IMPACT OF ENERGY GAIN AND SUBSYSTEM CHARACTERISTICS ON FUSION PROPULSION PERFORMANCE

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Abstract

Rapid transportation of large payloads and human crews to destinations throughout the solar system will require propulsion systems having not only very high exhaust velocities ($I_{sp} \geq 10^4$ to 10^5 sec) but also extremely low mass-power ratios ($\alpha \leq 10^{-1}$ kg/kW). Such low α are difficult to achieve with power-limited propulsion systems, but may be attainable with fusion and other high-I_{sp} nuclear concepts that produce energy within the propellant. The magnitude of this energy gain is of fundamental importance. It must be large enough to sustain the nuclear process while still providing a high jet power relative to the massive power-intensive subsystems associated with these types of concepts. This paper evaluates the energy gain and mass-power characteristics required for α consistent with 1-year roundtrip planetary missions ranging up to 100 AU. Central to this analysis is an equation for overall system α, which is derived from the power balance of a generalized "gain-limited" propulsion system. Results show that the gain required to achieve $\alpha \sim 10^{-1}$ kg/kW with foreseeable subsystem technology can vary from 50 to as high as 10,000, which is 2 to 5 orders of magnitude greater than current state-of-the art. However, order of magnitude improvements in propulsion subsystem mass and efficiency could reduce gain requirements to 10 to 1,000 — still a very challenging goal.

Nomenclature

Variables

I_{sp}	= Specific impulse
α	= Overall system mass-power ratio (similar but not same as specific mass)
G	= Energy gain in nuclear process
ΔV	= Mission velocity increment
$\hat{\alpha}$	= Subsystem mass-power ratio
η	= Subsystem efficiency
A	= Starting point of arbitrary straight-line trajectory
B	= Ending point of arbitrary straight-line trajectory
τ	= Trip time
D	= Distance (between A and B)
g	= Gravitational acceleration at Earth surface
T	= Thrust
m_{A1}	= Initial vehicle mass at point A
m_B	= Vehicle mass after trip from A to B
m_{A2}	= Final vehicle mass after round trip return to A
R_{AB1}	= Outbound mass ratio m_{A1}/m_B
R_{BA2}	= Inbound mass ratio m_B/m_{A2}
V	= Velocity
$\dot{m}$	= Mass flow rate
U	= Dimensionless velocity ratio
h,k	= Constants in trip time expressions
m_{pay}	= Payload mass
P	= Power
β	= Propellant tankage mass fraction
m_{prop}	= Propellant mass
V_e	= Exhaust velocity
λ_{pay}	= Payload to vehicle inert dry mass

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fraction

X, Y, Z = Parameter combinations in trip time expressions

e = Fractional power from onboard power supply

f = Fraction of power used for reaction driver subsystem

G_{MIN} = Minimum energy gain for net positive power

Φ = Mass-power ratio factor

g_G = Ratio of Gain to Minimum Gain

f_α = Driver to processor ratio

Subscripts

$(\hat{\alpha}, \eta, m, P)_S$ = power supply

$(\hat{\alpha}, \eta, m, P)_D$ = driver

$(\hat{\alpha}, \eta, m, P)_P$ = power processor

$(\hat{\alpha}, \eta, m, P)_T$ = thruster

$(\hat{\alpha}, \eta, m, P)_H$ = heat disposal subsystem

Introduction

The goal of rapid interplanetary space flight will require the development of new propulsion systems based on advanced forms of nuclear energy. Over the last several decades, many propulsion concepts have been studied which would enable multi-month round-trips to Mars and missions to the furthest outer planets on the order of a year.[1-7] The large ΔV and vehicle accelerations required for such missions demand propulsion systems having not only very high exhaust velocities ($I_{sp} \geq 10^4$ to 10^5 sec) but also extremely low mass-power ratios ($\alpha \leq 10^{-1}$ kg/kW).

High-energy electric propulsion systems could achieve the performance necessary for multi-month transits to Mars and near-earth asteroids. Such power-limited concepts can also provide the I_{sp} required for extremely large-ΔV missions. However, the α may be too high to achieve the accelerations necessary for rapid excursions to the outer solar system. Faster missions on the order of a year will probably require systems that produce more jet power in the exhaust than that provided from onboard sources.

