SOLAR SAIL OPTIMAL ORBIT TRANSFERS TO SYNCHRONOUS ORBITS

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A constant outward radial thrust acceleration can be used to reduce the radius of a circular orbit of specified period. Heliocentric circular orbits are designed to match the orbital period of Earth or Mars for various radial thrust accelerations and are defined as synchronous orbits. Minimum-time solar sail orbit transfers to these synchronous heliocentric orbits are presented.

INTRODUCTION

The concept of using a solar sail as a means of propulsion for a space vehicle was first introduced in 1924 by two Russian space pioneers, Konstantin Tsiolkovsky and Fridrickh Tsander. In their theory, the transfer of momentum that occurs when a photon of light bounces off the reflective surface of a solar sail should provide a mechanism for space propulsion. The effect of this sunlight pressure on the orbital motion of interplanetary spacecraft was first taken into account in 1974 with the Mariner 10 missions to Mercury and Venus. Solar panels on the Mariner 10 spacecraft when tilted at various angles to the sun experienced differences in sunlight pressure, thus inducing rotational motion of the spacecraft. [1]

In the late 1970's, the NASA Jet Propulsion Laboratory (JPL) initiated a team effort to study the feasibility of using a solar sail vehicle to rendezvous with Halley's Comet during its 1986 approach to the inner solar system. Due to the large energy requirements of the mission, both solar sails and solar electric propulsion were studied as feasible spacecraft propulsion techniques. Solar electric propulsion was ultimately chosen for the mission since solar sails were deemed to not be a sufficiently mature technology for a near term space mission. In the end, the U.S. Halley mission was cancelled and research efforts directed at producing an operational solar sail vehicle for NASA were halted in 1981. [1]

In an era of tight space mission budgets, solar sail spacecraft fit NASA's "better, faster, cheaper" paradigm and look even more promising than they did in the 1970's. New materials are accessible today for producing extremely thin, strong sheets of sail material. The booms and struts that stretch out and maintain a sail's shape could be made from composite material that is currently available. Also, microsatellite technology allows the solar sail payload to have a smaller mass. Together, these new technologies add up to feasible solar sail missions. [2]

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This paper considers the mission of transferring between a circular orbit of one astronomical unit (AU) and a circular synchronous orbit in minimum time using solar sail propulsion. A synchronous orbit is an orbit that matches the period of a celestial body but at a different orbit radius than would exist for an non-thrusting spacecraft in a two body system. The ability of a solar sail to provide a constant radial thrust allows the existence of these synchronous orbits and will be described in more detail in a later section.[3] Several investigations into solar sail optimal flight paths have been made in the past. Tsu [4] and London[5] performed pioneering work using logarithmic spirals to approximate the motion of the sail. Kelley[6], Cavoti[7], Zhukov and Lebedev[8] and Jayaraman[9] have formulated interplanetary minimum-time transfers for solar sail spacecraft using a variety of optimization approaches. Following the methodology of Zhukov and Lebedev, Sauer[10] used calculus-of-variations to formulate the necessary conditions for interplanetary transfers. Sauer however modeled three-dimensional transfers and included inclined, elliptic orbits for the launch and target planets. In this paper, calculus-of-variations is used to derive the first-order necessary conditions for a locally optimal minimum-time transfer using solar sail propulsion. The transfers are heliocentric beginning in a circular orbit of one AU and terminating in circular orbits that have the same period of the Earth or Mars. The transfers that will be presented are planar but the analysis could be generalized to three-dimensional transfers if desired.

SOLAR SAIL EQUATIONS OF MOTION

To simplify the analysis, two-dimensional planar transfers were modeled using the polar coordinate system shown in Figure 1. In the equations of motion, the symbol r denotes the distance from the sun to the solar sail center of mass, ϕ is the transfer angle between a reference axis and the sun-sail line, u is the radial velocity, v is the tangential velocity, and θ is the sail control angle between the sun-sail line and the sail normal measured positive in the counter-clockwise direction from the sun-sail line.

The force F generated by the sail depends on the total surface area S, the solar intensity W at one AU, the speed of light c, r, and the sail control angle θ.[11] The acceleration is given below where m is the spacecraft mass.

$$F/m = 2WS\cos^2\theta/cr^2 = k\cos^2\theta\,/\,r^2 \tag{1}$$

As a further simplification, the constants W, S, and c have been combined into the characteristic acceleration constant denoted by $k = 2WS/c$. Equation (1) shows that the constant k is the maximum acceleration that the sail can produce at a reference distance unit of 1 AU.

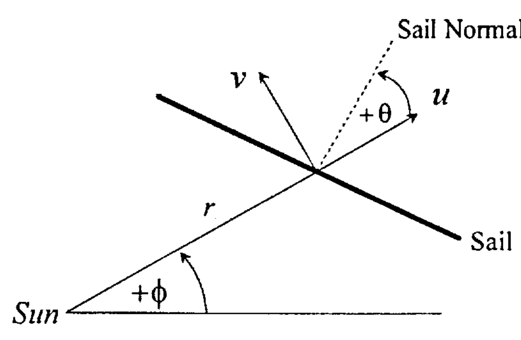

Figure 1. Coordinate System

The variable $\bar{x}$ is used for the state vector and is defined in Eq. (2). The equations of motion for the solar sail in a two-body heliocentric system can then be written as shown in Eq. (3).

