Title: Further Investigation of Receding Horizon-Based Controllers and Neural Network-Based GPC Systems

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Summary

This report provides a comprehensive summary of the research work performed over the entire duration of the co-operative research agreement NAG-1-2234 between NASA Langley Research Center and Kansas State University.

This summary briefly lists the findings and also suggests possible future directions for the continuation of the subject research in the area of GPC and NGPC.

Brief Summary

Model Predictive Control

History of Model-based Predictive Controllers(MPC) goes back to late 70's when the process industry showed a keen interest in using these control methods. The control formulation at the time was mainly heuristic and algorithmic [1-2], and exploited the increasing potential of digital processors. These controllers were closely related to the minimum time optimal control methodology. The receding-horizon principle which is central to the most of the MPC algorithms came about as early as early 60's [3]. MPCs became quite popular in the process industries where computational speed was not a major concern. Also, many MPC algorithms were used on multivariable systems with constraints but no formal proofs of stability or robustness were available. Another parallel development took place using ideas from adaptive control which led to the development of self-tuning controllers [4] and extended horizon adaptive controllers (EHAC)[5]. This continued evolution of MPCs led to the emergence of the Generalized Predictive Control (GPC) methodology in late 80's [6] which incorporates all major features of the predictive controllers in a unified framework. The various versions of the same common idea give rise to the following different types of predictive controllers: Multistep Multivariable Adaptive Control (MUSMAR)[7], Multipredictor Receding Horizon Adaptive Control (MURHAC) [8], Predictive Functional Control (PFC) [9], and Unified Predictive Control (UPC)[10].

MPC has also been formulated in the state-space setting [11], which not only allows the use of well established state-space theories for analysis but also provides the ease for extensions to multivariable systems. Moreover, it facilitates the use of stochastic theories and treatment of actuator/sensor noise. The well developed estimation theory from state space methods can be easily incorporated without much complication. The perspective gained by working in these different domains made it possible to devise some simple tuning rules for ensuring stability and robustness for MPC systems. *As a simple analogy, MPC controller can be viewed as an observer-based controller wherein its stability, performance, and robustness is determined by the observer dynamics, which can be fixed by adjustable parameters, and regulator dynamics, determined by MPC parameters such as weightings, horizon lengths, etc.* Although, in [12],

some specific stability theorems are given for GPC using the state-space setting the general stability results for GPC were lacking. Recently, in 90's, stability of GPC under end-point constraints was shown in [13],[14], where the equality constraints were imposed on the output after a finite horizon. The work in the robustness area for GPC has mainly hinged on the explicit modeling of the uncertainties and designing the controller for the worst-case scenario.

II. Research issues in MPC

The existing technology of MPCs is not matured enough even for the case of linear systems for aerospace industry to use in its current form. The limitations of industrial MPC technology are summarized in [15]. Much work still needs to be done in this area. Following are some of the main issues that need further investigation.

Over-parameterization:

Guidelines for choosing a "minimal" representation of the system (in the parameter space) is still an issue which needs further investigation. Most of the commercial products use the step or impulse response models of the system that are known to be over parameterized. Moreover, such models are not valid for unstable systems and systems with integrators.

Optimization of cost function:

The high computational cost is one of the inherent drawbacks of the MPCs. In an effort to minimize this cost many optimization routines used in MPC computations are designed to yield sub-optimal solutions rather than the optimal solution. Also, it is not known how close this sub-optimal solution is to the optimal solution. In certain situations these sub-optimal solution may not be acceptable. In high-speed application on the other hand there may not be any other choice than to accept such solutions.

Uncertainties:

Since the crux of the MPC lies in the accuracy of the predictor model it is very important that the predictor model is obtained very carefully and as accurately as possible. Since, in most cases, analytically derived models have high degree of approximation, system identification techniques are used to minimize these errors. However, it is difficult to use such techniques for open-loop unstable systems. Moreover, despite the use of best available modeling techniques existence of modeling errors and parametric uncertainties is unavoidable. This necessitates

systematic methodology for handling uncertainties in the system. The techniques of handling uncertainties in MPC framework are still under development and warrant more work.

Tuning parameters:

Although there has been some attempt in devising rules for picking tuning parameters of MPC, the correlation of tuning parameters and closed-loop behavior is not very clear. Only empirical results are available (for example, see our earlier work in [16]). Use of tuning parameters for providing robustness and stability is still the subject of research. Moreover, tuning under constraints is another challenging problem that needs further investigation.

Stability and robustness analysis:

There are some recent results on the stability of MPCs, however, the results are limited to a very restricted class of systems under nominal conditions. The issue of stability robustness is wide open and needs much attention.

III. Nonlinear MPC and neural networks

The central idea of MPC doesn't assume that the system to be controlled has to be linear. That means, conceptually the idea of MPC architecture can be extended to nonlinear systems, as well. However, this extension is not that trivial. There are many open issues due to nonlinear nature of the plant:

Model: The availability of a "good" nonlinear plant model is the main problem. The identification techniques are not advanced for nonlinear systems as they are in the linear case. Neural networks offer one possible solution to this problem.

