A Fresh Look at Spatial Power Combining Oscillators

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(with liberal use of results of W. Wang, 1998)

Motivations for Presentation

- Spatial Power Combining is an enabling technology for achieving useful power levels from solid state devices at millimeter wavelengths
- Oscillator-based combining came first, historically, but has been passed over for what practitioners believe to be more reliable amplifier-based technology
- Recent results in oscillator-based combining systems offer possibilities for technology breakthrough

Historical Overview of Spatial Power Combining

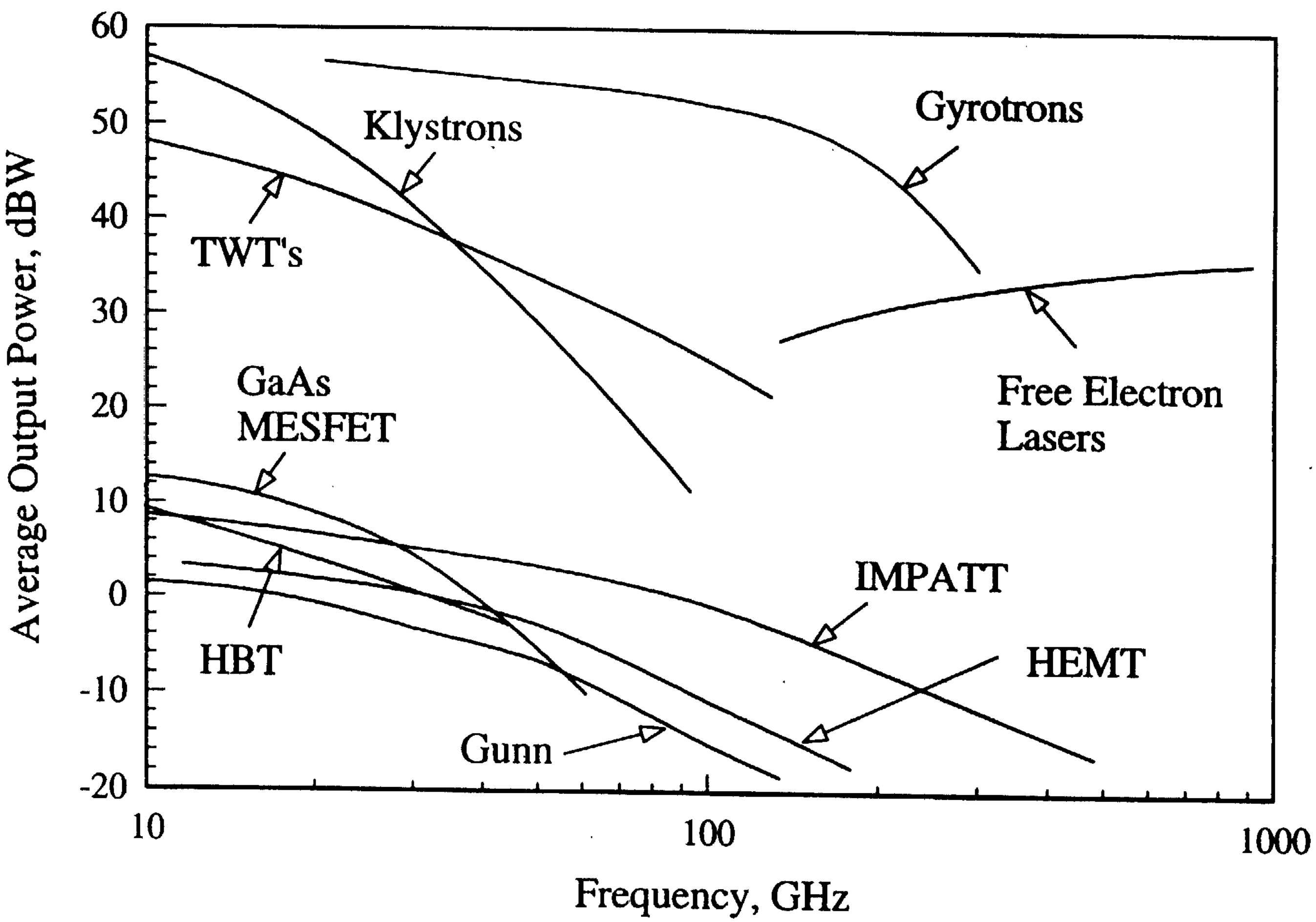
"Pre-History" (spatial)

- Saiman, Breese and Patton, 1968
- F. Durkin, 1981
- Hyltin, *et. al.*, 1968
- All of phased array practice

History (quasi-optic)

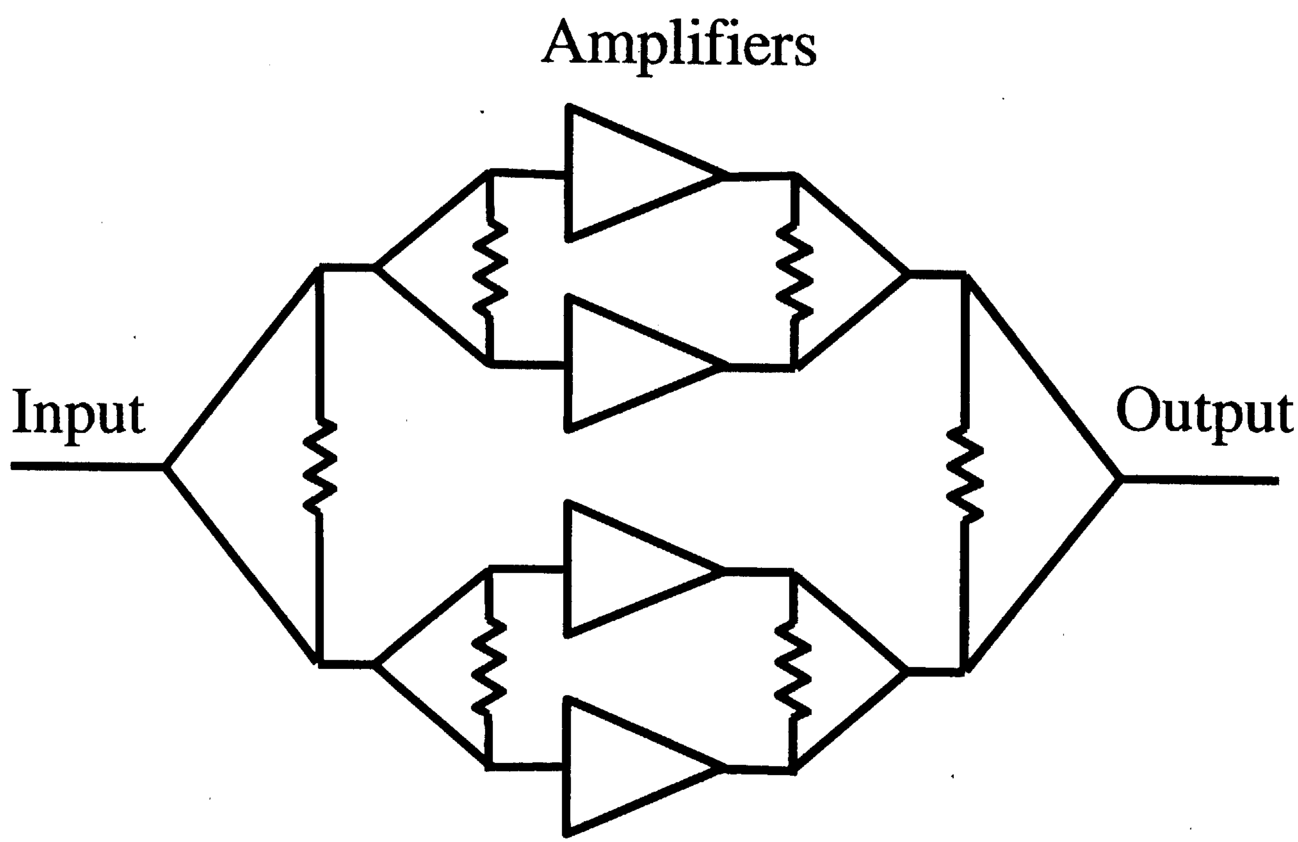
- L. Wandinger, V. Nalbandian 1983
- Mink, 1986
- Popovic and Rutledge, 1988
- ....

