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A CLUE TO THE EXTENT OF CONVECTIVE MIXING INSIDE MASSIVE STARS: THE SURFACE HYDROGEN ABUNDANCES OF LUMINOUS BLUE VARIABLES AND HYDROGEN-POOR WOLF-RAYET STARS

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ABSTRACT

Interior layers of stars that have been exposed by surface mass loss reveal aspects of their chemical and convective histories that are otherwise inaccessible to observation. It must be significant that the surface hydrogen abundances of luminous blue variables (LBVs) show a remarkable uniformity, specifically $X_{surf} = 0.3 - 0.4$, while those of hydrogen-poor Wolf-Rayet (WN) stars fall, almost without exception, below these values, ranging down to $X_{surf} = 0$. According to our stellar model calculations, most LBVs are post-red-supergiant objects in a late blue phase of dynamical instability, and most hydrogen-poor WN stars are their immediate descendants. If this is so, stellar models constructed with the Schwarzschild (temperature-gradient) criterion for convection account well for the observed hydrogen abundances, whereas models built with the Ledoux (density-gradient) criterion fail. At the brightest luminosities, the observed hydrogen abundances of LBVs are too large to be explained by any of our highly evolved stellar models, but these LBVs may occupy transient blue loops that exist during an earlier phase of dynamical instability when the star first becomes a yellow supergiant. Independent evidence concerning the criterion for convection, which is based mostly on traditional color distributions of less massive supergiants on the Hertzsprung-Russell diagram, tends to favor the Ledoux criterion. It is quite possible that the true criterion for convection changes over from something like the Ledoux criterion to something like the Schwarzschild criterion as the stellar mass increases.

Subject headings: convection-- stars: evolution-- stars: oscillations-- stars: variables: other (luminous blue variables)-- stars: Wolf-Rayet-- supergiants

INTRODUCTION

One of the most remarkable, but least remarked about, features of the classical luminous blue variables (LBVs) is the near uniformity of their surface hydrogen abundances. For all LBVs or candidate LBVs having accurate measurements of X_{surf} (the surface hydrogen abundance by mass fraction), the observed range of X_{surf} is found to be restricted to 0.3 - 0.4 (§4.1) This has been difficult to explain.

Standard theories of LBVs in which very massive stars are assumed to lose mass in some way at an enormously enhanced rate around the end of the main-sequence phase predict that the surface hydrogen abundance should fall through a considerable range during the LBV phase. Langer et al. (1994) and Pasquali et al. (1997a) found X_{surf} dropping from 0.4 to 0.1. Although their specific predictions are not very firm, their results do illustrate the range of X_{surf} that might be expected. The models that they computed, however, apply solely to initial stellar masses of 60 M_0 and greater. For smaller initial masses, which are representative of the fainter LBVs, X_{surf} should drop through an even larger range, starting from $X_{surf} = 0.7$, owing to the fact that the original hydrogen envelope in these less massive stars is not entirely removed by the ordinary stellar wind during the main-sequence phase (Schaller et al. 1992; Bressan et al. 1993).

To remedy the lack of any proven LBV mechanism, an alternative theory that we have proposed relies on a known, classical instability that is inevitably encountered by an evolving massive star when sufficient mass has been shed through the ordinary stellar wind: ionization-induced dynamical instability (Stothers & Chin 1993, 1996, 1999; Stothers 1999a,b). This instability takes place after the main-sequence phase has ended, when the star has become a yellow or red supergiant. Provided that the initial mass is not too high, the instability later reestablishes itself, for a longer period of time, when the star is again a blue supergiant. Predicted values of X_{surf} during the second phase of dynamical instability are 0.1 - 0.2. Although this range of values seems sufficiently narrow, the values themselves are too small to account for the hydrogen abundances that are actually observed in LBVs.

In our previous investigations of LBVs, however, one potentially important factor was never examined: the possibility of full convective mixing in the layers outside the helium core and below the hydrogen-rich envelope, where the mean molecular weight, μ, varies. During the main-sequence phase of evolution, the region with a μ gradient becomes convectively unstable, but the μ gradient remains little affected by the slow, semiconvective mixing that takes place there. After the stage of central hydrogen exhaustion, however, the region just above the newly ignited hydrogen shell becomes convectively even more unstable. If the Ledoux (density-gradient) criterion for convection is employed to treat semiconvective mixing, as has been done in all of our LBV studies to date, little additional change in the μ gradient occurs. By adopting the more lenient Schwarzschild (temperature-gradient) criterion for convection, though, semiconvective mixing near the hydrogen shell quickly turns into full convective mixing, producing a homogeneously mixed zone (FCZ) that extends all the way down to the base of the shell. Further surface ejection of mass from the star is then expected to eventually expose the FCZ. Thereafter, the surface hydrogen abundance remains the constant value characterizing the FCZ, until the hydrogen envelope is almost completely gone. This series of developments, in fact, happens in the evolutionary sequences of Schaller et al. (1992), who adopted the Schwarzschild criterion for convection, but its significance seems to have been overlooked.

The purpose of the present paper is to investigate more fully the consequences of adopting the Schwarzschild criterion for convection for interpreting the surface hydrogen abundances of LBVs and of their likely immediate descendants, the hydrogen-poor Wolf-Rayet stars. In §2, the physical input data to our stellar models are specified. Our new evolutionary sequences are presented in §3 and compared with observations in §4. Important new inferences concerning the two criteria for convection are drawn in §5.

For our present purposes we follow the simplified prescriptions for semiconvection as outlined elsewhere (Stothers & Chin 1975, 1976). The reason why we are explicitly considering only the Schwarzschild and Ledoux criteria is that they bracket the range of possible choices, while the available observational data probably do not allow a more refined choice to be made. Although possible prescriptions between these two extremes could conceivably lead to nonmonotonic behavior of the stellar evolutionary tracks, we tend to doubt that this would be the case.

2. PHYSICAL ASSUMPTIONS

2.1. *Initial Mass and Chemical Composition*

The initial stellar masses adopted here are 30, 60, and 90 M_0. They reflect the range of observed luminosities of classical LBVs, for which log $(L/L_0) = 5.4 - 6.3$. To cover the chemical compositions of LBVs in both the Galaxy and the Large Magellanic Cloud, we assign initial (hydrogen, metals) abundances by mass of $(X_e, Z_e) = (0.700, 0.030)$, (0.735, 0.015), and (0.756, 0.004).

