FORMATION FLYING IN HIGHLY ELLIPTICAL ORBITS
INITIALIZING THE FORMATION

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ABSTRACT *– In this paper several methods are examined for initializing formations in which all spacecraft start in a common elliptical orbit subsequent to separation from the launch vehicle. The tetrahedron formation used on missions such as the Magnetospheric Multiscale (MMS), Auroral Multiscale Midex (AMM), and Cluster is used as a test bed. Such a formation provides full three degrees-of-freedom in the relative motion about the reference orbit and is germane to several missions. The type of maneuver strategy that can be employed depends on the specific initial conditions of each member of the formation. Single-impulse maneuvers based on a Gaussian variation-of-parameters (VOP) approach, while operationally simple and intuitively-based, work only in a limited sense for a special class of initial conditions. These 'tailored' initial conditions are characterized as having only a few of the Keplerian elements different from the reference orbit. Attempts to achieve more generic initial conditions exceed the capabilities of the single impulse VOP. For these cases, multiple-impulse implementations are always possible but are generally less intuitive than the single-impulse case. The four-impulse VOP formalism discussed by Schaub is examined but smaller delta-V costs are achieved in our test problem by optimizing a Lambert solution.*

NOMENCLATURE

a	Semi-major axis (km)
e	Eccentricity
i	Inclination (radian)
Ω	Right Ascension of the Ascending Node (radian)
ω	Argument of Perigee (radian)
f	True Anomaly (radian)
E	Eccentric Anomaly (radian)
M	Mean Anomaly (radian)
n	Mean Motion (radian/s)
τ	Time of Perigee Passage (s)
μ	Earth's Gravitational Constant (3.986×10^5 km^3/s^2)
$\mathbf{Z}$	[6x3] Variation of Parameters Matrix.
$\mathbf{Z}^T$	[3x6] Z Transpose.
$\mathbf{L}^*$	[3x6] PseudoInverse Matrix

1 - INTRODUCTION

In recent years, the need for spacecraft flying in close formation has increased significantly. Its diverse applications range from synthetic aperture radar systems, like TechSat 21 to science missions such as EO-1 or LISA. Correspondingly, many studies have been performed on the relative motion of a spacecraft with respect to a reference orbit. Much of the literature, building on the early work of Clohessy and Wiltshire [Cloh 60], is focused on solving the relative motion between a spacecraft in a circular reference orbit and another spacecraft in a nearly circular orbit. Their solution works fairly well for low eccentricity missions. Recently, however, several missions have been proposed, designed or flown that need spacecraft flying in formation about a highly elliptical reference orbit. Most of these missions, such as the European Space Agency's (ESA) Cluster and NASA's Magnetospheric MultiScale (MMS), have space physics science objectives, which involve at least four spacecraft moving in a "tetrahedron" configuration at apogee. The shape and the separation of the configuration are designed to resolve spatial and temporal variations in essential regions of the Earth's magnetosphere.

One of the central issues associated with formation flying is the determination of initial conditions for each member spacecraft that gives desired cooperative dynamics and the maneuver strategies used to establish them. Several models of the relative motion about arbitrary-eccentricity orbits have been developed [Zare 90][Der 97]. These are, however, only a necessary first step in determining initial conditions. The full process for determining the initial conditions involves searching through all of the available initial-conditions to find the set that, when dynamically propagated, best meets the mission metrics. These metrics may involve constraints on range and range-rate between the member spacecraft, constraints on orientation of the formation, or requirements on the volume contained within the region bounded by the spacecraft. The definition of the available initial-conditions space is also a function of the on-board delta-V capability. Thus there is established a practical coupling between the family of initial conditions that can be achieved and the maneuver strategies used to achieve them.