Why Combining is Necessary

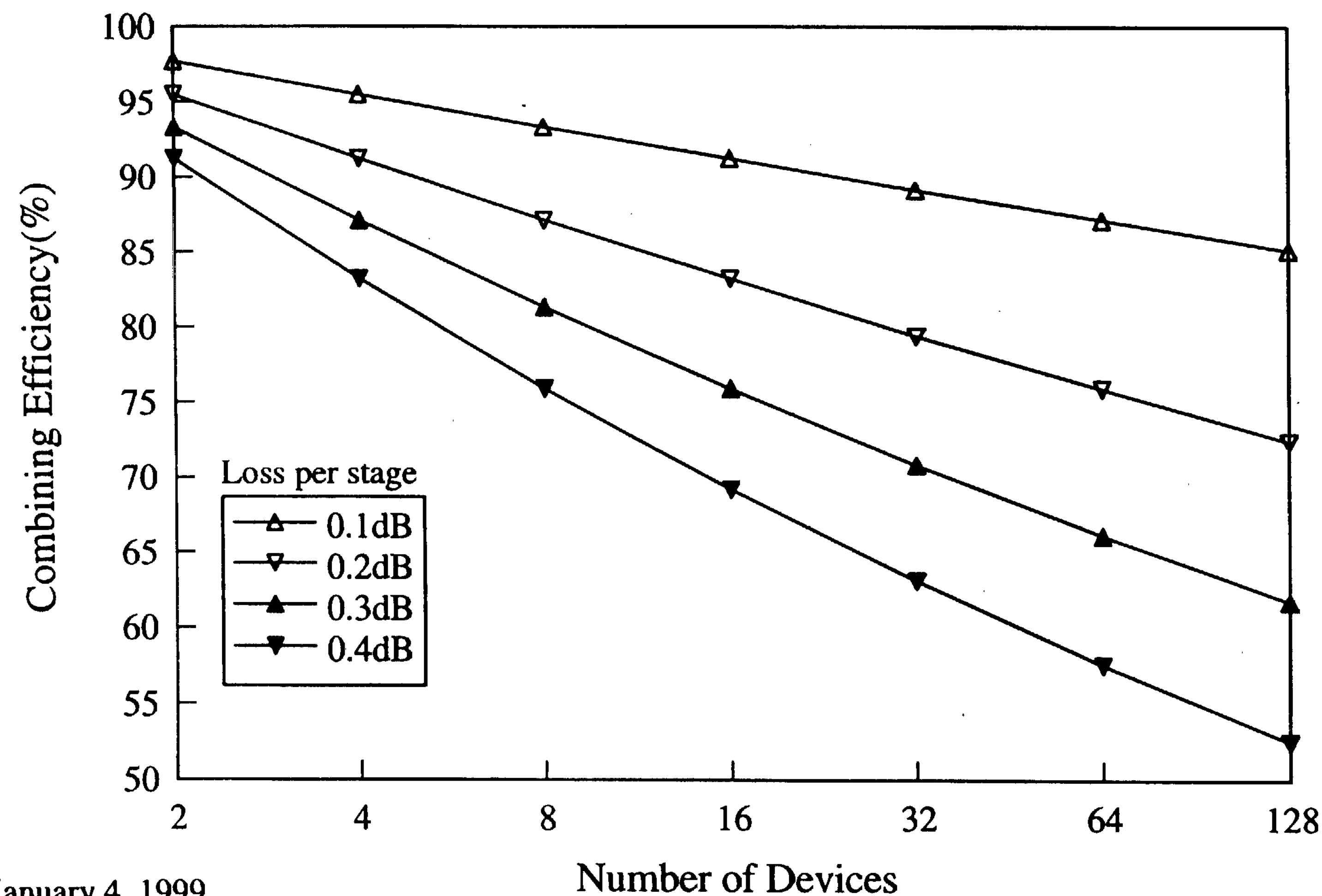


Attribution: Brown, Harvey, …, *et. al.*

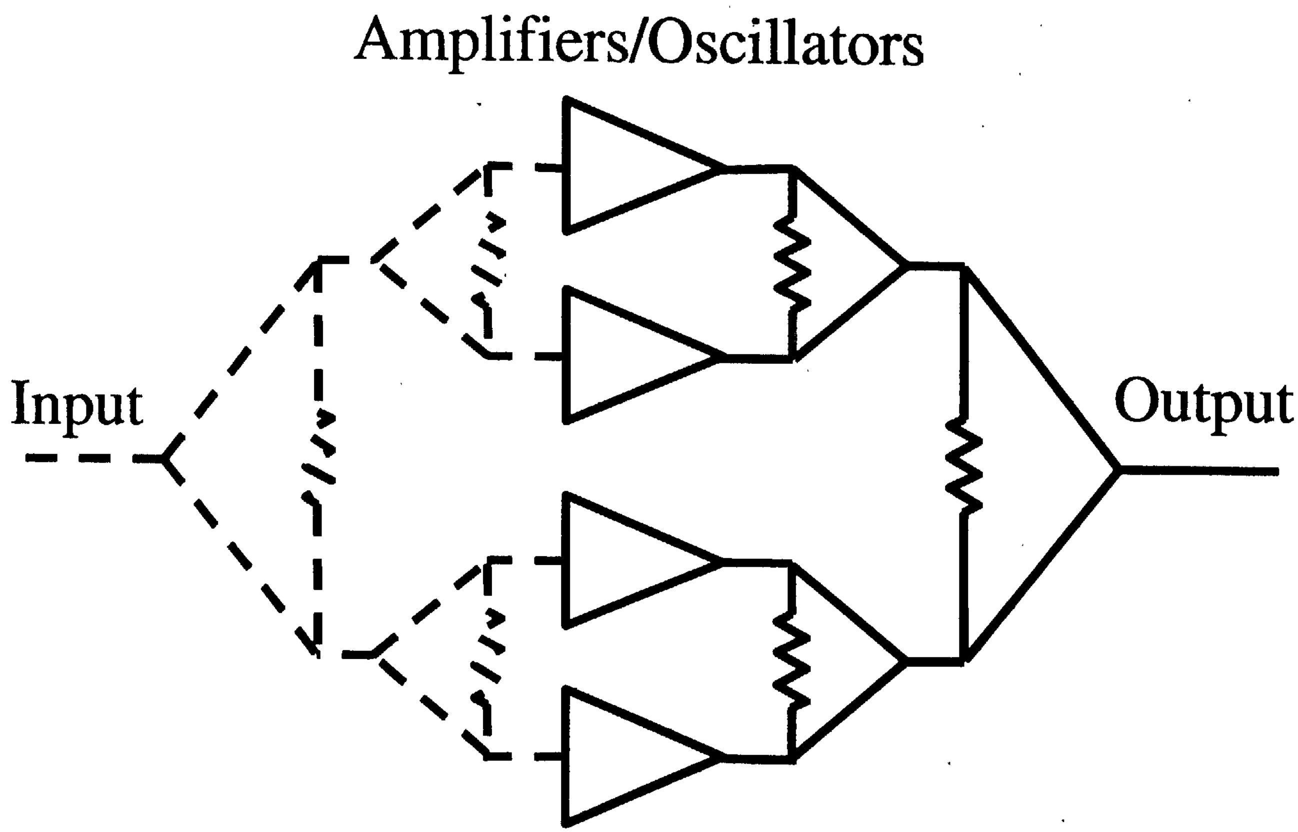
Classic Circuit Combining



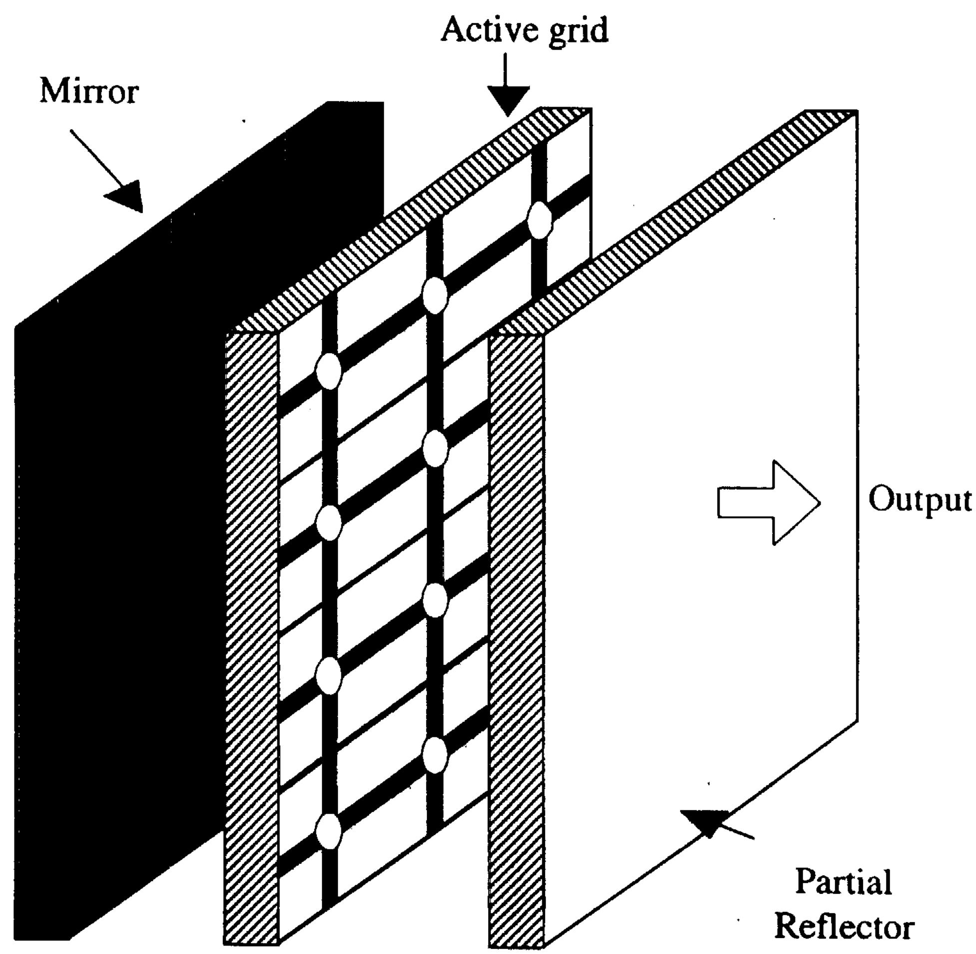
Efficiency Limitation in Wilkinson-Divider Tree