2.2. *Mass Loss Rate*

Mean rates of stellar wind mass loss are taken from the three-parameter formula fitted to a large body of observational data by Nieuwenhuijzen & de Jager (1990):

$$-dM/dt = 1.17 \times 10^{-8}\ w\ (L/L_0)^{1.64}\ (M/M_0)^{0.16}\ T_e^{-1.61}$$

where the units of the mass-loss rate are $M_0\ yr^{-1}$ and we have applied a multiplicative correction factor w. The standard mass-loss rates for Galactic and Large Magellanic Cloud O-type stars are approximately represented by w = 1 (Lamers & Leitherer 1993; Puls et al. 1996; Lamers et al. 1999). To accommodate the significantly larger rates derived by de

Koter, Heap, & Hubeny (1997) and Crowther & Bohannan (1997) for a number of Galactic and Large Magellanic Cloud O-type stars having log $(L/L_{0}) \geq 5.8$, we consider also the effect of adopting w = 3. The assumption of no mass loss, w = 0, will be used as a control case.

In all of our evolutionary sequences, the FCZ that develops at the base of the semiconvection zone reaches its full size before dynamical instability occurs in the outer envelope. Since our purpose here is only to determine the hydrogen abundance in the FCZ, the evolutionary tracks will be terminated before the onset of this rapid dynamical phase of mass loss. While the adopted cutoff ignores any subsequent flows of hydrogen into and out of the FCZ, these fluxes are known to be small from the results of earlier calculations performed both with and without heavy mass loss (e.g., Stothers & Chin 1976; Schaller et al. 1992; Mowlavi & Forestini 1994; see also §3).

2.3. *Other Assumptions about Convection*

In the outer envelope, we adopt for the ratio of convective mixing length to local pressure scale height $\alpha_P = 1.4$, but calculate also a few exploratory model sequences with $\alpha_P = 2.8$. At the convective core boundary, we allow for convective mixing of material beyond the layer with vanishing buoyancy by assigning a constant ratio of the overshoot distance, d, to the local pressure scale height, H_P. Two alternative assumptions will be made: no significant core overshooting ($d/H_P = 0$) and moderate core overshooting ($d/H_P = 0.2$).

3. THEORETICAL RESULTS

Beginning at the zero-age main sequence (ZAMS), our new evolutionary sequences show, as expected, that the star's surface initially expands, brightening and cooling until hydrogen in the core becomes low in abundance. Then the evolutionary track on the Hertzsprung-Russell (H-R) diagram halts its redward trajectory (TAMS stage) and moves back toward hotter effective temperatures. When central hydrogen finally disappears, a shell of hydrogen around the helium core ignites, and the surface begins to re-expand. At this stage, the FCZ quickly forms and soon reaches its full extent. Although chemical homogenization of the star's inner envelope normally tends to stop or to slow down the surface expansion, the opposing effect of a relatively high opacity in the outer envelope counters this tendency. In the very massive stars under consideration here, the opacity wins out and forces these stars directly into yellow and red supergiants. Figure 1 shows five of our evolutionary tracks for stars of initially 60 $M_\odot$ We have designated the track based on input parameters $Z_t = 0.03$, $w = 1$, $\alpha_P = 1.4$, and $d/H_P = 0$ as our "standard" track.

From their overt appearnace on the H-R diagram, these evolutionary tracks resemble very closely the ones based on the Ledoux criterion that we published previously. The chief physical differences in the models are internal, notably the hydrogen distribution at the bottom of the envelope, as depicted here in Figure 2 for post-main-sequence models based on the two convection criteria with "standard" input parameters.

Figure 1 reveals also the familiar result that the main-sequence effective temperature drops if Z_e is increased (Haselgrove & Hoyle 1959), if α_P is decreased (Demarque 1961), or if d/H_P is moderately increased (Stothers 1972). A lower effective temperature promotes more mass loss. Although the consequences are relatively minor in the case of any reasonable changes of α_P and Z_e, they can be substantial if moderate convective core overshooting applies, insofar as the most luminous stars are concerned. In Figure 3 we have plotted the hydrogen abundance in the FCZ, X_f, versus the mass of the star at the time when the FCZ has fully formed. The equivalent stellar models of Schaller et al. (1992), which included moderate convective core overshooting ($d/H_P = 0.2$) and other input parameters close to our "standard" ones, were computed for initial masses of 40, 60, and 85 $M_\odot$, and points representing them are added to Figure 3. Notice that the greater the mass loss, the smaller is X_f owing to the reduction in size of the FCZ.

For stars of initially 30-40 $M_\odot$, in which the main-sequence loss of mass is slight, little variation of X_f takes place. At 60 $M_\odot$, however, moderate convective core overshooting leads to such large mass loss that X_f drops by about a factor of 2, both in our evolutionary sequence and in the equivalent sequence of Schaller et al. (1992) (the two lowest open circles in Fig. 3). For initial stellar masses as high as 85-90M_0, main-sequence mass loss is so heavy, with or without convective core overshooting, that all of the models end up having very low X_f. Strictly speaking, our results are not new, since the suppressive effect of mass loss on convective instability in the envelope has long been known (Tanaka 1966a; Chiosi & Nasi 1974). But our detailed numerical predictions are new. Further properties of our evolutionary tracks are presented in Table 1, where "Final M/M_0" refers to the star's mass when the FCZ has just attained its full size.

In order to check our expectation that the later evolutionary phases based on the Schwarzschild criterion for convection closely resemble those based on the Ledoux criterion, we have continued the "standard" evolutionary track for 60 M_0 into and beyond the onset stages of dynamical instability, by using the mass loss prescriptions that we adopted previously (Stothers & Chin 1996, 1999). Soon after reaching the red-supergiant region, the star undergoes its first phase of dynamical instability and does not emerge from that state until most of its hydrogen envelope has been lost. Thereafter, X_{surf} remains equal to $X_f = 0.28$, because mass stripping has reached far into the FCZ. Only 2 M_0 of the star's hydrogen envelope is left.