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} r \\ \phi \\ u \\ v \end{bmatrix} \tag{2}$$

$$\dot{\bar{x}} = \begin{bmatrix} \dot{r} \\ \dot{\phi} \\ \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} u \\ v/r \\ v^2/r - \mu/r^2 + k\cos^3\theta/r^2 \\ -uv/r + k\sin\theta\cos^2\theta/r^2 \end{bmatrix} \tag{3}$$

In this paper, heliocentric transfers are modeled and canonical units used, therefore $\mu_{sun} = 1\ AU^3/TU^2$. Equations (2) and (3) are then combined to obtain the state space formulation of the equations of motion.

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4/x_1 \\ \dot{x}_3 &= -1/x_1^2 + k\cos^3\theta/x_1^2 + x_4^2/x_1 \\ \dot{x}_4 &= -x_3x_4/x_1 + k\sin\theta\cos^2\theta/x_1^2 \end{aligned} \tag{4a-d}$$

OPTIMAL CONTROL PROBLEM

Optimal control theory[12] will be introduced and then applied to the problem of minimum-time transfer to a specified orbit for a solar sail. The nonlinear differential equation describing the motion of the solar sail (Eq. (4)) can be written compactly as shown in Eq. (5).

$$\dot{\bar{x}} = \bar{f}\left[\bar{x}(t), \bar{\theta}(t), t\right] \quad \bar{x}(t_0)\text{ given, } t_0 \le t \le t_f \tag{5}$$

In Eq. (5), the state parameters $\bar{x}(t)$ are determined by the control parameters, $\bar{\theta}(t)$ where the dot specifies differentiation with respect to time, t . The general optimal control problem is then to determine the controls $\bar{\theta}(t)$ that will minimize a performance index J that is specific to the problem being solved. The performance index J is a scalar representation of the dynamic system's cost. Cost for achieving spacecraft orbits using impulsive propulsion systems can typically be defined by the amount of velocity change Δv or equivalently by the amount of fuel expended. For the case of the solar sail in which continuous thrust with no consumption of propellant is possible, the cost being minimized is the time of flight.

A general performance index J can be written in the form;

$$J = \eta\left[\bar{x}(t_f), t_f\right] + \int_{t_0}^{t_f} l\left(\bar{x}(t), \bar{\theta}(t), t\right) dt \tag{6}$$

In the above cost function, η represents the scalar terminal cost and I represents the integral cost accumulated over a particular time interval. For the transfer of a solar sail to a specified orbit in minimum time, $I = 0$, $\eta = t_f$, and therefore the cost function being minimized is the time of flight t_f. An equivalent formulation is to define I=1 and $\eta = 0$. The Hamiltonian can then be written as;

$$H = I + \bar{\lambda}^T \dot{\bar{x}} = \bar{\lambda}^T \dot{\bar{x}} \tag{7}$$

Here the variable $\bar{\lambda}$ refers to the vector of Lagrange multipliers, also known as the vector of adjoint variables. The Hamiltonian for this problem, using Eqs. (4a-d) and four Lagrange multipliers is;

$$H = \lambda_1(u) + \lambda_2(v/r) + \lambda_3\left(v^2/r - 1/r^2 + k\cos^3\theta/r^2\right) + \lambda_4\left(-uv/r + k\sin\theta\cos^2\theta/r^2\right) \tag{8}$$

Using the calculus-of-variations optimization technique, a set of necessary conditions can be determined. The solution to these necessary conditions provides the $\bar{\theta}(t)$ that produces a stationary value of the performance index J[12]. These first-order necessary conditions are the Euler-Lagrange equations. The first of these is the already stated differential equations of motion with the associated boundary condition from Eq. (5). The second is the differential equation governing the Lagrange multipliers with the associated boundary condition at the final time. These are

$$\dot{\bar{\lambda}}^T = -\partial H / \partial \bar{x}$$

$$\bar{\lambda}^T\Big|_{t_f} = \left(\partial\eta/\partial\bar{x} + \bar{v}^T\,\partial\bar{\Psi}/\partial\bar{x}\right)\Big|_{t_f} \tag{9}$$

In Eq. (9) $\bar{\lambda}^T\Big|_{t_f}$ denotes that the Lagrange multiplier is evaluated at the final time, as is the right hand side of (9). Also in Eq. (9), $\bar{\Psi}$ is the vector of specified terminal constraints and $\bar{v}$ is an additional set of constant multipliers. For transfers to circular synchronous orbits, the three chosen terminal constraints Ψ_1, Ψ_2, Ψ_3 are shown in Eq. (10). The values for $r_{specified}$ and $v_{specified}$ will be defined in later sections to correspond to desired terminal circular orbits.

$$\bar{\Psi} = \begin{bmatrix} r_f - r_{specified} \\ u_f - 0 \\ v_f - v_{specified} \end{bmatrix} \tag{10}$$

Since there are four states in the $\bar{x}$ vector, there are four differential equations for $\dot{\bar{\lambda}}$ according to Eqs. (8) and (9).

$$\dot{\lambda}_1 = -\partial H/\partial r = -2\lambda_3/r^3 + \lambda_3 v^2/r^2 + 2k\lambda_3\cos^3\theta/r^3 - \lambda_4 uv/r^2 + 2k\lambda_4\sin\theta\cos^2\theta/r^3 + \lambda_2 v/r^2 \tag{11a}$$

$$\lambda_1(t_f) = \partial\eta/\partial r_f + v_1\partial\Psi_1/\partial r_f + v_2\partial\Psi_2/\partial r_f + v_3\partial\Psi_3/\partial r_3 = v_1 = \text{constant} \tag{11b}$$

$$\dot{\lambda}_2 = -\partial H/\partial\phi = 0 \tag{12a}$$

$$\lambda_2(t_f) = \partial\eta / \partial\phi_f + \nu_1 \partial\Psi_1 / \partial\phi_f + \nu_2 \partial\Psi_2 / \partial\phi_f + \nu_3 \partial\Psi_3 / \partial\phi_f = 0 \tag{12b}$$