Feeding Incidental to PAE though Significant in Gain



Caltech Grid Oscillator Format



State of the Art in Grid Oscillators

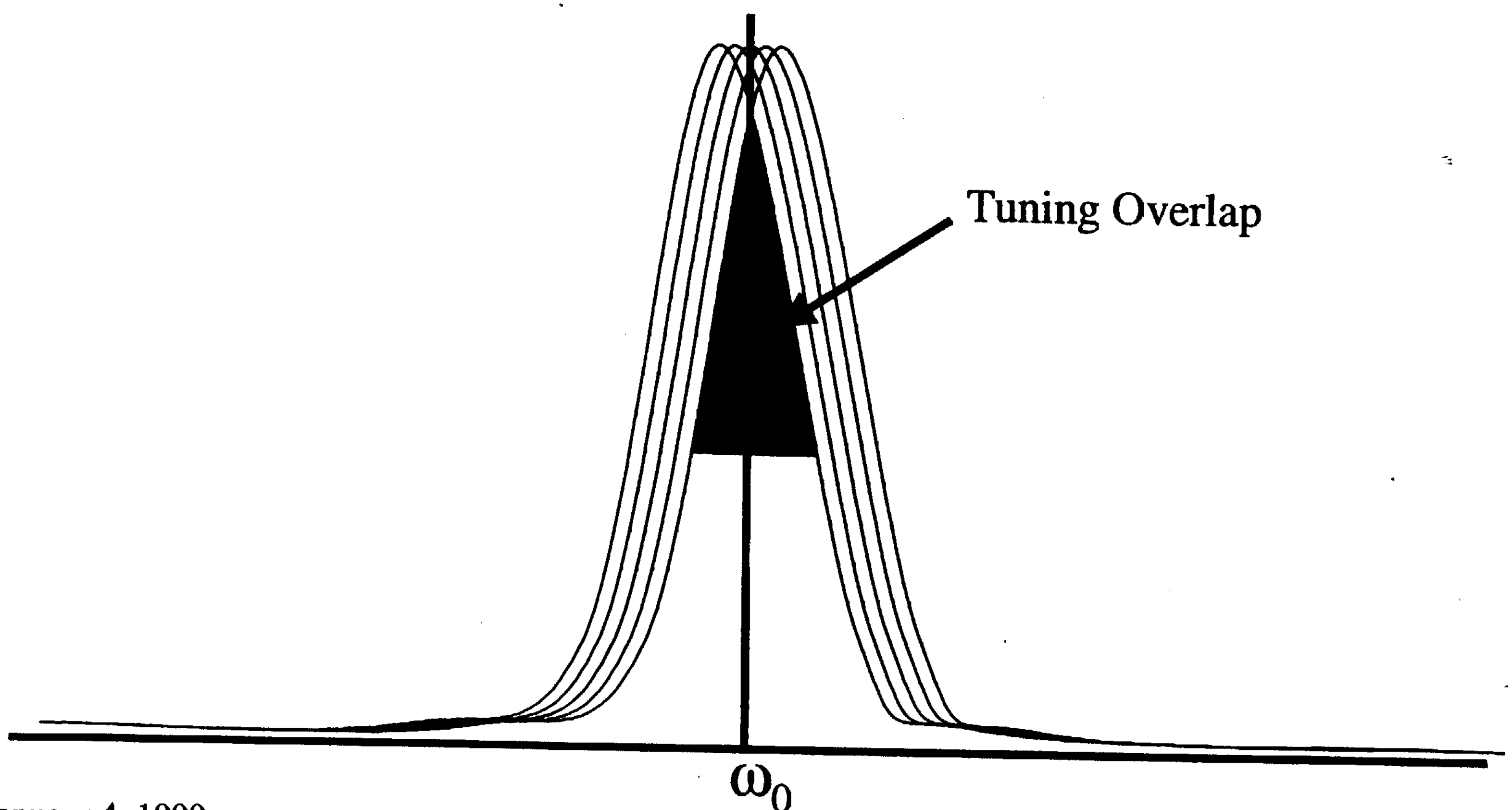
Size	Devices	Frequency	Power	Institute
10×10	FSC11LF	5.0GHz	550mW-ETP	Caltech
4×4	FSC11X	11.6GHz	335mW-ETP	Caltech
6×6	FSC11X	17.0GHz	235mW-ETP	Caltech
5×5	ATF35576	4.7GHz	1.6W-ERP	Colorado
10×10	FLK052 chip	9.8GHz	[redacted]	Caltech
2-10×10	ATF35576	5.0GHz	3.8W-ERP	Colorado
4-10×10	ATF35576	5.0GHz	8.0W-ERP	Colorado
2×3	MESFET	37.0GHz	1mW	Georgia Tech
6×6	InP-HEMT	[redacted]	200mW-ERP	Caltech
6×6	FHX35LG	4.4GHz	2.56W-ERP	Clemson

Compiled by W. Wang, 1998

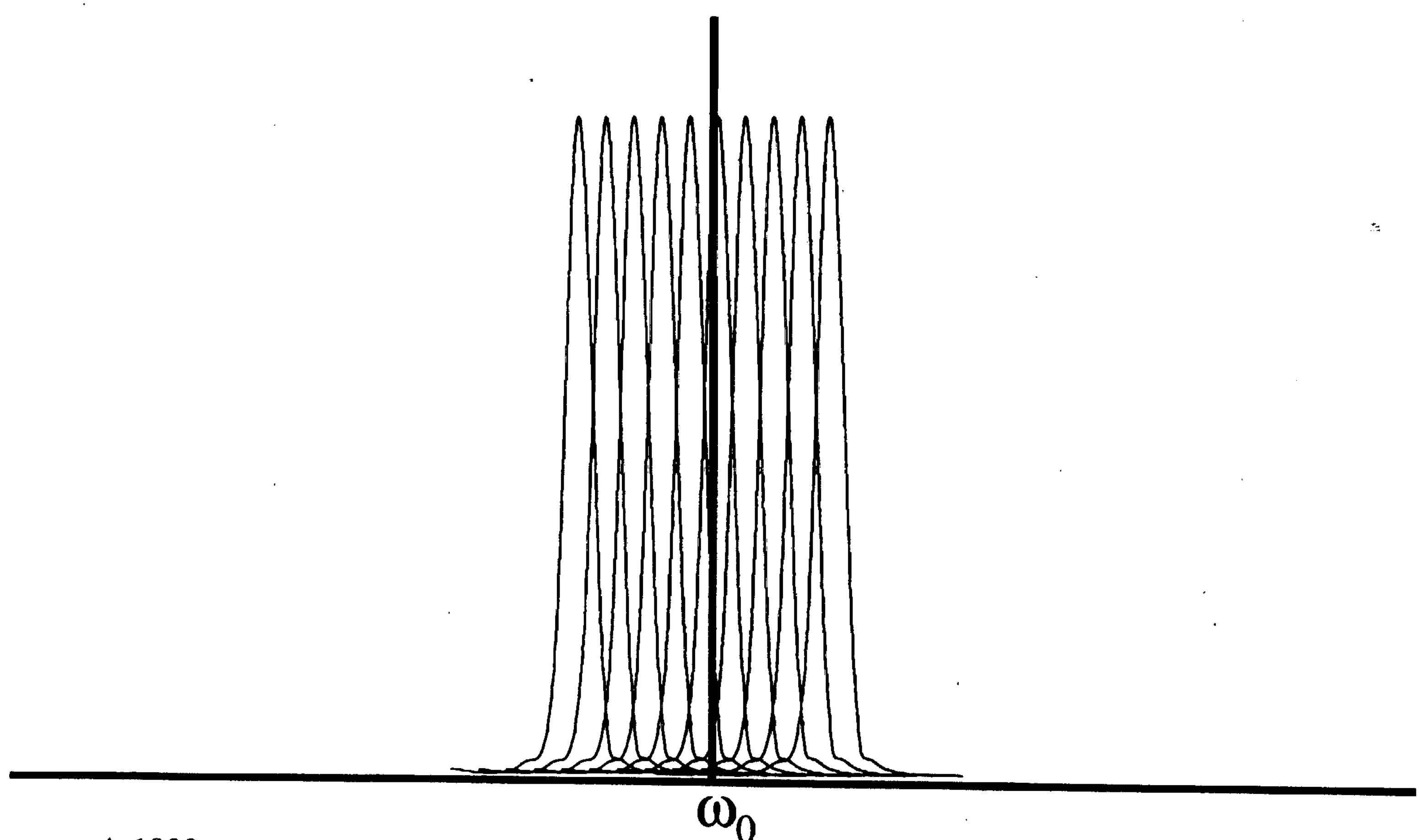
S.O.A. in Voltage Controlled Grid Oscillators