Although the Ledoux criterion yields a comparably small envelope mass, 3 M_0, it produces a much more hydrogen-depleted surface, $X_{surf} \approx 0.17$. In both cases, however, the luminosity remains close to the TAMS luminosity until the surface hydrogen finally vanishes.

4. COMPARISON WITH OBSERVATIONS

4.1. *Luminous Blue Variables*

Observed LBVs and LBV candidates whose surface hydrogen abundances have been measured are listed in Table 2. These stars are critically compared with our most realistic stellar models (those with $w \geq 1$) in a plot of X_{surf} versus log (L/L_0) in Figure 4, where we have included the three models from Schaller et al. (1992) that we discussed earlier. The theoretical values of X_{surf} and of log (L/L_0) refer to X_f and to the TAMS luminosity, respectively, in the case of the Schwarzschild criterion. For the Ledoux criterion, the data come from our previous evolutionary sequences for 30, 45, and 90 M_0 (Stothers & Chin 1996).

In our proposed scenario, two luminosity regimes for LBVs have to be distinguished. LBVs with log $(L/L_0) < 5.9$ are interpreted as post-red-supergiant objects belonging to the second phase of dynamical instability (Stothers & Chin 1996). In this case, X_{surf} obviously cannot exceed the surface hydrogen abundance that exists at the end of the first dynamical phase. LBVs with log $(L/L_0) > 5.9$, on the other hand, although they possibly belong to the second phase, more probably populate the transient blue loops that characterize the first phase, owing to the powerful stellar wind that may quickly remove the remaining hydrogen envelope before a second phase can begin (Stothers & Chin 1999). Although our previous calculations were based on the Ledoux criterion, our analogous calculations for the Schwarzschild criterion (§3) do not change this picture. Therefore, X_{surf} for LBVs with log $(L/L_0) > 5.9$ could conceivably be as high as the value existing at the start of the first phase, which is 0.23-0.49 depending on the particular choices of the physical input parameters, but essentially independent of the criterion for convection.

Considering the possible observational errors and the few stars involved, agreement with the theoretical models seems quite good over the fainter range of LBV luminosities if the Schwarzschild criterion for convection is correct (compare the open symbols). Models based on the Ledoux criterion (the filled symbols) fare quite badly. Further favoring the Schwarzschild criterion in this low range of luminosities is the fact that specially tailored envelope models of the six LBVs having the most accurately measured masses, luminosities, and effective temperatures show marginal, or close to marginal, dynamical instability if $X_{surf} \approx 0.35$, but appreciably greater stability if $X_{surf} \approx 0.17$ (Stothers 1999b). Likewise, models constructed with moderate convective core overshooting (the two lowest open circles in Fig. 4) might also be rejected on the grounds of their very small values of X_{surf}.

In the brighter range of LBV luminosities, observations suggest that these LBVs, too, have $X_{surf} \approx 0.35$. These stars could belong to the second phase of dynamical instability if the Schwarzschild criterion were correct and if w were very small (Table 1); however, the latter condition must be regarded as unacceptable. Most realistically, these stars belong to the first phase of dynamical instability. In that case, both of the criteria for convection predict that X_{surf} ought to populate the range between ~0.2-0.5 uniformly. However, only a value $X_{surf} \approx 0.35$ is observed. On the other hand, so few objects (four) have been sampled that the true variance of their X_{surf} values is unknown. If these very luminous LBVs do represent the immediate precursors of hydrogen-poor WN stars, X_{surf} may in fact range down to ~0.2, the value observed for the least hydrogen-poor WN stars of the same luminosity (§ 4.2). Because these brighter LBVs probably belong to the first phase of dynamical instability, no conclusion can be drawn from them concerning the correct criterion for convection.

4.2 *Hydrogen-Poor Wolf-Rayet Stars*

A large number of accurate hydrogen abundance determinations for WN stars in the Galaxy and in the Large Magellanic Cloud have recently been published (Hamann, Koesterke, & Wessolowski 1995; Crowther, Hillier, & Smith 1995a,b; Crowther & Smith 1997). These abundances, which are supported by independent determinations made by Nugis & Niedzielski (1995), are plotted against log $(L/L_{\odot})$ in Figure 5. For Galactic stars, we have used the revised luminosities published by Hamann & Koesterke (1998a), but have ignored stars with log $(L/L_0) < 5.2$, as luminosities this low lie outside the scope of our present study. We also ignore the more highly evolved, hydrogen-free WN stars. In this paper, "WN stars" will refer only to the hydrogen-poor objects.

In our scenario, most of the WN stars are to be regarded as post-red-supergiant stars, differing from LBVs only in being hotter, as well as more hydrogen-poor, and therefore avoiding dynamical instability (Stothers & Chin 1996). Another class of WN stars showing hydrogen deficiencies is expected to occur at very high luminosities. These are extremely massive main-sequence stars that have expelled so much matter that helium-enriched layers become exposed at the surface before the main-sequence phase ends (Tanaka 1966b; Conti 1976). The predicted threshold luminosity for such stars falls around log $(L/L_0) \approx 6.2$ (Schaller et al. 1992; Meynet et al. 1994; Pasquali et al. 1997a; Stothers & Chin 1999) and so is too bright to be relevant here.

With reasonable allowance for observational errors and for model uncertainties, Figure 5 shows that only the stellar models based on the Schwarzschild criterion for convection (open symbols) form an acceptable upper envelope around the observed distribution of WN stars. There are four apparent outliers found among the observed stars, but within the possible errors of at least ±0.05 in X_{surf} and ±0.1 in $\log(L/L_0)$ these stars may be simply misplaced in the figure. This finding suggests that, by and large, WN stars are more highly evolved than LBVs and belong to a phase where the steep hydrogen gradient at the base of the FCZ becomes exposed at the stellar surface.