By integration, it is seen that λ_2 = constant and by enforcing the boundary condition $\lambda_2 = 0$ for all time. This Lagrange multiplier can be eliminated from the problem formulation if desired leading to a reduced order problem. The third and fourth differential equations for $\dot{\bar{\lambda}}$ are

$$\dot{\lambda}_3 = -\partial \mathrm{H}/\partial \mathrm{u} = \lambda_4 v / r - \lambda_1 \tag{13a}$$

$$\lambda_3(t_f) = \partial\eta / \partial u_f + \nu_1 \partial\Psi_1 / \partial u_f + \nu_2 \partial\Psi_2 / \partial u_f + \nu_3 \partial\Psi_3 / \partial u_f = \nu_2 = \text{constant} \tag{13b}$$

$$\dot{\lambda}_4 = -\partial \mathrm{H}/ \partial \mathrm{v} = -2\lambda_3 v / r + \lambda_4 u / r \tag{14a}$$

$$\lambda_4(t_f) = \partial\eta / \partial v_f + \nu_1 \partial\Psi_1 / \partial v_f + \nu_2 \partial\Psi_2 / \partial v_f + \nu_3 \partial\Psi_3 / \partial v_f = \nu_3 = \text{constant} \tag{14b}$$

The third Euler-Lagrange condition is that the terminal boundary conditions specified in Eq. (10) must be satisfied. Since the final transfer time is unknown and free to be determined in a time open problem, the fourth Euler-Lagrange equation, the so called transversality condition, applies

$$[\partial\eta / \partial \mathrm{t} + H]\big|_{t_f} = 0 \tag{15}$$

In a time open formulation, $\partial\eta / \partial \mathrm{t_f} = 1$, which leads to the transversality condition on the Hamiltonian at the final time of $H(t_f) + 1 = 0$.

The optimality condition provides a formula for the one control θ .

$$\partial \mathrm{H} / \partial \theta = 0 \tag{16}$$

Applying Eq. (16) to the Hamiltonian given in Eq. (8) yields;

$$-3\lambda_3 k \cos^2\theta \sin\theta / r^2 + \lambda_4 k(-2\cos\theta \sin^2\theta + \cos^3\theta) / r^2 = 0 \tag{17}$$

After dividing Eq. (17) through by $k\cos^3\theta / r^2$ a quadratic in $\tan\theta$ results.

$$-2\lambda_4 \tan^2\theta - 3\lambda_3 \tan\theta + \lambda_4 = 0 \tag{18}$$

Equation (18) can be solved using the quadratic formula to yield the condition for the optimal control

$$\theta = \tan^{-1}\{+[(9\lambda_3^2 + 8\lambda_4^2)^{1/2} - 3\lambda_3] / 4\lambda_4\} \tag{19}$$

The control angle in Eq. (19) is uniquely determined by λ_3 and λ_4. The sign ambiguity of the square root term in the numerator was determined to be strictly positive by examining all possibilities of the signs of λ_3 and λ_4 along with the resultant quadrant where θ must reside.[13]

Summarizing, a solar sail minimum-time orbit transfer can be formulated as a two-point boundary-value problem defined by the eight differential equations and associated boundary conditions Eqs. (4) and (11) through (14) along with the control in Eq. (19). A FORTRAN algorithm was constructed to solve this two point boundary value problem. Results obtained from this software were compared with published results as described in the next section.

TEST CASES

Mars

In order to validate the computational methodology used to solve the solar sail two point boundary value problem, a minimum time solar sail transfer from Earth orbit, r = 1.0 AU, to a Martian orbit, r = 1.524 AU that is described in Reference 8 was generated. The vector of terminal constraints shown in Eq. (10) was driven to zero (within a prescribed tolerance) where r_f, u_f, and v_f are the final values of the radius, radial velocity, and tangential velocity and r_{spec} and v_{spec} are the specified values of radius and tangential velocity for the desired orbit. For the transfer to a Martian orbit, the values $r_{spec} = 1.524$ AU and $v_{spec} = 1/\sqrt{1.524}$ were used. A characteristic acceleration of $k = 2\,mm/s^2$ was used to correspond with the results given in Reference 8.

A multi-dimensional shooting method was used to solve the two-point boundary value problem. Initial guesses for the transfer time and the Lagrange multipliers are required with this technique. The values of a final time of 5 TU and $\lambda_1 = 1$, $\lambda_3 = 1$, and $\lambda_4 = 1$ were used. In the previous section, the variable λ_2 was shown to be zero for all time. By reinserting successive values of the initial values for the three Lagrange multipliers, a converged solution was obtained. This method is known as the "method of continuation". In the converged solution, the constraint vector $\overline{\Psi}$ achieved the final desired values for r, u, and v within a prescribed tolerance of 1×10^{-6}. This meant that the final radial velocity u was within 1×10^{-6} of its desired value of zero and that the final tangential velocity v was within 1×10^{-6} of its desired value of $1/\sqrt{1.524}$. Also, the final radius r was within 1×10^{-6} of its desired value of 1.524 and thus within the Mars sphere of influence (SOI) of 170 Mars radii. The accuracy of the final value of the transversality constraint was also on the order of 1×10^{-6} as compared to its ideal value of zero. The final transfer angle value was allowed to vary freely. A total minimum transfer time of 324 days or 5.57 TU was obtained for a converged solution, which compared favorably with the value of 5.54 TU from Reference 8 for transfer at a characteristic acceleration of $2\,mm/s^2$. Figure 2 illustrates the control angle state history. Figure 3 illustrates the control angle at each time interval on the resultant outward spiral trajectory. The state histories obtained essentially duplicated the solar sail Mars transfer state histories of Figures 2 and 3 of Reference 8.