This paper is an attempt to elucidate a portion of this coupling in a simplified setting. A simple formation metric is defined and a test problem is constructed to evaluated each maneuver strategy examined. The test problem formulation is discussed in Section 2. Two separate maneuver strategies are examined. The first strategy, discussed in Section 3, is based on Gauss' variation-of-parameters (VOP) equations. It provides us with an intuitive approach, which brings the physics to the fore. There are specific types of initial conditions, termed 'tailored', that allow for the multiple-impulse VOP method to work but that they are restricted to a sub-space of the initial-conditions parameter space. Initial conditions, which are not restricted in this fashion, are termed 'generic'. Generic initial conditions can always be established using a multiple-impulse VOP strategy but at the cost of physical intuition and generally greater delta-V. These results are also discussed in Section 3. Section 4 details a maneuver strategy based on solving Lambert's problem using the Der form of the state-transition matrix [Der 97]. The method is completely general but lacks the physical interpretation that can be attached to many of the VOP-equations strategies.

2 - FORMATION FLYING TEST PROBLEM

As discussed above, in a real mission design scenario, the formation initial conditions will be chosen by a trade-study which finds the best fit to the desired mission metrics subject to the mission- allocated delta-V. Depending on the goals of the mission and the number of spacecraft this can be an immensely complex task. To avoid this difficulty, a test problem is used for the work in this paper. We take as the only mission metric the establishment of a regular tetrahedron at apogee about a baseline orbit characterized by a semi-major axis, $a = 42095$ km, an eccentricity, $e = 0.81818$, an inclination, $i = 10$ degrees, and with zero right-ascension of ascending node and argument of perigee. Without loss of generality, the time of perigee passage was taken to be zero.

Using the geometric model detailed by Schiff *et al* [Schi 00], a regular tetrahedron was constructed about the baseline orbit's apogee. In the geometric model, the positions relative to the baseline orbit are specified in the corresponding apogee VBN frame. Requiring that the period of all the orbits be equal and that the velocity vectors be parallel at the moment the formation is established then completes the full state. A tetrahedron formation was chosen since it gave all three degrees of freedom in the relative motion and thus ensured a sample of in-plane and out-of-plane motion. For the test case each spacecraft was positioned 1000 km from the baseline orbit $\overline{S}_b$, as shown in Figure 1.
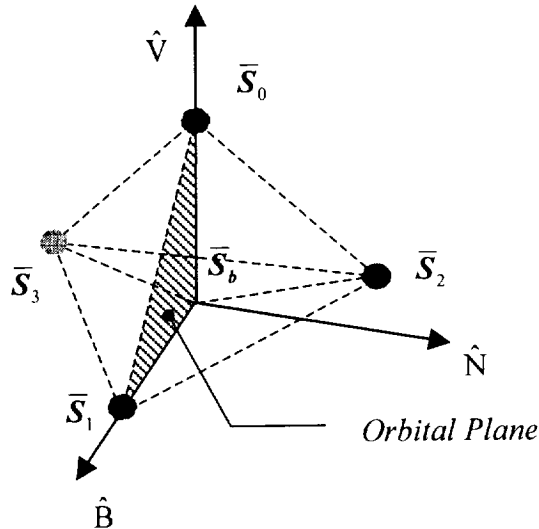
Fig. 1: Schematic of the Formation Configuration.

$\overline{S}_0$ and $\overline{S}_1$ are the in-plane components. $\overline{S}_0$ lies along the velocity direction and $\overline{S}_1$ lies on the binormal direction. $\overline{S}_2$ and $\overline{S}_3$ are the out-of-plane spacecraft. $\overline{S}_1$, $\overline{S}_2$ and $\overline{S}_3$ are evenly distributed (spacing of 120 degrees) in the $\hat{B} - \hat{N}$ plane and form a regular tetrahedron with $\overline{S}_0$. The orbital elements for the reference and the spacecraft in the formation are presented in Table 1. The semi-major axis of all the member spacecraft are equal to the baseline value, ensuring that all the spacecraft have the same period. Note that the out-of-plane spacecraft ($\overline{S}_2$, $\overline{S}_3$) exhibit change in all of the remaining orbital elements but the true anomaly. In addition, as they are symmetric about the orbit plane, their maneuver strategy will be identical, reducing the number of spacecraft to study to three.