To proceed more quantitatively, we note that the hydrogen gradient in the remnant stellar envelope, $dX/dM(r)$, and the rate of mass loss at the surface $-dM/dt$, imply a WN lifetime,

$$\tau = \left[\frac{1}{\Delta X_{surf}} \frac{dX}{dM(r)} \qquad \frac{dM}{dt} \right]^{-1}$$

By the time the WN phase is reached, the hydrogen-burning shell must have burned so far outward in mass that the hydrogen profile at the base of the FCZ would have acquired a nearly constant form. As an average value for stars of initially 30-60 M_0, we set dX/dM(r) = 0.5 M_0, which represents the hydrogen gradient in all of our models to within a factor of 2. The observationally derived standard mass-loss rates for WN stars (Nugis & Niedzielski 1995; Leitherer, Chapman, & Koribalski 1997; Hamann & Koesterke 1998a) are based on the assumption that the atmosphere has a smooth structure. Since clumpiness probably reduces these rates by a factor of ~2 (Nugis, Crowther, & Willis 1998; Hamann & Koesterke 1998b), we shall adopt $-dM/dt = 2 \times 10^{-5}\ M_0\,yr^{-1}$. Then using $\Delta X_{surf} = 0.35$ (Fig. 5), we obtain $\tau \approx 3 \times 10^4$ yr. This predicted lifetime is in satisfactory agreement with the measured kinematical expansion ages of the compact nebulae observed around several luminous WN stars, (2 - 6) x 10^4 yr (Marston 1995; Esteban & Rosado 1995; Chu, Weis, & Garnett 1999).

5, CONCLUSION

Parallel evolutionary sequences, calculated for the Schwarzschild and Ledoux criteria for convection, have been the subject of many previous investigations (Stothers & Chin 1968, 1975, 1976, 1979, 1992, 1994; Chiosi & Summa 1970; Simpson 1971; Robertson 1972; Sreenivasan & Ziebarth 1974; Varshavsky & Tutukov 1975; Chiosi & Nasi 1978; Sreenivasan & Wilson 1978; Langer, El Eid, & Fricke 1985; Staritsin & Tutukov 1989; Weiss 1989; Brocato & Castellani 1993; Mowlavi & Forestini 1994; El Eid 1995; Langer & Maeder 1995; Grossman & Taam 1996; Ritossa 1996). Generally speaking, comparisons of these parallel sequences have been made by plotting them in the H-R diagram. For a sufficiently high initial stellar mass, the large FCZ that is produced by using the Schwarzschild criterion can prevent a star from becoming a red supergiant during the early stages of core helium burning, and can thus lead to a very different predicted distribution of blue and red supergiants on the H-R diagram compared to what is obtained with the Ledoux criterion.

As we have seen, the allowable initial stellar masses for this to happen must be not too low and not too high, or the star will begin core helium burning as a red supergiant regardless of the criterion for convection. In the case of Galactic and Large Magellanic Cloud supergiants, the distinguishing range of initial stellar masses is roughly 13-25 M_0 for standard evolutionary assumptions. Observed supergiant distributions in the H-R diagram, as well as the presence of fossil dust shells around many luminous yellow supergiants, suggest that stellar evolution during core helium burning for these initial masses proceeds from red to blue. This finding apparently supports the Ledoux criterion (Stothers & Chin 1994).

How can this result be reconciled with our present conclusion that favors the Schwarzschild criterion in the case of stars with initial masses of 30-60 M_0? Assuming the approximate validity of the Schwarzschild criterion, it is quite possible that our earlier models for 13-25 M_0 based on this criterion have FCZs that are somewhat too large. The present Figures 4 and 5 suggest that even at 30 M_0 the derived FCZ is too large, yielding as it does values of X_{surf} that rather exceed the observational values. It is easily conceivable that a proper theory of semiconvection might lead to a solution in which the Schwarzschild criterion is somewhat modified by the μ gradient, although not as much as the Ledoux criterion is (e.g., Canuto 1999). Theoretical treatments of semiconvection as a diffusion process already are able to approximate this situation to some extent (Langer, El Eid, & Fricke 1985; Deng, Bressan, & Chiosi 1996; Grossman & Taam 1996).

Alternatively, the assumption of a moderate amount of convective core overshooting can also lead to a reduction in the size of the FCZ, even when the standard Schwarzschild criterion is adopted. However, the reduction turns out to be not sufficient in stars of ~20 M_0 to change their blue to red evolution during core helium burning (Schaller et al. 1992; Schaerer et al. 1993; Howard 1993; Mowlavi & Forestini 1994; Li & Gong 1994).

The inclusion of axial rotation is also unlikely to change this picture. If initial rates of rotation equal to the average value observed for slightly evolved O and B stars are applied to ZAMS stellar models, the mixing due to meridional circulation and to flows resulting from rotational instabilities apparently produces only a minor effect on the hydrogen distribution outside the core by the time the star leaves the main sequence. This conclusion has been reached on the basis of rotating stellar models calculated with the Schwarzschild criterion (Howard 1993) as well as with what amounts to the Ledoux criterion (Endal & Sofia 1976). For an assumed much stronger mixing due to rotational shear flow turbulence, however, the helium core mass can grow arbitrarily larger, mimicking the effects of semiconvection and even of convective core overshooting (e.g., Maeder 1987; Langer 1992; Denissenkov 1994; Weiss 1994; Eryurt et al. 1994; Talon et al. 1997; Heger et al. 1997; Pasquali et al. 1997a). Although all rotationally induced flows are strongly opposed by a gradient of mean molecular weight, extremely fast rotation could conceivably mix material more extensively. If the star were completely mixed, it would not become a LBV, because LBVs are observed to lie well off the main sequence. Even if it were less than fully mixed, it would be one of only a few such stars, because extremely fast rotators comprise a minority among observed O stars. Since, in any case, all known LBVs show nearly the same surface hydrogen abundance, their previous mixing conditions must have been very similar. This alone suggests that rotational mixing has not altered the hydrogen profile in these stars by a large amount, as their original rotational velocities on the main sequence are likely to have been quite different. However, the fact remains that rotation is still very much an uncertain factor in stellar evolution.

All in all, the present evidence seems to suggest that a modified Schwarzschild criterion is needed. Nevertheless, our previous general conclusions concerning the evolution of stars of 30-90 M_0 based on the Ledoux criterion (Stothers & Chin 1996) can be taken over almost bodily, because the evolutionary tracks based on these two standard convective criteria, for such high initial masses, so closely resemble each other. Only one detail, which is specifically related to the exposed FCZ, in the case of the Schwarzchild criterion, needs to be altered in our proposed scheme of evolution. We now predict:

Blue LBV$\rightarrow$ H-poor WN$\rightarrow$ H-free WN.

