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New Third–Order Moments for the PBL

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Abstract

Turbulent convection is inherently non–local and a primary condition for a successful treatment of the PBL is a reliable model of non–locality. In the dynamic equations governing the convective flux, turbulent kinetic energy, etc, non–locality enters through the third–order moments, TOMs. Since the simplest form, the so–called down gradient approximation (DGA), severely underestimates the TOMs (by up to an order of magnitude), a more physical model is needed. In 1994, an analytical model was presented which was derived directly from the dynamical equations for the TOMs. It considerably improved the DGA but was a bit cumbersome to use.

Here, we present a new analytic expression for the TOMs which is considerably simpler than the 1994 expression and which at the same time yields a much better fit to the LES data.

I. Introduction.

The search for a reliable expression for the third–order moments to be used in the dynamic equations for the second–order moments such as the turbulent kinetic energy, convective fluxes, etc, has a long history. For many years, people used the so–called down gradient approximation but the advent of LES (large eddy simulation) showed that said model severely underestimates the true value of the TOMs (Moeng and Wyngaard, 1989)

Prompted by those results, Canuto et al.(1994) undertook the task of solving directly the dynamic equations for the TOMs themselves thus avoiding the use of phenomenological expressions. The first merit of the new expressions was to show that all the TOMs are a linear combination of the gradients of all the second–order moments and not only of selected ones, as assumed in the down–gradient approximation. From the performance viewpoint, the TOMs were tested against the results from three LES and the general agreement was satisfactory. Zilitinkevich et al. (1999) have however pointed out that the performance of $\overline{w^2\theta}$ and $\overline{w\theta^2}$ was not as good as that of the other TOMs and thus improvements were needed. In addition, the new TOMs though analytic, were a bit cumbersome to use. For these reasons, we have been motivated to find new expressions for the TOMs which are at the same time simpler and which yield better results than the 1994 expressions.

II. The New Third–Order Moments

Even though the down–gradient approximation gives rise to a non–local convective flux, it is known that it poorly represents the full TOM's and thus a better model is needed. The general dynamic equations for the six TOMs

$$\overline{w^3},\ \overline{q^2w},\ \overline{w^2\theta},\ \overline{w\theta^2},\ \overline{\theta^3},\ \overline{q^2\theta} \tag{1}$$

where $q^2=u^2+v^2+w^2$, u,v,w and θ are the fluctuating velocity and temperature fields, are given by Eqs.(37a–g) of CD98. A first solution, obtained with symbolic algebra, was presented in Canuto et al. (1994)..

In this paper, we present a new solution of the stationary limit of the dynamic equation for the TOMs given by Eq.(17–22) of Canuto et al.(1994). Since the different TOMs in (8a) have different dimensions, we shall multiply the first four by appropriate factors so that all TOMs have the dimensions of a velocity cubed. The solutions can be expressed as follows:

$$x_1 \equiv g\alpha\tau_V\overline{w^2\theta} = X_0 z - X_1 \tag{2}$$

$$x_2 \equiv (g\alpha\tau_V)^2\overline{w\theta^2} = Y_0 z - Y_1 \tag{3}$$

$$x_3 \equiv (g\alpha\tau_V)^3\overline{\theta^3} = Z_0 x_2 - Z_1 \tag{4}$$

$$x_4 \equiv g\alpha\tau_V\overline{q^2\theta} = \lambda x_5 + \mu x_2 + \nu \tag{5}$$

$$x_5 \equiv \overline{wq^2} = \Omega_0 z - \Omega_1 \tag{6}$$

$$z \equiv \overline{w^3} = \frac{A}{B} \tag{7}$$

with

$$X_0 = \gamma_2 N^2(1-\gamma_3 N^2)\delta \tag{8a}$$

$$X_1 = [\gamma_0 f_0 + \gamma_1 f_1 + \gamma_2(1-\gamma_3 N^2)f_2]\delta \tag{8b}$$

$$\delta = [1-(\gamma_1+\gamma_3)N^2]^{-1}, \tag{8c}$$

$$\gamma_0 = \frac{3}{8c^2}\frac{de}{c-2},\quad \gamma_1 = \frac{d}{2c^2},\quad \gamma_2 = \frac{1}{2c},\quad \gamma_3 = \frac{2c}{d}\gamma_0 \tag{8d}$$

$$N^2 \equiv g\alpha\beta\tau_V^2,\qquad \tau_V \equiv \tau[1+\lambda_0 g\alpha\beta\tau^2]^{-1},\quad \tau = 2K\epsilon^{-1} \tag{8e}$$

where K and ϵ are the kinetic energy and its rate of dissipation and where $\lambda_0=0.04$ for $\beta>0$ and $\lambda_0=0$ if $\beta<0$, with $\beta=-\partial T/\partial z$. The constants have the values c=7, d=26/15, e=4/5. Furthermore:

$$Y_0 = 2\gamma_2 N^2(1-\gamma_3 N^2)^{-1}X_0,\quad Y_1 = 2\gamma_2(1-\gamma_3 N^2)^{-1}(N^2 X_1 + y_1) \tag{9a}$$

with

$$y_1 = \gamma_0\gamma_1^{-1}f_0 + f_1 \tag{9b}$$

$$Z_0 = \frac{3}{2}(c-2)^{-1}N^2,\quad Z_1 = \frac{3}{2}(c-2)^{-1}f_0 \tag{9c}$$

$$\lambda = \frac{1}{2c}N^2,\quad \mu = c^{-1},\quad \nu = -c^{-1}f_3 \tag{9d}$$

$$\Omega_0 = \omega_0 X_0 + \omega_1 Y_0 \tag{10a}$$

$$\Omega_1 = \omega_0 X_1 + \omega_1 Y_1 + \omega_2 \tag{10b}$$

with

$$\omega_0 = \gamma_4 (1-\gamma_5 N^2)^{-1}, \quad \omega_1 = (2c)^{-1}\omega_0, \quad \omega_2 = \omega_1 f_3 + e^{-1}\omega_0 f_4 \tag{10c}$$

where

$$\gamma_4 = \frac{e}{c+5/3}, \quad \gamma_5 = \frac{1}{4c}\gamma_4 \tag{10d}$$

Finally,

$$A = \Omega_1 - \frac{3e}{2} X_1 - \frac{3}{2} f_5 \tag{11a}$$

$$B = c - \frac{3e}{2} X_0 + \Omega_0 \tag{11b}$$

The functions $f_{0,..,5}$ depend on the second–order moments and are given by:

$$f_0 = (g\alpha)^3 \tau_V^4 J \frac{\partial \overline{\theta^2}}{\partial z} \tag{12a}$$

$$f_1 = (g\alpha)^2 \tau_V^3 \left(J \frac{\partial J}{\partial z} + \tfrac{1}{2}\overline{w^2}\frac{\partial \overline{\theta^2}}{\partial z}\right) \tag{12b}$$

$$f_2 = g\alpha\tau_V^2 J \frac{\partial \overline{w^2}}{\partial z} + 2g\alpha\tau_V^2 \overline{w^2}\frac{\partial J}{\partial z} \tag{12c}$$

$$f_3 = g\alpha\tau_V^2 \left(\overline{w^2}\frac{\partial J}{\partial z} + J \frac{\partial K}{\partial z}\right) \tag{12d}$$

$$f_4 = \tau_V \overline{w^2}\left(\frac{\partial \overline{w^2}}{\partial z} + \frac{\partial K}{\partial z}\right) \tag{12e}$$

$$f_5 = \tau \overline{w^2} \frac{\partial \overline{w^2}}{\partial z} \tag{12f}$$

All the functions $f_{0\cdots,5}$ have dimensions of velocity cubed.

III. Test of the new TOM vs LES data.

In Fig.1 we present the comparison of the new TOMs vs. LES data. We also show the down–gradient approximation to highlight its difference with the full treatment. The results in Fig.1 were obtained using the LES data to compute the second–order moments Eq.(12a–f), K and τ=2K/ϵ and β(z). The new TOMs were then obtained using Eqs.(2)–(7). The agreement between the model results and the LES data is quite satisfactory.

IV. New Physical ingredient

The superior agreement obtained with respect to the 19994 model is of course only partly due to the simpler analytical form of the TOM which has allowed us to easily find the best set of parameters (especially c) to fit the data. Due to its rather rigid nature, the 1994 model did not allow the same freedom. However, the main reason is one of physical origin: we have abandoned the quasi–normal approximation used to treat the fourth–order moments in favor of

$$\overline{abcd} = f(\overline{ad}\,\overline{cd} + \overline{ac}\,\overline{bd} + \overline{ad}\,\overline{bc}) \tag{12g}$$

where f is no longer a constant as in 1994. This has two advantages: since the fourth–order moments enter the problem under a divergence, even when the second–order moments are constant, the non–zero derivative of the function f makes the TOM different than zero, a behavior that was not possible within the standard approximation. Second, by choosing the function f to be a function of the Brunt–Vaisala frequency, which is the most obvious suggestion and in its simplest form, one indeed obtains a considerable improvement in the behavior of the TOM. The new treatment of the fourth–order moments is represented here by the function τ_V given in (8e): taking λ_0=0 reduces the problem to the f=1 case.