Size	Frequency	Tuning Range	ERP	Power Variation	Authors
7×7D	2.8GHz	200MHz/7.1%	N/A	24.6dB	Colorado
7×7B	6.0GHz	616MHz/10.3%	N/A	12.0dB	Colorado
4×6B	4.9GHz	486MHz/9.9%	N/A	2.0dB	Colorado
4×4D	12.4GHz	200MHz/(1.5%)	N/A	N/A	Virginia
4×4D	4.9GHz	[redacted]	300mW	8.0dB	Clemson
6×6D	6.3GHz	350MHz/(5.5%)	1.4W	N/A	Virginia
4×4D	4.7GHz	330MHz/7%	900mW	10dB	Clemson

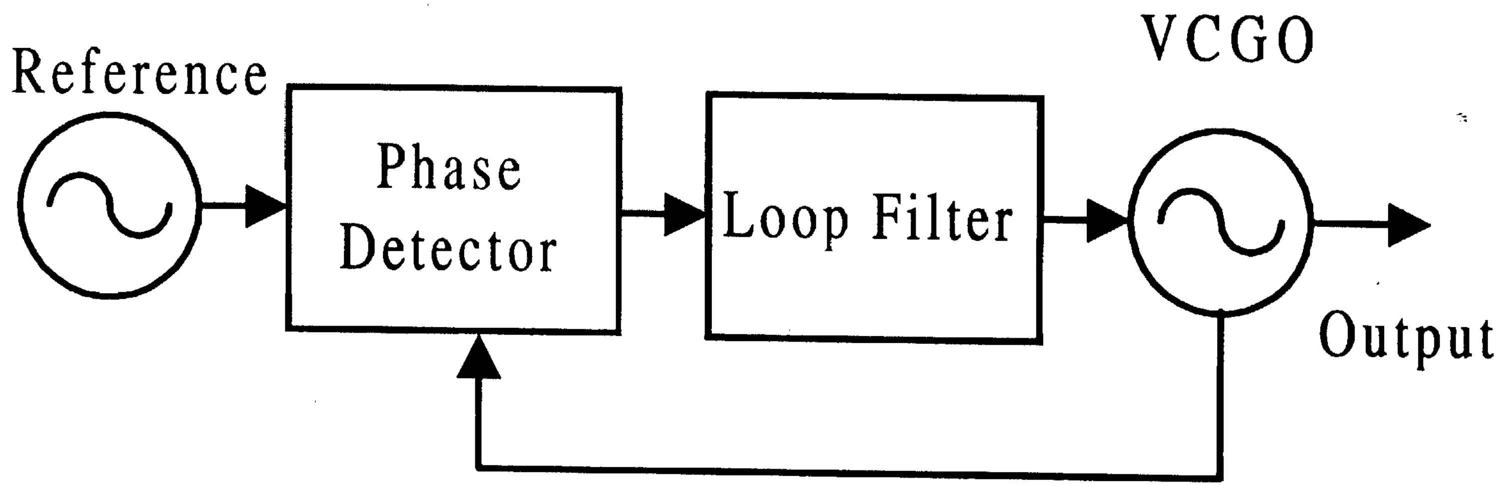
Assemble & Go Coupled Oscillators



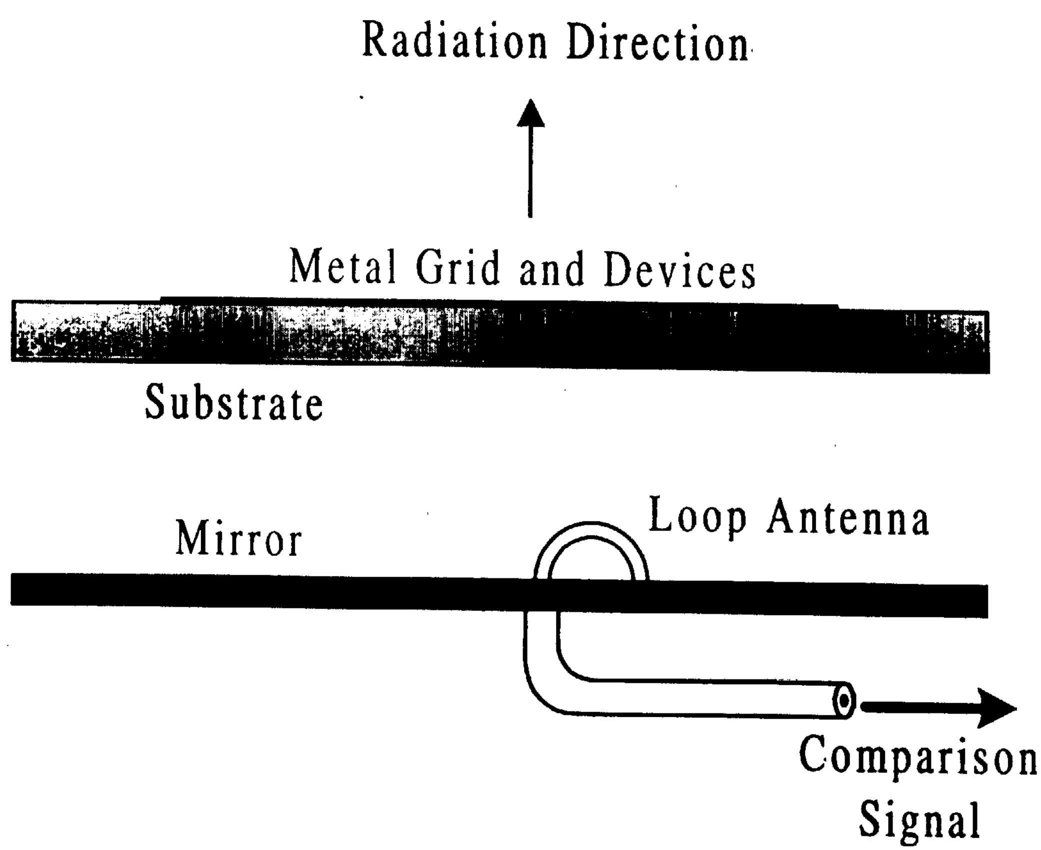
High-Q Resonators Require Tuning



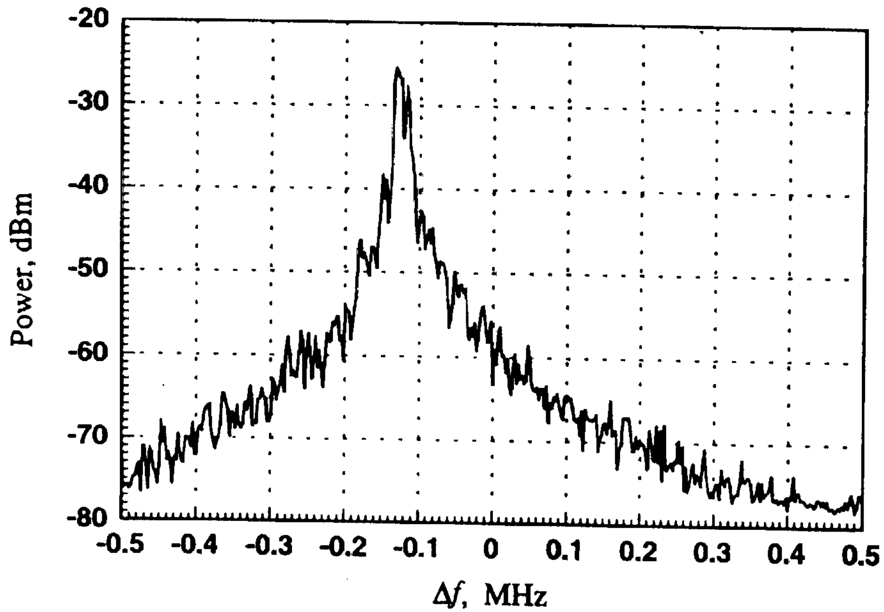
Phase-locked Loop



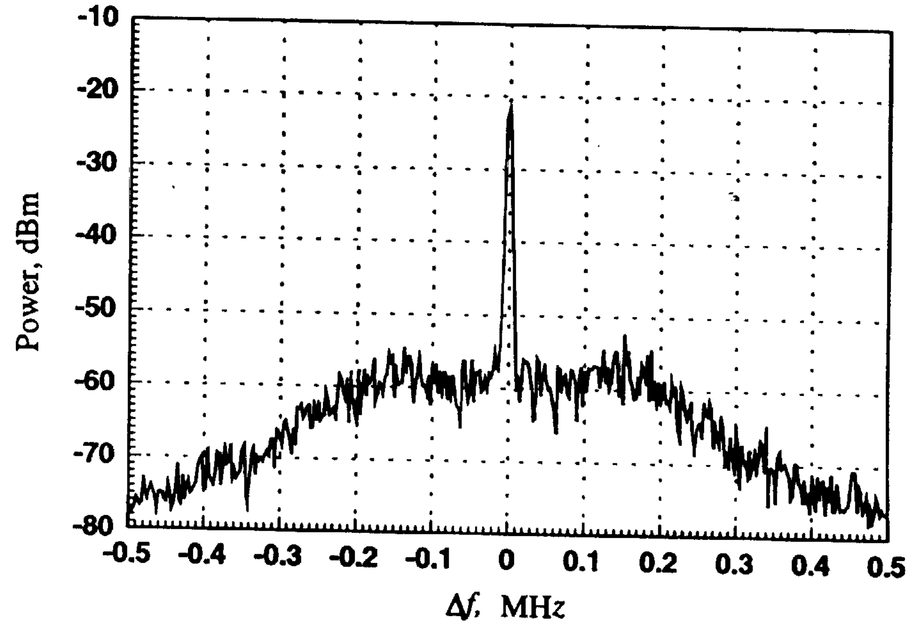
Detector Pickup



Frequency Stabilization



Unlocked



Locked

$f_0 = 4.643$ GHz

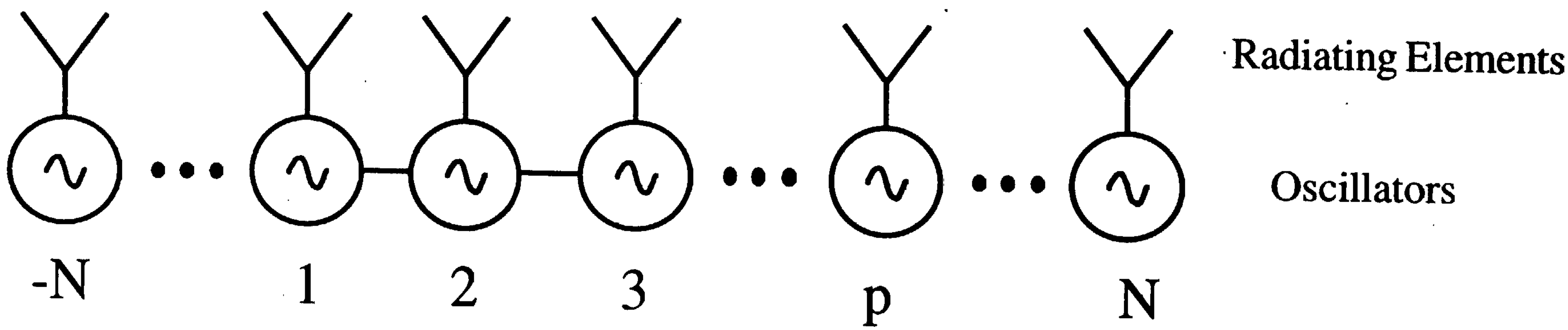
Other Results Bearing on Future Work

- High-Combining-Efficiency Microstrip Structure (Mortazawi)
