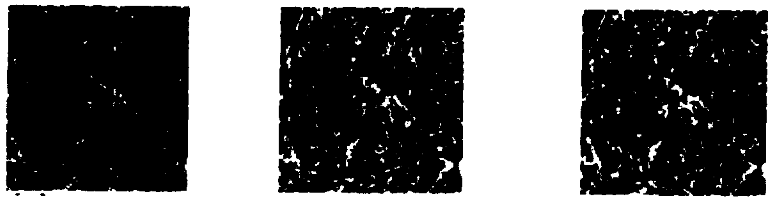
A Comparitive Study of Interferometric Regridding Algorithms



by

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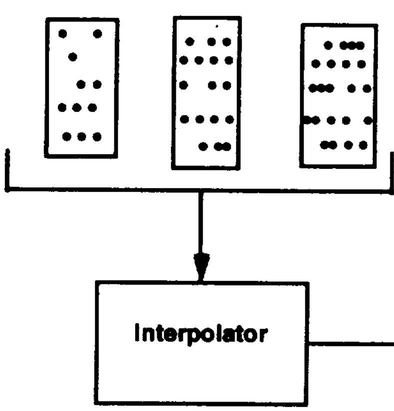
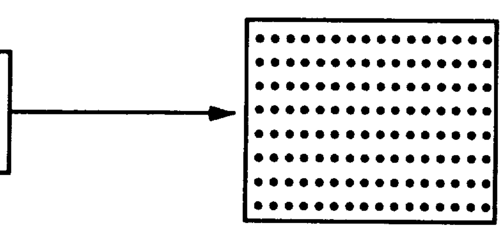
Regridding



- Position vectors are not uniformly distributed in the plane following the height reconstruction process and thus need to be resampled to a uniform output grid. This process is called regridding.



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Regridding Options

- The problem of interpolating data that is not sampled on a uniform grid, that is noisy, and contains gaps is a difficult problem.
- Several interpolation algorithms have been implemented
  - Nearest neighbor - Fast and easy but shows some artifacts in shaded relief images.
  - Simplical interpolator - uses plane going through three points containing point where interpolation is required. Reasonably fast and accurate.
  - Convolutional - uses a windowed Gaussian approximating the optimal prolate spheroidal weighting function for a specified bandwidth.
  - First or second order surface fitting - Uses the height data centered in a box about a given point and does a weighted least squares surface fit.

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Regridding Options Status

Method	Code	Testing	Throughput
Nearest Neighbor	√	√	√
Simplical	√	√	√
Adaptive Surface Fitting	√	√	√
Adaptive Convolutional	√	√	√

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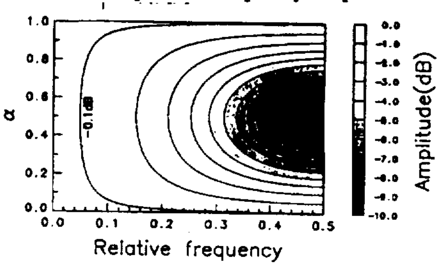
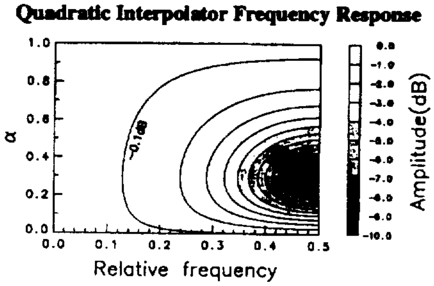
Linear and Quadratic Interpolators

- In order to fully reconstruct a band limited signal the Nyquist Theorem says the signal must be sampled at twice the bandwidth. Basically, we must sample the signal so that we catch all the zero crossings of the function on the domain of interest.
- The maximal number of real zeros for a polynomial function is bounded by the degree of the polynomial, hence low order polynomial interpolators are inherently low pass filters, that is they can faithfully reconstruct only slowly varying functions on the interval of interest. The frequency response of these filters is

