ABSTRACT

This short paper is in response to one that appeared in this journal a few years ago [2]. The article was a comment on a previous paper [1], which presented the transformation equations between the standard two-port parameters. The equations were stated to be valid for complex terminations; which are useful when S-parameters are treated. The authors in [2] made some incorrect conclusions concerning the concept of " generalized scattering parameters" , and this paper seeks to clarify the somewhat confusing area of generalized scattering parameters.

On the Interpretation of Scattering Parameters

In 1994, Frickey [1] presented equations for the conversions between two-port parameters. The paper's contribution was that the new equations were valid for complex source and load impedances. Prior to this, all the published conversion equations (in explicit form) were restricted to real terminating impedances. Later a short paper by Marks and Williams [2] appeared, which discussed both the measurement and calculation of S-parameters. They claimed the conversions stated in [1] could lead to incorrect results. The objective of this note is to show that the equations in [1] are correct and useful, and that the objections in [2] are due to a misconception concerning S-parameters. To indicate the causes for the differences in opinion between [1] and [2], a brief review of S-parameters is in order.

The voltage and current on a lossless TEM transmission line (TL) with real characteristic impedance (Z_0), are expressed as

$$V(\chi) = V_f e^{-j\beta x} + V_r e^{j\beta x} = V^+ + V^- \tag{1a}$$

$$I(\chi) = \frac{1}{Z_0}\left(V_f e^{-j\beta x} - V_r e^{j\beta x}\right) = I^+ + I^- \tag{1b}$$

The voltage reflection coefficient at the load is

$$\Gamma_L = \frac{V^-(\chi = 0)}{V^+(\chi = 0)} = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{2}$$

The basic equations require

$$V^+ = Z_0 I^+ \tag{3a}$$

$$V^- = -Z_0 I^- \tag{3b}$$

See Figure 1 for the notation. Notice that the current is referenced positive to the right (for both the forward I^+ and reflected I^- parts). The S-parameters may be developed by connecting a two-port between two TLs with the same real characteristic impedance Z_0. When one line is excited by a generator of internal impedance Z_0, and the other terminated in Z_0, the incident and reflected waves on the driven line yield s_{11}. The exiting wave on the terminated line is used with the incident one on the generator side to develop s_{21}. Placing the generator on the other side provides for the determination of s_{22} and s_{12}. This is the way the scattering parameters are determined using a network analyzer. Figure 2 shows a general two-port with waves in both directions on the connecting TLs. The S-parameters may be expressed as [3], [4]

$$V_1^- = s_{11}V_1^+ + s_{12}V_2^+ \tag{4a}$$

$$V_2^- = s_{21}V_1^+ + s_{22}V_2^+ \tag{4b}$$

More involved (but equivalent) definitions follow; and in many of them, <u>total</u> voltages and currents at a given reference plane are used. The reason for this is that the incident and reflected waves are easily measured, but most circuit theory deals with total quantities. The <u>total</u> voltage and current at either reference port is

$$V_k = V_k^+ + V_k^- \tag{5a}$$

$$I_k = \frac{V_k^+}{Z_0} - \frac{V_k^-}{Z_0} \qquad k = 1,2 \tag{5b}$$

Rearrange these to obtain

$$V_k^+ = \tfrac{1}{2}(V_k + Z_0 I_k) \tag{6a}$$

$$V_k^- = \tfrac{1}{2}(V_k - Z_0 I_k) \tag{6b}$$

Which expresses the incident and reflected variables in terms of the total quantities. Now recall that the average power in an incident traveling wave (V_1^+ or V_2^+) is

$$P_{AV} = \tfrac{1}{2} Re \left\{ V_k^+ (I_k^+)^* \right\} = \frac{\left|V_k^+\right|^2}{2Z_0} \tag{7}$$

Where peak quantities are used. With this observation we can define incident and reflected complex amplitudes as

$$a_k = \frac{V_k^+}{\sqrt{Z_0}} \tag{8a}$$

$$b_k = \frac{V_k^-}{\sqrt{Z_0}} \tag{8b}$$

This choice is made so that

$$\frac{aa^*}{2} = \frac{\left|V^+\right|^2}{2Z_0} \tag{8c}$$

$$\frac{bb^*}{2} = \frac{\left|V^-\right|^2}{2Z_0} \tag{8d}$$

are equal to the incident and reflected average real power respectively. Then from eqns. (6a) and (6b)