Such a net energy gain can be achieved in systems where nuclear reactions occur within a portion of the propellant. This energy gain is expressed in terms of G, defined as the effective power output from the local nuclear process divided by the power required to drive it. A significant amount of power is needed to "drive" such reactions, and the total produced must be sufficient to provide thrust and sustain the nuclear process. This type of system is "gain-limited" in that the driver can consume a significant fraction of the total power produced. Nuclear fusion is the most familiar example of a "gain-limited" system, although some concepts involving fission also fit the definition.

A major question often raised is: what is the magnitude of energy gain required to realize the benefits of fusion and gain-limited propulsion? This is important because estimating the upper limits of energy gain has been a central issue in fusion research over the last several decades, and it certainly plays a major part in dictating the near-term viability of fusion-based propulsion.

In recent years, high-energy plasma experiments have achieved "scientific" gains Q of up to 0.5 for very short times with large, specialized facilities. For ground-based commercial power, it is generally accepted that "engineering" Q of over 50 will be required to make fusion economically competitive with other technologies. However for space propulsion, there is much less consensus on what values of gain will ultimately be required to make fusion superior to other high-I_{sp} alternatives, such as electric propulsion. We attempt to answer this question by investigating the effect of gain and propulsion subsystem characteristics on the overall α of fusion-propelled spacecraft, and comparing these values against the α requirements for ambitious interplanetary missions.

Definition of Gain-Limited Propulsion

Almost all propulsion systems in use today are classified as being either energy- or power-limited. The former type, which is best represented by chemical rockets, derives *all* of its propulsive energy from exothermic reactions within the propellant. Total impulse and overall performance is limited by the quantity of propellant carried onboard the spacecraft. These systems typically exhibit very low exhaust velocities, but can deliver high accelerations due to their ability to heat large amounts of propellant quickly and efficiently.

Power-limited systems, such as electric propulsion, utilize a separate onboard power source to impart energy to the propellant. Although the jet power is always less than that provided from the source, power-limited concepts can achieve high exhaust velocities and I_{sp}. Unlike chemical systems, the total impulse is limited by available power, and performance is further constrained by the lower limits of attainable α. High values of α translate into more massive systems, which yield correspondingly lower accelerations and longer trip times.

Fusion and other gain-based systems share some of the features of energy and power-limited systems, but are ultimately restricted by the net energy produced by the nuclear process, hence the term "gain-limited." Because of the small burnup fraction of reactants – even at high gain $G > 100$ – only a small portion of the available binding energy in the nuclear fuel is utilized, and the system is relatively independent of the total energy content of the reactants. Similarly, the total power system mass is not directly proportional to exhaust power.

As gain increases, the fraction of delivered power that must be diverted back into the driver decreases. In effect, this reduces the impact of driver mass and efficiency on the system. This is important because the driver usually has a higher mass-power ratio $\hat{\alpha}_D$ and lower efficiency η_D than the other subsystems. The performance of a gain-limited system can be improved by either reducing the $\hat{\alpha}$ of its various power-intensive subsystems or increasing G in the nuclear process.

Mission Requirements and Performance

The performance of power- and gain-limited propulsion systems is characterized by the parameters α, I_{sp} and vehicle acceleration. These parameters are interdependent in that only two can be independently specified at once. If vehicle acceleration is treated as a dependent variable, then the objective is to understand how different combinations of α and I_{sp} affect mission performance.

We limit our consideration to performance requirements representative of fast, crewed interplanetary missions throughout the solar system. We are primarily interested in conducting round-trip missions between points A and B in as short of time as possible. These requirements are embodied in the two parameters of trip time, τ_{RT}, and the distance between points A and B, D_{AB}. Simple equations for τ_{RT} as a function of D_{AB}, α and I_{sp} are obtained by assuming (1) an instantaneous vehicle acceleration much greater than the local acceleration of the sun, (2) a constant thrust acceleration and deceleration of the vehicle, and (3) a vehicle velocity of zero at A and B for both legs of the mission.[2,3] The latter assumption implies that the vehicle thrusts at a constant I_{sp} and mass flow rate to a point approximately midway in the trajectory, whereupon it turns around and decelerates to zero velocity at the destination. The same type of maneuver is conducted on the return leg. These assumptions permit us to apply a straight-line trajectory, which is quite desirable from the standpoint of mission flexibility and planning. The relevant parameters and assumptions are summarized in Figure 1.