Table 1 Orbital Elements of the Formation Flying at f_{ref} =180 °.

Orbital Elements	$\overline{S}_b$	Off-Set from $\overline{S}_b$	$\overline{S}_0$	$\overline{S}_1$	$\overline{S}_2$	$\overline{S}_3$
a (km)	42095	Δa (km)	0	0	0	0
e	0.818181	Δe	0.000189	-0.023756	0.011993	0.011993
i (deg)	10	Δi (deg)	0	0	0.0205	0.0205
Ω (deg)	0	$\Delta\Omega$ (deg)	0	0	3.704	-3.704
ω (deg)	0	$\Delta\omega$ (deg)	0.915	0	-3.648	3.648
f (deg)	180	Δf (deg)	-0.166	0	0	0

The final ingredient in the test problem is the application of a maneuver strategy to take the spacecraft from the baseline orbit to the initial state on the tetrahedron. To avoid the timing problems that would arise otherwise, it was required that all the spacecraft must reach their corresponding tetrahedron state at the same time, independent of where they start on the baseline orbit. This requirement ensures that each member of the formation begins its dynamics in concert

with the others. While the maneuver strategies examined are based on analytic approximations, *FreeFlyer®* was used to confirm the approximations in a fully modeled simulation. Details of such simulations are discussed in [Schi 00]. In all of the simulations, the *FreeFlyer®* targeter was run to achieve an accuracy in the desired goals of 0.1% and the results were used as a benchmark for a comparison between the different methods. For simplicity, the propagation force-model consisted only of the point-mass gravity due to the Earth. This is a good approximation, since it is envisioned that all maneuver scenarios will be performed over one orbit, during which the effects due to the orbital perturbations are minimal.

3 - VOP METHODS

In this section maneuver strategies are discussed for initializing the formation based on the Gaussian variation-of-parameters (VOP) equations. The reason for employing a VOP method was that it is inherently based on Keplerian elements. Since the Keplerian elements offer more physical insight into orbital mechanics than the corresponding Cartesian formulation, one might expect that the VOP equations would bring the same type of physical insight to what the maneuver (or maneuvers) are needed to initialize each spacecraft efficiently. However, this expectation generally proves false for reasons discussed below.

To begin, a basic single-impulse formalism is presented. The Gaussian VOP equations can be modeled as a [6x3] matrix where the rows represent the six Keplerian elements and the column the radial (R), tangential (T) and normal (N) thrust acceleration components [Prus 93]:

$$
\begin{pmatrix} \dot{a} \\ \dot{e} \\ di/dt \\ \dot{\Omega} \\ \dot{\omega} \\ \dot{\chi} \end{pmatrix} = Z \cdot \begin{pmatrix} R \\ T \\ N \end{pmatrix} = \begin{bmatrix} \dfrac{2 \cdot e \cdot \sin f}{n \cdot a} & \dfrac{2}{n \cdot a} \cdot (1 + e \cdot \cos f) & 0 \\[2mm] \sqrt{p/\mu} \cdot \sin f & \sqrt{p/\mu} \cdot (\cos f + \cos E) & 0 \\[2mm] 0 & 0 & \sqrt{p/\mu} \cdot \dfrac{\cos(\omega + f)}{1 + e \cdot \cos f} \\[2mm] 0 & 0 & \sqrt{p/\mu} \cdot \dfrac{\sin(\omega + f)}{\sin i \cdot (1 + e \cdot \cos f)} \\[2mm] -\sqrt{p/\mu} \cdot \dfrac{\cos f}{e} & \sqrt{p/\mu} \cdot \dfrac{\sin f \cdot (2 + e \cdot \cos f)}{e \cdot (1 + e \cdot \cos f)} & -\sqrt{p/\mu} \cdot \dfrac{\cos i \cdot \sin(\omega + f)}{\sin i \cdot (1 + e \cdot \cos f)} \\[2mm] \dfrac{\beta \cdot (2e - \cos f - e \cdot \cos^2 f)}{(1 + e \cdot \cos f)} & \dfrac{\beta \cdot \sin f \cdot (2 + e \cdot \cos f)}{(1 + e \cdot \cos f)} & 0 \end{bmatrix} \cdot \begin{pmatrix} R \\ T \\ N \end{pmatrix}
\tag{1a}
$$

