Aero–Structural Interaction, Analysis, and Shape Sensitivity

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Abstract

A multidisciplinary sensitivity analysis technique that has been shown to be independent of step–size selection is examined further. The accuracy of this step–size independent technique, which uses complex variables for determining sensitivity derivatives, has been previously established. The primary focus of this work is to validate the aero–structural analysis procedure currently being used. This validation consists of comparing computed and experimental data obtained for an Aeroelastic Research Wing (ARW–2). Since the aero–structural analysis procedure has the complex variable modifications already included into the software, sensitivity derivatives can automatically be computed. Other than for design purposes, sensitivity derivatives can be used for predicting the solution at nearby conditions. The use of sensitivity derivatives for predicting the aero–structural characteristics of this configuration is demonstrated.

Introduction

Over the last half–century, through relentless experimental and computational studies, resourceful design engineers have produced near optimal aerospace configurations. To further improve these designs, where the margin for improvement is small, designers will require additional information such as sensitivity derivatives. This additional information may also be used to expedite the design of new engineering systems for which there is no vast experimental or computational data base. These needs are the impetus for the development of efficient and accurate multidisciplinary analysis and sensitivity analysis procedures. To maximize the benefits of these procedures, they must have the capability of resolving both the physics and the geometric complexities of practical configurations.

In the mid 1970s, researchers began exploring the use of numerical optimization techniques for the design of aircraft components. These early studies primarily focused on airfoil and wing design using low fidelity fluid models for the analyses and finite–difference calculations for gradient information. The inability of these fluid models to accurately predict nonlinear phenomena limited their applicability. By the mid 1980s, computational resources were available that permitted aerodynamic simulations using the higher fidelity Euler and Navier–Stokes equations about isolated components and moderately complex configurations. Then Sobieski[43] challenged the aerodynamics community to extend these algorithms to include the shape sensitivity analysis for the geometry. This plea ignited studies aimed at developing methods that would allow for the use of nonlinear aerodynamics in shape optimization. A review on the early aerodynamic shape optimization work has been reviewed by Labrujere and Sloof[25] and a concise review on the use of sensitivity analysis in aerodynamic shape optimization has been reported by Taylor et al.[49] and by Newman et al.[32]. A recent paper by Jameson[22], furthermore, delineates the evolution of computational fluid dynamics as a design tool.

For aerodynamic optimization, the state equation is a system of nonlinear partial differential equations (PDE) expressing the conservation of mass, momentum, and energy. Differentiation of this system of PDE (i.e., sensitivity analysis) can be performed at one of two levels. In the first method, termed the *continuous* or variational approach, the PDE are differentiated prior to discretization, either directly or by introducing Lagrange multipliers which are defined as a set of continuous linear equations *adjoint* to the governing PDE. Subsequently, these directly differentiated or adjoint equations are discretized and solved. In the second method, termed the *discrete* approach, the PDE are differentiated after discretization. The discrete approach may also be cast in either a *direct* or an *adjoint* formulation, and the reader should refer to Hou et al.[20] for a comprehensive presentation of both discrete formulations. For more detailed recent discussions of the continuous approaches to aerodynamic design optimization, the interested reader is directed, for example, to Refs.

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4, 9, 21, 23, and 39 for the adjoint formulations and to Ref. 11 for the direct formulation.

The task of constructing exactly or analytically all of the required linearizations and derivatives by hand for either the direct or adjoint approach and then building the software for evaluating these terms can be extremely tedious. This problem is compounded by the inclusion of even the most elementary turbulence model (for viscous flow) or the use of a sophisticated grid generation package for adapting (or regenerating) the computational mesh to the latest design. One solution to this problem has been found in the use of a technique known as automatic differentiation. Application of this technique to an existing source code, that evaluates output functions, automatically generates another source code that evaluates both output functions and derivatives of those functions with respect to specified code input or internal parameters. A precompiler software tool, called ADIFOR (Automatic Differentiation of FORtran, Bischof et al.[10]), has been developed and utilized with much success to obtain complicated derivatives from advanced CFD and grid generation codes, for use within aerodynamic design optimization procedures[15,16,48,50]. The use of ADIFOR produces code that, when executed, evaluates these derivatives of the output functions via a discrete–direct approach, referred to as *forward–mode* automatic differentiation. More recently, automatic differentiation software has emerged that enables the derivatives to be evaluated with a discrete–adjoint approach[13,28]. This type of automatic differentiation is known as *reverse–mode*[29].

The best known method for computing the sensitivity information between coupled systems is via the solution of the global sensitivity equations derived by Sobieski[45]. This system of equations, which is obtained by directly differentiating the state vector of each discipline, may sometimes be ill–conditioned and the memory requirements associated with the storage of the coefficient matrix may be prohibitive. The ill–conditioned global sensitivity equations, as well as those associated with higher–order spatially accurate aerodynamic sensitivity analysis, may be reformulated and solved by the incremental iterative technique[24,36]. The incremental iterative technique allows the linear sensitivity equations to be solved iteratively where the coefficient matrix may be any convenient approximation that will converge the system. This allows a better conditioned, and reduced memory requirement, coefficient matrix to be used. Examples on the use of this technique for aerodynamic shape sensitivity analysis may be found in Refs. 18, 21, and 44, and for multidisciplinary sensitivity analysis in Ref. 6. Furthermore, Arslan and Carlson[6] demonstrated the need for multidisciplinary sensitivity analysis, within the design optimization process, by showing that the sensitivity derivatives produced by an aerodynamic–only calculation had different magnitudes, and in some cases different signs, from that obtained with the coupled aero–structural sensitivity analysis. Similar findings have been reported by Barthelemy and Bergen[7] and by Newman et al.[31] A detailed survey of the research being conducted in the field of multidisciplinary sensitivity analysis and optimization may be found in Ref. 44.