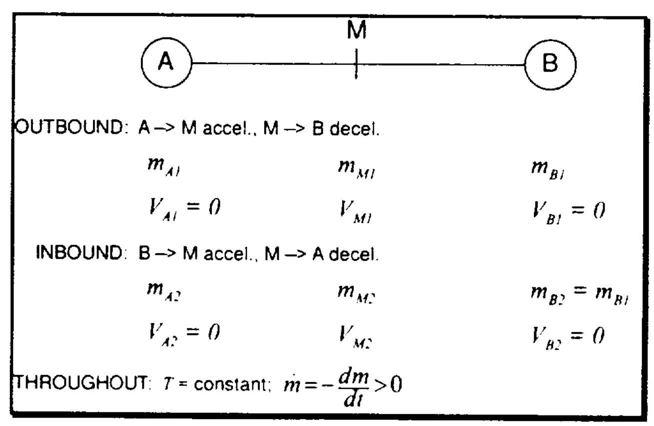

Figure 1: Parameters for round-trip from point A to point B and back.

The time taken to travel the outbound and inbound legs of a continuous thrust mission is nothing more than the propellant mass expended during that period divided by the mass flow rate. This yields the following expressions for transit times for the outbound and inbound legs of the mission, respectively:

$$\tau_{AB} = \frac{gI_{sp}}{T/m_{A2}} R_{BA2}\left(R_{AB1} - 1\right) \ , \tag{1}$$

$$\tau_{BA} = \frac{gI_{sp}}{T/m_{A2}}\left(R_{BA2} - 1\right) \ . \tag{2}$$

Since vehicle velocity is a function of time and the propellant flow rate is assumed constant $\dot{m} = T/\left(gI_{sp}\right)$, the distance traveled in each leg of the mission can be expressed in terms of the integral $D = \frac{1}{\dot{m}}\int_{m_i}^{m_f} V dm$. Substituting into this the instantaneous form of the rocket equation $V = gI_{sp} \ln \frac{m_i}{m(t)}$ and integrating yields the following equations for distance traveled:

$$D_{AB} = \frac{\left(gI_{sp}\right)^2}{T/m_{A2}} R_{BA2}\left(\sqrt{R_{AB1}} - 1\right)^2 \ , \tag{3}$$

$$D_{BA} = \frac{\left(gI_{sp}\right)^2}{T/m_{A2}}\left(\sqrt{R_{BA2}} - 1\right)^2 \ . \tag{4}$$

Equations (3) and (4) are two distinct expressions for the mass ratios R_{AB1} and R_{BA2}. Recognizing that $D_{AB} = D_{BA}$, R_{AB1} and R_{BA2} can be solved in terms of D_{AB} and then substituted into Eqs. (1) and (2) to yield the following generalized expression for trip time:

$$\tau = \frac{D_{AB}}{gI_{sp}} \cdot (h + kU) \ , \tag{5}$$

where,

$$U = \frac{gI_{sp}}{\sqrt{D_{AB}\left(T/m_{A2}\right)}} \ .$$

For the outbound leg from A to B, $(h,k) = (3,2)$, while for the return leg from B to A, $(h,k) = (1,2)$. Therefore $(h,k) = (4,4)$ for the total round trip. The travel time is expressed as a function of distance D_{AB}, I_{sp} and the final vehicle acceleration T/m_{A2}.

In order to express τ in terms of I_{sp} and α, we derive an equation for T/m_{A2} from a general relationship for final vehicle mass and substitute it into Eq. (5). The dry mass of both gain- and power-limited systems contain the basic elements of power-sensitive equipment, propellant-sensitive structure, and payload (all non-power and propellant items). Thus, the equation for final mass of the vehicle at mission completion is:

$$m_{A2} = m_{pay} + \alpha P_{out} + \beta m_{prop} \ . \tag{6}$$

The mass of power-sensitive components αP_{out} is proportional to the power delivered by the system, while the mass of the propellant subsystem βm_{prop} is scaled to the total propellant quantity. The total power output of the propulsion system is nothing more than the jet power $P_{out} = TV_e/2$, while the total propellant is a function of total impulse and exhaust velocity $m_{prop} = T\tau/V_e$. To simplify the derivation, we assume a constant ratio between payload and dry mass $m_{pay} = m_{A2}\lambda_{pay}$. Substituting these definitions into Eq. (6) and rearranging yields an equation for final vehicle acceleration:

$$\frac{1}{T/m_{A2}} = \frac{1}{1 - \lambda_{pay}}\left(\alpha\frac{gI_{sp}}{2} + \beta\frac{\tau}{gI_{sp}}\right) \ . \tag{7}$$

