MAGNETOMETER-ONLY ATTITUDE AND RATE ESTIMATES FOR SPINNING SPACECRAFT *

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ABSTRACT

A deterministic algorithm and a Kalman filter for gyroless spacecraft are used independently to estimate the three-axis attitude and rates of rapidly spinning spacecraft using only magnetometer data. In-flight data from the Wide-Field Infrared Explorer (WIRE) during its tumble, and the Fast Auroral Snapshot Explorer (FAST) during its nominal mission mode are used to show that the algorithms can successfully estimate the above in spite of the high rates. Results using simulated data are used to illustrate the importance of accurate and frequent data.

INTRODUCTION AND OBJECTIVE

We have demonstrated earlier that two algorithms - Deterministic Attitude Determination from Magnetometer-Only Data (DARDMOD)[1-3] and the Real Time Sequential Filter (RTSF)[4,5] - can successfully[6-11] estimate the three-axis attitude and rates of spacecraft using only three-axis magnetometer (TAM) data in spite of no a priori knowledge of the spacecraft state. Our past studies have included spacecraft such as the Solar, Anomalous, and Magnetospheric Particle Explorer (SAMPEX), the Earth Radiation Budget Satellite (ERBS), the Rossi X-ray Timing Explorer (RXTE). In particular, we have shown that: SAMPEX requirements can be met[6,7,9] in both nominal and safehold modes if the Sun sensor fails, and that the ERBS attitude and rates could be successfully estimated[10] during its 1987 control anomaly[12] when it tumbled at about 2 degrees/second (deg/sec) resulting in the TAM being the only functional attitude sensor. The accuracies obtained in such work are 0.5 - 2.0 deg in attitude, and 0.002 - 0.01 deg/sec in rates. Thus we had shown that a *single TAM can be a surprisingly effective low-cost attitude-and-rate sensor.*

A feature of our past studies has been the relatively low spacecraft spin rate: 1 rotation per orbit (0.06 deg/sec)

during SAMPEX and ERBS nominal modes, and about 2 deg/sec during the ERBS tumble.

The present study aims to evaluate the TAM-only performance of the above algorithms in an important alternate scenario: *when the rates are high, of the order of tens of deg/sec.* A successful outcome would allow two important capabilities in spacecraft guidance for three-axis stabilized *and* spin stabilized spacecraft:

- an early warning/recovery system for spacecraft tumbles
- a magnetometer-only attitude determination system for cheaper spacecraft

The present study uses high-rate in-flight data from the following two scenarios:

- The Wide-Field Infrared Explorer (WIRE)[13] post-release tumble[14], when the rates eventually reached 360 deg/sec.
- Nominal mode data from the Fast Auroral Snapshot Explorer (FAST)[15] which spins at 12 rpm (72 deg/sec).

The rest of this paper is organized into sections dealing with the following topics: brief theoretical descriptions of the DARDMOD and the RTSF, applications to the WIRE tumble, applications to the FAST, and conclusions.

THEORY

Deterministic Attitude and Rate Determination Using Magnetometer-Only Data (DARDMOD)

The DARDMOD algorithm solves for the attitude and rates using a batch of TAM measurements and control law data such as wheel angular momentum and magnetic torquer dipole moments. The resulting mathematics can be complex[1,3,11], involving the

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solution of up to 8th-order polynomials. In view of space limitations, we present here only new DARDMOD results showing that the attitude and the rate-component about $\vec{B}$ can not be determined deterministically when the rates are high.

Let $\vec{\omega}$ denote the spacecraft angular velocity, superscripts "B" and "I" denote resolution of vector components in the Geocentric Inertial frame (GCI) and spacecraft body frames respectively, A denote the GCI-to-body attitude matrix, and $\dot{\vec{B}}$ denote the time derivative of $\vec{B}$. We then have the following kinematic relationship between the rates of change of $\vec{B}$ in the two frames:

$$\dot{\vec{B}}^B + [\,\vec{\omega}^B \times \vec{B}^B\,] = A\,\dot{\vec{B}}^I \quad . \tag{1}$$

For large $|\vec{\omega}|$, one can neglect the rate of change of $\vec{B}$ in the body frame which is much larger than that in GCI. Thus, (1) reduces to:

$$\dot{\vec{B}}^B + [\,\vec{\omega}^B \times \vec{B}^B\,] \approx \vec{0} \tag{2}$$

does not contain the attitude matrix. Thus DARDMOD can be used only for computing $\vec{\omega}$ when the rates are high.