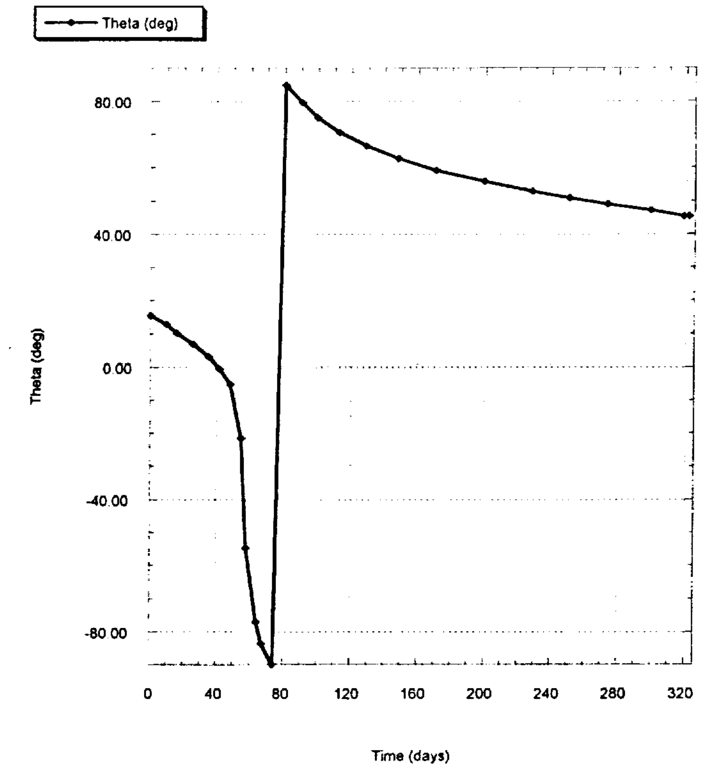

Figure 2. Time Open Control Angle History for Solar Sail Earth to Mars Minimum Time Transfer

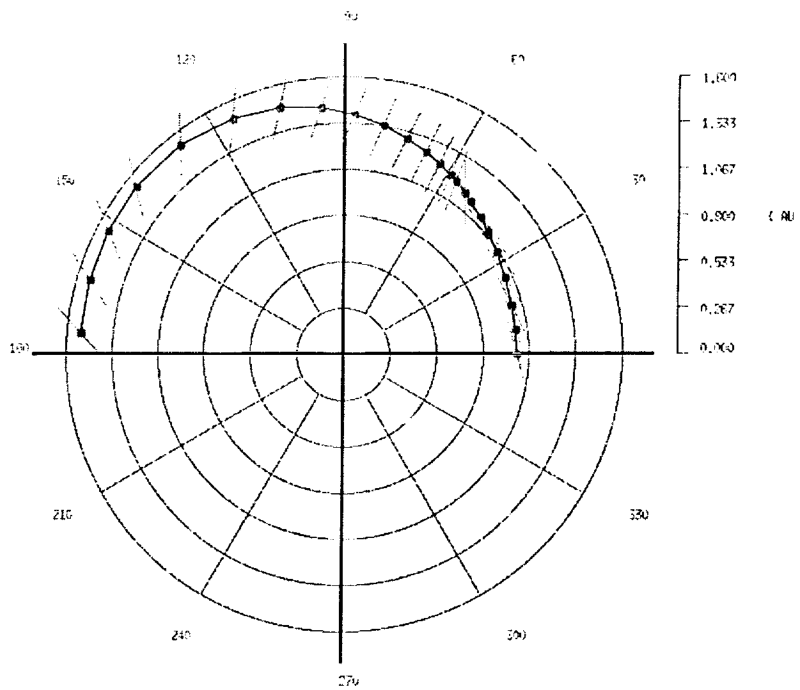

Figure 3. Control Angle Theta at Time Intervals for Solar Sail Earth to Mars Minimum Time Transfer

Venus

For a time open transfer to Venus orbit, r_{spec}=0.7233 AU , $u_f = 0$, and $v_{spec} = 1/\sqrt{0.7233}$ were used in the terminal constraint vector. A characteristic acceleration of $k = 2\,mm/s^2$ was again used for comparison with results from Reference 8. Lagrange multipliers corresponding to a converged solution were obtained once again using the method of continuation. In the converged solution, the constraint vector $\overline{\Psi}$ achieved the final desired values for r, u, and v to within a prescribed tolerance of 1×10^{-6} in a manner similar to how the constraints were satisfied for the Martian transfer described above. A total minimum transfer time of 163.65 days or 2.815 TU was obtained for a converged solution, which again compared very favorably with the value of 164 days from Reference 8 for transfer at a characteristic acceleration of $2\,mm/s^2$. Figure 4 illustrates the control angle state history. Figure 5 illustrates the control angle at each time interval along the resultant inward spiral trajectory. The state histories obtained essentially duplicated the solar sail Venus transfer state histories of Figures 4 and 5 of Reference 8.

The optimization software was able to achieve converged solutions for both the Earth-to-Mars and Earth-to Venus solar sail transfers with transfer times and state histories that duplicated the results of Reference 8 , the software was considered validated. Next, the software was used to investigate inward or outward minimum time spirals from Earth to synchronous orbits of any specified radius.