Recently, a new method for performing aerodynamic, structural, and multidisciplinary sensitivity analysis has been developed [35]. This method is based on ideas that where explored over three decades ago by Lyness and Moler[27] and Lyness[26], and recently revisited by Squire and Trapp[46]. This technique uses complex variables to approximate derivatives of real functions. In Ref. 35, the advantages and disadvantages of the complex variable method were discussed and compared with the existing methods presented above. This method was then demonstrated via the computation of aerodynamic (inviscid), structural, and multidisciplinary sensitivity derivatives with respect to design variables appropriate for both aerodynamic and structural design optimizations. The configuration used in Ref. 35 for investigation was a low aspect ratio ONERA M6 wing immersed in a transonic flow. More recently, the work of Ref. 35 has been extended to obtain sensitivity derivatives for turbulent flows [5]. Furthermore, Ref. 5 closely examines and discusses various features and drawbacks of the complex variable technique.

In the current work, the aero–structural analysis procedure used in Ref. 35 is validated and multidisciplinary sensitivity derivatives computed. The validation is comprised of comparing computed pressure and deflection values with experimental data for an Aeroelastic Research Wing (ARW–2). Furthermore, the use of sensitivity derivatives for predicting the aero–structural characteristics of this configuration is demonstrated.

Sensitivity Derivatives Using Complex Variables

For a central finite–difference approximation to the derivative, one may expand the function in a Taylor series about the point x using a forward step and a backward step, and then subtracting to yield the formula,

$$f'(x) = (f(x+h) - f(x-h))/2h \qquad (1)$$

This expression for the derivative has a truncation error of $O(h^2)$. The advantage of the finite–difference approximation to obtain sensitivity derivatives, is that any existing code may be used without modification. The disadvantages of this method are the computational time required and the possible inaccuracy of the derivatives. The former is due to the fact that for every derivative, Eq. 1 requires two well–converged solutions for the function evaluations. In the case of nonlinear aerodynamics, these solutions may become extremely expensive. The latter is attributed to the sensitivity of the derivatives to the choice of the step size. To minimize the truncation error one selects a smaller step size, however, an exceedingly small step size may produce significant subtractive cancellation errors. The optimal choice for the step size is not known a priori, and may vary from one function to another, and from one design variable to the next.Instead, if the function is expanded in a Taylor series using a complex step as

$$f(x+hi) = f(x) + hi\,f'(x) - h^2 f''(x)/2! - h^3 i\,f'''(x)/3! \quad (2)$$

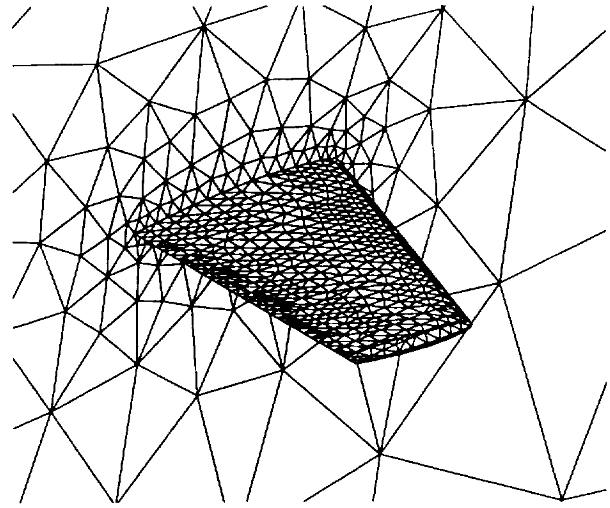
where $i=\sqrt{-1}$. Solving this equation for the imaginary part of the function yields

$$f'(x) = Im[f(x+hi)]/h \quad (3)$$

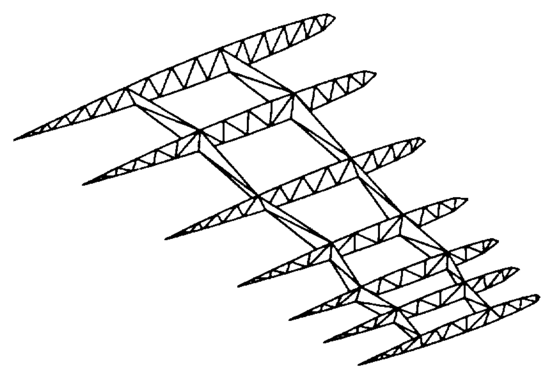
This expression for the derivative also has a truncation error of $O(h^2)$. By evaluating the function at a complex argument, both the function and its derivative are obtained, without subtractive terms, and thus cancellation errors are avoided. The real part is the function value.