The hydrogen-poor WN phase *follows* the blue LBV phase. For the brightest (and hottest) LBVs, however, these two phases are expected to partially overlap.

Although the speculation that mass loss from yellow and red supergiants produces blue LBVs (Lamers, de Groot, & Cassatella 1983) and WN stars (Bisnovatyi-Kogan & Nadyoshin 1972; Chiosi, Nasi, & Sreenivasan 1978; Stothers & Chin 1979; Maeder 1981a,b) is of long standing, the role of dynamical instability in the evolutionary history of these stars is of course new. It may now be sufficient to evolve models of helium cores surrounded by small homogeneous hydrogen envelopes (with $X \approx 0.35$) in order to obtain complete stellar models that would be representative enough of real LBVs for use in improved theoretical tests for dynamical instability.

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TABLE 1

Summary of the Stellar Evolutionary Sequences

Initial					ZAMS		TAMS		Final	
$M/M_\odot$	Z_e	w	α_P	d/H_P	$\log(L/L_\odot)$	$\log T_e$	$\log(L/L_\odot)$	$\log T_e$	$M/M_\odot$	X_f
30 ...	0.004	1	1.4	0	5.04	4.62	5.33	4.49	28	0.44
	0.015	1	1.4	0	5.05	4.60	5.32	4.45	28	0.45
	0.030	0	1.4	0	5.06	4.58	5.38	4.42	30	0.44
	0.030	1	1.4	0	5.06	4.58	5.33	4.41	28	0.46
	0.030	1	2.8	0	5.06	4.58	5.33	4.42	28	0.43
	0.030	1	1.4	0.2	5.06	4.58	5.36	4.36	28	0.44
60 ...	0.004	1	1.4	0	5.68	4.70	5.88	4.46	51	0.35
	0.015	1	1.4	0	5.69	4.67	5.87	4.33	49	0.32
	0.030	0	1.4	0	5.70	4.66	5.96	4.32	60	0.35
	0.030	1	1.4	0	5.70	4.66	5.87	4.13	47	0.28
	0.030	1	2.8	0	5.70	4.66	5.87	4.23	48	0.28
	0.030	1	1.4	0.2	5.70	4.66	5.87	3.95	38	0.17
90 ...	0.004	1	1.4	0	6.00	4.73	6.14	4.40	65	0.19
	0.015	1	1.4	0	6.01	4.70	6.11	4.23	55	0.19
	0.030	0	1.4	0	6.02	4.67	6.24	4.11	90	0.31
	0.030	1	1.4	0	6.02	4.67	6.09	4.09	49	0.11
	0.030	3	1.4	0	6.02	4.67	5.91	4.40	34	0.09
	0.030	1	2.8	0	6.02	4.68	6.11	4.16	55	0.15
	0.030	1	1.4	0.2	6.02	4.67	6.11	4.27	48	0.12

TABLE 2

Observed LBVs and LBV Candidates

Variable	Galaxy	$\log(L/L_\odot)$	X_{surf}	References
AG Car	MW	6.24	0.35	1, 2, 3, 4
He 3-519	MW	6.22	0.33	1, 4
HDE 269445	LMC	6.20	0.33	5
R127	LMC	6.10	0.33	1, 6
BE294	LMC	5.89	0.32	7, 8
P Cyg	MW	5.86	0.38	1, 2, 9, 10
R84	LMC	5.81	0.41	8, 11, 12
S119	LMC	5.90	0.29	7, 8
R71	LMC	5.42	0.36	1, 13

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FIGURE CAPTIONS

Fig. 1. -- Theoretical H-R diagram showing evolutionary tracks for a star of initially 60 M_o, running from the ZAMS to a stage shortly after the full development of the FCZ. "Standard" input parameters are $Z_e = 0.03$, $w = 1$, $\alpha_P = 1.4$, and $d/H_P = 0$. All models shown are based on the Schwarzschild criterion.

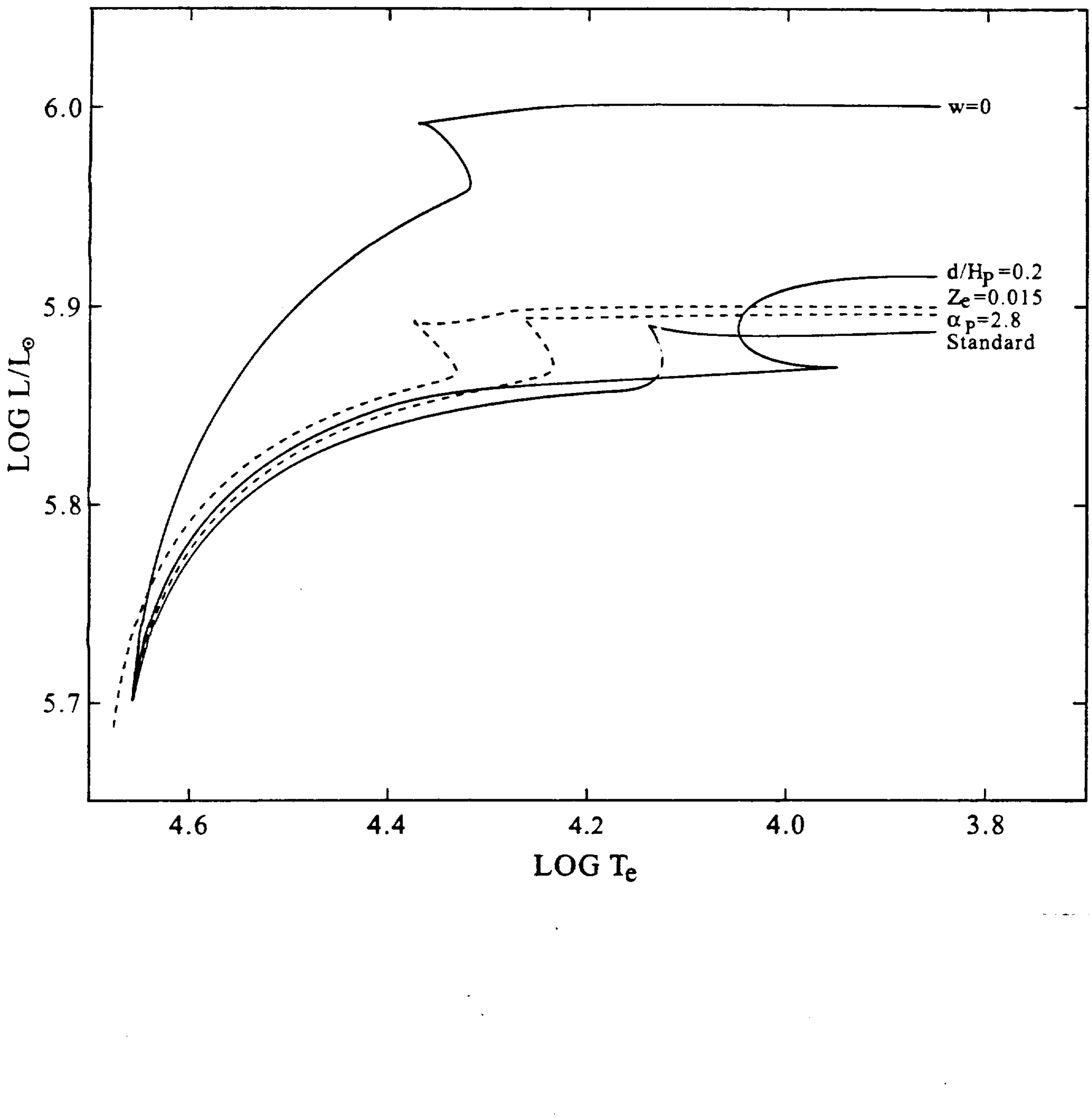
Fig. 2. -- Hydrogen profiles in models of stars of initially 60 M_o constructed with "standard" input parameters and with the Ledoux criterion (top) and the Schwarzschild criterion (bottom). The stage of evolution shown occurs shortly after central hydrogen exhaustion, when convective instability in the μ-gradient region reaches its greatest extent.