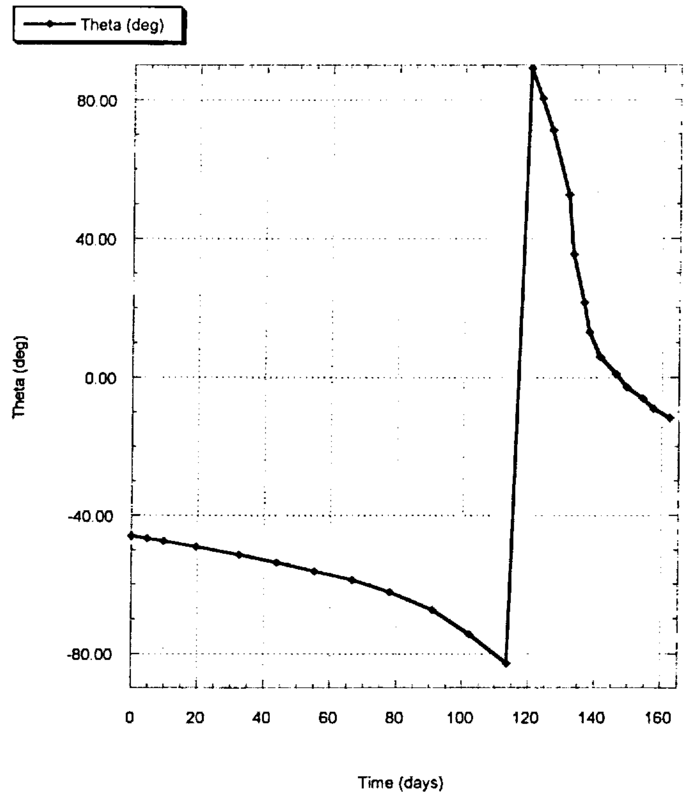

Figure 4. Time Open Control Angle History for Solar Sail Earth to Venus Minimum Time Transfer

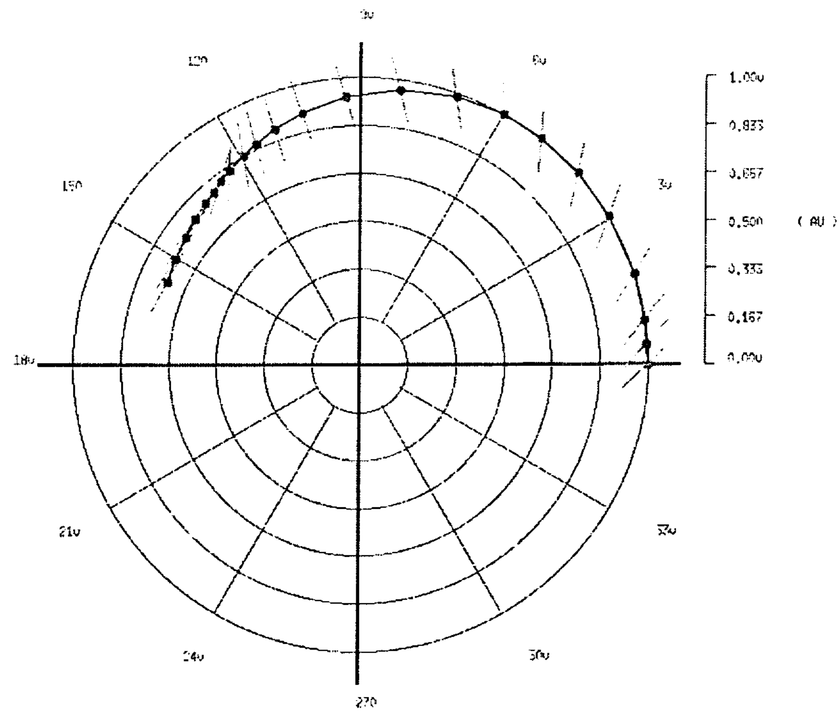

Figure 5. Control Angle Theta at Time Intervals for Solar Sail Earth to Venus Minimum Time Transfer

SYNCHRONOUS CIRCULAR ORBITS

The boundary conditions on the radius and velocity magnitudes for synchronous orbits will be derived in this section. The solar sail mission statement is to determine the control profile to transfer from a circular orbit of 1 AU to a synchronous orbit of a given period in minimum time. The sail control authority is governed by the solar sail constant $k = 2WS/c$. Using Eq. (1), k can be shown to be equivalent to the maximum acceleration that the sail can provide at 1 AU. To design a synchronous orbit, a desired period $P_{desired}$ for a circular orbit is chosen and is written in terms of the radius (r_{syn}) and velocity (v_{syn}) of the circular synchronous orbit.

$$P_{desired} = 2\pi r_{syn} / v_{syn} \tag{20}$$

The period $P_{desired}$ is next related to the earth orbital period by the factor R.

$$R = P_{desired} / P_{earth} \tag{21}$$

Using canonical units with distance referenced to 1 AU, the period of the earth is $P_{earth} = 2\pi$ TU and Eq. (20) and (21) are combined to yield a simple relationship between r_{syn} and v_{syn}.

$$v_{syn} = r_{syn} / R \tag{22}$$

Next, the constant radial acceleration that the solar sail provides at the synchronous distance must be related with the synchronous velocity. This is done through the equations of motion by enforcing zero radial velocity and acceleration on the synchronous orbit.

$$v_{syn}^2 = (\mu - k)/r_{syn} = (1-k)/r_{syn} \tag{23}$$

Note that by the spacecraft thrusting in only in the outward radial direction once in the synchronous orbit, the sail is in essence reducing the local gravitational constant, μ. Finally, by combining Eqs. (22) and (23), an expression relating the maximum sail acceleration at one AU (k) and the desired period is determined.

$$r_{syn} = R^{2/3}(1-k)^{1/3} \tag{24}$$

The radius of the synchronous orbit supplied by Eq. (24) is used for $r_{specified}$ in Eq. (10). The velocity is specified by Eq. (22) and is set equal to $v_{specified}$ in Eq. (10). Equations (22) and (24) serve as terminal boundary conditions for the transfers along with the sail having zero radial velocity at the end of the transfer (u=0).