Linear Interpolator Frequency Response



Quadratic Interpolator Frequency Response



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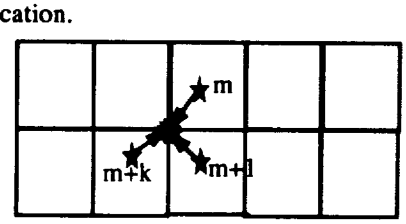
Some Bookkeeping Details

- After height reconstruction each unwrapped phase point consists of a triple of numbers, the SCH coordinates of that point. Note that this point does not necessarily lie on an output grid point. To preserve the full information of the reconstructed target and have a convenient referencing frame relative to the output grid each reconstructed point is assigned a number which is stored in an array indexed by output grid location.



- Multiple points can be assigned to the same location (until the buffer is full - currently set to five points)

- Location in output grid (s_l,c_l) is determined by nearest neighbor location

$$c_l = \left[\frac{c - c_o}{\Delta c}\right] \qquad s_l = \left[\frac{s - s_o}{\Delta s}\right]$$

where (s,c) are the target s,c coordinates, c_o,s_o are the map offsets and Δs, Δc are the pixel dimensions.

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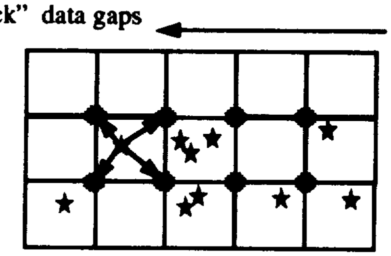
Nearest Neighbor

- This is the simplest algorithm considered for regridding the height and other data layers.
- Very efficient and easy to implement.
- Used very successfully in the TOPSAR and IFSARE processors.
- Drawbacks include
  - no further reduction of height noise
  - occasional regridding artifacts such as
    - terracing of amplitude and height data
    - height ramps due to range dependent noise levels
    - valleys can be rounded out or filled
    - "Pin prick" data gaps



- Data written from near to far range

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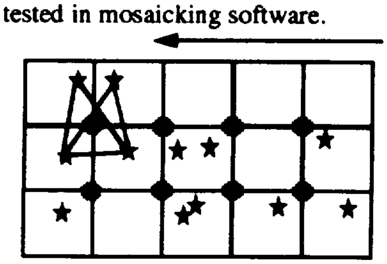
Simplical Regridding

- Regrids the data using a planar fit with three points enclosing the point where regridded value is desired.
- Several criteria are considered for selecting which triangle (or simplex) to use when obtaining height value at regrid point
  - simplex of minimal area containing regrid point
  - simplex with vertex closest to regrid point
  - simplex with minimal height error
  - simplex with large isoperimetric ratio
- More robust than nearest neighbor in avoiding pin pricks
- Some reduction in height noise with this method.
- Used and well tested in mosaicking software.



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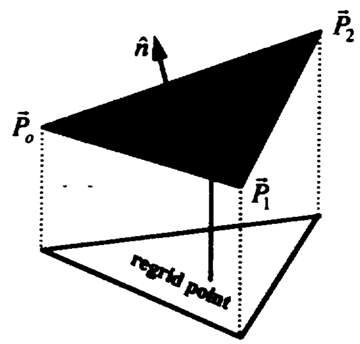
Simplical Regridding Formulas



- Plane passing through three points satisfies the equation

$$\left(\vec{P}-\vec{P}_o\right)\bullet\hat{n}=0$$

where $\vec{P}_o,\vec{P}_1,\vec{P}_2$ are three known points in the plane and $\hat{n}$ is the normal to the plane given by

$$\hat{n}=\left(\vec{P}_1-\vec{P}_o\right)\times\left(\vec{P}_2-\vec{P}_o\right)$$