where

$$\chi = n \cdot \tau \tag{1b}$$

$$p = a \cdot (1 - e^2) \tag{1c}$$

$$\alpha = \sqrt{1 - e^2} \tag{1d}$$

$$\beta = \frac{-\alpha \cdot p}{\sqrt{\mu \cdot p \cdot e}} \tag{1e}$$

The maneuvers are impulsive and it is assumed that the VOP matrix (Z) is constant during the burn and that it depends only on the initial (*i.e.* baseline) orbital elements.

Different approaches using these equations can be envisioned. In general, the VOP equations will not provide the exact final state for a single impulse strategy, as only three control variables are available to target six final orbital elements. In the cases where the exact final state can be

achieved, portions of the **Z** matrix become identically zero, effectively decoupling some set of the Keplerian elements from the others. As an example, consider the classic case where only a normal component is performed and only change in i is desired. An arbitrary normal maneuver will cause changes in i, Ω, and ω unless the true anomaly is chosen such that $\sin(\omega + f)$ vanishes. These cases physically correspond to performing the single impulse at the point on the initial orbit where it is intersected by the final orbit. To successfully exploit this, special initial conditions for the formation, termed 'tailored', must be chosen that ensure that this intersection condition is true. While it may be possible to achieve these tailored conditions, it is not physically intuitive how to proceed. In addition, restriction of the initial conditions to satisfy the intersection condition generally implies that the probability that the final orbit also satisfies the formation metrics is smaller than if sampled from 'generic' initial conditions (where 'generic' is defined as not restricted). The reason the probability is necessarily equal to or smaller is that if solutions exist which satisfy the metrics, then the locus of these solutions must be a sub-space of the initial-conditions parameter space. Insisting that the solutions must also be a part of another sub-space to satisfy the intersection constraint requires that the two sub-spaces intersect. Obviously, if the initial conditions are not restricted, then the sub-space for the maneuver strategy is the entire initial-parameter space and intersection is assured by the above argument. As a result of this reasoning the intersection condition was dismissed as an avenue of further research.

The next avenue explored, referred to here as VOP1, was based on using the pseudo-inverse matrix (L^*) of the VOP **Z**. This least-square type inverse is the closest solution for a one-burn strategy to the system of equations expressed in (1a). The resulting ΔV is:

$$\Delta \overline{V} = \begin{pmatrix} R \cdot \Delta t \\ T \cdot \Delta t \\ N \cdot \Delta t \end{pmatrix} = L^* \cdot \begin{pmatrix} \Delta a \\ \Delta e \\ \Delta i \\ \Delta \Omega \\ \Delta \omega \\ \Delta \chi \end{pmatrix}_{target} \tag{2}$$

$$L^* = \left(Z^T \cdot Z \right)^{-1} \cdot Z^T \tag{3}$$

Note that the pseudo-inverse matrix exists only and only if the product $Z^T \cdot Z$ is non-singular. Also it should be noted that this method practically guarantees that all of the elements of the final state will only be partially achieved.

Before reporting the resulting delta-Vs for the VOP1 model, two multiple-impulse solutions are investigated. In principle, (1a) should have at least locally unique solutions for two-impulse strategies. In this case, there would be six control values (two sets of R, T, and N components) and six changes in the Keplerian elements. Solutions to this system of nonlinear, coupled equations could then be obtained using standard numerical techniques. However, this type of approach offers no physical insight into the problem nor does it offer the simplicity of the single-impulse VOP1 strategy. Since well-known techniques for solving the Lambert problem using two-impulse strategies exist, this approach was not pursued.