TRANSFERS TO EARTH SYNCHRONOUS ORBITS

Figure 6 is a plot of Eq. (24) for Earth synchronous orbits, i.e. R=1. Values of k between 0-0.34 AU/TU^2 (0-2.0162 mm/s^2) are presented. This range of sail accelerations was selected due to the fact that they are believed achievable using current technology. Figure 6 shows that as the thrust acceleration is increased, the synchronous orbit radius moves closer to the sun. For Earth synchronous orbits $v_{syn}=r_{syn}$. The transfer times to the earth synchronous orbits decrease as the thrust acceleration increases even though the radial difference between the initial and terminal orbits increases. Points A and B have been labeled on Figure 6 corresponding to an acceleration of 0.006 mm/s^2 and 2 mm/s^2 respectively. Transfers to Earth synchronous orbits at these two acceleration levels are presented in Figure 7. Converged solutions for thrust levels between points A and B were obtained with a tolerance of 10^{-6} as in the test cases.

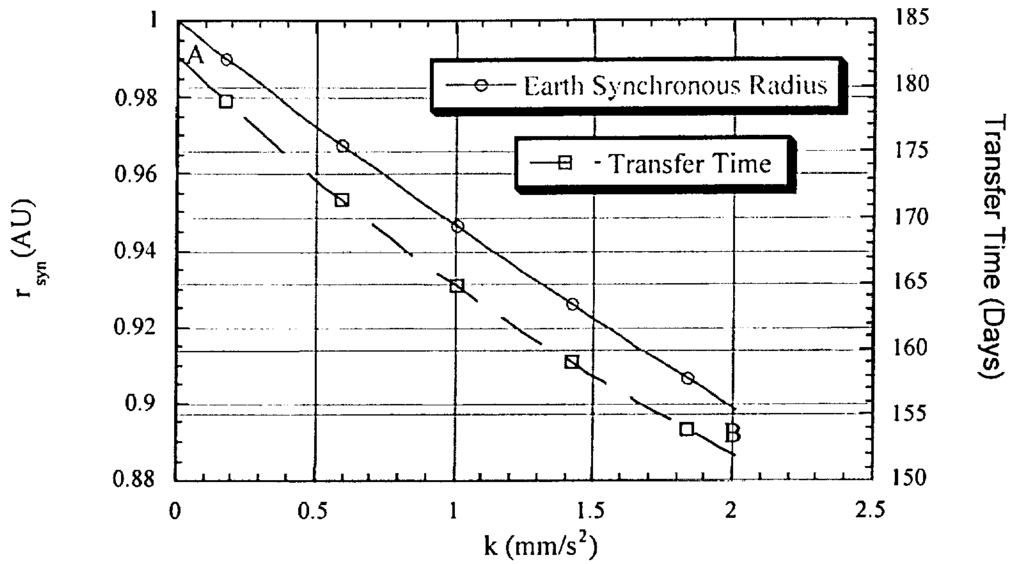

Figure 6 Earth Synchronous Radius and Transfer Times Versus Sail Acceleration

The transfer to an Earth synchronous orbit corresponding to a 0.006 mm/s^2 is barely distinguishable from the circular reference orbit at 1 AU. The transfer requires 182 days, approximately one-half of the period of the circular reference orbit. The transfer at the higher acceleration (2 mm/s^2) requires only 152 days to complete the transfer.

The control profiles for these two transfers are displayed in Figure 8. Each transfer shows similar characteristics. A large slew maneuver is required to match the desired boundary conditions on the velocity. The maneuver for the lower acceleration occurs at approximately the middle of the transfer while the maneuver in the larger acceleration transfer is slightly delayed. Note that the control angle is always negative corresponding to the acceleration being applied in the direction opposite to the velocity. This control decreases the orbital energy of the spacecraft causing an inward spiral to occur.