Consider now the triad

$$\vec{D}_1^B \equiv \vec{B}^B \,, \quad \vec{D}_2^B \equiv \vec{D}_1^B \times \dot{\vec{B}}^B \,, \quad \hat{D}_3^B \equiv \vec{D}_1^B \times \vec{D}_2^B \tag{3}$$

One can easily verify that the rate component, $\omega_3 \equiv \vec{\omega}^B \cdot \hat{D}_3^B$, vanishes at high rates. In fact, multiplying Eq. (2) by $\vec{D}_2^B$, one finds

$$(\,\vec{D}_2^B \cdot [\,\vec{\omega}^B \times \vec{B}^B\,]\,) = \omega_3\,|\,\hat{D}_3^B\,|\,|\,\vec{B}^B\,| \approx 0. \tag{4}$$

Therefore, we have another important result:

$$\omega_3 \approx 0 \,. \tag{5}$$

For spinning spacecraft one can avoid the problems caused by the high rate of change of $\vec{B}$ in the body frame compared with its change in GCI by rotating the TAM measurements back with a spin $\vec{\omega}^0$. The spin rate can be directly determined from TAM measurements by analyzing the change in the phase angle of the projection of the measured field on the plane perpendicular to the spin axis.

Assuming that the spacecraft spins in the positive direction around the body axis z, the rotated magnetic field can be represented as:

$$\vec{B}'(t) = O^T(t)\,\vec{B}^B(t) \tag{6}$$

where

$$O(t) = A_3(\vec{\omega}^0 t) \,, \tag{7}$$

with the matrix $A_3(\Phi)$ given by Eq. (12-6a) in Ref. 15. The equations for the first and second derivatives of the geomagnetic field in the rotating reference frame formally look similar to those in the body frame, with the only difference that the angular acceleration vector $\dot{\vec{\omega}}'(t)$ relative to the rotating frame is related to the angular velocity vector sought for, $\vec{\omega}'(t)$, via:

$$\dot{\vec{\omega}}'(t) = O^T(t)\,\dot{\vec{\omega}}^B(t) + [O^T(t)\,\vec{\omega}_0 \times \vec{\omega}'(t)] \,. \tag{8}$$

In (8) the angular acceleration vector $\dot{\vec{\omega}}^B(t)$ relative to GCI is related to the angular velocity vector

$$\vec{\omega}^B(t) = \vec{\omega}_0 + O(t)\,\vec{\omega}'(t) \tag{9}$$

via the conventional dynamic equation of motion.

By neglecting terms quadratic in $\vec{\omega}'(t)$ in the dynamical equations of motion one comes to the quadratic algebraic equation similar to Eq. (3-11) in Ref. 1, which was derived by setting the angular acceleration to zero.

Real Time Sequential Filter (RTSF)

The RTSF is a Kalman filter whose state vector, $\vec{X}$, consists of the four components of the GCI-to-spacecraft body frame attitude quaternion, $\vec{q}$, and the three components of the rate-correction, $\vec{b}$, to $\vec{\omega}$:

$$\vec{X} = \begin{bmatrix} \vec{q}^T & \vec{b}^T \end{bmatrix}^T \,, \tag{10}$$

where the superscript T denotes matrix transpose. Note that $\vec{b}$ are *filter estimates* of errors in its Euler-propagated rates[17], and should not be confused be with true errors in rates which would be the differences between filter rate estimates and the truth model values.

The filter uses sensor data to estimate $\vec{q}$ as well as $\vec{b}$, with $\vec{b}$ being estimated kinematically in the same manner as gyro biases for a gyro-based spacecraft, that is, by attributing differences between the measured and propagated attitudes to errors in $\vec{\omega}$. The $\vec{b}$ estimates are then used to correct $\vec{\omega}$, and these corrected rates

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are used as initial conditions to propagate Euler's equation to the next measurement time. The propagation of $\vec{b}$ is modeled via the following first-order Markov model[18]:

$$\frac{d\vec{b}}{dt} = -\tau^{-1}\vec{b} + \vec{\eta}_b \ , \qquad (11)$$

where $\vec{\eta}_b$ is a white noise term, and τ is a finite time constant. The distinguishing feature of this Kalman filter is that $\vec{b}$ represents rate errors accumulated during propagation between measurements.

APPLICATIONS TO THE WIRE TUMBLE

WIRE Spacecraft and Tumble Description

The WIRE (launched: March 4, 1999) nominally is an inertially-pointing spacecraft with the following attitude hardware: a three-axis gyro package, a star tracker, a digital Sun sensor (DSS) of 0.5 deg accuracy, six coarse Sun sensors (CSS) of approximately 10 deg accuracy, a TAM, a wide-angle Earth sensor,. a four-wheel reaction wheel assembly, and three magnetic torque bars. The nominal orbit of the WIRE is near circular, with an altitude of 540 km and an inclination of 97 deg.

The WIRE instrument's telescope cover was ejected prematurely soon after release from the launch vehicle because of an incorrectly designed electronics box[14]. This led to the rapid and total venting of the frozen hydrogen used to cool the telescope, and thereby to the loss of the scientific mission. In addition, the venting produced a thrust that threw the spacecraft into a tumble. The rates increased continuously, and eventually stabilized at 360 deg/sec after the cryogen was fully vented.

The WIRE tumble began after only one orbit, at which point spacecraft attitude could be computed only from the DSS and TAM measurements input to the single-frame algorithm known as QUEST[19]. Since the gyros were not yet powered on, the onset of the tumble could be recognized only by examining the computed $\vec{\omega}$, i. e., rates computed by numerically differentiating the QUEST attitude history. These single-frame rates are shown in Figure 1 where we see that WIRE was spinning at approximately (75, 65, 30) deg/sec towards the end of the data span used here, the magnitudes of the rates then slightly exceeding 100 deg/sec.