- Three points are chosen such that
  - Regrid point lies in the interior of a triangle of three points for which the position vectors are known
  - Triangle has isoperimetric ratio larger than .3 (prevents using long skinny triangles)
  - Points are within a specified distance to regrid point
  - Smallest height variance

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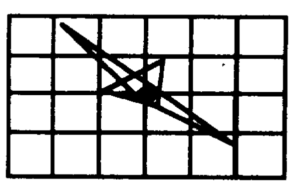
Details on Simplex Selection

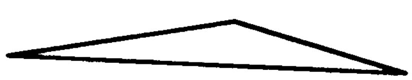


—— Selected Simplex

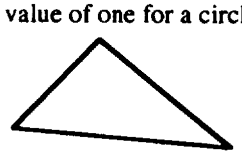
—— Rejected Simplex

- Take point of minimal height variance at each lattice point in output grid.
- Loop over all set of three vertices and eliminate simplicies
  - do not contain regrid point
  - are two thin and narrow (use isoperimetric ratio)
  - distance of closest vertex in simplex to regrid point is two large
- If multiple candidates exist take one with minimal height variance

Isoperimetric ratio = $\frac{4\pi A}{P^2}$ Note: This value is always less than or equal to one. Attains the value of one for a circle.



Smaller isoperimetric ratio



Larger isoperimetric ratio

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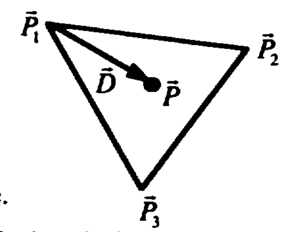
Height Variance Computation

- From the equation of the plane used for simplical interpolation the regridded height, h_f in term of the heights at the three vertices has the form

$$h_f = \left(1 + \frac{(\vec{D}_{32} \times \vec{D})_z}{(\vec{D}_{21} \times \vec{D}_{31})_z}\right) h_1 + \left(\frac{(\vec{D}_{31} \times \vec{D})_z}{(\vec{D}_{21} \times \vec{D}_{31})_z}\right) h_2 + \left(\frac{(\vec{D}_{21} \times \vec{D})_z}{(\vec{D}_{21} \times \vec{D}_{31})_z}\right) h_3$$

where $\vec{D}_{ij} = \vec{P}_i - \vec{P}_j$ and $\vec{D} = \vec{P} - \vec{P}_1$



- The height variance of the regrid point is given by

$$\sigma_{h_f} = \sqrt{\sum_{i=1}^{3} k_i^2 \sigma_{h_i}^2}$$

where the k_i are the coefficients of the h_i given above.

- Note if the regrid point is at the median of the triangle then the k_i are all equal to 1/3. Assuming height variances are all equal there is a reduction in the height variance by $\sqrt{3}$.

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Surface Fitting and Adaptive Regridding

- The surface fitting regrid method fits a quadratic surface to points within a specified region containing the regrid point. This method has several advantages
  - reduction in height noise that depends roughly on the inverse of the square root of the number of points in the region used to make the fit
  - helps eliminate pin prick holes in the DEM
  - can simultaneously estimate slope and curvature information
- The region used for fitting plus the weights can be adjusted to adaptively smooth the noise at the expense of spatial resolution
  - standard weighting is by the by the expected height variance, σ, determined from the interferometric correlation
  - to reduce the effect of points far from the regrid point the weighting can be increased, using a simple distance dependent additional weighting

$$w(\bar{x} - \bar{x}_o) = \frac{k}{\sigma} |\bar{x} - \bar{x}_o|^{\alpha}$$

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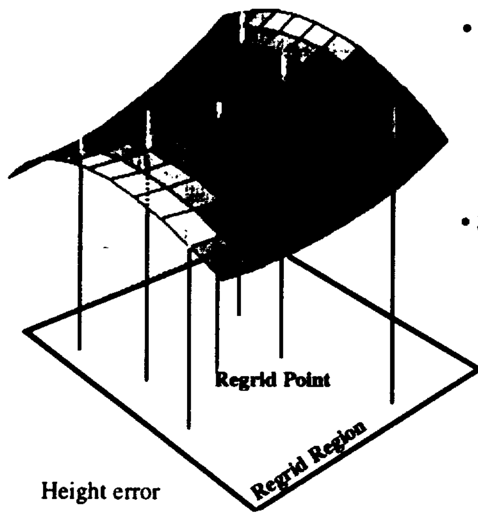
Surface Fitting Regrid Geometry