$$a_j = \frac{V_j + Z_0 I_j}{2\sqrt{Z_0}} \tag{9a}$$

$$b_j = \frac{V_j - Z_0 I_j}{2\sqrt{Z_0}} \tag{9b}$$

Then the S-parameters are defined by

$$b_1 = s_{11}a_1 + s_{12}a_2 \tag{10a}$$

$$b_2 = s_{21}a_1 + s_{22}a_2 \tag{10b}$$

Which is equivalent to the first definition in eq. (4). This is the most often used definition and will be called the " standard" . See Figure 2 for details. If the transmission lines connected to a general two-port have different (real) characteristic impedances, say Z_{01} and Z_{02}, then eq. (9) is modified accordingly, see Figure 3. In this case the resulting S-parameters are sometimes called " generalized scattering parameters," see [4], p. 205. For this situation s_{11} and s_{22} are the same as the standard, but the transfer terms s_{12} and s_{21} differ from the standards by

$$s_{12} = \sqrt{\frac{Z_{02}}{Z_{01}}} \quad s_{12} \quad \text{(standard)} \tag{11a}$$

$$s_{21} = \sqrt{\frac{Z_{01}}{Z_{02}}} \quad s_{21} \quad \text{(standard)} \tag{11b}$$

Recall that at a reflection plane in TL theory

$$\Gamma^I = \frac{I^-}{I^+} = -\frac{V^-}{V^+} = -\Gamma^V \tag{12}$$

Which says the current reflection coefficient is the negative of the voltage reflection coefficient. This concludes the transmission line portion of the S-parameter development.

Now we give the development of " scattering parameters" based on strictly lumped circuits [5] - [10] . Consider a source and arbitrary load as given in Figure 4. The source with complex internal impedance

Z_0 is connected successively to a matched load $Z_0{}^*$ and an arbitrary complex load. When connected to the load $Z_0{}^*$ we have

$$V_i = \frac{V_s Z_0{}^*}{Z_0 + Z_0{}^*} \tag{13a}$$

$$I_i = \frac{V_s}{Z_o + Z_0{}^*} \tag{13b}$$

Even though the circuit is lumped, we call V_i and I_i incident voltage and current. The rationale is that in the matched case, all the power the generator can develop is absorbed in the load. Therefore no reflection of voltage and current exists. For the general load Z_L we have

$$V = \frac{V_s Z_L}{Z_0 + Z_L} \stackrel{\Delta}{=} V_i + V_r \tag{14a}$$

$$I = \frac{V_s}{Z_0 + Z_L} \stackrel{\Delta}{=} I_i - I_r \tag{14b}$$

and we define reflected quantities V_r and I_r; since for the general load, maximum power is not absorbed, so some must be reflected back into the generator. These terms are arbitrarily introduced to give a sense of motivation for " the reader who is already familiar with TL theory." Carlin and Giordano [5] explain this on p. 226 of their text. Manipulating equations (13) and (14) yield

$$I_r = \left(\frac{Z_L - Z_0{}^*}{Z_L + Z_0} \right) I_i \stackrel{\Delta}{=} S^I I_i \tag{15a}$$

$$V_r = \frac{Z_0}{Z_0^*}\left(\frac{Z_L - Z_0^*}{Z_L + Z_0}\right)V_i \stackrel{\Delta}{=} S^V V_i \quad (15b)$$

So

$$S^V = \frac{Z_0}{Z_0^*} S^I \quad (15c)$$

and if Z_0 is real, $S^V = S^I$; which is never the case in TL theory (see eq. (12)). From eq. (13) we can show

$$V_i = Z_0^* I_i \quad (16a)$$

and with eq. (14) we find

$$V_r = Z_0 I_r \quad (16b)$$

Notice eq. (16) relates incident and reflected quantities associated with a lumped network under different loading conditions (i.e., either Z_0^* or Z_L). They are similar to the incident and reflected quantities for a transmission line (eqns. (3a) and (3b)). When the source impedance is pure real, equations (3b) and (16b) always differ by a negative sign. The sign of the reflected current in eq. (14b) is discussed in [6] p. 573.

With this background we now introduce the scattering variables as in [5], eq. (4.3), p. 225.

$$a = \frac{1}{2}\left[\frac{\bar{V}}{\sqrt{r_0}} + \bar{I}\sqrt{r_0}\right] \quad (17a)$$

$$b = \frac{1}{2}\left[\frac{\bar{V}}{\sqrt{r_0}} - \bar{I}\sqrt{r_0}\right] \quad (17b)$$