- Phase Distribution Control (York, Pogorzelski, *et. al.*)
- Modulation (Wang)

Pogorzelski's Continuum Model

- Begins with Adler's difference equations, which describe a system of coupled oscillators
- Extends Adler's equations to a continuum, resulting in a Poisson's equation
- Demonstrates that Phase Perturbations Diffuse through an Array
- Intentional end perturbations lead to progressive phase shift

Coupled Oscillator Array



Coupled Oscillators (Continued)

Define the phase of the ith oscillator, ϕ_i ,by:

$$\theta_i = \omega_{ref} t + \phi_i$$

Then, the continuum model yields,

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial \phi}{\partial \tau} = -\frac{\omega_{tune} - \omega_{ref}}{\Delta \omega_{lock}} = -Cu(\tau)\delta(x-b)$$

Beamsteering Dynamics

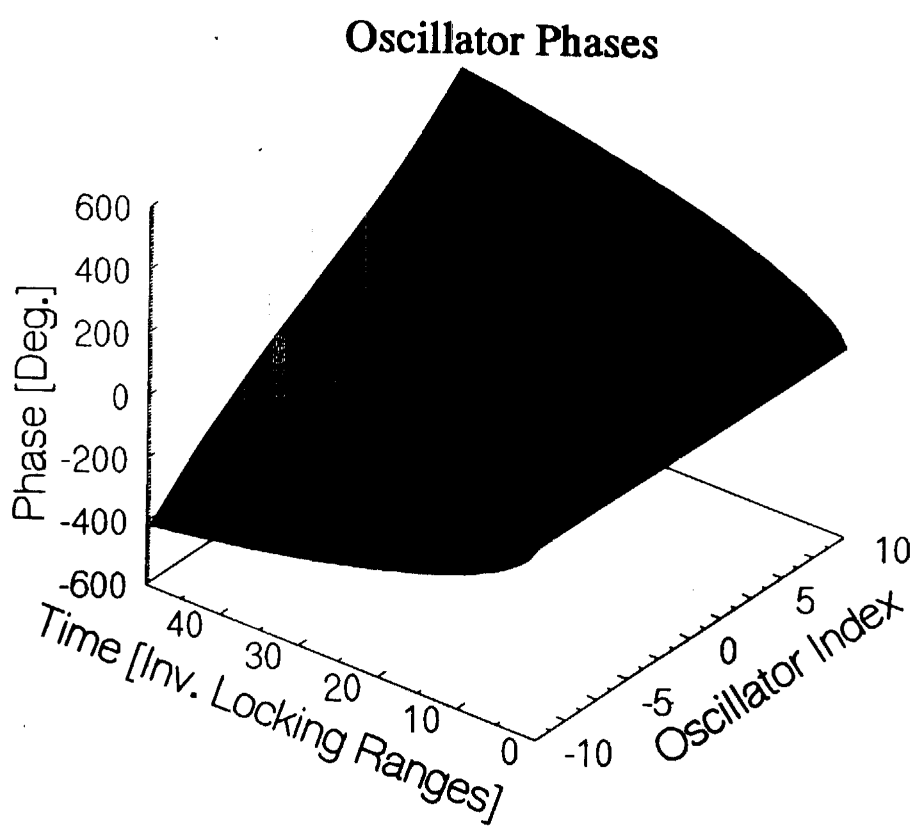
Equal and opposite detuning of the end oscillators; i.e.,