The WIRE in-flight data used in this study is approximately 7.5 hours long (about 4.5 orbits), from the epoch of 19990305.030531 (in YYYYMMDD.

HHMMSS format), which is only a few seconds after spacecraft separation. (Henceforth the times will usually be referred to in seconds with respect to the above epoch.) The attitude data were collected at 1.0-sec intervals.

TAM-Only Results

In Figure 2 the RTSF TAM-only rate results are compared with the true rates for about one half-orbit after release and power-on. The filter was initialized assuming that the spacecraft is at rest in GCI with its axes coinciding with the GCI axes, that is, with large initial errors. Nevertheless, we find that the filter rate estimates have converged to the truth in only 200 sec. The RTSF results deteriorate at the start of the tumble (1396 sec) because the thrust due to the cryogen venting has not been included in the Kalman filter's torque model. However, the filter rate corrections. $\vec{b}$ in Eq. (7), allow the estimates to converge to the truth about 600 sec later. RTSF results from a separate run starting about 1 orbit after release are presented in Figure 3. Again we again see excellent convergence of the rates to the true values in about 500 sec in spite of rate magnitudes of the order of 30 deg/sec and no a priori knowledge.

Figure 4 compares the TAM-only rate results from the DARDMOD theory of Section 3.1, with those of the RTSF and the true rates. Note that the DARDMOD theory in the high-rate limit always gives $\omega_3 = 0$, in excellent agreement with the true values; for this reason the appropriate subplot of Fig. 4 contains only true and Kalman filter values. We see generally good agreement among all three results, although unexplained anomalies arise in the RTSF results during 12000 - 13000 sec. A similar anomalous behavior is observed in the DARDMOD results for the rate ω_1, which computation requires knowledge of torques. On the other hand, DARDMOD predictions for ω_2 and especially ω_3 (which use only the magnetic field and its first time derivatives) remain in reasonable agreement with true values. This indicates that cited anomalies are caused by some unmodeled torques.

APPLICATIONS TO THE FAST MISSION MODE

FAST Spacecraft Description

The FAST (launched: August 1996) is a spin-stabilized spacecraft with a nominal spin rate of 12 rpm about the spacecraft +Z axis. The FAST has an eccentric 350x4200-km orbit around the Earth, with an 83-deg inclination. The nominal attitude for FAST is with the

+Z axis pointing 3.5 deg south of negative orbit normal. Because the ascending node precesses at a rate of 0.5 deg/day, a nominal spin axis precession of 0.5 deg/day is required. The onboard sensors include one horizon-crossing indicator (HCI), one spin sun sensor (SSS) of 0.5 deg resolution, one TAM, and two electromagnetic torquers. The attitude determination requirements are 1.0 deg (3σ) for definitive attitude determination and 0.5 % spin rate.

The data used in this study were a four-orbit span from 09^h30^m to 18^h54^m Universal Time (UT) on March 28, 2000. The spin period during this time (as measured from successive Sun crossings) varied from 5.014 to 5.017 sec. 30 minute (min) spans of data centered around the apogees at 10^h57^m, 13^h08^m, 15^h20^m, and 17^h32^m UT were input to a differential correction algorithm in the FAST ground attitude determination system. The differential correction algorithm was then invoked using the following five sensor models: Sun angle, Earth-In, Earth-Out, Earth Width, and Earth Mid-scan, and yielded the following estimate of the average spin axis attitude in GCI:

right ascension (RA) = 12.7 ± 0.5 deg

declination (DEC) = -10.9 ± 0.5 deg

These results are taken as the attitude truth model, and an average rate of 71.78 deg/sec using the Sun-crossing data was taken as the spin rate truth model.

TAM-Only Results Using Rotated TAM Data

One of the most serious obstacles encountered by DARDMOD is a low frequency of telemetered TAM measurements. The higher a spacecraft rate is, the smaller time interval between sequential measurements should be used to compute time derivatives of the measured magnetic field. For FAST the time interval between two sequential measurements is four seconds, while the spin period is 5 seconds. With so huge change of the phase angle from one TAM measurement to another, it becomes practically impossible to compute the derivatives of the measured magnetic field with a required accuracy.

To go around this problem, the measured magnetic field was projected on the axes of the intermediate reference frame rotating about the body axis z with a constant rate. To evaluate the FAST spinning rate, we analyzed the change in the phase angle of the projection of the measured field on the body plane xy. A certain complication comes from the fact that one cannot distinguish whether the changes of the phase angle from one TAM measurement to another was over $\phi = 288$ degrees or $\phi = -72$ degrees, corresponds to spin rates 72 deg/sec and -18 deg/sec. (In particular, the Kalman

filter accidentally picked up the solution with the negative rate.) To determine the spin rate, we thus had to explicitly take into account that the FAST rotates in the positive direction.