Where " a" is the incident voltage or current, " b" is the reflected current or voltage. These are very similar to eq. (9), but differ in two ways. Firstly, here rms quantities are used, and secondly, the normalizing constant r_0 is an arbitrarily chosen pure real positive number. It is usually chosen to be equal to the real source impedance, out of which the network is to operate. When the normalizing constant is complex, a modification to the above is needed. See [5], sec. 4.13, or [6], sec. 8.5, for a complete discussion. To retain many of the desirable properties of the " standard" scattering matrix when real normalization is used, the definitions for complex normalization turn out to be

$$a_k = \frac{1}{2}\left[\frac{V_k}{\sqrt{r_k}} + \frac{i_k Z_k}{\sqrt{r_k}}\right] \tag{18a}$$

$$b_k = \frac{1}{2}\left[\frac{V_k}{\sqrt{r_k}} - \frac{i_k Z_k{}^*}{\sqrt{r_k}}\right] \tag{18b}$$

These are equations (4.230a,b) in [5], (the equation given for b_k has a typographical sign error). These equations are also those in [6]; eqns. (91a,b), p. 608. Actually one can define either voltage S^V or current S^I scattering parameters, and it turns out the above is that for currents. This happens to be the best choice, since this set reduces to the " standard" when the terminations become pure real.

A further generalization for complex normalization is referred to as the " power wave" approach [11], [12]. The power waves are defined by

$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{Re|Z_i|}} \tag{19a}$$

$$b_i = \frac{V_i - Z_i * I_i}{2\sqrt{Re|Z_i|}} \tag{19b}$$

and a power wave reflection coefficient is defined by

$$S = \frac{b_i}{a_i} = \frac{Z_L - Z_i *}{Z_L + Z_i} \tag{20}$$

Which reduces to a TL voltage reflection coefficient when Z_i is real and positive. Notice the real part of Z_i may be negative in the power wave realm. In all cases before, however, the real part of Z_i was always positive. Kurokawa [11], [12], clearly states the difference between power waves and the voltage traveling waves of transmission lines.

All S-parameters depend on both the two-port in question, and the termination impedances upon which they are defined. This is unlike the [Z] or [Y] parameters which depend only on the two-port. A practical result of this fact is as follows. Given that the S-parameters are determined by measurement in a 50 ohm system, how does one modify these for analysis purposes where the source now has a complex internal impedance, and the load is also complex? Perhaps the first (easily accessible) published method was that of Kurokawa [11], where he showed how to transform S-parameters from one complex normalization Z_i to another complex normalization $Z_i^{'}$. The transformation was given as

$$[s'] = [A]^{-1}[s - \Gamma^+][I - \Gamma s]^{-1}[A^+] \tag{21a}$$

$[\Gamma]$ and $[A]$ are diagonal matrices with elements given by r_i and

$$(1-r_i{}^*)\frac{\sqrt{|1-r_i r_i{}^*|}}{|1-r_i|}$$

where

$$r_i = \frac{Z_i' - Z_i}{Z_i' + Z_i{}^*} \tag{21b}$$

Bodway [13] also published the transformations for complex normalization, and his formulas are in a more useful form. They are:

$$s_{11}' = \frac{A_1{}^*}{A_1}\frac{\left[(1-r_2 s_{22})(s_{11}-r_1{}^*)+r_2 s_{12} s_{21}\right]}{\left[(1-r_1 s_{11})(1-r_2 s_{22})-r_1 r_2 s_{12} s_{21}\right]} = \frac{A_1{}^*}{A_1}\frac{N}{D} \tag{22a}$$

$$s_{12}' = \frac{A_2{}^*}{A_1} \quad \frac{s_{12}\left[1-|r_1|^2\right]}{D} \tag{22b}$$

$$s_{21}' = \frac{A_1{}^*}{A_2} \quad \frac{s_{21}\left[1-|r_2|^2\right]}{D} \tag{22c}$$

$$s_{22}' = \frac{A_2{}^*}{A_2} \quad \frac{\left[(1-r_1 s_{11})(s_{22}-r_2{}^*)+r_1 s_{12} s_{21}\right]}{D} \tag{22d}$$

$$A_i = \frac{(1-r_i{}^*)}{|1-r_i|}\left(1-|\ r_i\ |^2\right)^{\frac{1}{2}} \tag{22e}$$

$$r_i = \frac{Z_i' - Z_i}{Z_i' + Z_i{}^*} \tag{22f}$$

Later, Carson [8] gave the details of the development of eq. (21) in his Chapter 7 and Appendix A; however, he restricted the original set of parameters to be with respect to a real termination. Gonzalez [14] has also given the equations with the same restriction.