V. Conclusions.

The results presented in Fig.1 satisfy the two requirements set out at the beginning: the expressions for the TOMs are considerably simpler than those of 1994 and their performance is better. In fact, the large values of $\overline{w^2\theta}$ and $\overline{w\theta^2}$ that characterized the 1994 solutions are no longer present and a much better fit is obtained.

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Figure Caption

Fig.1. The third–order moments Eq.(1) vs. z/z_i. The LES results of Zilitinkevich et al. (1999) are plotted as dotted lines (the LES data did not contain the $\overline{q^2\theta}$ value); the down–gradient approximation (DGA) model is plotted as dashed lines while the present model, Eqs.(2–7), yields the results plotted as solid lines. As well known, the DGA severely underestimates the third–order moments. All TOMs are normalized with Deardorff convective scales w_*(=2ms^{-1}) and θ_*(=0.12k). The value of z_i is 1010m.

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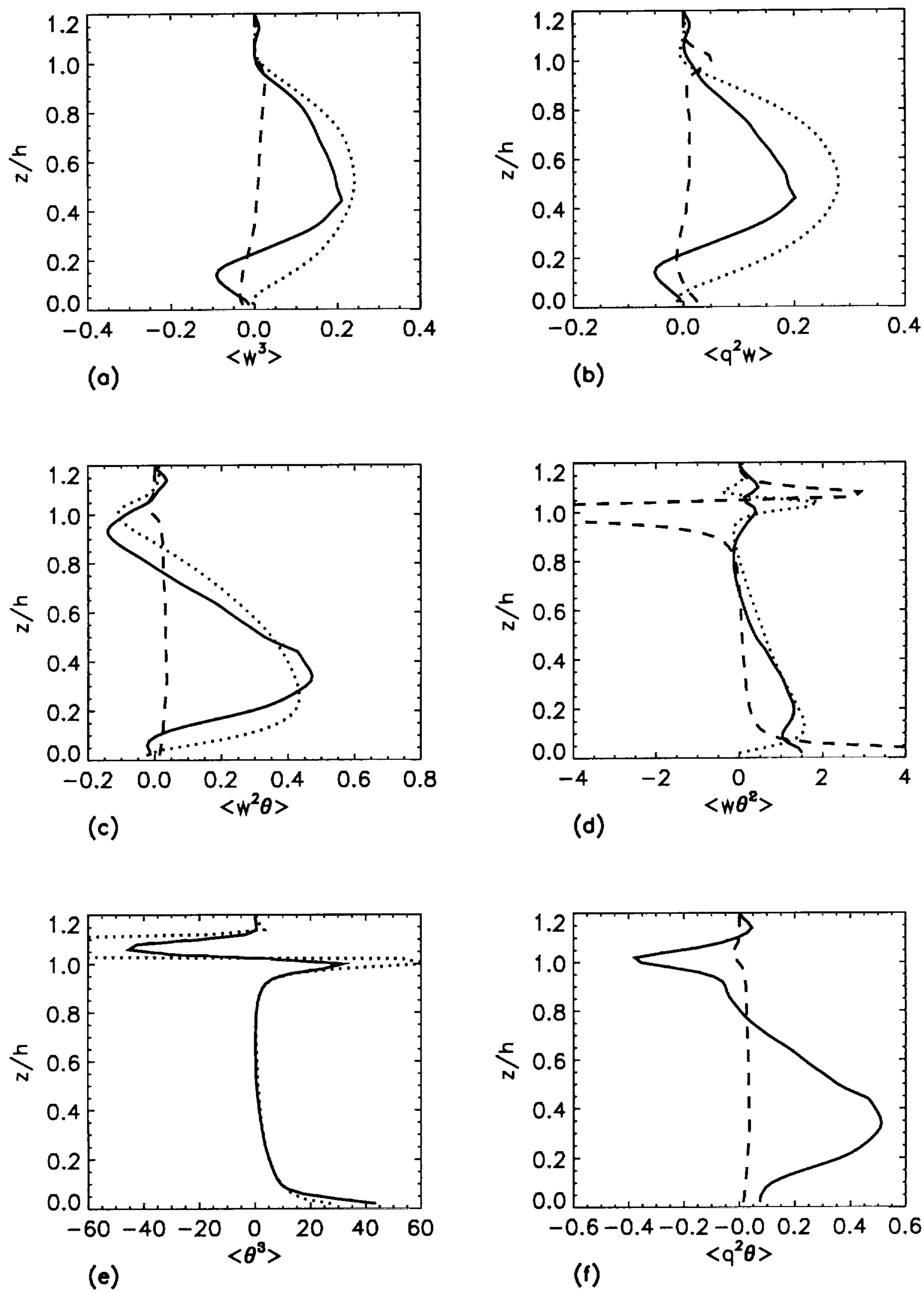

Fig.1