Equation (7) can now be substituted into Eq. (5) and rearranged to yield the following generalized expression for trip time:

$$\tau = \frac{1}{I_{sp}}\left[X \pm \sqrt{Y + Z\alpha I_{sp}^3}\right] \ . \tag{8}$$

where X, Y, and Z are functions independent of I_{sp} and α, that is:

$$X = \left(\frac{D_{AB}}{2g}\right)\left(2h + k^2 \frac{\beta}{1-\lambda_{pay}}\right) \; ,$$

$$Y = \left(\frac{kD_{AB}}{2g}\right)^2 \left(4h + k^2 \frac{\beta}{1-\lambda_{pay}}\right)\frac{\beta}{1-\lambda_{pay}} \; ,$$

$$Z = k^2 \frac{gD_{AB}}{1-\lambda_{pay}} \; .$$

We neglect the negative branch of Eq. (8) since it implies a negative trip time for large I_{sp}. The placement of I_{sp} in Eq. (8) indicates that an optimum value of I_{sp} exists which minimizes trip time. Differentiating Eq. (8) by I_{sp} and setting to zero eventually results in a quadratic equation for I_{sp}^3 that can be solved to yield:

$$I_{sp}\big)_{opt} = \left(\frac{2}{Z\alpha}\right)^{1/3}\left[\left(Y + X^2\right) \pm X\sqrt{X^2 + 3Y}\right]^{1/3} \; . \quad (9)$$

Again neglecting the negative branch, we substitute this optimum value of I_{sp} into Eq. (8) to obtain minimum trip times for a specified α. This substitution also yields the interesting result that $\tau \propto \alpha^{1/3}$, and confirms that lower mass-power ratios translate to shorter trip times. Although I_{sp} is important, any system that relies on an onboard power source to provide propulsive energy will have an optimum I_{sp} that is a function of α and mission requirements.

A plot illustrating the sensitivity of round-trip time to D_{AB} and α is shown in Fig. 2, where the optimum values of I_{sp} are superimposed on lines of constant α. In assessing the requirements for interplanetary missions, we consider the parameter values necessary to complete round-trip transits between earth and destination planets within 1 year. For missions between earth and other inner planets, which are represented by the middle band in Fig. 2, a system with $\alpha \leq 10$ kg/kW is adequate. The optimum I_{sp} values range from 3,000 to 5,000 sec. Improved technology in spacecraft subsystems – as reflected by lower subsystem mass and α – tends to increase the optimum I_{sp} for a given destination. This is especially true for electric propulsion systems, since reducing α decreases the negative mass impact associated with the αP_{out} term in Eq. (6). This permits a higher vehicle acceleration and shorter trip time.

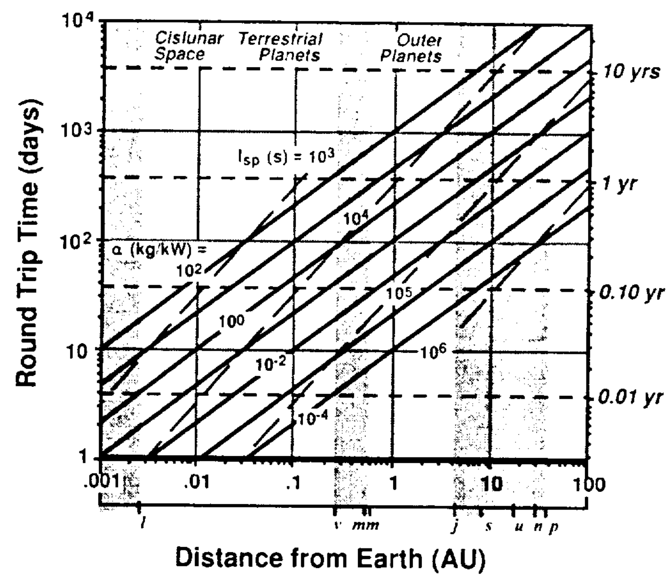

Figure 2: Round-trip time vs. distance from earth.