Now we show how complex normalization may be introduced into the traveling waves of transmission lines. From eq. (7) we find

$$
\begin{aligned}
P_{AV} &= \tfrac{1}{2} Re\{V_k^+ (I_k^+)^*\} \\
&= \tfrac{1}{2}\tfrac{1}{2}\{Z_0 I^+ (I^+)^* + Z_0^* (I^+)^* (I^+)\} \\
&= \tfrac{1}{4}[Z_0 + Z_0^*](I^+)(I^+)^*
\end{aligned}
$$

Now define

$$\tfrac{1}{2} aa^* = \tfrac{1}{4}[Z + Z_0^*] I^+ (I^+)^* \tag{23a}$$

Then we can assign

$$a = \sqrt{\frac{Z_0 + Z_0^*}{2}} I^+ \tag{23b}$$

Notice we have assigned $V_k^+ = Z_0 I_k^+$ which is in agreement with eq. (3a). Now eq. (23b) is eq. (7a) of [1]. Now for a reflected wave we find

$$
\begin{aligned}
P_{AV} &= \tfrac{1}{2} Re\{V_k^- (I_k^-)^*\} \\
&= \tfrac{1}{2}\tfrac{1}{2}\{-Z_0 I_k^- (I_k^-)^* - Z_0^* (I_k^-)^* (I_k^-)\} \\
&= -\tfrac{1}{4}[Z_0 + Z_0^*](I_k^-)(I_k^-)^*
\end{aligned} \tag{24a}
$$

Now assign

$$b_k = \sqrt{\frac{Z_0 + Z_0^*}{2}} I_k^- \tag{24b}$$

Which is eq. (7b) of [1]. With a and b so defined, we find

$$a_j = \frac{V_j + Z_{0j} I_j}{[2(Z_{0j} + Z_{0j}^*)]^{1/2}} \tag{25a}$$

$$b_j = \frac{V_j - Z_{0j}^* I_j}{[2(Z_{0j} + Z_{0j}^*)]^{1/2}} \tag{25b}$$

(see [1], eqns. (12) and (13)). They are also eqns. (3) and (4) in [7]. Observe only two difficulties appear in equations (23) through (25). The negative sign in eq. (24a) for the power is due to the TL requirement that $V^- = -Z_0 I^-$, and secondly the V_j and I_j in eq. (25)

are rms, whereas those in eq. (23) (V_k, I_k) are peak values. Since the S-parameters are ratios, the peak vs. rms condition is not an issue. The negative sign may be dropped, since there is no direction in a lumped circuit. The sign appears in TL equations due to the variation of the wave with distance, see [15], p. 44. Equation (25) reduces exactly to eq. (9) when Z_0 is real; which demonstrates the validity of the assignments for " a" and " b" .

Returning to eq. (25), we may define the generalized scattering parameters for complex normalization; which was done by Frickey [1]. To verify the consistency between the equations given so far, we will analyze a simple network. Consider the circuit in Figure 5a; straightforward calculations show

$$\begin{aligned} &Z_{11} = 3 - j \qquad Z_{22} = 7 + j \qquad Z_{12} = Z_{21} = 3 + j \\ &s_{11} = .168/\underline{-59.4^\circ} \qquad s_{22} = .375/\underline{-27.8^\circ} \\ &s_{12} = s_{21} = .357/\underline{33.1^\circ} \end{aligned} \tag{26}$$

If we calculate the Z-parameters from the above S-parameters based on the complex normalization, and use the equations in [1]

$$Z_{11} = \frac{(Z_{01}{}^* + s_{11} Z_{01})(1 - s_{22}) + s_{12} s_{21} Z_{01}}{D} \tag{27a}$$

$$Z_{12} = \frac{2\, s_{12} \sqrt{R_{01} R_{02}}}{D} \tag{27b}$$

$$Z_{21} = \frac{2\, s_{21} \sqrt{R_{01} R_{02}}}{D} \tag{27c}$$

$$Z_{22} = \frac{(1 - s_{11})(Z_{02}{}^* + s_{22} Z_{02}) + s_{12} s_{21} Z_{02}}{D} \tag{27d}$$

$$D = (1 - s_{11})(1 - s_{22}) - s_{12} s_{21}$$

We indeed obtain the Z-parameters of the network, which are independent of the terminations. If we remove the complex terminations, i.e. let $Z_{01} = 2\Omega$ and $Z_{02} = 3\Omega$, (see Figure 5b); then the S-parameters become (again by circuit analysis using eqns. (9) and (10))

$$
\begin{aligned}
s_{11} &= .345/\underline{-64.3^\circ} \\
s_{12} = s_{21} &= .349/\underline{32.8^\circ} \\
s_{22} &= .314/\underline{-6.7^\circ}
\end{aligned}
\tag{28}
$$

Which we see are different from those determined earlier. Using eq. (27) again, we recover the Z-parameters as before. Notice the S-parameters depend on both the two-port and the reference terminations; whereas the

Z-parameters depend only on the two-port. Now assume a second set of complex terminations as given in Figure 5c. The S-parameters are (by circuit analysis)

$$
\begin{gathered}
s'_{11} = .726/\underline{-26.2^\circ} \qquad s'_{22} = .765/\underline{-7.77^\circ} \\
s'_{12} = s'_{21} = .186/\underline{68.2^\circ}
\end{gathered}
\tag{29}
$$

If we use the equations of Bodway [13], and start with the first [S] set defined on their complex normalization (circuit in part a), we should be able to develop those just given above $[S']$, which are defined with complex normalization Z_i'. Performing the calculations in eq. (22) shows that this is indeed the case.