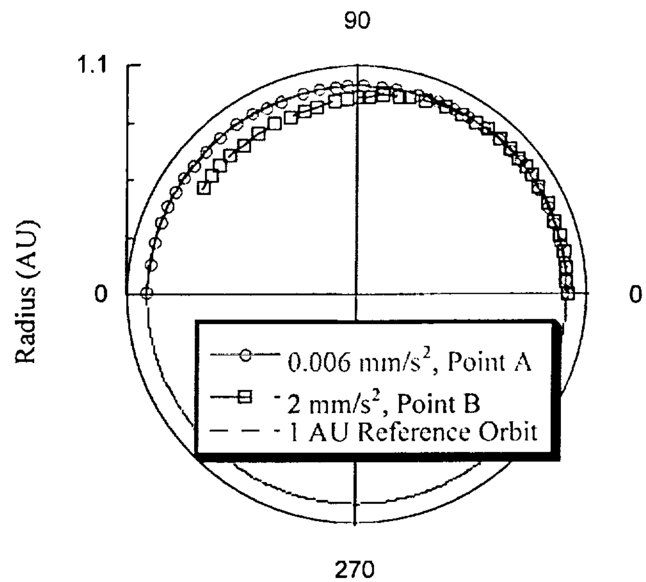

Figure 7 Transfers to Two Earth Synchronous Orbits

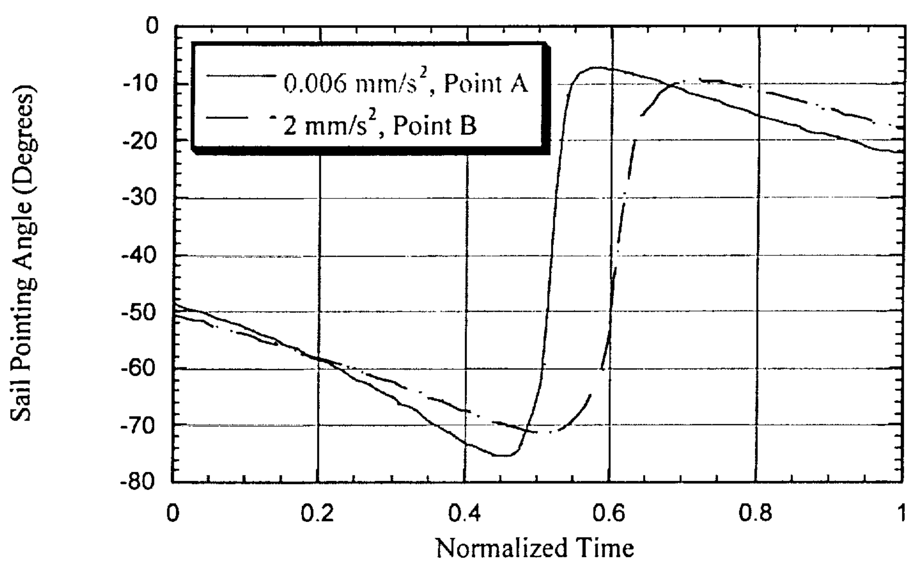

Figure 8 Control Angle to Two Earth Synchronous Orbits

TRANSFERS TO MARS SYNCHRONOUS ORBITS

Figure 9 is a plot of Eq. (24) for Mars synchronous orbits, i.e. R=(1y 321.73d)/(1y)=1.88083.[14] Figure 9 shows that as the thrust acceleration is increased, the synchronous orbit radius moves closer to the Earth. Also, recall that for synchronous orbits $v_{syn}=r_{syn}/R$. The transfer times to the Mars synchronous orbits rapidly decrease as the thrust acceleration increases (See Figure 10) since the radial difference between the initial and terminal orbits decreases along with the increased control authority. Points C and D have been labeled on Figure 9 corresponding to an acceleration of 0.38 mm/s^2 and 2 mm/s^2 respectively. Converged solutions for thrust levels between points C and D were obtained with a tolerance of 10^{-6} as in the test cases. Minimum-time transfers for characteristic accelerations less than 0.38 mm/s^2 were not obtained due to the strict convergence requirements and the fact that the sail has less control authority and larger energy changes than the higher acceleration cases.