Fig. 3. -- Hydrogen abundance in the FCZ versus the mass of the star when the FCZ has attained its full size. The initial stellar mass: 30 M_o, *triangles*; 40-45 M_o, *diamond*; 60 M_o, *circles*; and 85-90 M_o, *squares*. Each point refers to a different evolutionary sequence based on different physical input parameters.

Fig. 4. -- Surface hydrogen abundance versus luminosity for classical LBVs. Observed stars, *asterisks*; stellar models based on the Schwarzschild criterion, *open symbols*; and stellar models based on the Ledoux criterion, *filled symbols*. The

shapes of the symbols refer to the initial stellar mass, as per Fig. 3.

Fig. 5. -- Surface hydrogen abundance versus luminosity for hydrogen-poor WN stars. The symbols are as in Fig. 4.



6.0
5.9
5.8
5.7
LOG L/L$_\odot$
4.6
4.4
4.2
4.0
3.8
LOG T$_e$
w=0
d/H$_p$=0.2
Z$_e$=0.015
α_p=2.8
Standard

Fig. 1

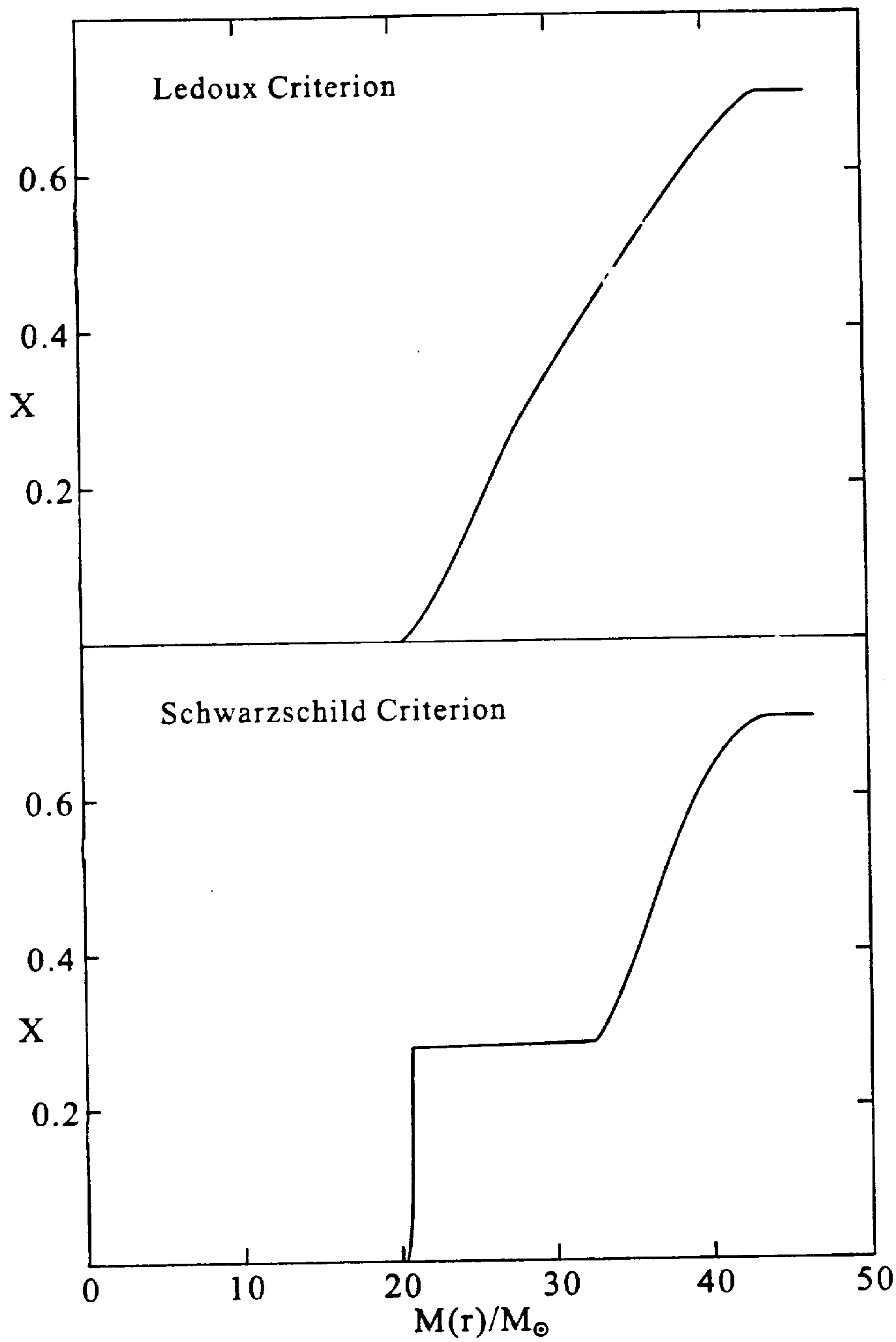

Ledoux Criterion
0.6
0.4
X
0.2
Schwarzschild Criterion
0.6
0.4
X
0.2
0
10
20
30
40
50
M(r)/M⊙