We now address [2]. Their eq. (2) is

$$
\hat{\Gamma} = \frac{Z_L - Z_{ref}^*}{Z_L + Z_{ref}}
\tag{30}
$$

Which is the current reflection coefficient, eq. (15a), or the power wave reflection coefficient, eq. (20). They incurred errors by assuming $\hat{\Gamma}$ is actually the reflection coefficient determined by the network analyzer. The instrument uses real ($Z_0 = 50\Omega$) reference impedances and measures $V^- / V^+ = (Z_L - Z_0)/(Z_L + Z_0)$. The S-parameters it measures are for 50Ω references. If the set is to be transformed into one for complex normalization, then eqns. (21) and (22) are applicable. Recall that the S-parameters (and all other two-port sets) are used in lumped circuit analysis, even though [S] may be developed (and measured) using traveling wave procedures. The fact that Frickey reported his equations were verified by using both PSPICE and Microwave Harmonica, means the routines are also without error for this type of calculation.

The main points to make are that the " generalized scattering parameters" (where generalized means complex normalization) reduce to the measured " microwave" scattering parameters when the normalization Z_0 is pure real. In the microwave case, Z_0 is the real characteristic impedance of the TL. For the generalized case, Z_0 is an arbitrary constant. Slight differences in definition exist for the variables " a" and " b" under real or complex Z_0 cases. The HP application note #95 [13] clearly explains this on pages 1-3.

From the equations developed earlier, we may summarize as follows. The word generalized may mean complex terminations or just different (real) terminations at the input and output ports. We have real traveling waves on TLs which spawn the " standard set" . We have incident and reflected variables V_i, I_i, V_r, I_r in lumped circuit theory, from which the generalized case emerges. There are also " power waves" , which are close to the generalized case (difference is that Re{ Z_i } may be negative for power waves). Also Z_0 is called either the characteristic, or reference, impedance. In TL theory the variables are related by

$$\begin{gathered} V^+ = Z_0 I^+ \qquad V^- = -Z_0 I^- \qquad \Gamma^V = -\Gamma^I \\ V = V^+ + V^- \qquad I = I^+ + I^- \end{gathered} \tag{31}$$

and in many cases peak values are used. The characteristic impedance Z_0 is real, and is coupled to V, I, and the power flow along the line. A complex characteristic impedance $Z_0 = R + jX$ is inherently lossy, and is not of practical use; (omit the special " distortionless line" case). The complex normalization case has the following group of equations:

$$\begin{gathered} V_i = Z_0 * I_i \qquad V_r = Z_0 I_r \qquad \Gamma^V = \frac{Z_0}{Z_0 *} \Gamma^I \\ V = V_i + V_r \qquad I = I_i - I_r \end{gathered} \tag{32}$$

here Z_0 is arbitrary, and rms values are used. The variables " a" and " b" are incident and reflected current variables. A reflection factor has the form

$$s = \frac{Z_L - Z_0 *}{Z_L + Z_0} \tag{33}$$

Which is the same as the power wave reflection factor.

There exist more definitions of " scattering variables" , see for example Ishii [16]. There he relates incident and reflected electric field intensities (p.120). Woods [17], [18] extends the case to actual transmission lines with complex Z_0. See references [15], pg. 202, and [19], [20] for discussions of power flow on lossy lines with complex Z_0, and their analysis on the Smith Chart. The remaining references augment the presentation here.