The disadvantage of the complex variable approximation is the increased runtime required by the evaluating routines when run with complex arguments. With current compiler options, this run time is on the order of three times the cost of the original solver. However, automatic differentiated versions of analysis software, using say ADIFOR, incur about the same time penalties. A detailed comparison of sensitivity derivative calculations from automatic differentiated software and using the complex variable technique can be found in Ref. 5. The reader is directed to this source for a more complete discussion on the competing methods.

The advantages of the complex variable approximation for obtaining the derivatives are numerous. First, like the finite–difference approximation to the derivatives, very little modification to the original software is required. All the original features and capabilities of the software are retained. Thus, user experience with the current software is not lost, and ongoing advancements and enhancements can be readily introduced into subsequent versions without extensive modifications or re–differentiation. This is in direct contrast to hand or automatically differentiated codes where any modification to the original software will require re–differentiation. This advantage is extremely useful in the problem formulation stages of the design process when new objective functions and constraints are being explored. Second, this method is equivalent to a discrete–direct approach, either from automatic differentiation or hand differentiated codes solved in incremental iterative form, in the way that the state vector and its derivatives are being solved for simultaneously. When solving the state equation, the state vector resides in the real part and the derivatives in the imaginary part. Hence, fully converged flow solutions are not required to obtain derivatives of sufficient accuracy for design. Finally, the complex variable approximation to the derivative is not sensitive to the step size selection and only requires step size that avoids excessive truncation error. In addition, the complex variable technique can be used to compute second derivative information using available data [5]; however, these computations are subject to cancellation errors.



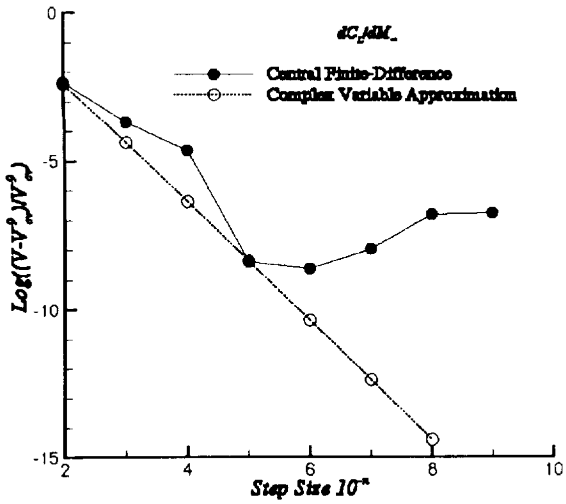
(a) Unstructured aerodynamic mesh.



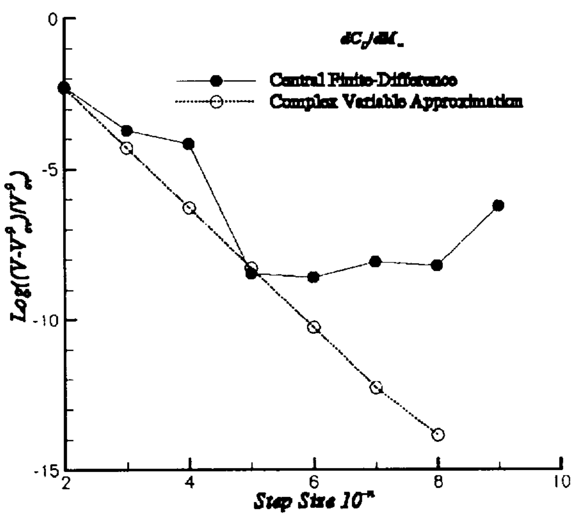
(b) Structural mesh; only ribs and spars shown.

Figure 1. ONERA M6 wing discretizations.

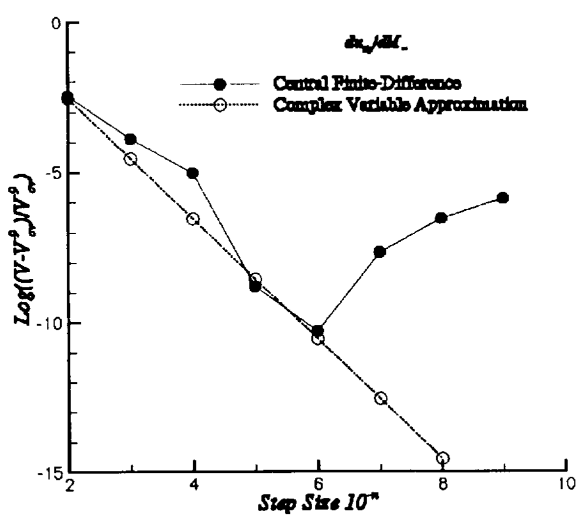
The accuracy of this complex variable approach for obtaining multidisciplinary (aero–structural) sensitivity derivatives has been previously established in Ref. 35. In that work an inviscid flow solver was coupled with a structural analysis code to perform the aero–structural analysis and sensitivity analysis of a low aspect–ratio ONERA M6 wing. The aerodynamic mesh and the spars and ribs of the structure are shown in Fig. 1. It should be noted that the spars of this wing are not straight due to the fact that the structure was created from a subset of the aerodynamic nodes. Further details of that analysis and the interdisciplinary transfer of information may be found in Ref. 35. The accuracy of complex variable approach, as compared with central finite–differences, is illustrated in Fig. 2. In this figure, the sensitivity derivatives of lift, drag, and wing tip deflection with respect to free–stream Mach number are shown for various step sizes. Furthermore, plotted is the Log of the difference between the finite–difference and the complex variable approach; values have been normalized by derivatives obtained using the complex variable approximation at the smallest step size. As seen, the accuracy of the finite–difference



(a) Sensitivity of lift coefficient.