DARDMOD computations were performed in two steps. First, we determined the spin rate using the first two sequential measurements, which was found to be about 72.03 deg/s. TAM measurements were then rotated back with this rate using Eqs. (6) and (7). The rotated magnetic field was then used as an input for both DARDMOD and Kalman filter. After obtaining the DARDMOD solitions we found that the average spin rate is smaller than the reference spin rate by 0.33 deg/s. To check the sensitivity of DARDMOD to the choice of the reference rate, we rotated TAM measurements once again using the new reference rate of 71.7 deg/s. It turned out that the second step indeed improved an agreement between the 'TAM-only' spin rates and those extracted from DSS measurements. For this reason only results from this final step will be analyzed here

An essential deficiency of DARDMOD is existence of multiple (up to 8) solutions which nullify residuals of both TAM measurements and their rates of change. As mentioned in Section 3.1, the attitude/rate solutions for spinning spacecraft can be rather accurately described by roots of a quadratic equation, which implies that there are two solutions. As initially suggested in Ref. 3, the correct root can be selected by analyzing global changes in the total angular momentum $\bar{L}^{\,1}$ in GCI. Since all torques are assumed to be negligibly small, the vector $\bar{L}^{\,1}$ must remain practically unchanged. In fact, comparison of Figs. 5a and 5b, which depict the projection L_z^1 of $\bar{L}^{\,1}$ onto the GCI axis z *for the first and second solution, shows that the physical solution is described by the second roots.*

Sharp spikes in both solutions happen when the discriminant of the quadratic equation becomes dangerously small. It can be shown that at the zero rate ($\bar{\omega}' \approx \bar{0}$) and zero acceleration($\dot{\bar{\omega}}' \approx \bar{0}$) limit the discriminant vanishes when the magnetic field vector in GCI, $\bar{B}^{\,1}$, becomes coplanar with its first and second time derivatives, $\dot{\bar{B}}^{\,1}$ and $\ddot{\bar{B}}^{\,1}$. Fig. Xc depicts cosine of the angle between the vectors $\ddot{\bar{B}}^{\,1}$ and the cross product $\bar{B}^{\,1} \times \dot{\bar{B}}^{\,1}$. (To easier trace the correlation between the spikes and zero values of the coplanarity cosine, the figure depicts only points from coplanarity regions.) An analysis of Figs. 5a, 5b, and 5c, shows that nearly all spikes in Fig. 5a and 5b (except the one at the very

beginning of the run) happen within the coplanarity regions. A more accurate match could be done by directly evaluating the discriminant for each point and flagging the point if the discriminant is too small.

Fig. 6 depicts the right ascension, declination, and spinning rate computed using the second root. Only points with angular momentum residuals smaller than standard deviations for each of the GCI axes are kept in the figure, which resulted in some gaps in the data. Disregarding spikes within the coplanarity regions, both right ascension and declination vary only within a few degrees, which is near the edge of the DARDMOD potential.

As seen from Fig. 6c, the DARDMOD solution for the z-projection of the angular velocity vector $\vec{\omega}^B$ is in excellent agreement with the spin rates determined from DSS measurements.

TAM-Only Results Using Simulated Data

The solution to resolving the ambiguity in rate determination is simple: downlink TAM measurements as frequently as possible, say every 0.5 sec. To illustrate this, simulated FAST data were generated with the following characteristics:
- TAM data every 0.5 sec
- RA = 12.7 deg, DEC = -10.9 deg
- $\vec{\omega} = (0, 0, 72)$ deg/sec

The simulated data were input to the filter and the results are shown in Fig. 8, where we see that the filter easily converges to within 0.006 deg of the true spin-axis attitude and to within 0.006 deg/sec of the true spin rate. This clearly illustrates the importance of accurate data when working with rapidly spinning spacecraft. (Note that, on most spacecraft, it is possible to telemeter data every 0.125 sec. Also, the times of the TAM measurements can usually be resolved to within 0.01 sec.)

CONCLUSIONS

We find see that both RTSF and DARDMOD yield excellent rate estimates during the WIRE tumble even when the rate magnitudes were nearly 60 deg/sec. Notable features are: (1) the filter's rapid convergence to the truth (within 500 sec), and (2) success of the recently developed DARDMOD high-rate theory.

Note that the Sun sensors were active during the WIRE tumble, and, in conjunction with the TAM, provided the true attitude and rates used in this study. However, it is

not inconceivable that a future spacecraft may experience a similar contingency even before the Sun sensors were powered on, or that the Sun sensor configuration may allow reliable single-frame attitude computations only very infrequently. A TAM-only attitude determination and control system - with the RTSF as a real-time back-up/early warning algorithm complemented by a near-real-time RTSF+DARDMOD magnetometer-only utility - would be very useful in such scenarios

The results using FAST data, together with our past work on three-axis stabilized spacecraft, show that using only magnetometers can vastly simplify attitude determination and control systems for a variety of spacecraft. Estimation accuracies of 0.5 deg in attitude and 0.01 deg/sec in rates can be routinely attained. Of course, it is important to have frequent and accurate TAM data, as we have shown using simulated data.

