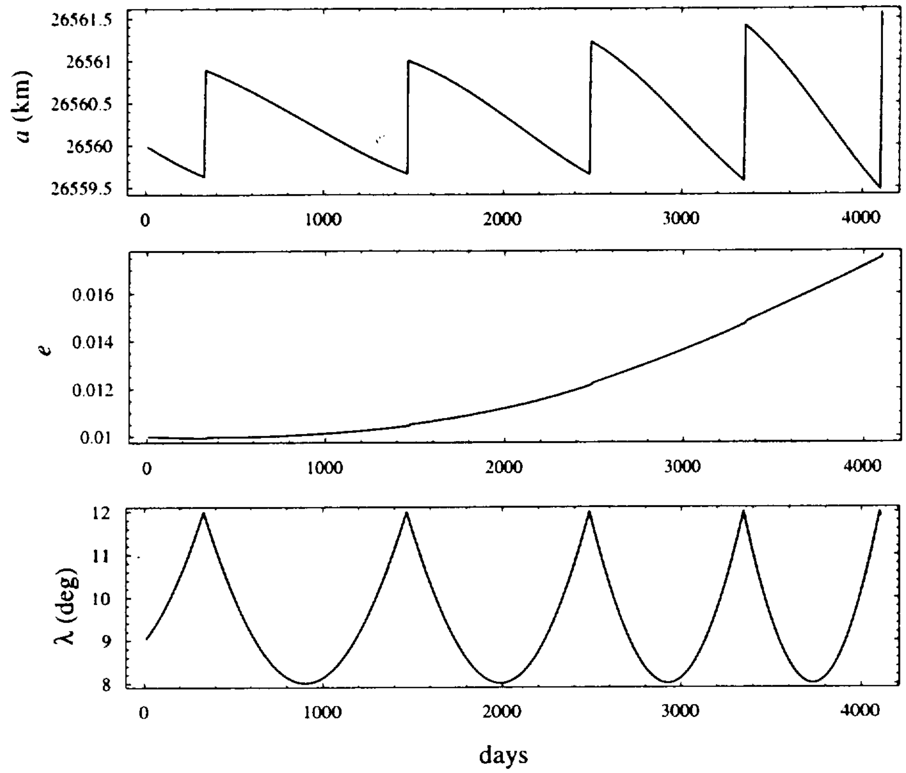

Figure 3: Stationkeeping cycle for the case with e = .01. No action rate zeros encountered.

values of eccentricity. Note that the behavior of the location of the zeros increases in complexity with modest changes in eccentricity. Recall, that for an inclination of 55°, perigee varies in a nearly secular fashion and, for a mission lifetime of 10 years, perigee can be expected to change by ~100°. Hence, if a satellite initial condition is placed near a line of zeros with an irregular shape, there exists an increased possibility of encountering the zero as one moves to higher eccentricities.

Two cases are considered as examples, one with an initial eccentricity at .01 and the other with an eccentricity of .02. All other initial mean elements are equal, and have values of $(a, i, \Omega, \omega, M) = (26560\,\text{km}, 55°, 0°, 65°, -47°)$. The selected deadband region has a width of 4° and a nominal nodal value of $\lambda_n = 10°$. The boxes shown in Figure2 for e = .01, .015, and .02 have sides with lengths equal to the deadband and a 100° change in perigee (i.e., a 10 yr mission). The boxes indicate the possibility of encountering an action rate zero during the lifetime of a mission. Examination of Figure 2 shows that the selected initial are such that the 1st case, with e = .01, does not encounter the SK instability over the satellite lifetime, and the 2nd case, with e = .02, does. Both cases employ the Eccentric Orbit Stationkeeping (EOSK) algorithm developed in reference [3] for use with orbits that encounter action rate zeros. Both cases are propagated for several thousands days with the results for semi-major axis, eccentricity, and node shown in Figures 3 and 4. The force model used in the propagation includes tesseral, zonal, lunar, and solar effects. In Figure 3, as expected, the trajectory does not encounter an action rate equal to zero, thus the nodal history exhibits the traditional 'scallop' shape. It is