Theoretical basis: The theoretical foundation for analysis of stability and robustness of nonlinear MPCs is still in its infancy and needs significant work.

Computational complexity: The computational burden for nonlinear MPC is considerably higher compared to linear case which prohibits their use in real-time applications.

Neural networks as predictors:

Over the last decade, Artificial Neural Networks (ANN) have gained increasing attention of researchers from various fields. In past, neural networks were mainly used in pattern recognition and function approximation problems. However, in subsequent years, the range of neural networks applications has considerably expanded. The application area of our interest

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is the control of dynamical systems. In particular, the focus will be on the use of neural networks in system identification, modeling, and control. Applicability of neural networks in these areas is due to their approximation capabilities. The ability of ANN to approximate any nonlinear function to arbitrary precision is central to their use in controls. It is this key property of neural networks which makes ANNs a viable tool for identification, modeling, and control of dynamical systems. The structure of multilayer neural network comprising nonlinear activation functions, feedback mechanism, and use of delay nodes makes it possible to model any dynamical system. The key result, which states that the multilayer feedforward networks with only one hidden layer are capable of approximating any continuous function on a compact set in a very precise sense, was proved independently by many researchers. This result is based on the famous Stone-Weierestrass theorem for approximation of a function. Although neural networks with only one hidden layer were proved to be the universal approximators, no result exists to date which gives the theoretical basis for selecting the number of hidden nodes required (or equivalently the number of basis functions required) in the hidden layer.

Artificial neural networks have been shown to perform well in the identification and control of linear time-invariant (LTI) systems. Because of their ability to learn any nonlinear map, they can be effectively used for identification and control of nonlinear dynamical systems as well. In general, for nonlinear systems, the control theory is not as developed as it is for the linear systems; only systematic methods available to date for analysis of such systems are Lyapunov-based techniques. In recent years, some researchers have demonstrated the use of neural networks in the control of nonlinear systems. One drawback with the existing neural network literature for nonlinear systems is that the results are based mostly on empirical methods and no comprehensive theoretical foundation is available. In addition, most of the reported methods are ad-hoc and are applicable for a small class of systems.

IV. Research issues in ANN

The function approximation capability of ANNs can be exploited to use them as predictors in MPC architecture for nonlinear systems. When MPC uses neural network as a plant model, the resulting MPC architecture will be called as Neural MPC (NMPC). In particular, if ANN is used in the GPC framework it will be referred to as NGPC. In our earlier work [16],

control of flexible joint link using NGPC was accomplished. Some theoretical as well as some empirical results were also given for tuning of NGPC parameters. A methodology for modeling uncertainty was also presented for linear as well as nonlinear systems. Use of ANNs in MPC architecture has several issues that need to be researched. Some of these issues are specific to ANNs as predictors and some others relate to assessment of their stability and their role in MPC framework.

Type and size of ANN:

There doesn't exist any result which gives a clear choice of the type and size of ANN for a particular application which will give a minimal representation of the system with prescribed prediction accuracy. The choice of number of hidden layers/nodes and activation function also becomes important factor in ensuring the stability of learning dynamics of the network. The problem becomes even more involved if the on-line learning is required.

Stability of MPC with ANN as predictor:

Stability of NMPC is probably the most difficult challenge at this stage of research in MPC and ANN. It is important that the stability issues related to ANN and MPC are thoroughly understood before the stability of NMPC can be assessed.

IV Accomplishments and future directions

During the period of one year (06/13/99-06/12/00) following tasks were accomplished.
(a) Stability analysis: the work completed includes characterization of stability of receding horizon-based MPC in the setting of LQ paradigm. It has been shown that finite horizon LQ formulation can be used to analyze unconstrained and constrained receding horizon MPC problems. In the case of constrained problem, however, stability is dependent on solution trajectories of finite horizon problem reaching into the feasible region where the infinite horizon LQ problem has finite solution [17].

The current work-in-progress includes analyzing local as well as global stability of the closed-loop system under various nonlinearities; for example, (i) actuator nonlinearities (ii) sensor nonlinearities, and (iii) other plant nonlinearities. Actuator nonlinearities include three major types of nonlinearities: saturation, dead-zone, and $(0, \infty)$ sector.
(b) Robustness analysis: It is shown that receding horizon parameters such as input and

output horizon lengths have direct effect on the robustness of the system. It has been shown empirically that in most cases parametric uncertainties can be handled by increasing the output horizon.

(c) Code development: A matlab code has been developed which can simulate various MPC formulations. This code can facilitate comparison of various MPCs and will also serve as a validation tool for NNET software. The current effort is to generalize the code to include ability to handle all plant types and all MPC types.

(d) Improved predictor: It is shown that MPC design using better predictors that can minimize prediction errors. It is shown analytically and numerically that Smith predictor can provide closed-loop stability under GPC operation for plants with dead times where standard optimal predictor fails.

(e) Neural network predictors: When neural network is used as predictor it can be shown that neural network predicts the plant output within some finite error bound under certain conditions. Our preliminary study shows that with proper choice of update laws and network architectures such bound can be obtained. However, much work needs to be done to obtain a similar result in general case. Future work will address this issue.

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