TURBULENT EDDY VISCOSITY AND LARGE-SCALE CONVECTION IN THE SUN

Richard B. Stothers

Institute for Space Studies, NASA/Goddard Space Flight Center,

2880 Broadway, New York, NY 10025, USA

ABSTRACT

It is suggested here that the laminar character of the large-scale deep convective flows appearing in numerical simulations of the Sun's convective envelope arises from the effect of turbulent eddy viscosity. Previously, M. Schwarzchild suggested the same idea to explain the observed surface granulation in the Sun.

Key words: convection-- Sun: granulation-- Sun: interior-- turbulence

1 INTRODUCTION

The problem of how physically to describe thermal convection in the Sun's envelope has a long pedigree, going back to the middle of the nineteenth century. Recent attempts to solve this problem by performing numerical simulations have revealed the presence of unexpected structures in the deep turbulent flows, including large-scale laminar features (Spruit, Nordlund & Title 1990; Cattaneo et al. 1991; Brummell, Cattaneo & Toomre 1995; Stein & Nordlund 1998). Although the numerical simulations are affected by enormously restricted space and time scales and refer only to the Sun's uppermost few scale heights, they apparently are able to reproduce fairly well the main observed surface convective patterns, such as the photospheric granulation (Stein & Nordlund 1989, 1998; Nordlund et al. 1997). This lends some confidence in their verisimilitude. To explain simultaneously the deep plume-like convective flows and the surface cellular patterns is, however, difficult, and Nordlund et al. have doubted whether the granulation is actually turbulence. The purpose here is to offer an alternative possible explanation by invoking the notion of turbulent eddy viscosity. This leads to a more clearly unifying picture, which is a simple and inevitable consequence of basic turbulence theory.

2 VISCOUS CONTROL OF SOLAR ENVELOPE CONVECTION ?

The basic equations of fluid hydrodynamics can be nondimensionalized and the problem of convection can be parameterized in terms of a few dimensionless numbers, such as the Rayleigh (Ra) and Prandtl (Pr) numbers. Since Ra and Pr both depend on ν, the kinematic viscosity, it becomes obvious that the parameters of the convective flows must be very different in the gaseous solar envelope and in the semisolid terrestrial mantle. Despite this formal difference, the resulting flows are found in numerical simulations to have large-scale similarities that are imposed by the physical boundary conditions and by some additional factor in the case of the Sun that must involve more than the highly non-Boussinesque flow conditions.

The Reynolds number, which describes the apparent resistance of smooth laminar flow to the outbreak of disordered motions, is defined by

$$Re = V\Lambda/\nu$$

where V represents the mean flow velocity and Λ is a characteristic length associated with the mean flow. In laboratory experiments as well as numerical simulations, if Re exceeds ~2000 the flow becomes turbulent (Davies 1972; Brummell, Hurlburt & Toomre 1998).

Since Re is of order 10^{-20} in the Earth's mantle (Bercovici, Schubert & Glatzmaier 1992), convection there is believed to be of laminar, or cellular, type, although the situation is complicated by the possible concomitant presence of thermal plumes, indicative of local soft or even hard turbulence due to very high Rayleigh numbers (Yuen et al. 1993).

In the solar convection zone, the molecular (and radiative) viscosities are negligible in the context of the larger scale of the energy-containing eddies. Since Re formally attains 10^{10} (Canuto & Christensen-Dalsgaard 1998) the convective flow is expected to be highly turbulent, with a wide spectrum of eddy sizes $\boldsymbol{l}$ and velocities v. External energy is supplied to the system from the envelope bottom at mostly very large scales. Subsequently, nonlinear interactions determine the transfer of kinetic energy to and from the eddies of all sizes, with the consequence that the kinetic energy of the major energy-containing eddies moves in a cascade down to the smallest eddies, where it is dissipated by molecular viscosity as heat. In the case of very large eddies, there is no feeding in of energy (except by the external sources), and so these eddies experience only the drain due to turbulent eddy viscosity. This process is fundamental in any turbulence phenomenon and is not specific to the Sun (Batchelor 1970; Tennekes & Lumley 1972). Since turbulence generates stresses that formally resemble viscous stresses, a coefficient of turbulent eddy viscosity can be defined very roughly by

$$\nu_t = C\, v\, \boldsymbol{l}$$

in analogy with molecular viscosity. Laboratory experiments indicate that the constant C is of order unity (Davies 1972).