Fig. 2

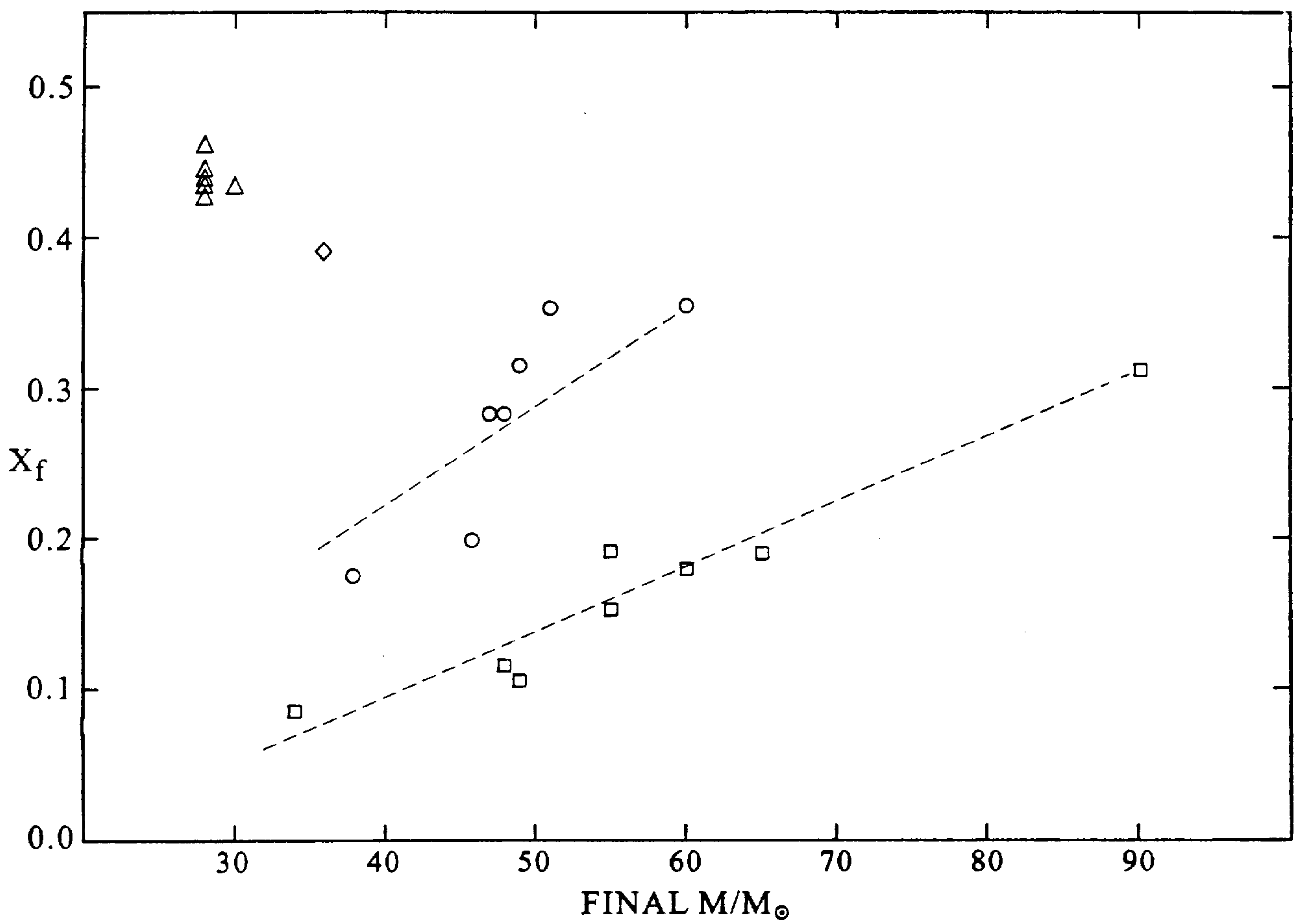


Fig. 3

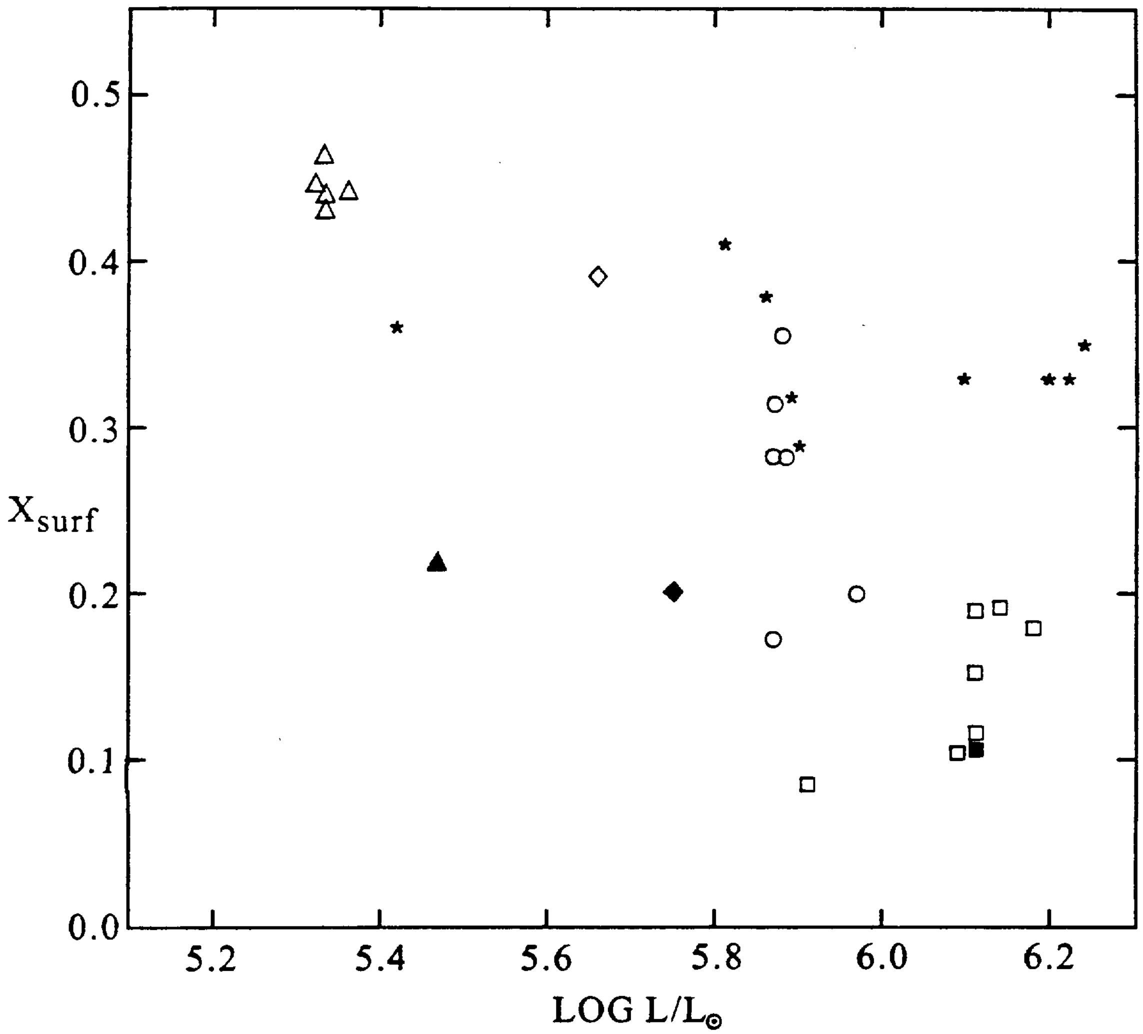