There is a well-defined four-impulsive burn strategy that employs some physical reasoning to set the spacecraft on the exact desired final target [Scha 00]. This method will be referred to as VOP2 in this paper. The concept elaborated by Schaub *et. al.* was to restrict the influence of each burn component (radial, tangential and normal) to only two orbital elements. The inclination change was performed by a normal burn at latitude 0° or 180° leaving both the argument of perigee and right

ascension of the ascending node unchanged. The variation in right ascension of the ascending node maneuver occurred at latitude 90°. The latter burn induced a change in argument of perigee that was corrected during the argument of perigee burn. The argument of perigee and mean anomaly were varied using two impulsive burns along the radial direction, one at perigee and one at apogee. Finally, the semi-major axis and eccentricity were modified using the same two impulsive burn strategy but along the tangential direction. In the case where the initial argument of perigee is zero, the total number of maneuvers reduces to three.

Table 2 presents a comparison of the optimal ΔV estimated by the different VOP approaches. Overall, it was observed that the pseudo-inverse method VOP1 gives a better ΔV cost than the multiple-impulse VOP2 method at the expense of not exactly achieving the desired final state. This is primarily due to the fact that making changes to specific orbital elements in a physically intuitive sequence, as in VOP2, forces the dynamics move along several orthogonal directions in the VOP parameter space. Moving in a general space in such a fashion is usually more costly and inefficient than moving along a single curve connecting the initial and final points. Note that when only one or two orbital elements were varied, the pseudo-inverse method did not provide consistent results. Indeed, the VOP matrix is highly coupled and a maneuver in one direction will vary up to four orbital elements. Therefore, the pseudo-inverse method is unlikely converge on any valid solution for simple cases and a more intuitive approach, like VOP2, is necessary.

## Table 2 Total ΔV comparison between VOP and *FreeFlyer*®

Method	$\overline{S}_o$	$\overline{S}_1$	$\overline{S}_2 / \overline{S}_3$
VOP1	20 m/s	N/A	40 m/s
VOP2	71 m/s	64 m/s	96 m/s

As discussed above, the VOP1 method generally fails to achieve any of the desired changes precisely. Thus a fair comparison between the two VOP methods can only be done after it was determined whether the deviation from the final state when using the VOP1 method was tolerable. For simplicity, only the out-of-plane spacecraft $\overline{S}_2$ was examined. The different deviations in orbital elements were plotted (Figs. 2A-B) and the pseudo-inverse method showed a "close" match with the final state for a true anomaly around 80° (i.e. all the deviations close to zero except for one orbital element). While there is no way of judging within the context of the model problem whether these inaccuracies are tolerable, in the context of the MMS mission they are unacceptable.