- Box size and additional weighting adjusted depending upon the ratio of the RMS surface elevation to the mean expected height error as determined from the interferometric correlation.
- Surface fitting acts as a low pass filter so certain higher frequency components will be lost in the resampling process.

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Adaptive Regridding Parameter Determination

- In the adpative regridding process it is desired to adjust the amount of smoothing depending on the amount of topographic compared to the intrinsic measurement noise.
- The amount of noise reduction and smoothing depends on the size of box used for fitting and the amount of weighting employed.
- For computational efficiency is deisred to have the weighting depend only on the measurement noise and not vary spatially with the data, however this reduces the flexibilty in controlling the amount of smoothing vs noise reduction.
- The box size for fitting is determined by comparing the χ^2 residual of the surface fit to the mean of the estimated height noise as determined from the correlation in the box.
    - large residuals compared to the instrinsic noise level means that surface fit is not a good model for the local topography and therefore a smaller box size should be use.
    - for smaller box sizes (1 and possibly 2) better to use a linear fit since a better covariance estimate is possible fitting a smaller number of parameters.

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Surface Fitting Regridding Equations

- The least squares fit to a quadratic surface (degree of surface, N =2) requires the estimation of six parameters

$$q(x,y) = a_{oo} + a_{10}x + a_{01}y + a_{11}xy + a_{20}x^2 + a_{02}y^2$$

which are obtained by solving the 6x6 linear system given below

$$\begin{bmatrix} a_{oo} \\ a_{10} \\ a_{01} \\ a_{20} \\ a_{11} \\ a_{02} \end{bmatrix} = \begin{bmatrix} \sum_i \frac{1}{\sigma_i^2} & \sum_i \frac{x_i}{\sigma_i^2} & \sum_i \frac{y_i}{\sigma_i^2} & \sum_i \frac{x_i^2}{\sigma_i^2} & \sum_i \frac{x_i y_i}{\sigma_i^2} & \sum_i \frac{y_i^2}{\sigma_i^2} \\ \sum_i \frac{x_i}{\sigma_i^2} & \sum_i \frac{x_i^2}{\sigma_i^2} & \sum_i \frac{x_i y_i}{\sigma_i^2} & \sum_i \frac{x_i^3}{\sigma_i^2} & \sum_i \frac{x_i^2 y_i}{\sigma_i^2} & \sum_i \frac{x_i y_i^2}{\sigma_i^2} \\ \sum_i \frac{y_i}{\sigma_i^2} & \sum_i \frac{x_i y_i}{\sigma_i^2} & \sum_i \frac{y_i^2}{\sigma_i^2} & \sum_i \frac{x_i^2 y_i}{\sigma_i^2} & \sum_i \frac{x_i y_i^2}{\sigma_i^2} & \sum_i \frac{y_i^3}{\sigma_i^2} \\ \sum_i \frac{x_i^2}{\sigma_i^2} & \sum_i \frac{x_i^3}{\sigma_i^2} & \sum_i \frac{x_i^2 y_i}{\sigma_i^2} & \sum_i \frac{x_i^4}{\sigma_i^2} & \sum_i \frac{x_i^3 y_i}{\sigma_i^2} & \sum_i \frac{x_i^2 y_i^2}{\sigma_i^2} \\ \sum_i \frac{x_i y_i}{\sigma_i^2} & \sum_i \frac{x_i^2 y_i}{\sigma_i^2} & \sum_i \frac{x_i y_i^2}{\sigma_i^2} & \sum_i \frac{x_i^3 y_i}{\sigma_i^2} & \sum_i \frac{x_i^2 y_i^2}{\sigma_i^2} & \sum_i \frac{x_i y_i^3}{\sigma_i^2} \\ \sum_i \frac{y_i^2}{\sigma_i^2} & \sum_i \frac{x_i y_i^2}{\sigma_i^2} & \sum_i \frac{y_i^3}{\sigma_i^2} & \sum_i \frac{x_i^2 y_i^2}{\sigma_i^2} & \sum_i \frac{x_i y_i^3}{\sigma_i^2} & \sum_i \frac{y_i^4}{\sigma_i^2} \end{bmatrix} \begin{bmatrix} \sum_i \frac{h_i}{\sigma_i^2} \\ \sum_i \frac{x_i h_i}{\sigma_i^2} \\ \sum_i \frac{y_i h_i}{\sigma_i^2} \\ \sum_i \frac{x_i^2 h_i}{\sigma_i^2} \\ \sum_i \frac{x_i y_i h_i}{\sigma_i^2} \\ \sum_i \frac{y_i^2 h_i}{\sigma_i^2} \end{bmatrix}$$