(b) Sensitivity of drag coefficient.



(c) Sensitivity of wing tip deflection.

Figure 2. Sensitivity with respect to Mach number.

formulation depends on the step size and suffers from subtractive cancellation errors as the step size is reduced. The complex variable approach demonstrates true second–order accuracy; each time the step size is reduced by one order of magnitude, the accuracy is increased by two digits.

Aero–Structural Analysis

Briefly described are the aerodynamic and structural analysis codes used in the current work. In addition, previous sensitivity analysis research performed with these codes are discussed. Finally, some issues concerning the interdisciplinary transfer of information across boundary interfaces are presented.

Aerodynamic Analysis

The aerodynamic analysis is conducted using the three–dimensional unstructured Euler/Navier–Stokes code described in Refs. 1–4 and known as FUN3D. This is an implicit, upwind, finite–volume code in which the dependent variables are stored at the vertices of the mesh. Both compressible and incompressible versions of this code exist, and have been previously "hand differentiated" using an adjoint approach for inviscid, laminar, and turbulent flows[2,4,37].

Note that in an adjoint approach, the sensitivity derivatives are evaluated by performing a product of the costate variables with the derivatives of the residual with respect to the design variables. Because the complex variable technique can be easily applied to obtain the derivatives of the residuals with respect to each of the design variables, the current methodology can be applied as part of an adjoint method for obtaining sensitivity derivatives.

For complex flow physics, such as chemically reacting or time dependent flows, the complex variable approach may be utilized and may provide the best approach for determining sensitivity information for these complicated codes. The current methodology may also be employed for a wide range of applications such as obtaining numerical flux Jacobians for complex flux functions or for Newton–Krylov schemes [52].

Structural Analysis

The finite–element structural analysis program used in the present work has been documented in Ref. 30. Since the stiffness matrix for linear static structural analysis is symmetric and positive–definite, a Choleski factorization is used to solve the system of equations. Further details of the solution algorithm may be found in Ref. 47. The solution to this system of equations produces the vector of nodal displacements. From this deformation field, element stresses can be computed. Furthermore, the available element types consist of truss members, beam elements, constant strain triangles with the ability to model multi–layer composites, triangular and quadrilateral plate/shell elements.

This finite–element structural analysis code has been previously differentiated using the automatic differentiation software tool ADIFOR. Details on the usage and development of the supplementary code are documented in Ref. 19. The new software is capable of computing the displacement derivatives with respect to shape design variables. These displacement derivatives may then be used to compute element stress derivatives.

In the current work, the structural derivatives are obtained using the complex variable approximation. The computational time required to compute structural derivatives using the code from Ref. 19 and the current method are comparable. In addition to shape and sizing derivatives, derivatives with respect to material properties can be evaluated without code modification. This is not the case for ADIFOR versions of this software, which would require re–differentiation. An interesting example that would use material properties as design variables, is the aeroelastic tailoring of composite wings. The design variables could be the fiber orientation in the composite layers.

Interdisciplinary Data Transfer

To resolve the nonlinear fluid flow around an arbitrary object, both the surface and the volume exterior to that surface must be discretized. For the structural analysis, the discretization encompasses the surface of the object and the volume interior to the surface. In practice the nonlinear aerodynamic analysis requires a higher degree of resolution than linear structural analysis. Therefore, the interdisciplinary transfer of data between the fluid and the structure becomes an important concern in the aeroelastic analysis of a flexible wing. This aero–structures interaction has been an active area of research[14,17,34,40,51].

In performing the aeroelastic analysis to determine the static equilibrium position of a wing, structural properties can be lumped into sectional quantities, and a reduced resolution model (e.g., classical beam theory) can be used. When element stresses are required for constraints or when structural optimization is performed, a more detailed model is necessary. Because multidisciplinary analysis and optimization is the focus of this work, a detailed model is used even though only the static equilibrium position is sought here.