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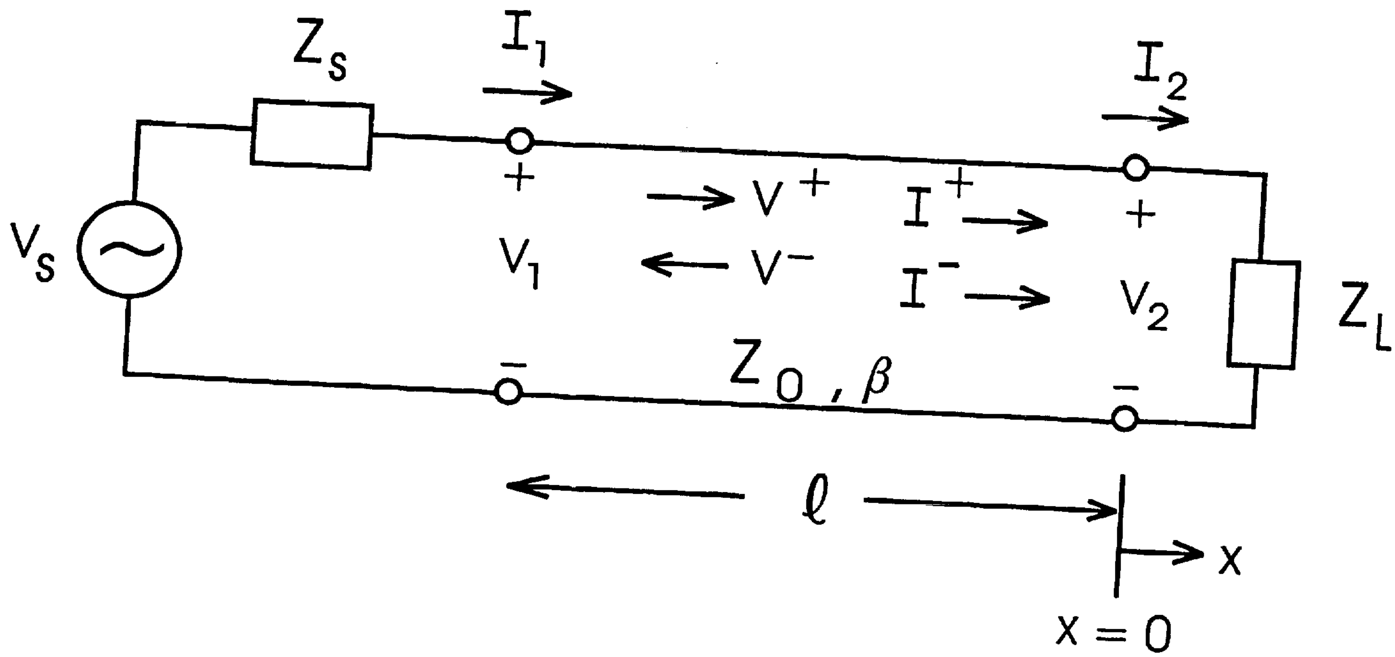