Fig. 4

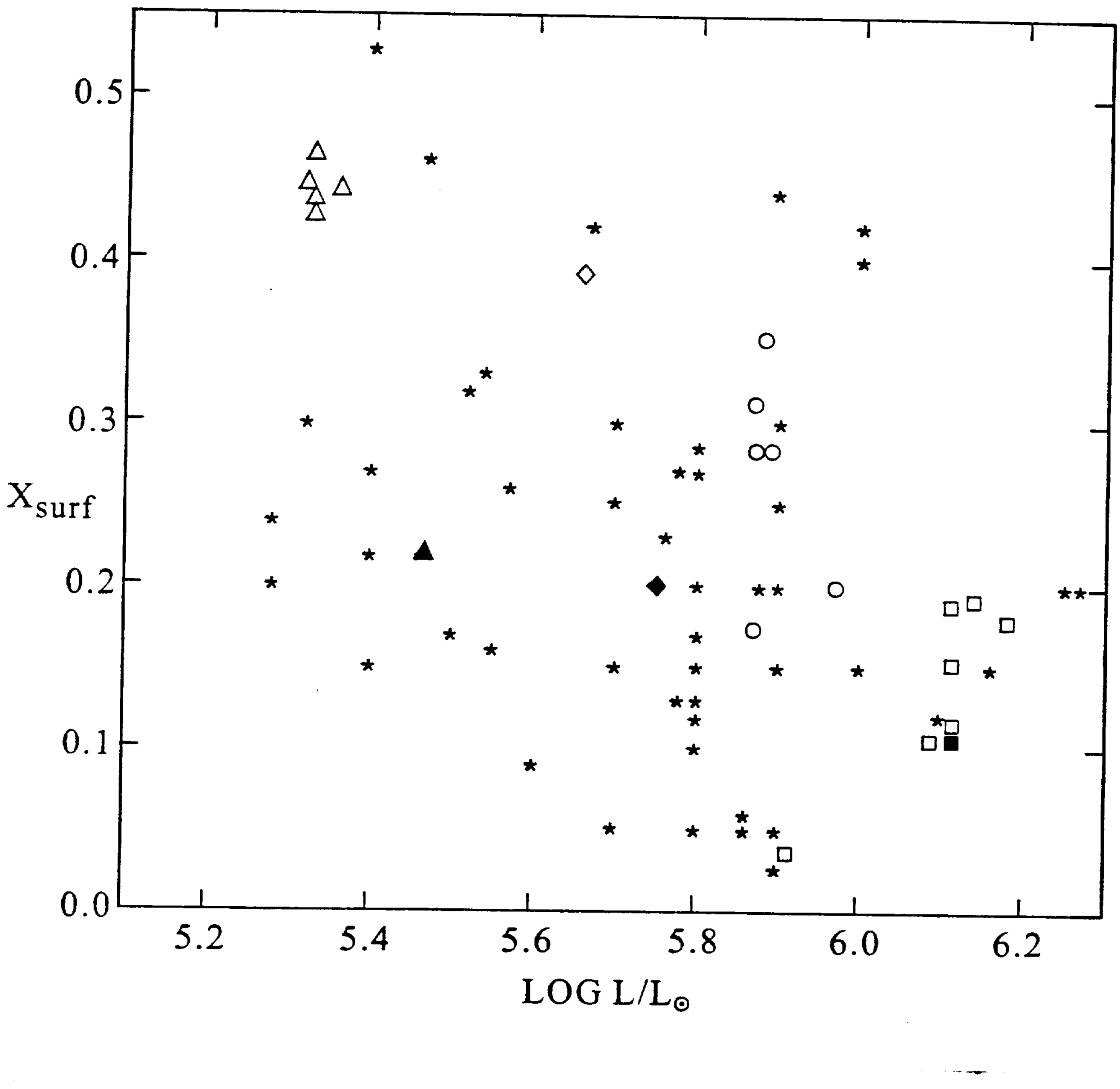


Fig. 5