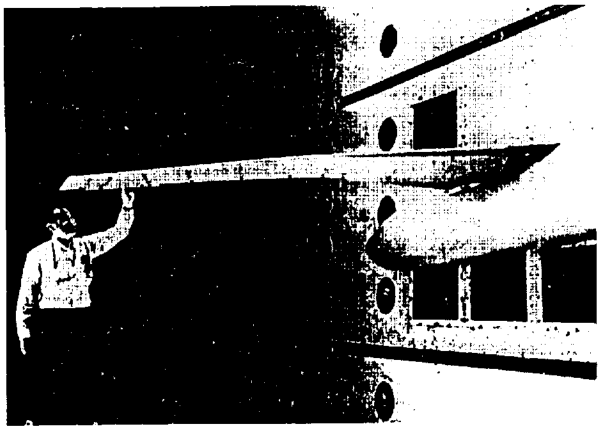
Once deformations have been determined from the structural analysis, these deflections must be represented on the aerodynamic surface. Similarly, aerodynamic loads must also be transferred to the structural nodes. The simplest approach uses bilinear interpolation to transfer the disciplinary information across the interface boundary. This approach, commonly referred to as load–lumping has been successfully used by numerous researchers[17,34]. An alternative to this procedure was developed by Guruswamy and Byun [17], who introduce a virtual surface, between the aerodynamic surface and the structural finite–element mesh. This virtual surface is then used to transfer the structural deformations to the aerodynamic mesh, and the principle of virtual work employed to obtain the loads at the structural nodes from the aerodynamic analysis. In a similar fashion, Tzong et al. [51] introduce a virtual surface based on finite–element technology to transfer the deflections, and virtual work (reciprocal theorem) to obtain the structural loads. Samareh [40] has demonstrated the ability to transfer interdisciplinary information across interface boundaries using Non–Uniform Rational B–Splines. The emphasis of that work was to demonstrate that parameterization techniques, consistent with the CAD definition of the geometry, can be used to perform the coupling between the aerodynamic and structures disciplines.

In the current work, the interaction between the fluid and the structure is accomplished by lumping the aerodynamic forces at the surface structural nodes. After the static structural equilibrium equations have been solved, using the aforementioned loads, the corresponding aerodynamic surface mesh must be updated to the computed deformations. Because the structural deformations at each node are three dimensional, changes in sectional properties are possible. Herein, it is assumed that the in–plane deformations of the sections are limited to rigid–body translation and rotation. Furthermore, for this wing these section deformations are modeled approximately with an equivalent vertical translation and a twist angle (i.e., deformations consistent with beam theory). This approximation is justified by the experimental findings in Ref. 12, which revealed sufficient chordwise rigidity for the ARW–2 wing.

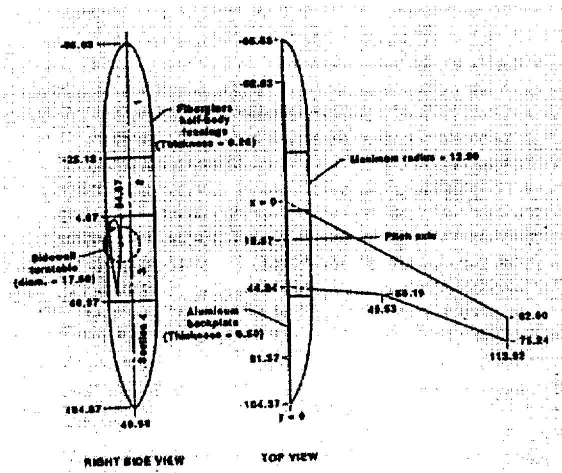
Once the surface deformations have been determined, either from the structural analysis or when the design process requires modifications to the surface geometry, the interior volume of the aerodynamic mesh must be adapted to reflect these changes. For the inviscid computations in the current work, the spring analogy method described in Ref. 37 is used. For discrete approaches to shape design optimization, the sensitivity of the mesh to the geometric design variables are required. In the design studies of Refs. 2, 4, 5, and 37 these terms were achieved by directly differentiating the mesh movement algorithm. In Ref. 33, these terms were obtained via automatic differentiation of the mesh movement algorithm. In the current work and that of Ref. 35, the grid sensitivity derivatives are computed with the complex variable approximation.

Aeroelastic Research Wing (ARW–2)

The ARW–2 was the second of a series of aeroelastic research wings developed at NASA Langley Research Center to experimentally study transonic steady and unsteady phenomenon in the Transonic Dynamics Tunnel (TDT). This research wing has an aspect ratio of 10.3, a leading edge sweep of 28.8°, and supercritical airfoil sections. The test configuration consisted of the ARW–2 and a rigid half–body fuselage and is shown in Fig. 3a. Figure 3b depicts a sketch of the complete wind tunnel model. Further details about this configuration can be found in Ref. 41.

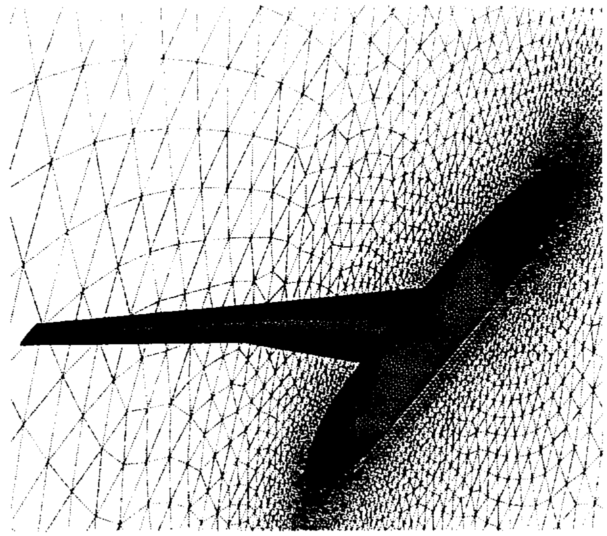


(a) ARW–2 mounted in wind tunnel.