Up to this point, the pros and cons associated with the VOP methods for initializing individual spacecraft have been discussed. This section is concluded with a note about a cooperative timing in the VOP methods. Some reflection on the nature of the VOP methods points out that there is a problem associated with initializing the formation as a whole. Since the time of flight does not figure into the VOP equations, it is unclear how to conveniently specify the desired final state (i.e. the initial state of the formation) simultaneously with the desired final time. This means that although the desired final orbit state with the correct orbital elements may be obtained, the state may not be achieved at the appropriate epoch. Since the cooperative dynamics of a formation depends on the spacecraft all being in a specific point on their respective orbits at a given time this presents a serious problem to which we currently have no general solution or mitigation for VOP related methods.
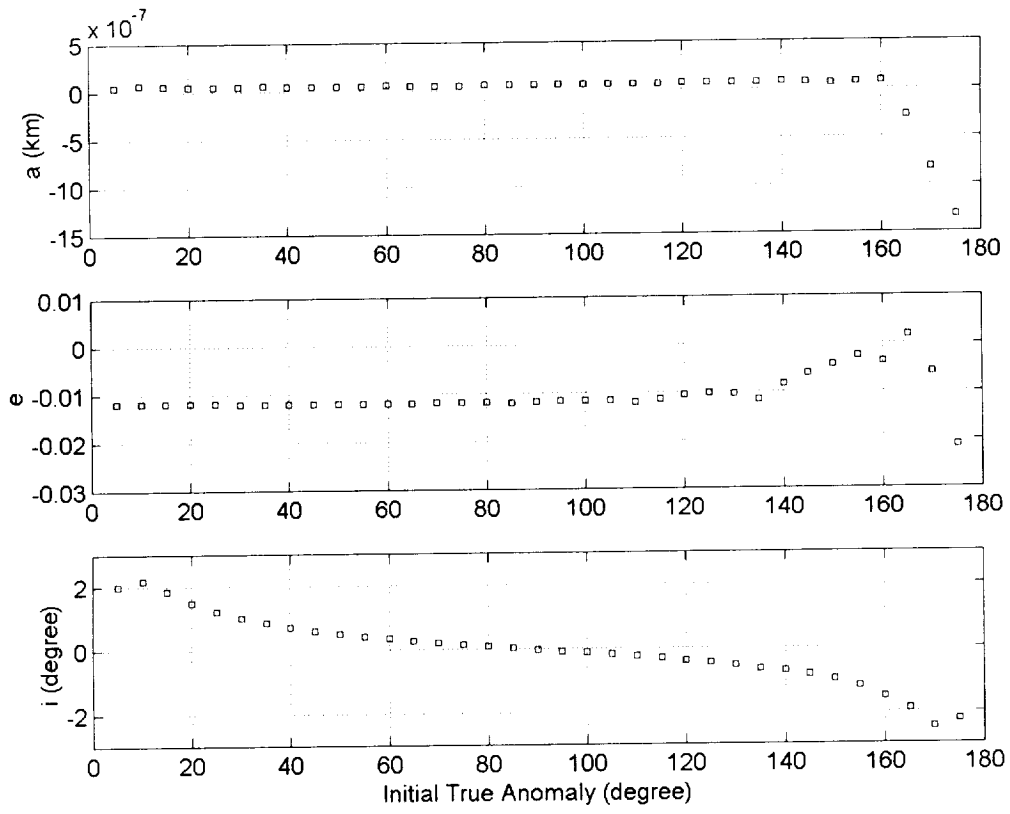
Fig. 2A: Deviation from Desired Change in Orbital Elements vs. f_o using VOP1. *Upper Plot*: Semi-Major Axis (km); *Middle Plot*: Eccentricity; *Bottom Plot*: Inclination (degree)
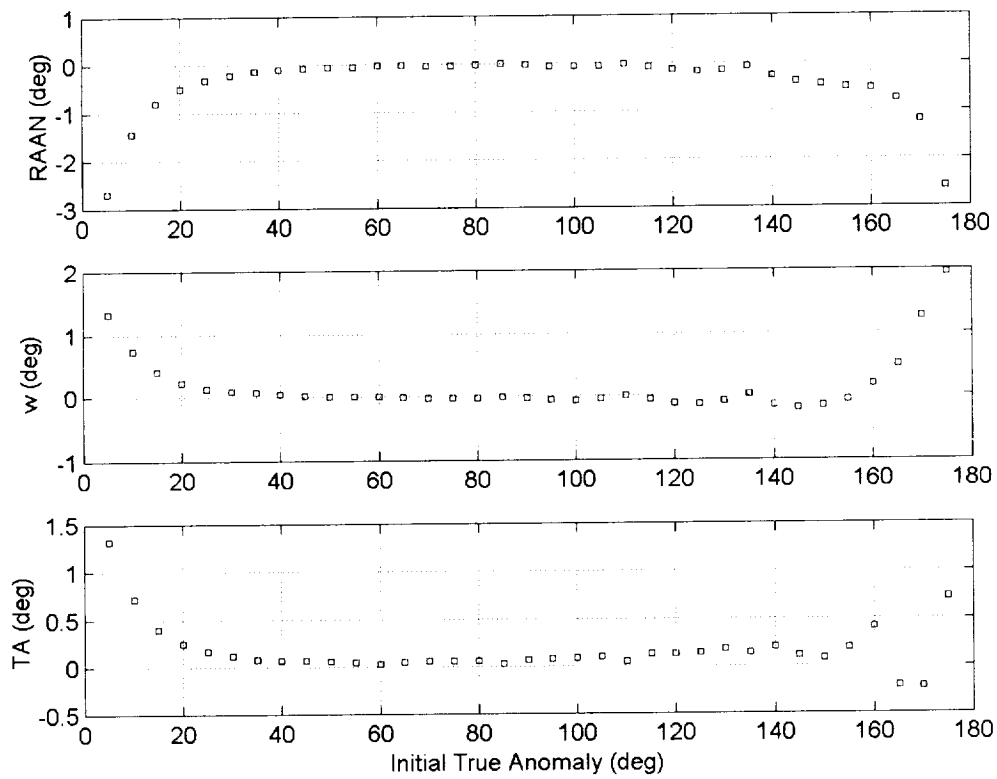
Fig. 2B: Deviation from Desired Change in Orbital Elements vs. f_o using VOP1. *Upper Plot*: Right Ascension of the Ascending Node (deg); *Middle Plot*: Argument of Perigee (deg); *Bottom Plot*: True Anomaly (deg).