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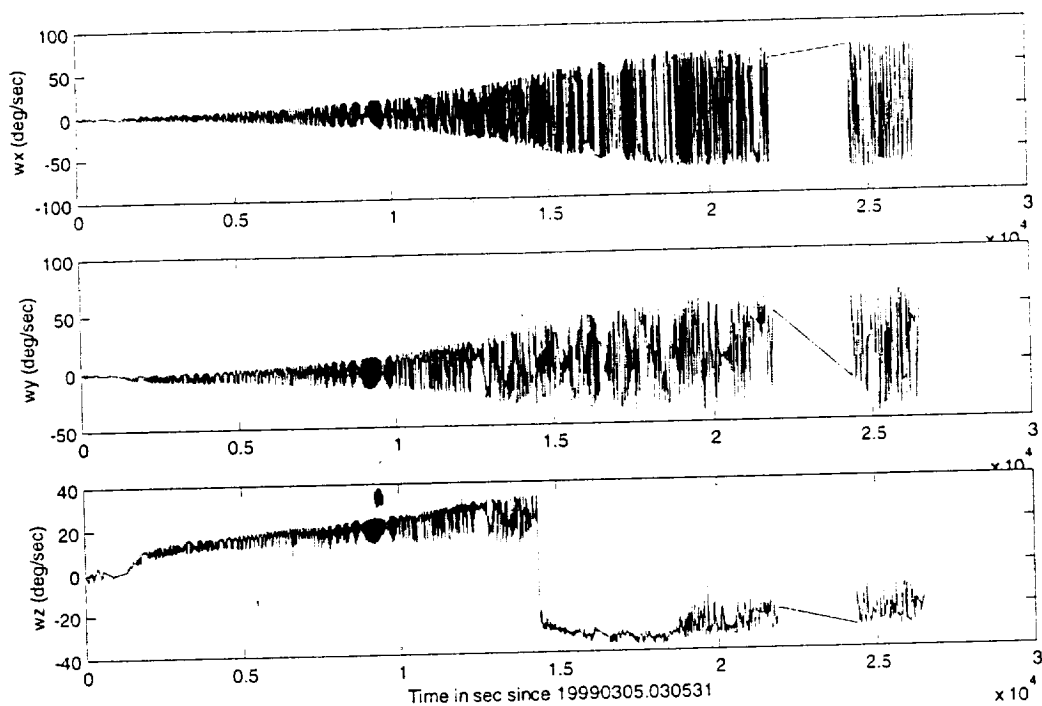
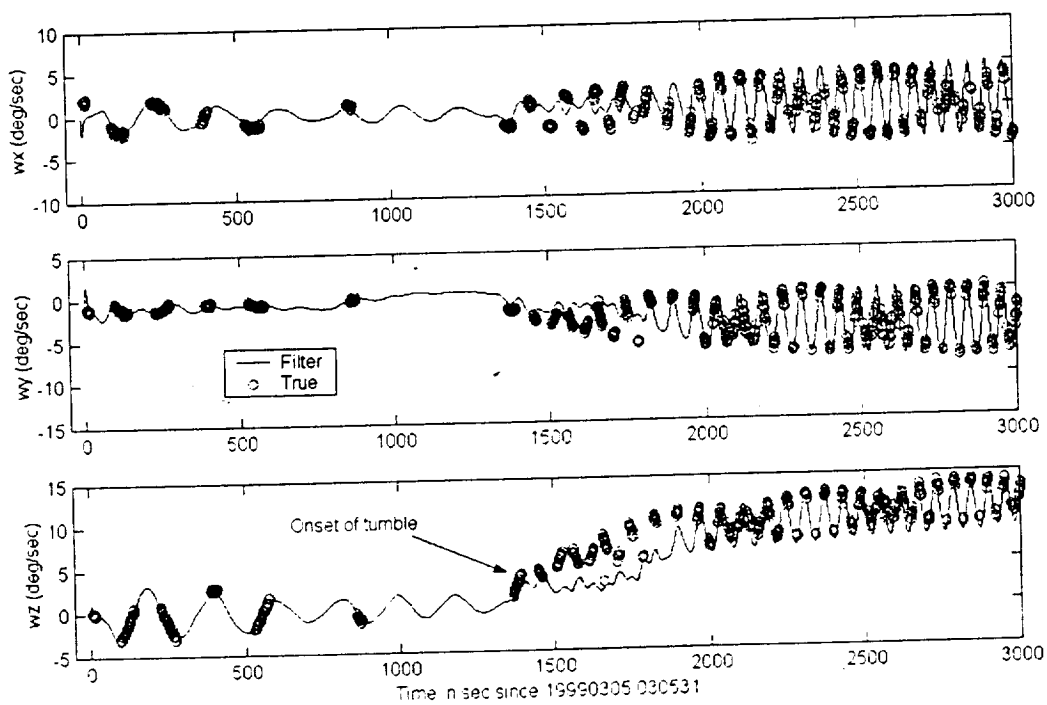**Fig. 1   True WIRE Rates over the Data Span Used in Present Study**