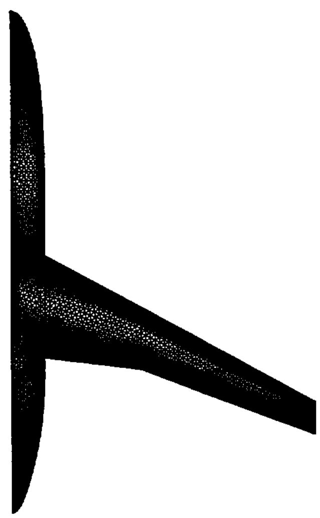


(b) Shetch of wind tunnel model.

Figure 3. ARW–2 test configuration [41].



(a) Aerodynamic grid for the ARW–2.



(b) Detailed view of wing and fuselage surfaces.

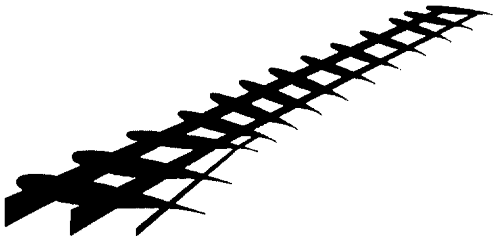
Figure 4. Aerodynamic grid for the ARW–2 test configuration.

For the aerodynamic analysis, the field around the test configuration contains 769,798 tetrahedron (146,397 nodes). The fuselage and wing surface has 52,784 triangular faces (26,565 surface nodes), and is shown in Fig. 4a with the plane of symmetry. Figure 4b illustrates a close–up view of the wing and fuselage surface mesh. This unstructured aerodynamic grid for the ARW–2 test configuration was generated using SolidMesh [53]. The structural model used for the ARW–2 wing is shown in Fig. 5. This figure only depicts the spars and ribs; for clarity the wing skin and stringers are not shown. The model contains 2,042 degrees–of–freedom.

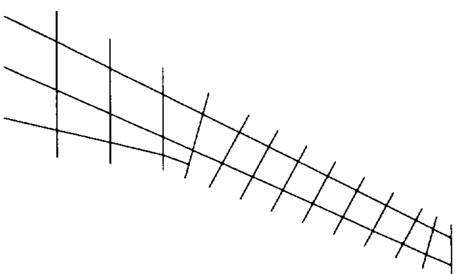
The ARW–2 test model was constructed with fiberglass skins. To avoid the need of simulating this composite skin, an isotropic wing model was developed that possessed nearly the same bending and torsional properties; which was also the approach taken by Bhardwaj et al.[8] to model the ARW–2 composite wing. In the current work, this was accomplished via an inverse design optimization whereby the difference in the computed and experimentally measured [41] bending and torsional rigidities was minimized. The design variables were the properties of the isotropic spars, ribs, stringers, and skin, and the sensitivity derivatives were computed using the complex variable technique.

Results

Examined is the aero–structural analysis of the ARW–2 configuration discussed in the proceeding section. The experimental study, presented in Refs. 12 and 42, consisted of an enormous number of test conditions. These test conditions studied variations in free–stream Mach number, angle–of–attack, dynamic pressure, and control surface deflections. Furthermore, tests were conducted in both a heavy gas and an air medium. The results presented herein correspond to a free stream Mach number of 0.8, a dynamic pressure 102.5 psf, an air medium, and angle–of–attack variations from –2.0 to 3.0 degrees. Control surface deflections were not considered.



(a) Structural mesh for the ARW–2.



(b) Upper surface view of the ribs and spars.

Figure 5. Structural mesh for the ARW–2 wing.

Shown in Fig. 6, are the computed (inviscid) aero–structural and experimentally measured [12] deflections of the front and rear spars for various free stream angles of attack. In addition, the wind–off–positions (W.O.P.) are also included in each figure to help quantify the static deflections taking place. The W.O.P. are the measured coordinates of the wing when the tunnel is not in operation. As seen, reasonable accuracy is obtained for each of the angles of attack considered. The greatest deviations occur at an angle of attack of 3.0°; which corresponds to the highest wing loading. This discrepancy is attributed to the inability to precisely model the bending and torsional properties of the composite wing, and possibly to increased viscous effects at this higher angle of attack.

Experimentally measured [42], computed rigid wing, and computed aeroelastic wing pressure coefficient data are illustrated in Fig. 7 for various spanwise stations at 0.0° angle of attack. The twist angle and bending deflections, as previously seen in Fig. 6, are relatively small for the inboard stations of the wing; hence, only minor differences can be seen between rigid and aerodynamic calculations. However, at the outboard stations of the wing, it can be seen that the aeroelastic calculations compare better with the experimental data. This is especially true for the station nearest the tip where the largest twist angle and bending deflections occur. It should also be noted that in Fig. 7a, the lower surface pressure distribution compares well with the experimental data. This may indicate that the aerodynamic interference between the high mounted wing and the fuselage is being captured. The discrepancy seen on the upper surface may indicate that viscous effects are significant. Moreover, an over prediction of the pressure on the suction side is typical of inviscid solutions.