$$\Delta\omega_L = -\Delta\omega_R = \Delta\omega_T$$

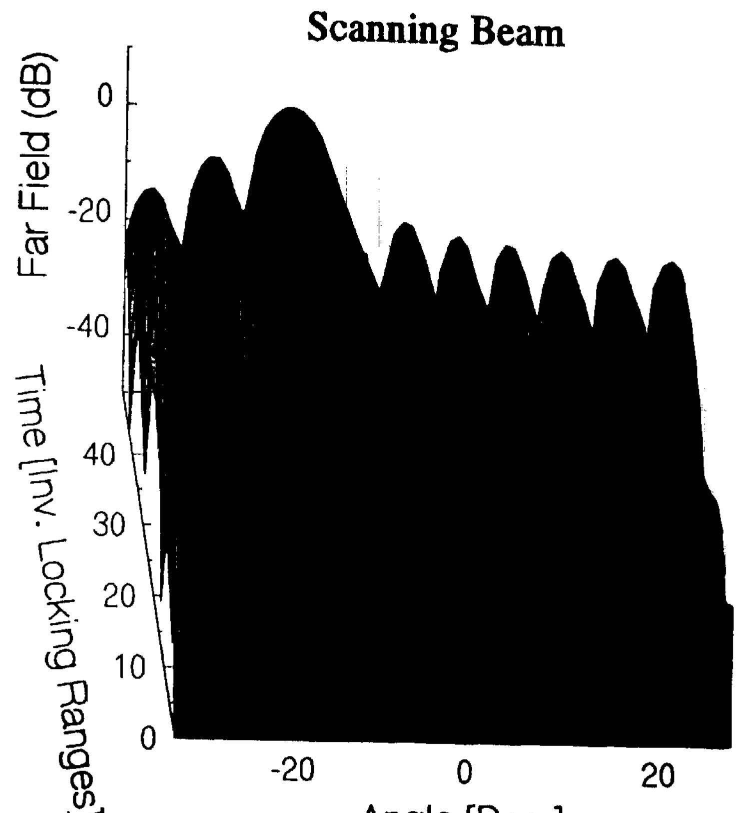
yields,

$$\phi(x,\tau) = \frac{\Delta\omega_T}{\Delta\omega_{lock}} \sum_{m=0}^{\infty} \frac{2\sin\left(b\sqrt{\sigma_m}\right)\sin\left(x\sqrt{\sigma_m}\right)}{(2a+1)\sigma_m}\left(1-e^{-\sigma_m\tau}\right)$$

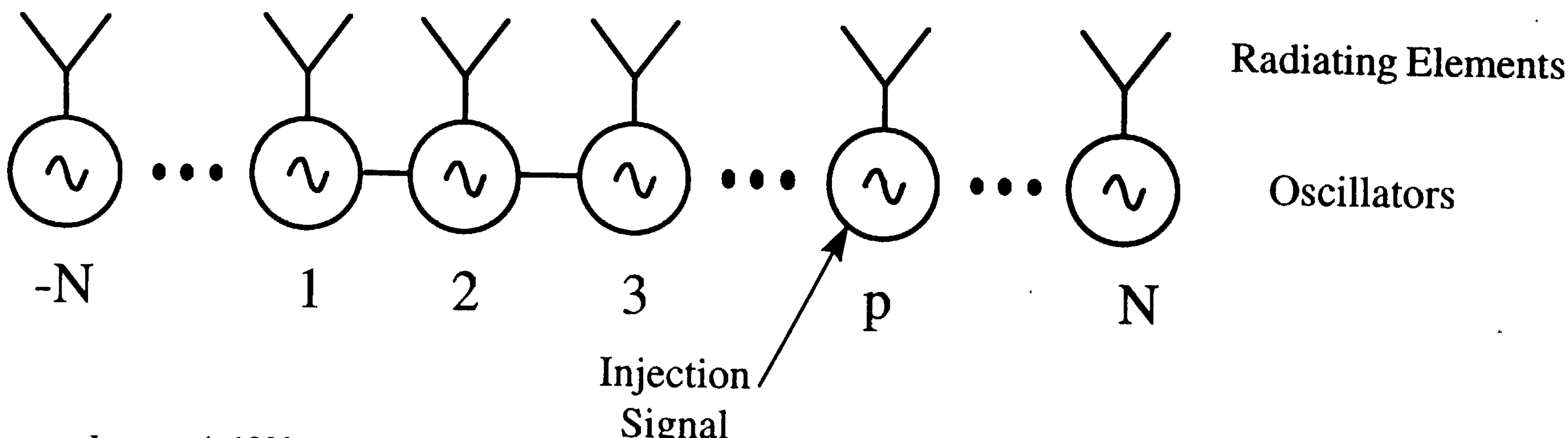
Beamsteering Phase



Far Zone Radiation Pattern



Injection Locked Coupled Oscillator Array



Beamsteering via Injection

Define the phase by: $\theta_i = \omega_{ref} t + \phi_i$

$$\frac{d\phi_i}{dt} = \omega_{tune,i} - \omega_{ref} + \Delta\omega_{lock}(\phi_{i+1} - 2\phi_i + \phi_{i-1}) - \delta_{ip}\Delta\omega_{lock,p,inj}(\phi_p - \phi_{inj})$$

$$\frac{\partial^2\phi}{\partial x^2} - \frac{\partial\phi}{\partial\tau} = -\frac{\omega_{tune} - \omega_{ref}}{\Delta\omega_{lock}} + \delta_{ip}\frac{\Delta\omega_{lock,p,inj}}{\Delta\omega_{lock}}(\phi - \phi_{inj})$$

Beam Steering

We injection lock two oscillators. The differential equation becomes

$$\frac{\partial^2 \phi}{\partial x^2} - \left[B_1 \delta(x-b_1) + B_2 \delta(x-b_2)\right]\phi - \frac{\partial \phi}{\partial \tau} = -B_1 \delta(x-b_1) p_1 u(\tau) - B_2 \delta(x-b_1) p_2 u(\tau)$$

The Laplace transform of the equation is,

$$\frac{\partial^2 F}{\partial x^2} - \left[B_1 \delta(x-b_1) + B_2 \delta(x-b_2)\right]F - sF = -B_1 \delta(x-b_1)\frac{p_1}{s} - B_2 \delta(x-b_1)\frac{p_2}{s}$$

Postulate,

$$F(x,s) = C_1 e^{-|x-b_1|\sqrt{s}} + C_2 e^{-|x-b_2|\sqrt{s}} + C_R e^{-x\sqrt{s}} + C_L e^{x\sqrt{s}}$$

Beam Steering Solution

The boundary conditions at the ends and the two injection points yield four equations for the four unknown constants and,

$$
\begin{aligned}
F(x,s) = \frac{1}{s\Delta}\Big\{ & 2B_2 p_2 \cosh\left[\sqrt{s}\left(2h - |b_2 - x|\right)\right] + 2B_2 p_2 \cosh\left[\sqrt{s}\left(b_2 + x\right)\right] \\
& + 2B_1 p_1 \cosh\left[\sqrt{s}\left(2h - |b_1 - x|\right)\right] + 2B_1 p_1 \cosh\left[\sqrt{s}\left(b_1 + x\right)\right] \\
& + \frac{B_1 B_2 p_2}{\sqrt{s}} \sinh\left[\sqrt{s}\left(2h - |b_2 - x|\right)\right] - \frac{B_1 B_2 p_1}{\sqrt{s}} \sinh\left[\sqrt{s}\left(2h - (b_2 - b_1) - |b_2 - x|\right)\right] \\
& - \frac{B_1 B_2 p_2}{\sqrt{s}} \sinh\left[\sqrt{s}\left(2b_1 - |b_2 - x|\right)\right] - \frac{B_1 B_2 p_1}{\sqrt{s}} \sinh\left[\sqrt{s}\left((b_2 + b_1) - |b_2 - x|\right)\right] \\
& + \frac{B_1 B_2 p_1}{\sqrt{s}} \sinh\left[\sqrt{s}\left(2h - |b_1 - x|\right)\right] - \frac{B_1 B_2 p_2}{\sqrt{s}} \sinh\left[\sqrt{s}\left(2h - (b_2 - b_1) - |b_1 - x|\right)\right] \\
& + \frac{B_1 B_2 p_1}{\sqrt{s}} \sinh\left[\sqrt{s}\left(2b_2 - |b_1 - x|\right)\right] + \frac{B_1 B_2 p_2}{\sqrt{s}} \sinh\left[\sqrt{s}\left((b_2 + b_1) + |b_1 - x|\right)\right]\Big\}
\end{aligned}
$$

Beam Steering Continued

where,

$$\Delta = 4\sqrt{s}\sinh\left[\sqrt{s}(2h)\right]$$
$$+2B_2\cosh\left[\sqrt{s}\left(2b_2\right)\right]+2B_1\cosh\left[\sqrt{s}\left(2b_1\right)\right]+2\left(B_2+B_1\right)\cosh\left[\sqrt{s}(2h)\right]$$
$$+\frac{B_1B_2}{\sqrt{s}}\left\{\sinh\left[\sqrt{s}(2h)\right]-\sinh\left[\sqrt{s}\left(2b_1\right)\right]+\sinh\left[\sqrt{s}\left(2b_2\right)\right]-\sinh\left[\sqrt{s}\left(2h-2\left(b_2-b_1\right)\right)\right]\right\}$$