Fig. 2   Filter Rate Results for WIRE From Release Through Tumble Onset

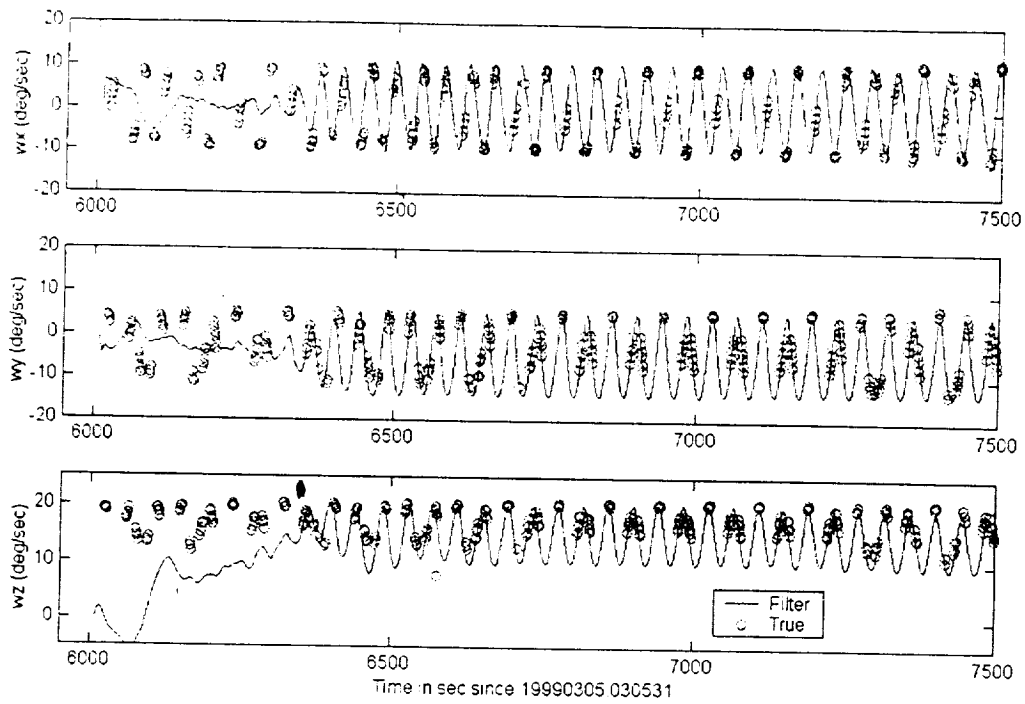American Institute of Aeronautics and Astronautics

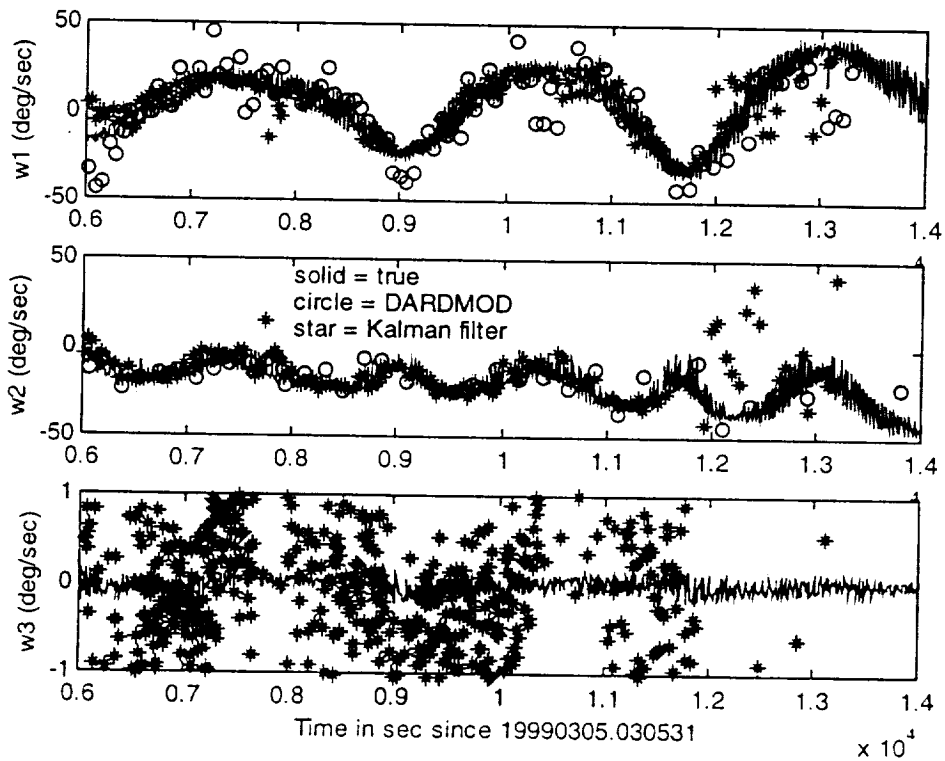Fig. 3   Filter Rate Results for WIRE at Intermediate Tumble Rates