Sensitivity derivatives, other than for design purposes, may be used to predict or estimate a solution at a nearby condition. This results from the fact that given a baseline solution and the gradient of that solution with respect to an independent variable, one may use a Taylor series to obtain a first order estimate of the solution at a neighboring value of this variable. To illustrate this point, the aforementioned complex variable technique was used to compute the sensitivity derivative of wing tip twist with respect to free–stream angle–of–attack. The angle–of–attack about which the derivative was taken is 0°. The comparison of the experimentally measured [42], computed, and predicted wing tip twist versus angle of attack are shown in Fig. 8. As expected, the wing tip twist is estimated reasonably well around the baseline solution, but deviate significantly at higher angles–of–attack.

Conclusions

In the current work the aero–structural analysis procedure, previously used to establish the accuracy of the complex variable method for obtaining multidisciplinary sensitivity derivatives in Ref. 35, was validated. This validation took the form of comparing computed deflection and pressure data with experimentally measured values for an Aeroelastic Research Wing (ARW–2). Since the aero–structural analysis procedure has the complex variable modifications already included into the software, multidisciplinary sensitivity derivatives can be computed automatically. To demonstrate the use of these sensitivity derivatives, the gradient of wing tip twist with respect to free–stream angle–of–attack was computed and used to estimate the tip twist at neighboring angles. Currently underway, is the multidisciplinary optimization of the current configuration using design variables appropriate for both aerodynamic and structural design. These results are forthcoming.

Acknowledgements

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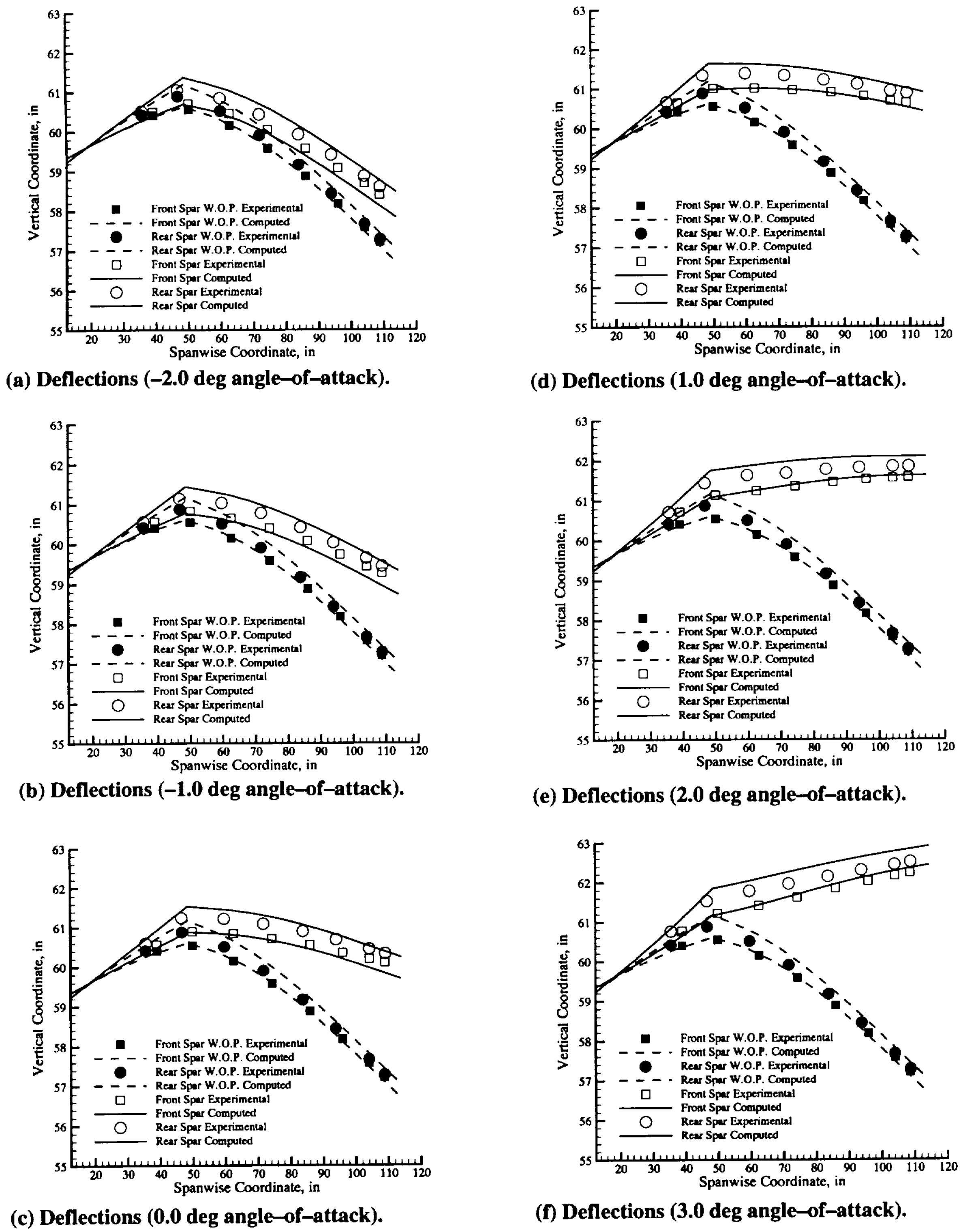

(a) Deflections (–2.0 deg angle–of–attack).