The final value theorem yields,

$$\phi(x,\infty)=\frac{B_2p_2+B_1p_1+\frac{1}{2}B_1B_2\left[\left(b_2-b_1\right)\left(p_2+p_1\right)+\left(\left|b_1-x\right|-\left|b_2-x\right|\right)\left(p_2-p_1\right)\right]}{B_2+B_1+B_1B_2\left(b_2-b_1\right)}$$

Beam Steering Example

$$B_1 = B_2 = 1$$

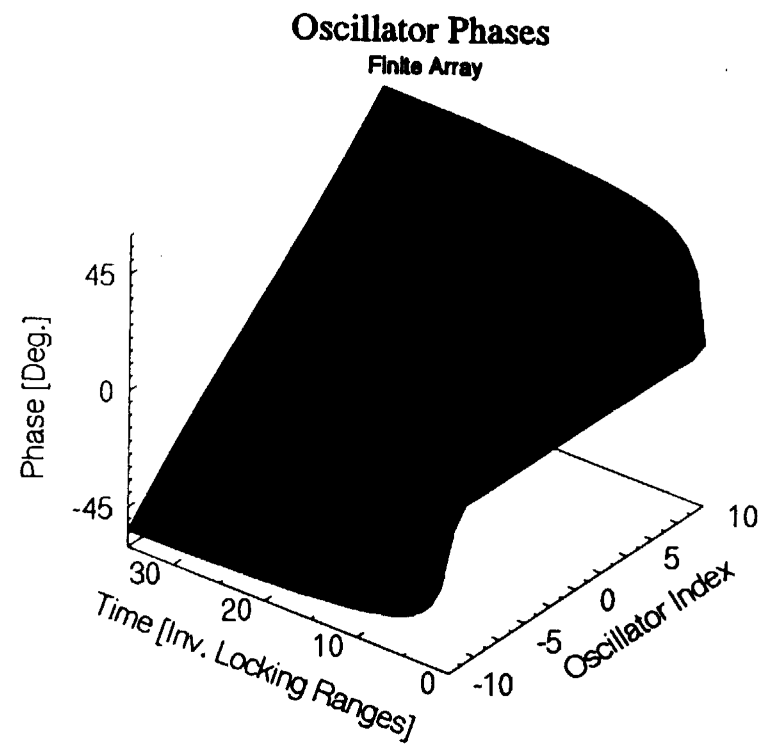
$$b_1 = -h$$

$$b_2 = h$$

$$p_1 = -60^0$$

$$p_2 = 60^0$$

Beam Steering Example



Gradual Phase Change

- Step injection phase change limited to less than ninety degrees.
  - Yields extremely limited beam steering angles.
  - Can be mitigated by gradual phase change.
- Gradual change result can be obtained by convolution with a Gaussian.
  - Time domain solution is expressed as a sum of exponentials.
  - Convolution of a Gaussian and an exponential can be expressed as multiplication by a function involving complementary error functions.

Convolution with a Gaussian

Let, $$g(\tau) = e^{-\alpha(\tau-\tau_0)^2}$$

Then,

$$A_n e^{-\sigma_n \tau} * g(\tau) = A_n e^{-\sigma_n \tau} \left\{ e^{-\sigma_n \tau_0} e^{\sigma_n^2/(4\alpha)} \frac{1}{\sqrt{\pi\alpha}} \left[erfc(\upsilon_1) - erfc(\upsilon_2) \right] \right\}$$

where,

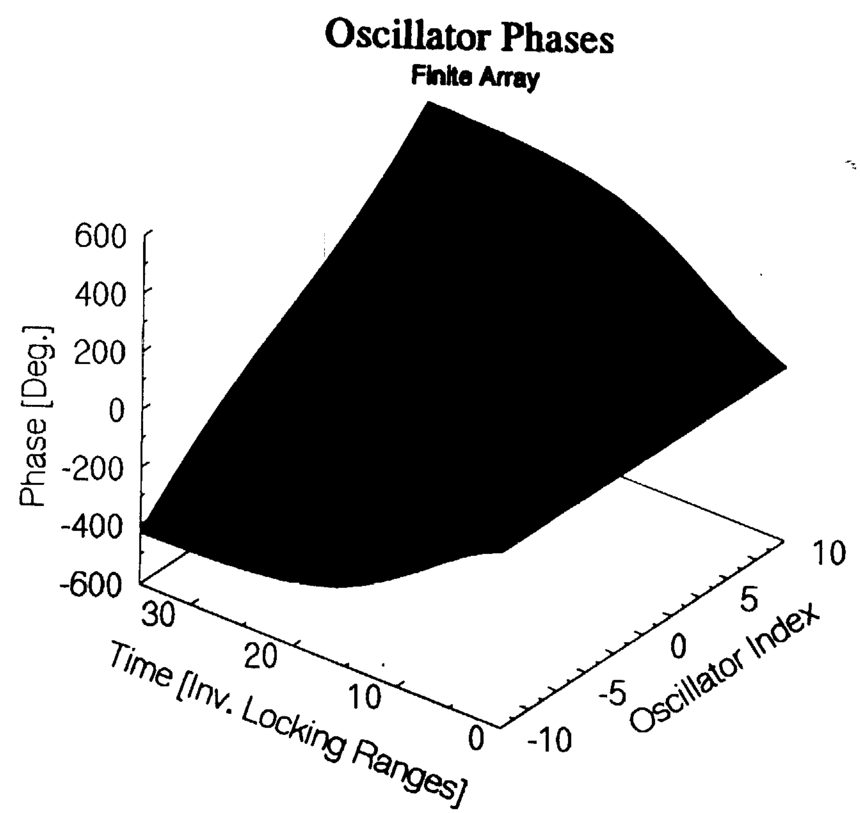
$$\upsilon_1 = -\sqrt{\alpha}\left(\tau_0 + \frac{\sigma_n}{2\alpha}\right)$$

$$\upsilon_2 = \sqrt{\alpha}\left[\tau - \left(\tau_0 + \frac{\sigma_n}{2\alpha}\right)\right]$$