Fig. 4   Comparison of DARDMOD (circle), RTSF (star), and True Rates (solid) for WIRE, in the instantaneous $\bar{B}$ frames used in DARDMOD
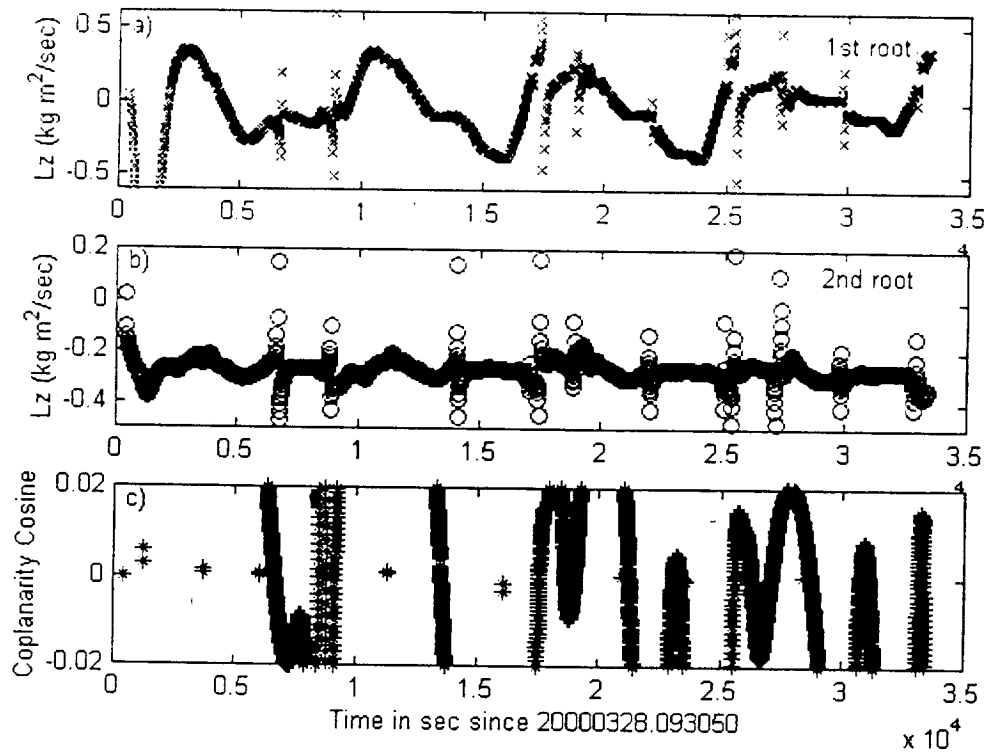
Fig. 5 Selection of the Physical DARDMOD Root for WIRE Using the Conservation of Angular Momentum
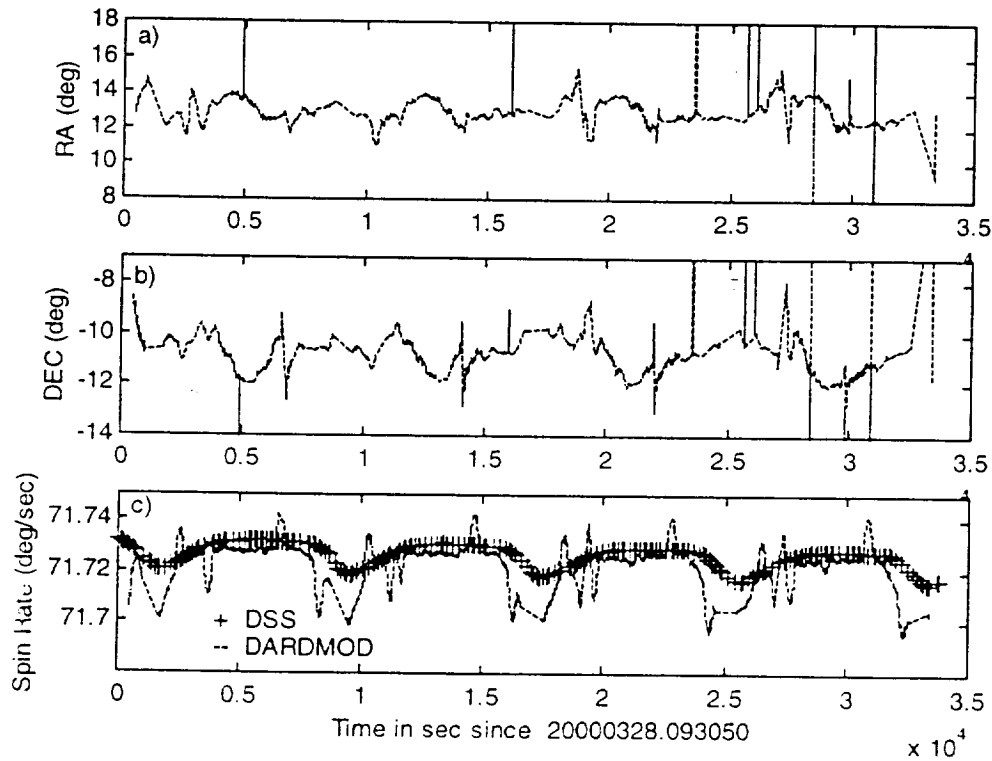