(b) Deflections (–1.0 deg angle–of–attack).

(c) Deflections (0.0 deg angle–of–attack).

(d) Deflections (1.0 deg angle–of–attack).

(e) Deflections (2.0 deg angle–of–attack).

(f) Deflections (3.0 deg angle–of–attack).

Figure 6. Computed and measured [12] front and rear spar deflections; M = 0.8, q = 102.5 psf.

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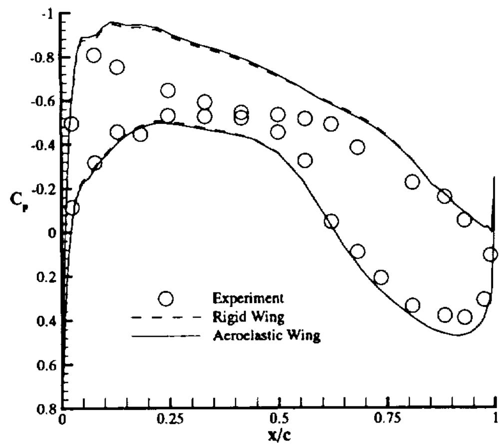
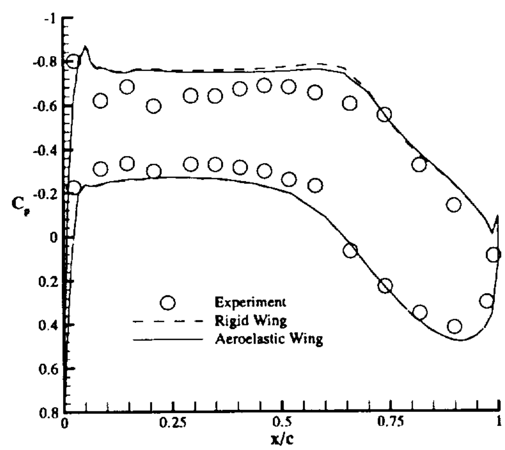
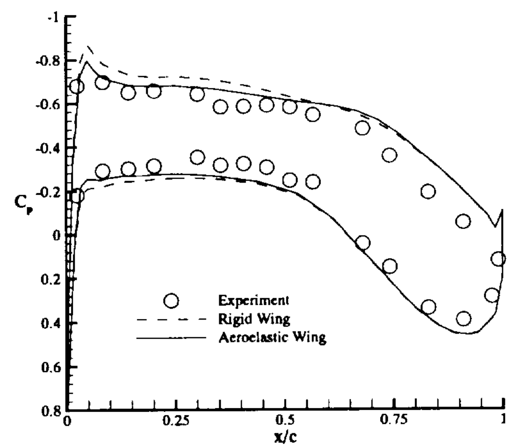


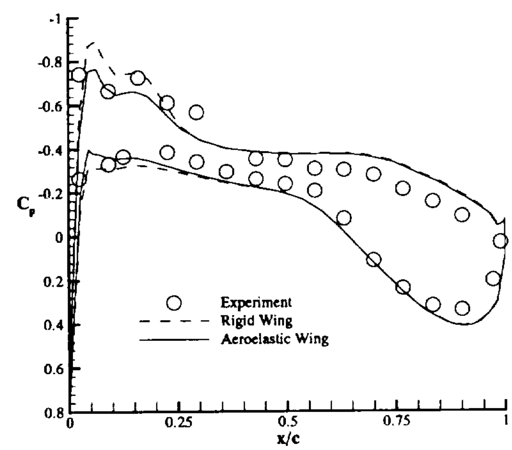
(a) Pressure coefficient at 27.4% semi–span.



(b) Pressure coefficient at 70.7% semi–span.



(c) Pressure coefficient at 87.1% semi–span.



(d) Pressure coefficient at 97.2% semi–span.

Figure 7. Computed and experimental [42] pressure coefficient data for M = 0.8, q = 102.5 psf, and 0.0 deg angle–of–attack.

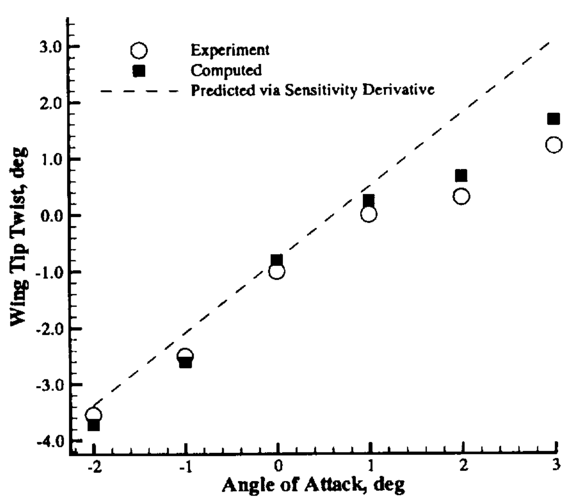


Figure 8. Computed, experimental [42], and predicted wing tip twist.