Z_S
I_1
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+
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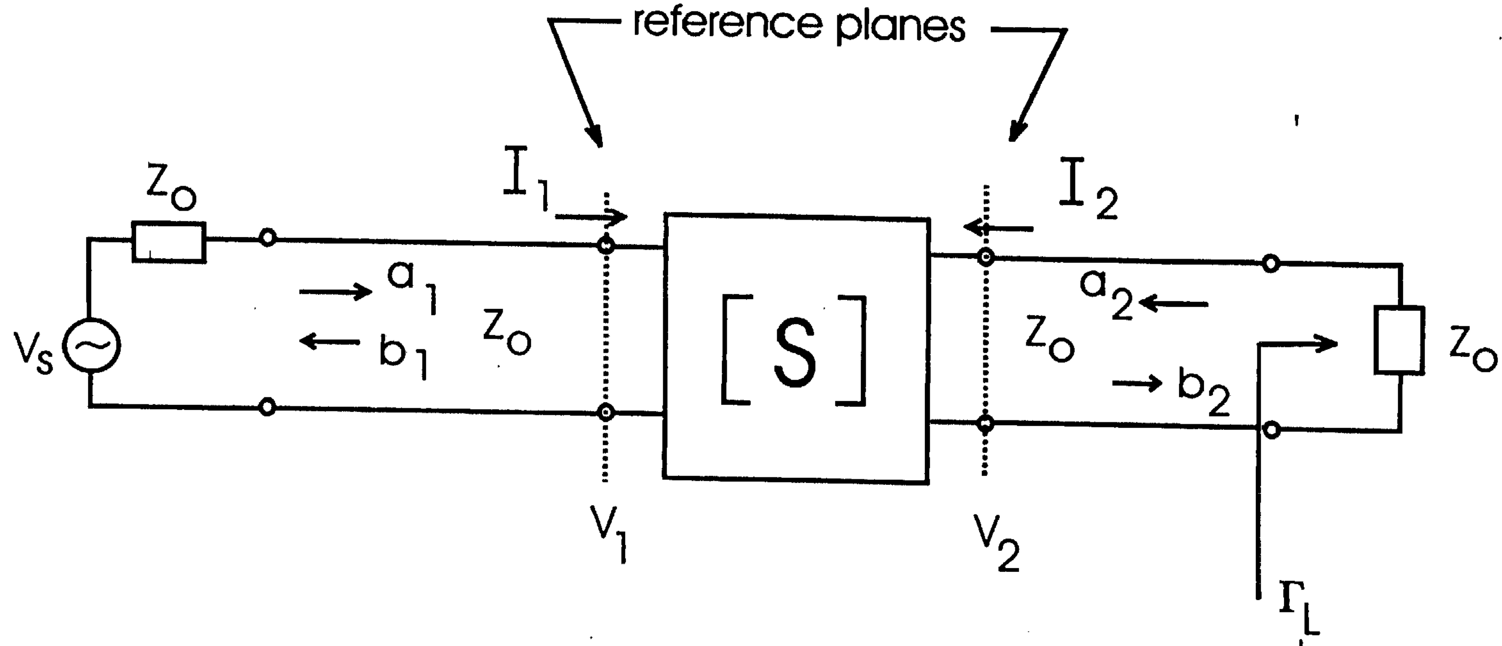
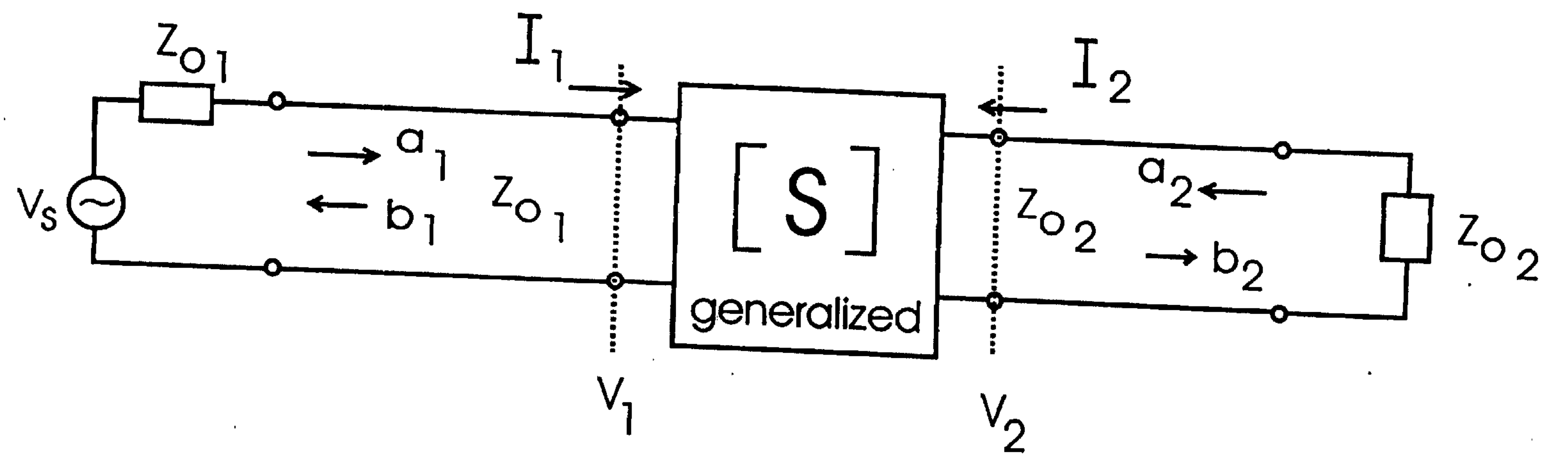


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



Zo1
I1
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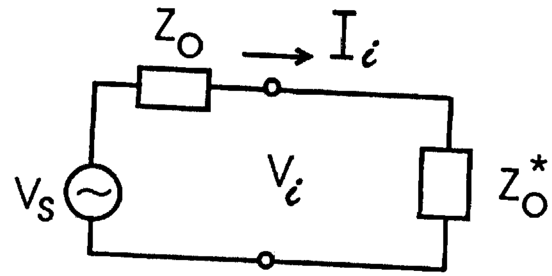
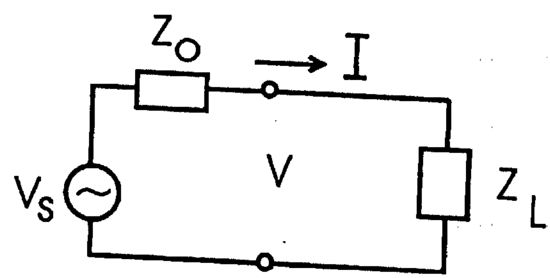
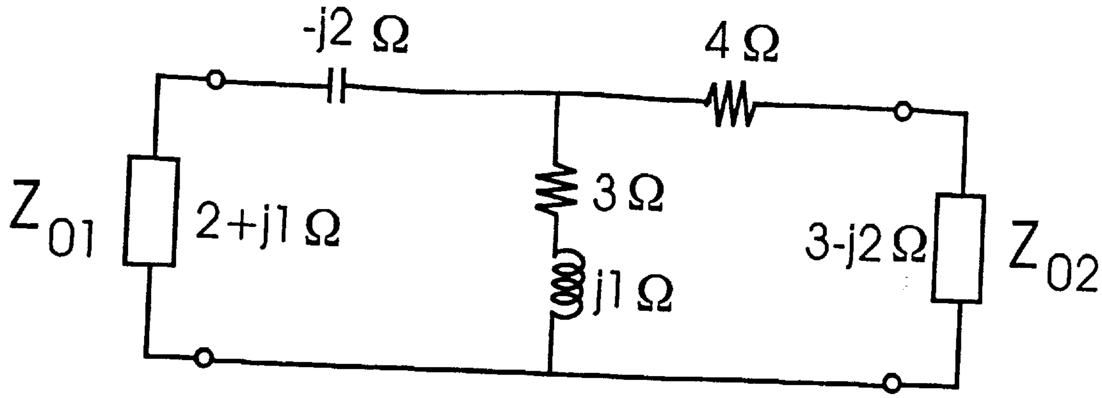