4 - THE STATE TRANSITION MATRIX

In this section a general two-impulse maneuver strategy based on the Lambert problem expressed in a Cartesian state formulation [Pru 93] is investigated. Under the general theory, Cartesian offsets δr and δv about the baseline orbit at an arbitrary time are related to the initial offsets δr_0 and δv_0 by:

$$\begin{pmatrix} \delta r \\ \delta v \end{pmatrix} = \phi(t, t_o) \cdot \begin{pmatrix} \delta r_o \\ \delta v_o \end{pmatrix} \tag{4}$$

where $\phi(t, t_0)$ is the state transition matrix. To solve the Lambert problem, the matrix is partitioned into four [3x3] matrices:

$$\phi(t, t_o) = \begin{pmatrix} M & N \\ S & T \end{pmatrix} \tag{5}$$

The final relative position and velocity are expressed as functions of the initial relative position and velocity:

$$\delta r = M \cdot \delta r_o + N \cdot \delta v_o \tag{6}$$

$$\delta v = S \cdot \delta r_o + T \cdot \delta v_o \tag{7}$$

If the spacecraft is assumed to initially lie in the baseline orbit then $\delta r_o \big|_{init} = 0$. The two impulses needed to rendezvous with the final state are then immediately calculated to be:

$$\Delta V_{initial} = N^{-1} \cdot \delta r_f - \delta v_o \big|_{init} \tag{8}$$

and

$$\Delta V_{final} = \delta v_f \big|_{final} - T \cdot N^{-1} \cdot \delta r_f \tag{9}$$

Since our aim is to initialize a formation about an elliptical baseline orbit, the Hill-Clohessy-Wiltshire approximation to the state transistion matrix (STM) is not applicable. However, there exist many forms for general STMs which are valid for elliptical orbits. An STM formulation by Der [Der 97] was used in this work. Der's formulation is expressed in terms of universal variables, and has the advatage that it is valid for arbitrary conics. Figure 3 presents a comparison of the ΔV between the estimate based on the Der analytic solution to the Lambert problem and the corresponding *FreeFlyer*® targeter simulation for all four spacecraft. Despite the linearization used to solve the Lambert problem the analytic model using the Der STM matched the *FreeFlyer*® results almost perfectly. Thus, the Der matrix could be used stand-alone as a relatively accurate tool to provide the ΔV budget for initializing the formation to arbitrary initial conditions.