- Using Cholesky decomposition to invert the matrix and some careful bookkeeping in computing the matrix , yields the required operations per point of

$$\underbrace{\frac{1}{6}\left(\frac{(N+1)(N+2)}{2}\right)^3}_{\text{Matrix Inversion}} + \underbrace{M(N+1)(2N+1)}_{\text{Matrix Computation}} \qquad \text{M = \# points in fit}$$

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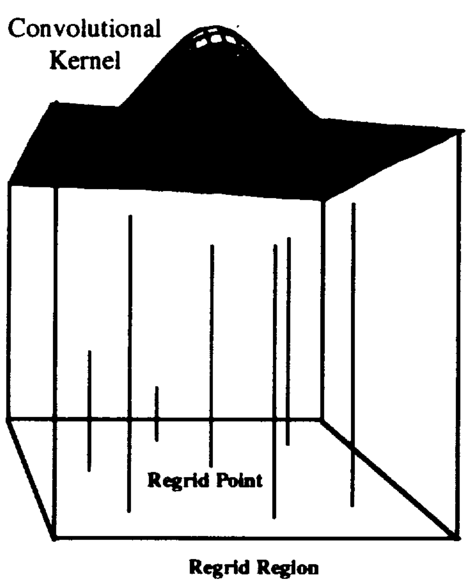
Convolutional Regrid Geometry



- Box size and additional weighting adjusted depending upon the ratio of the RMS surface elevation to the mean expected height error as determined from the interferometric correlation.
- Convolutional regridding kernel is a windowed Gaussian approximating the optimal prolate spheroidal weighting function for a specified bandwidth.

—— Measured Height

—— Regridded Height

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Convolutional Regridding Formulas

- The convolutional regridder determines the height at point as sum of all heights points within a box, B, centered at the desired regid point weighted by the convolutional kernel weights which a function of the distance from the regrid point.

$$h(\vec{p}_0) = \sum_{\vec{p} \in B} w(\vec{p} - \vec{p}_o)\, h(\vec{p})$$

- The height error estimate is obtained from the local height errors estimates for each point in the box weighted by the derviatives of the kernel with respect to the spatial variables.

$$\sigma_h(\vec{p}_0) = \sqrt{\sum_{\vec{p} \in B} \left(\frac{\partial w}{\partial \vec{p}} (\vec{p} - \vec{p}_o) \right)^2 \sigma_h^2(\vec{p})}$$

- As with the surface fitting algorithm an estimate of the slope and curvature can be obtained using the first and second derivatives of the convolutional kernel.