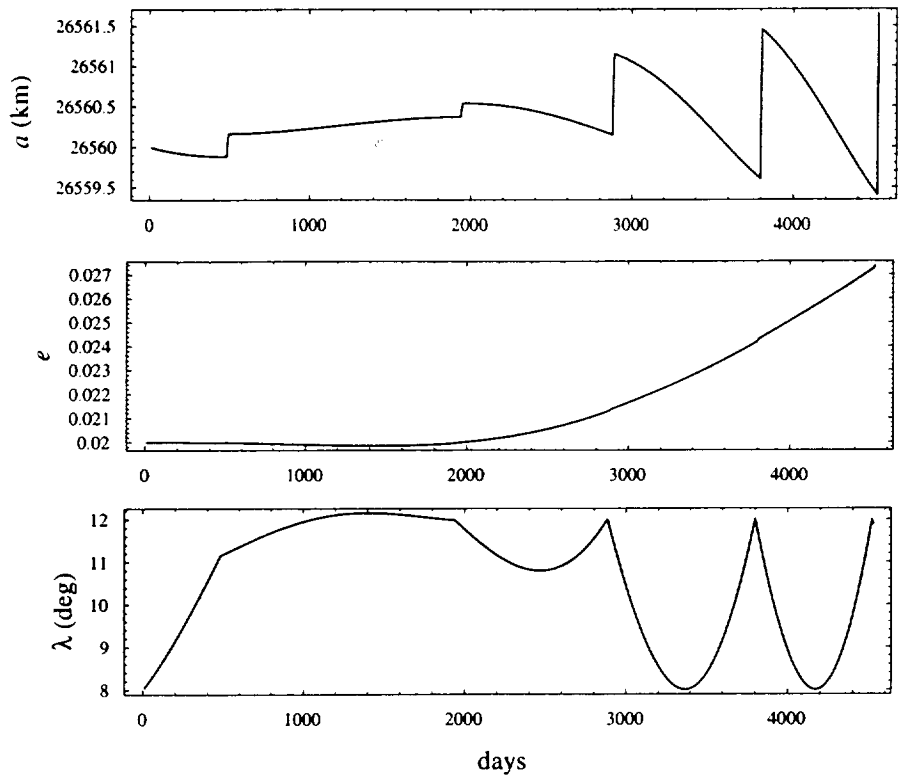

Figure 4: Stationkeeping cycle for the case with e = .02. Action rate zeros encountered in the second (500 to 2000 days) and third cycles (2000 to 2900 days).

noteworthy that the eccentricity grows significantly to around .015 after 10 years. The increase is due primarily to luni-solar effects. This behavior implies that the contours shown in Figure 2, that applied to the initial conditions with e = .01, are no longer relevant after 10 years. In this case, the contours for e = .015 are more appropriate. It can be seen in Figure 2 that, for e = .015, the action rate still does not pass through a zero for the selected nominal node (although it is close). Because of this behavior, it would be possible, even for the initial e = .01 case, to encounter an instability if a slightly larger nominal node had been selected, say 20°. Now, in the second case, the trajectory passes through an action rate equal to zero several times, as seen in Figure 4. Evidence of this can be seen in the irregular nodal histories during the second (500 to 2000 days) and third cycles (2000 to 2900 days). This is typical of the EOSK algorithm as it changes its grazing and burn boundaries to accommodate the sign change in the nodal acceleration. The EOSK algorithm is successful at keeping the node within the desired deadband region during the entire mission. It is important to note that, for this set of initial conditions, the traditional east-west stationkeeping algorithm would have become unstable during the 2nd cycle, and would have been unable to maintain the node with the desired deadband region.

CONCLUSION

These results indicate that a GPS constellation with modest orbital eccentricities can encounter significant regions of phase space with action rate zeros. They also show eccentricity is strongly influenced by luni-solar perturbations, continuing efforts will focus on characterizing the nature of a changing eccentricity, and how these changes impact the action rate zero regions (i.e., Figure

2) over a mission lifetime. Furthermore, the next phase of GPS orbits are to be delivered into their orbits with eccentricities not to exceed $e = .02$.[4] The preceding results show that future GPS satellites injected with borderline eccentricities, and using the current stationkeeping implementation, may run the risk of encountering SK instabilities. Continued work on this problem will consider the specific orbits proposed for the next phase of GPS, and assess their likelihood of encountering the stability issue.

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