Gradual Steering Example

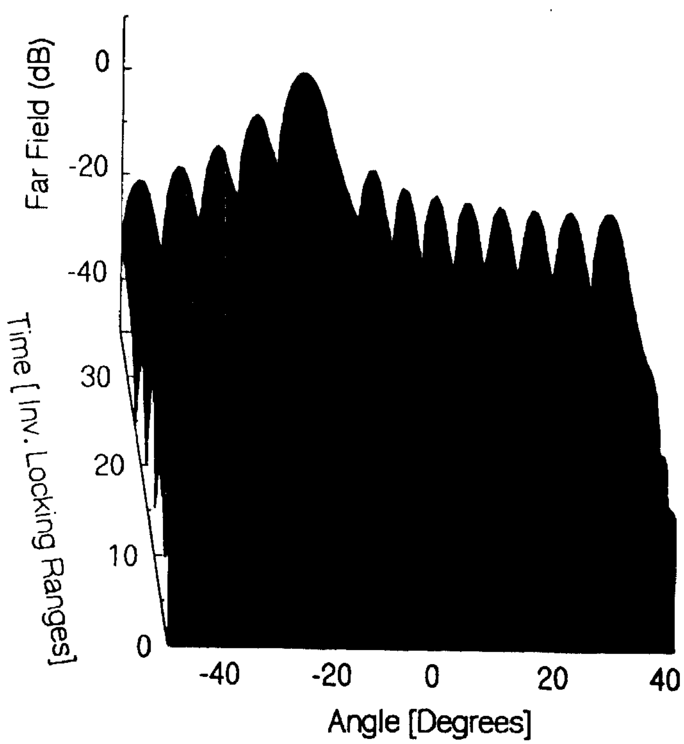
Choose, $\tau_0 = 6.0$

$\alpha = 0.01$

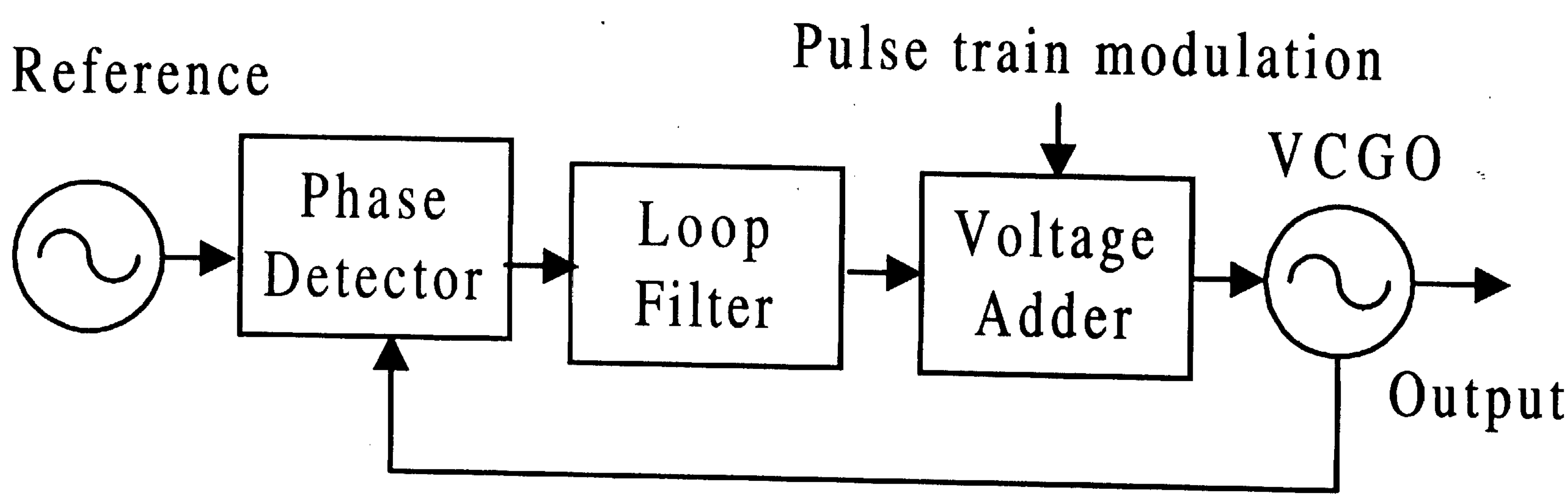


Far Zone Field

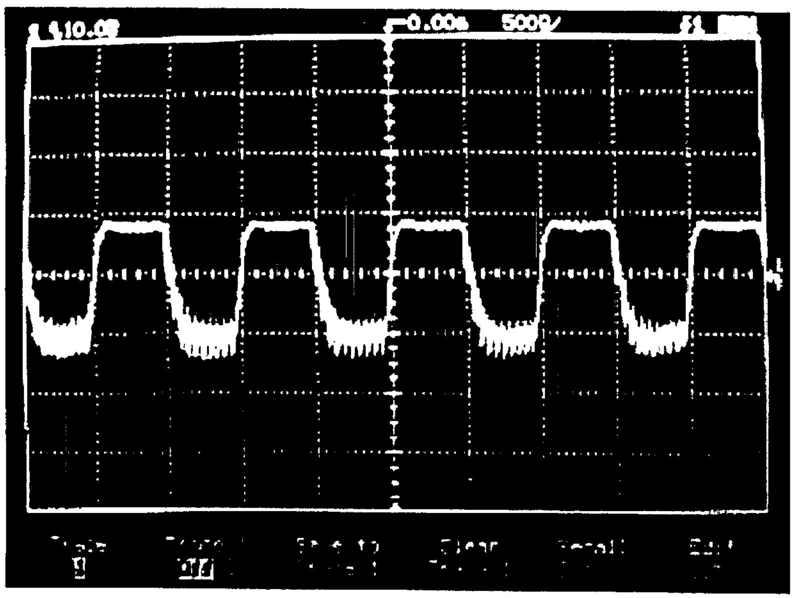
Scanning Beam



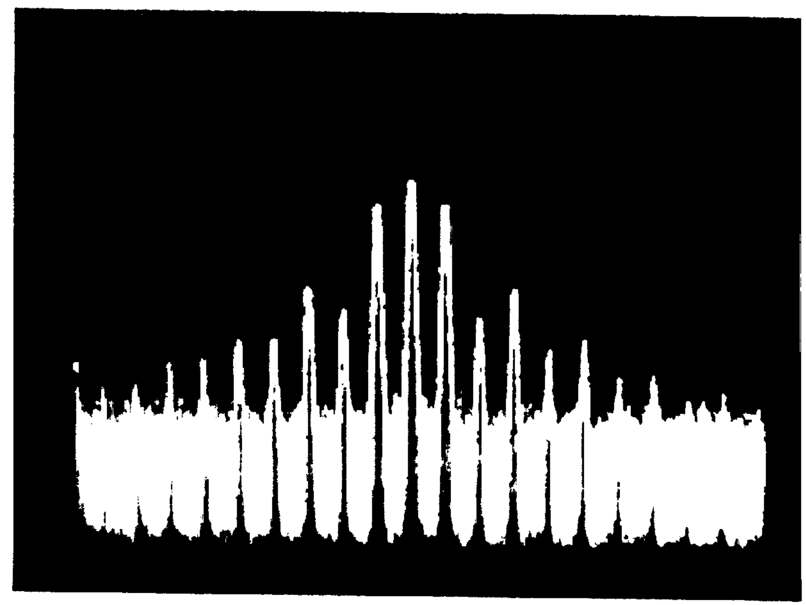
Phase Modulation



Modulation Out of Band to PLL

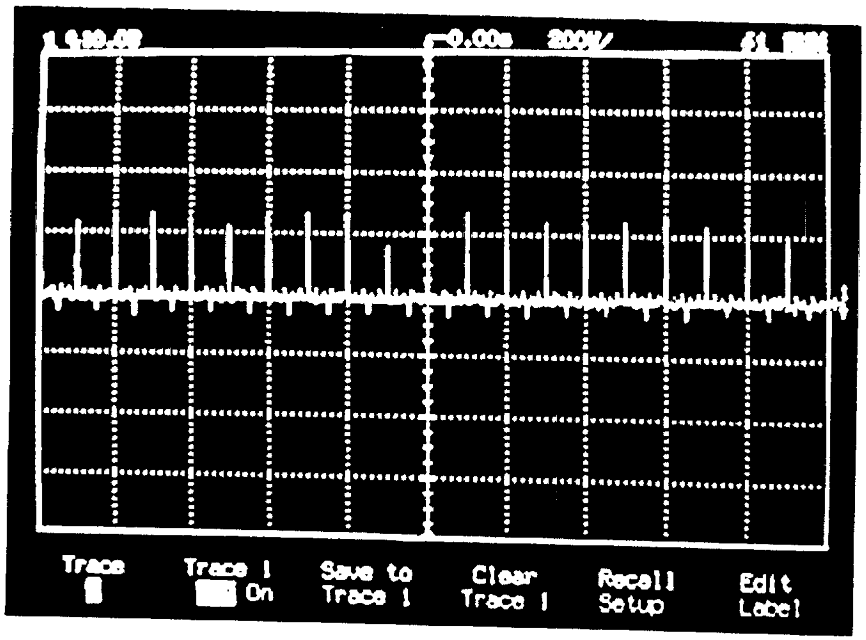


Received 1MHz signal

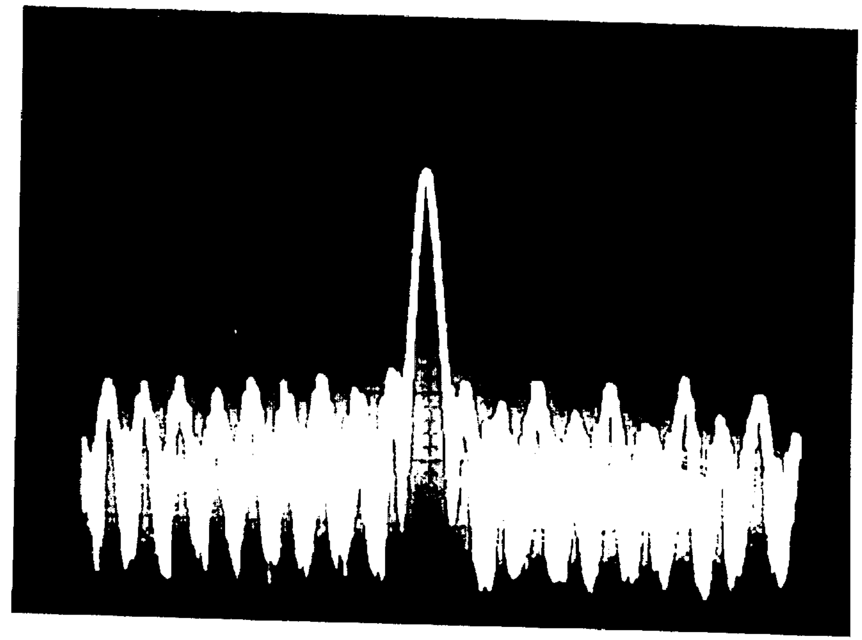


Spectrum of 1MHz modulation

Modulation Frequency In Band



Received 10kHz signal



Spectrum of 10kHz modulation

Constraints on Pulse Train

- The PLL integrates the modulation train and drifts according to the mean value of the train. => Zero-mean sequence

- The data rate in the pulse train must be much larger than the bandwidth of the PLL

Generic Replacement for a Phased Array