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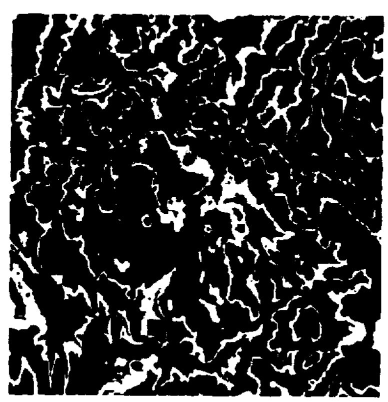
Longvalley DEM

Colorwrap = 100 m



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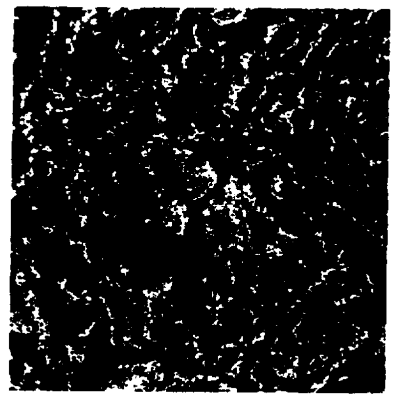
LongValley Nearest Neighbor

Noise = 10 m
Colorwrap = 100 m
r.m.s = 12.3 m
bias = -2.17 m
Coverage = 82%



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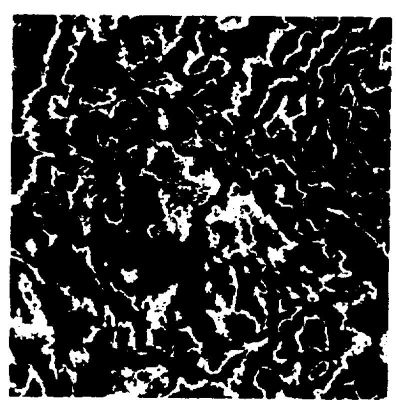
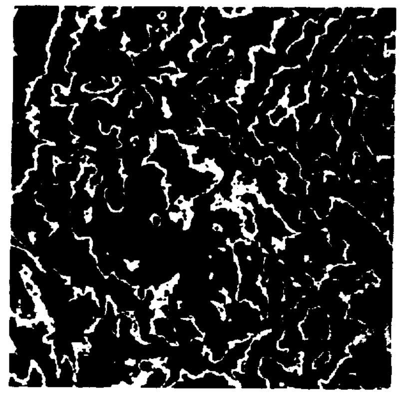
LongValley Simplical

Noise = 10 m
Colorwrap = 100 m
r.m.s = 7.87 m
bias = 2.44 m
Coverage = 82%



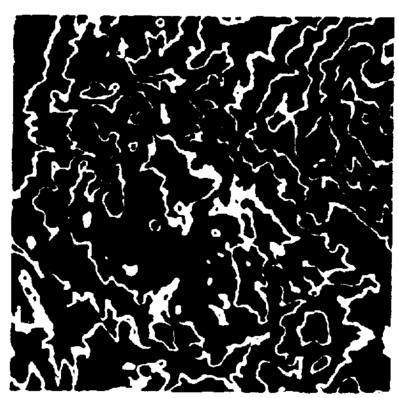
LongValley Convolutional

Noise = 10 m
Colorwrap = 100 m
r.m.s = 6.6 m
bias = 2.4E-2 m
Coverage = 85%



LongValley Surface Fit

Noise = 10 m
Colorwrap = 100 m
r.m.s = 7.6 m
bias = -2.2E-3 m
Coverage = 84%



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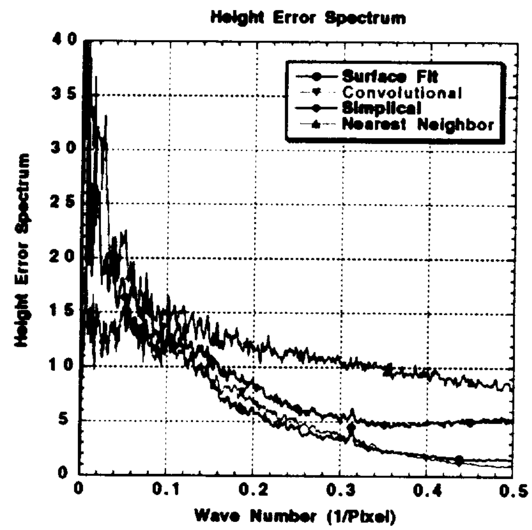
Height Error Spectrum for LongValley