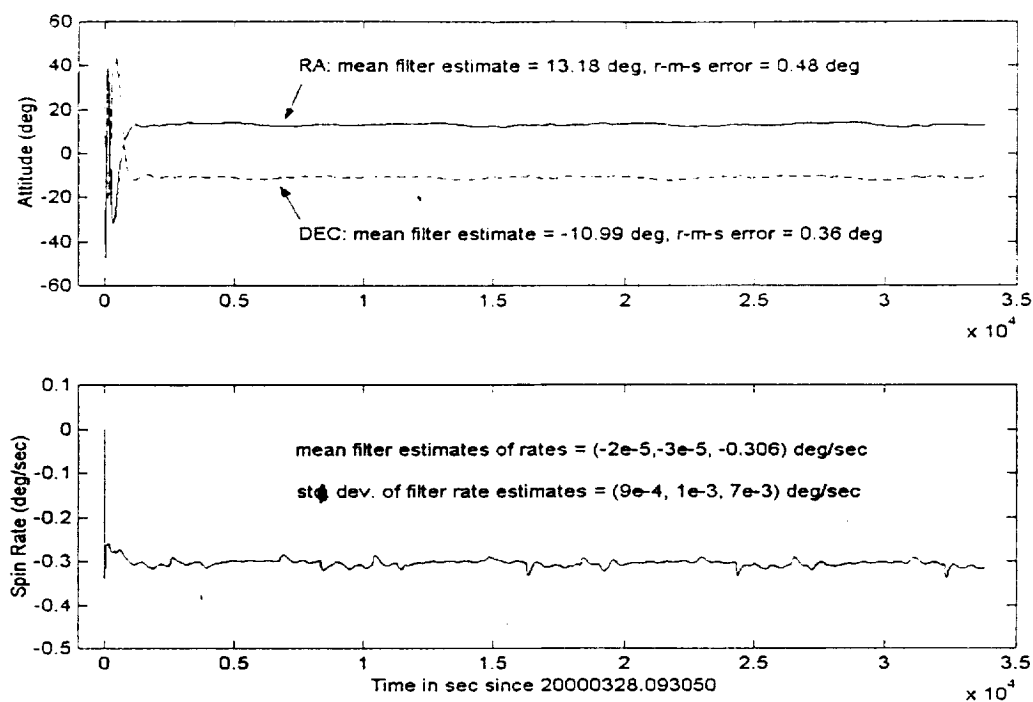Fig. 6 DARDMOD Attitude and Rate Solutions and True (DSS) Rates for WIRE
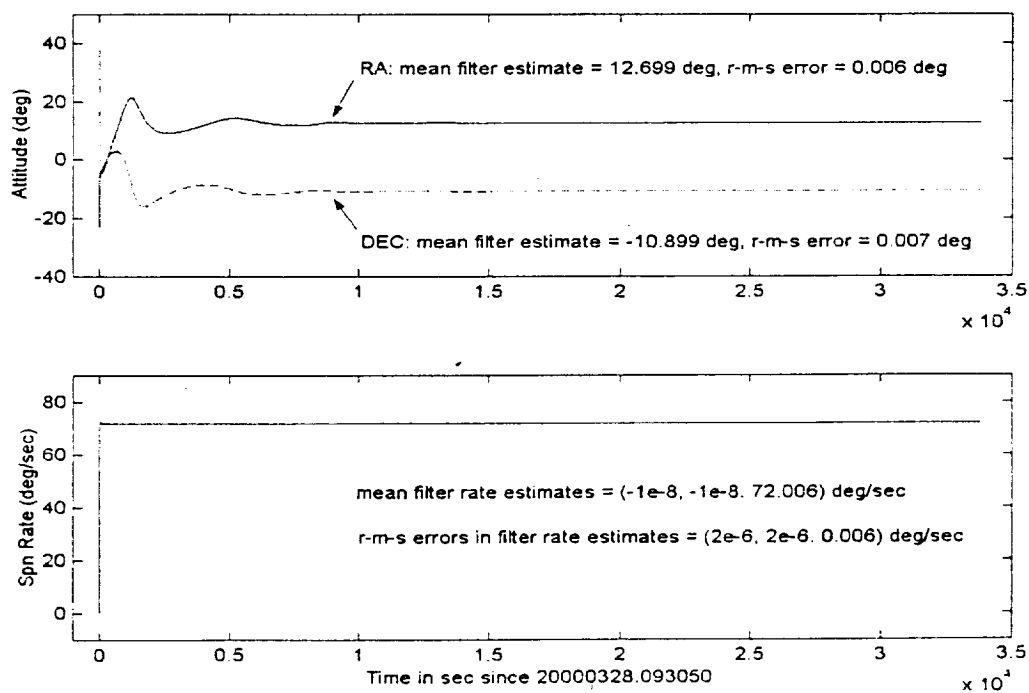
Fig. 7   Filter TAM-ONLY Results Using FAST Rotated-TAM Data

Fig. 8 Filter TAM-Only Results Using Simulated FAST Data