As previously mentioned, valid formation dynamics depend on getting the member spacecraft into the correct orbits at the correct time. A constraint was applied to the time of flight and the targeted positions to ensure the proper relative motion is achieved. The strategy required the same fixed epoch for all the final states, regardless of the epoch of the initial state. This puts a constraint on the time of flight that is dependent on the epoch of the initial maneuver. This method guaranteed that the desired formation metrics were unambiguously achieved. It is worth noting that another possible strategy exists, in which the time associated with the final states 'floats'. Each final state would then be specified at an independent epoch and when propagated to a common time would then form the desired formation. The formation would not be considered 'initialized' until the last spacecraft joined the array. Such a strategy may afford delta-V savings but is beyond the scope of the current study.
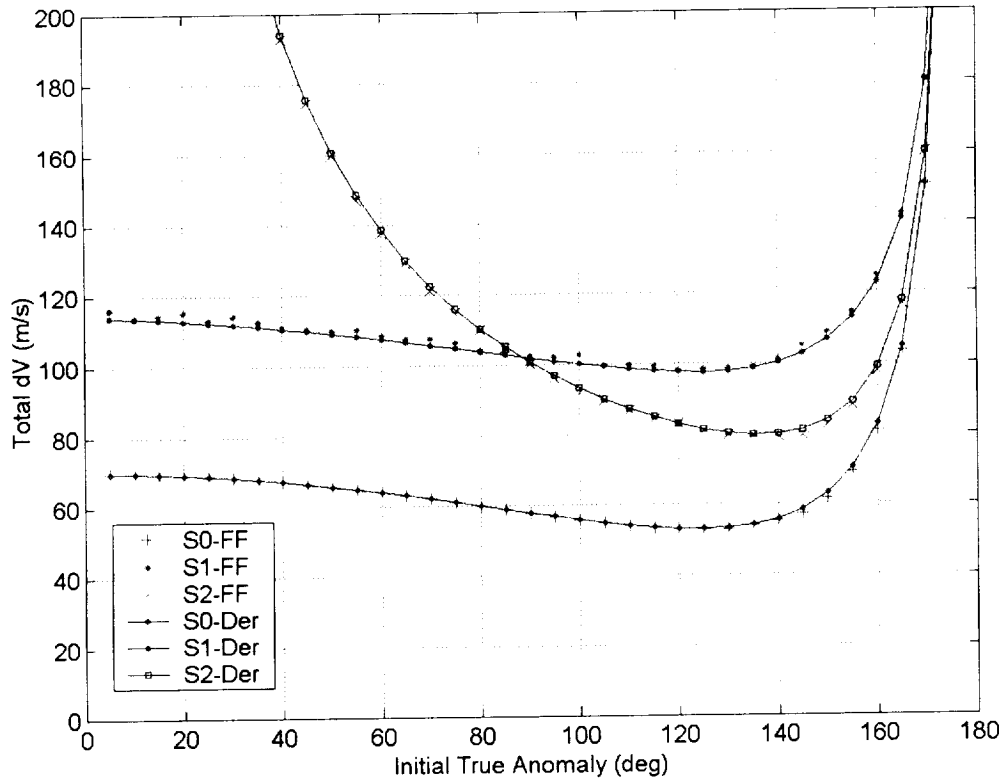
Fig. 3: ΔV vs. f_o comparison between Der's STM and *FreeFlyer*®.

5 - CONCLUSIONS

Two methods were investigated to estimate the maneuvers to initialize the formation. Strategies based on the Gaussian variation of parameters (VOP) were highly dependent on the initial conditions. Single-impulse VOP strategies lacked the ability to deal with general initial conditions directly. An attempt to use a pseudo-inverse method (VOP1) showed some success but the method was generally too inefficient to use in practice. Multiple-impulse VOP methods also exist but are more complex, lacking a great deal of physical intuition. In particular a four-impulse method (VOP2) based on the work of Schaub was examined. The method was found best suited for a select class of formation initial conditions in which only a few of the Keplerian elements changed relative to the baseline. Because the initial conditions in this class were subject to severe constraints, they had a lower probability of achieving the mission imposed formation metric as well. For more general initial states, the VOP2 method led to a fairly high ΔV cost. In addition, all the VOP methods suffered from too few restrictions on the time-of-flight. Ambiguities arose as to how ensure that each member of the formation arrived at the right state at the correct time. The second method considered involved solving a Lambert problem using the Der state transition matrix. In general, it exhibited good accuracy at a relatively low ΔV expense and the timing constraints associated with initializing the formation were easily satisfied. In addition, this closed form solution showed a good match with the *FreeFlyer*® numerical targeter.

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