It is untrue that turbulence theory implies an absence of structure in the turbulent flows. Using very simple arguments, Petrovay (1990) has characterized the four basic morphological features of turbulent convection: upflows, downflows, fibrillar structures, and cellular structures. He has estimated the horizontal filling factors of the moving parcels and the types of flows at various depths in the solar envelope; for example, cellular upflows prevail near the surface, and plume-like downflows in the bulk of the convection zone. More recently, Kupka (1999) has numerically solved the set of turbulent-convection moment equations of Canuto & Dubovikov (1998), which yield convective fluxes and horizontal filling factors in good agreement with large eddy simulations of idealized solar envelope models. The self-organization achieved by the convective flows cannot be all due to the compressibility and density stratification of the gases, because similar plume-like features appear also under Boussinesque laboratory conditions (Siggia 1994). Something else must be occuring.

Deep inside the solar convection zone, the mixing length of large eddies, Λ, is probably of the order of the distance to the nearest convective boundary (Canuto & Christensen-Dalsgaard 1998). According to numerical simulations the turbulent eddies that transport most of the net heat flux outward in the Sun are quite small compared to the coexisting large-scale motions (Cattaneo et al. 1991). In this situation, turbulence is not choked off by the substantial eddy viscosity it generates, at least not on the small flux-carrying scales (Unno 1961; Nakano et al. 1979). Turbulence, however, will have a bigger dissipative effect on the larger-scale convective motions. At the largest scales, the turbulent Reynolds number Re_t must be of order unity, because the turbulent flows and the mean flow become comparable in size. Therefore, the largest flows ought to be laminar-like in structure. In the numerical simulations the enormous upflows and downdrafts that occur (Brummell et al. 1995) do appear to be quasilaminar flows. These flows may well be related to the giant convection cells proposed on other grounds by Simon & Weiss (1968). If they overshoot into the surface boundary layer, their manifestation could conceivably be the huge velocity cells (Beck, Duvall, & Scherrer 1998) and large active regions (Bai 1988) that have been observed as long-lived background features.

To some extent, our argument about convective self-regulation can be verified by applying it more directly to the solar surface, where the coarsely regular granulation appears. There, $\Lambda \approx H$ (Canuto & Christensen-Dalsgaard 1998) and $l \approx H$ (Parker 1991; Stein & Nordlund 1998), H being the density or pressure scale height. Furthermore, both V and v reach a sizable fraction of sonic velocity near the surface. Hence $Re_t \sim 1$, and the flow is expected to be laminar, as Schwarzschild (1959) first pointed out. Smaller scales of motion at the surface doubtless exist, but are difficult to resolve observationally and numerically. A Kolmogorov energy spectrum, $E(k) \sim k^{-5/3}$, is not needed to indicate turbulence, because such a spectrum would show up only for the rather limited inertial subrange of wavenumbers, k. This range can be relatively small in many applications (Chandrasekhar 1949), and, not surprisingly, its exact location for solar surface conditions is still uncertain (Espagnet et al. 1993; Nordlund et al. 1997).

Schwarzschild's (1959) simple argument about why the Sun's granulation probably consists of turbulent eddies can be carried further. Lamb (1932) showed theoretically that, in an incompressible fluid, an imposed array of two parallel rows of equidistant vortical eddies (say, granules) separated horizontally by a distance a and vertically by a distance b becomes stable when $b/a = 0.281$. Although the upper boundary layer of the Sun bears only a coarse resemblance to Lamb's idealized situation, the eddy (granule) size anisotropy factor can be expected to be roughly equal to b/a. Consequently, if the solar granulation is loosely interpreted as a laminar phenomenon with a vertical dimension H, the anticipated diameter of a typical granule is $\sim H/0.281$ or ~1500 km. Other approaches give similar results (Simon & Weiss 1968). These predictions agree satisfactorily with the observed average diameter of ~1000 km.