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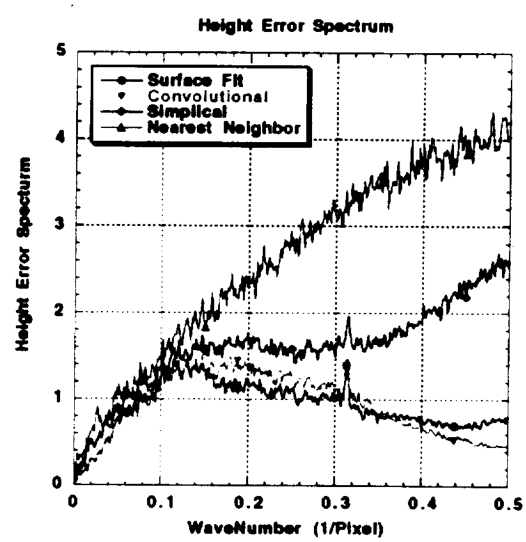
Height Error Spectrum for LongValley



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Evaluation Results: Convolutional Adaptive Method

Long Valley

Noise Level	No Noise	2 m	5 m	10 m
Error	2.58	2.82	3.70	5.85
Coverage	85%	85%	85%	85%
Bias	2.62E-2	2.78E-2	3.58E-2	5.4E-2

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Evaluation Results: Fixed Window 5 x 5

Long Valley

Noise Level	No Noise	2 m	5 m	10 m
Error	2.39	2.64	3.66	6.06
Coverage	82%	82%	84%	82%
Bias	2.75E-2	2.50E-2	2.67E-2	3.80E-2

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Evaluation Results: Fixed Window 7 x 7

Long Valley

Noise Level	No Noise	2 m	5 m	10 m
Error	2.6	2.78	3.59	5.69
Coverage	83%	84%	84%	84%
Bias	3.71E-2	3.37E-2	3.42E-2	4.53E-2

Convolutional

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Evaluation Results: Fixed Window 9 x 9

Long Valley

Noise Level	No Noise	2 m	5 m	10 m
Error	2.78	2.91	3.55	5.38
Coverage	80%	80%	80%	81%
Bias	3.93E-2	3.54E-2	3.49E-2	4.39E-2

Convolutional

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Evaluation Results: Surface Fit Adaptive Method

Long Valley

Noise Level	No Noise	2 m	5 m	10 m
Error	3.03	3.27	4.07	5.83
Coverage	85%	85%	85%	85%
Bias	1.60E-2	-1.06E-3	2.19E-2	3.75E-2

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Evaluation Results: Fixed Window 5 x 5

Long Valley

Noise Level	No Noise	2 m	5 m	10 m
Error	2.52	2.75	3.74	6.36
Coverage	81%	81%	82%	82%
Bias	1.79E-2	1.91E-2	2.18E-2	2.77E-2

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Evaluation Results: Fixed Window 7 x 7

Long Valley

Noise Level	No Noise	2 m	5 m	10 m
Error	3.05	3.15	3.67	5.28
Coverage	84%	84%	84%	85%
Bias	3.03E-2	2.80E-2	2.80E-2	3.32E-2

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Evaluation Results: Fixed Window 9 x 9

Long Valley

Noise Level	No Noise	2 m	5 m	10 m
Error	3.73	3.79	4.09	5.19
Coverage	81%	81%	81%	81%
Bias	3.02E-2	2.8E-2	2.60E-2	2.60E-2

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Surface Fit & Convolutional Mt. Everest

Coverage: 85%-90%