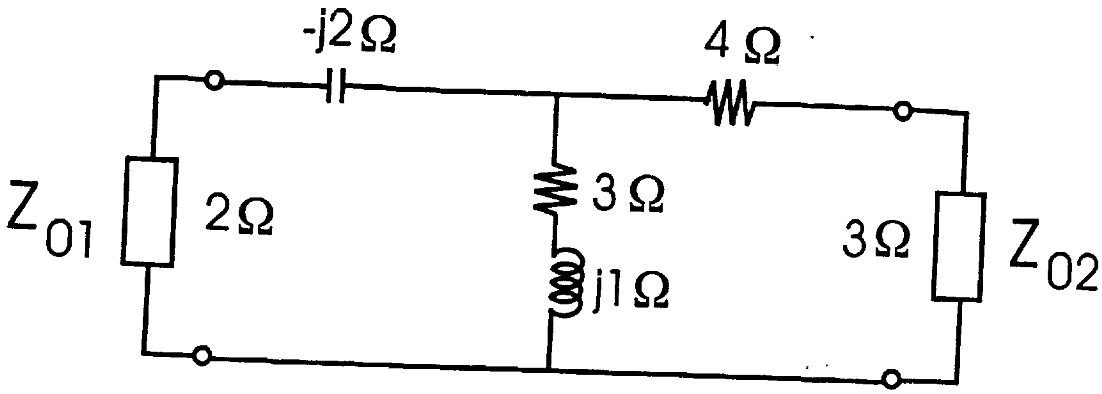
Z_O
I_i
V_S
V_i
Z_O^*



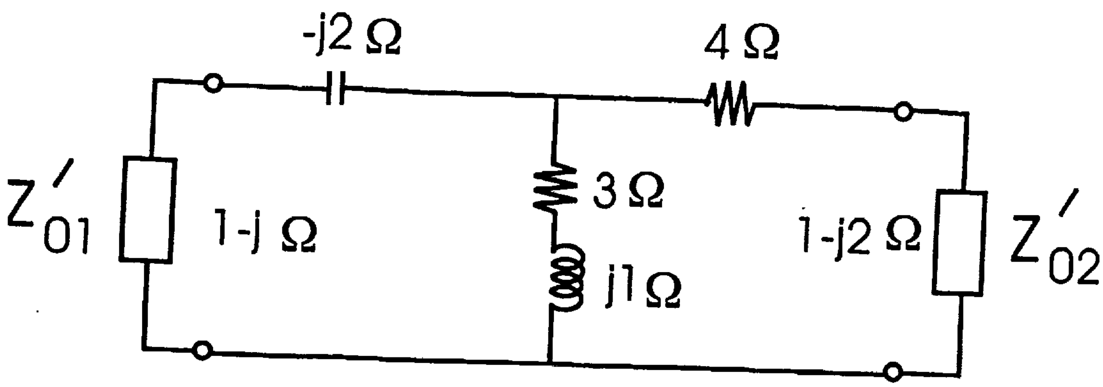
Z_O
I
V_S
V
Z_L


-j2 Ω
4 Ω
Z01
2+j1 Ω
3 Ω
j1 Ω
3-j2 Ω
Z02

a)


-j2 Ω
4 Ω
Z01
2 Ω
3 Ω
j1 Ω
3 Ω
Z02

b)


-j2 Ω
4 Ω
Z′01
1-j Ω
3 Ω
j1 Ω
1-j2 Ω
Z′02

c)