There exists considerable evidence from observations and also from numerical simulations that the convective flows near the solar surface are in fact essentially laminar (Nordlund et al. 1997). The origin of this behavior has been speculatively attributed by the numerical modelers to the rapid expansion of the large adiabatic upflows from deeper levels, leading to weak turbulence in the overturning fluid at the surface. Even the very existence of turbulence there has been questioned. This extreme view, as we have seen, need not be taken literally. In our view and Schwarzschild's, the granulation is a *consequence* of turbulence.

3 CONCLUSION

If the hypothesis offered here is correct, turbulent eddy viscosity controls large-scale convection in the Sun and presumably in other stars. A consistent interpretation of the various laminar-like features – the large-scale deep convective flows and the surface granulation – which appear in numerical simulations of solar convection is thus achievable in the framework of turbulence theory. Canuto (1999) has recently presented further justification of the present ideas. It follows that turbulent eddy viscosity may play a role in the Sun that is analogous to the role played by molecular viscosity in the case of the Earth.

ACKNOWLEDGMENTS

Many helpful discussions on turbulence theory were pursued with V. M. Canuto. This work was supported by the NASA Climate Research Program.

REFERENCES

Bai T., 1988, ApJ, 328, 860

Batchelor G. K., 1970, The Theory of Homogeneous Turbulence. Cambridge Univ. Press, Cambridge

Beck J. G., Duvall T. L., Jr., Scherrer P. H., 1998, Nat, 394, 653

Bercovici D., Schubert G., Glatzmaier G. A., 1992, J. Fluid Mech., 239, 683

Brummell N., Cattaneo F., Toomre J., 1995, Science, 269, 1370

Brummell N., Hurlburt N. E., Toomre J., 1998, ApJ, 493, 955

Canuto V.M., 1999, preprint

Canuto V. M., Christensen-Dalsgaard J., 1998, Annu. Rev. Fluid Mech., 30, 167

Canuto V.M., Dubovikov M.S., 1998, ApJ, 493, 834

Cattaneo F., Brummell N. H., Toomre J., Malagoli A., Hurlburt, N. E., 1991, ApJ, 370, 282

Chandrasekhar S., 1949, ApJ, 110, 329

Davies J. T., 1972, Turbulence Phenomena. Academic, New York

Espagnet O., Muller R., Roudier T., Mein N., 1993, A&A, 271, 589

Kupka F., 1999, in Giménez A., Guinan E.F., Montesinos B., eds, Theory and Tests of Convection in Stellar Structure. ASP, San Francisco, p. 157

Lamb H., 1932, Hydrodynamics. Cambridge Univ. Press, Cambridge, Sec. 156

Nakano T., Fukushima T., Unno W., Kondo M., 1979, PASJ, 31, 713

Nordlund A., Spruit H. C., Ludwig H.-G., Trampedach R., 1997, A&A, 328, 229

Parker E. N., 1991, ApJ, 380, 251

Petrovay K. G., 1990, ApJ, 362, 722

Schwarzschild M., 1959, ApJ, 130, 345

Siggia E. D., 1994, Annu. Rev. Fluid Mech., 26, 137

Simon G. W., Weiss N. O., 1968, Z. Astrophys., 69, 435

Spruit H. C., Nordlund A., Title A. M., 1990, ARA&A, 28, 263

Stein R., Nordlund A., 1989, ApJ, 342, L95

Stein R., Norlund A., 1998, ApJ, 499, 914

Tennekes H., Lumley J. L., 1972, A First Course in Turbulence. MIT Press, Cambridge, MA

Unno W., 1961, PASJ, 13, 276

Yuen D. A., Hansen U., Zhao W., Vincent A. P., Malevsky A. V., 1993, J. Geophys. Res., 98, 5355