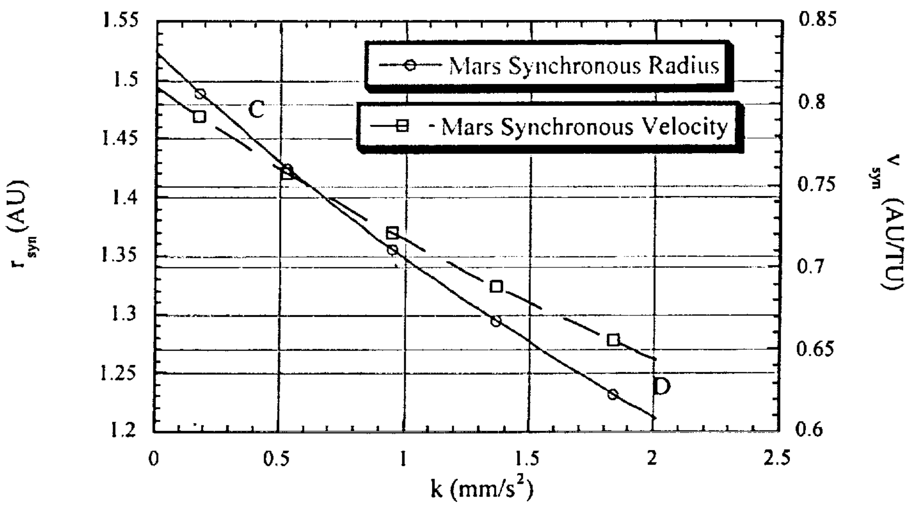

Figure 9 Mars Synchronous Radius and Velocity Versus Sail Acceleration

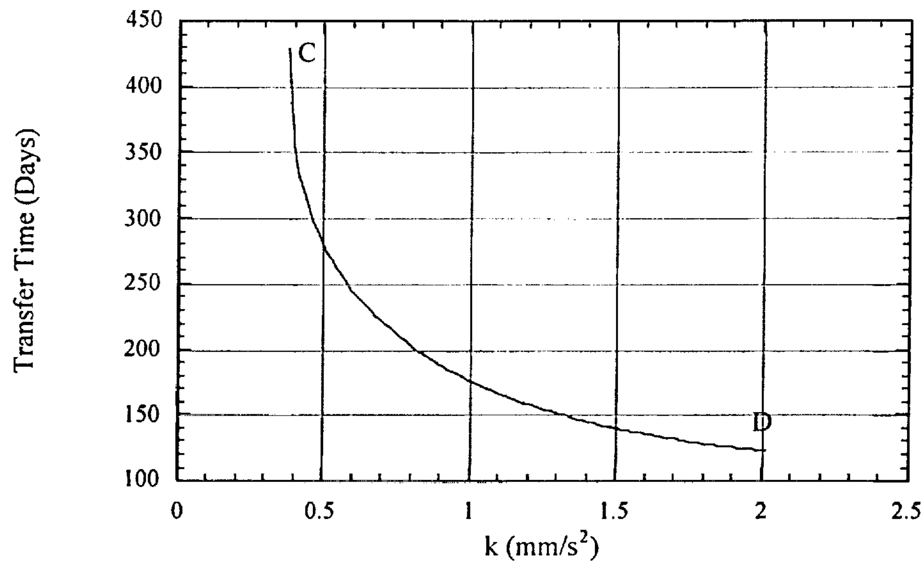

Figure 10 Mars Synchronous Transfer Times Versus Sail Acceleration

Figure 11 displays the trajectories corresponding to points C and D. The transfer to an Mars synchronous orbit corresponding to a 0.38 mm/s^2 travels a transfer angle of 320 degrees and requires 429 days complete. The transfer at the higher acceleration (2 mm/s^2) requires only 123 days to complete the transfer.

The control profiles for these two transfers are displayed in Figure 12. Due to the small acceleration of point C, the angle between the sun-sail line and the sail normal is always positive (thrusting in the positive velocity direction) in order to increase the energy. The control angle history for point D is quite different. Due to the larger available control, the boundary conditions are reached in minimum time by thrusting away from the velocity direction.

CONCLUSIONS

Minimum time transfers for solar sail spacecraft to Earth and Mars synchronous heliocentric orbits have been determined for sail accelerations that are achievable using current technology. Necessary conditions for local optimality were derived using calculus-of-variations and boundary conditions for synchronous circular orbits were presented. A multi-dimensional shooting method was implemented to solve the boundary value problem. The optimization algorithm was verified on two published interplanetary transfers and then applied to the minimum-time synchronous transfers. Solar sail spacecraft in heliocentric Earth synchronous orbits could be used to warn spacecraft of solar flare events that could have significant effects for power generation and communication on Earth. Mars synchronous orbits would provide locations for communication satellites with constant Earth-Mars viewing angles. Solar sail spacecraft have a great deal of potential for advancing the state-of-the-art in spacecraft propulsion technologies and the further exploration of the solar system.

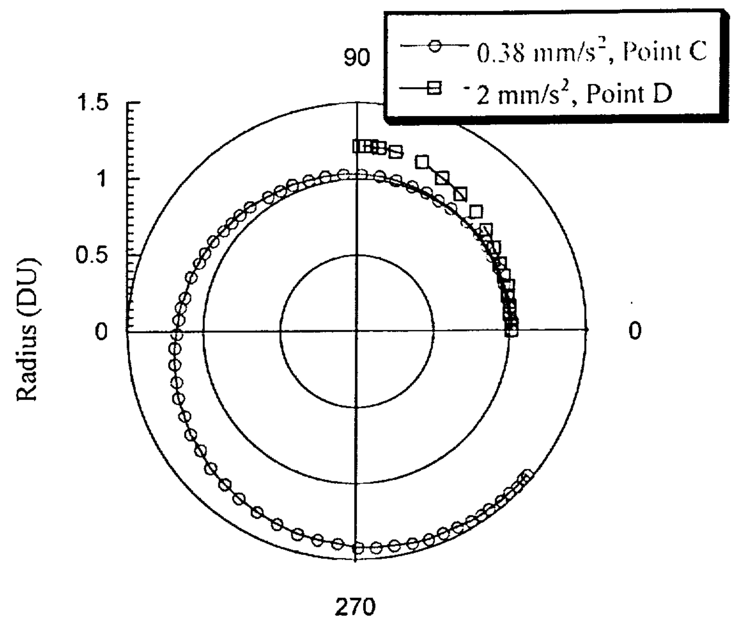

Figure 11 Transfers to Two Mars Synchronous Orbits

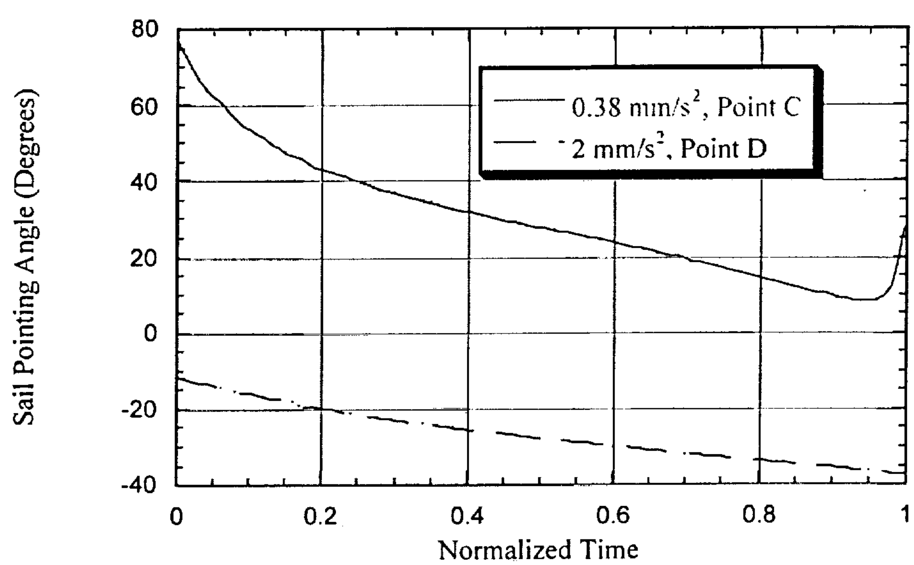

Figure 12 Control Angle to Two Mars Synchronous Orbits

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