The α and I_{sp} requirements become much more challenging when we consider the ambitious distances within the band representing the outer planets. The range of these distances is quite broad, extending from 4 AU to nearly 40 AU. The ambitious case of a 1-year round-trip mission between Earth and Pluto (37 AU) would require an $\alpha \sim 2 \times 10^{-3}$ kg/kW and I_{sp} of nearly 3×10^5 sec. The requirements become much less severe if we consider 1-year round-trip missions to Jupiter, where the required α increases to $\sim 10^{-1}$ kg/kW, and the I_{sp} is ~70,000 sec.

The fact that lower α translates to higher vehicle accelerations and shorter trip times can be seen in Fig. 3, which shows the relationship between α and the required final vehicle acceleration for different values of travel distance. Note that the axis for α is expressed in terms of power density $1/\alpha$ to facilitate the viewing of performance trends.

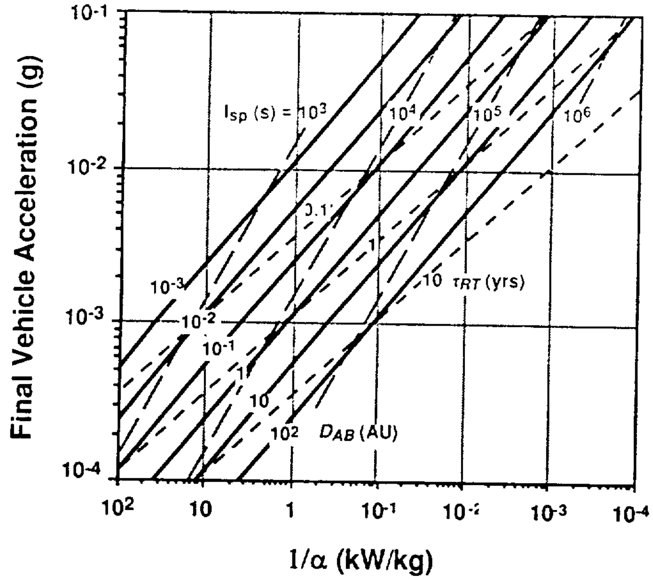

Figure 3: Final vehicle acceleration vs. $1/\alpha$ for round-trip trajectories.

For the missions that we are considering, the required final accelerations are fairly modest. Only at the most ambitious limits, that is round-trip missions to the outermost planets, do the accelerations become large – around 10^{-1} g. Figure 3 illustrates the limitation of pure power-limited concepts for advanced missions. Reasonable trip times require large accelerations, hence the need for less massive onboard power systems.

In summary, human interplanetary-class missions will require α ~ 3 orders of magnitude lower than the current state of the art for electric systems. Although electric propulsion can meet the high I_{sp} requirements, it will be difficult for this technology to meet the α requirements for missions beyond the innermost outer planets, which have been determined to be about 10^{-1} kg/kW. Ultimately, systems with α ranging from 10^{-1} kg/kW to as low as 10^{-3} kg/kW will be needed to open up the solar system to ambitious human exploration. This will require tremendous breakthroughs in power system technologies and reduction in effective component mass.

System Power Balance and Gain

Power- and gain-limited propulsion concepts are similar in that a large portion of the mass can be treated as being proportional to the power delivered by the system. In addition, the sensitivity of α can be derived directly from the total power balance of the system. Power and gain-limited concepts also contain many of the same basic functions and subsystems. Therefore, in order to compare α for both types of systems, it is reasonable to begin with a generic power balance that reflects the unique and common features of both. The model used here is shown in Fig. 4.

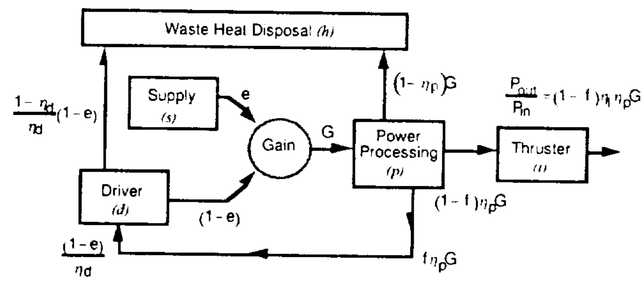

Figure 4: Generalized Power Balance.

The total power needed to heat, ignite or accelerate the propellant P_{in} is obtained from two sources — e represents the fraction of P_{in} delivered from an onboard source, while the remaining portion $1-e$ comes from a driver powered by energy extracted from the heated products. The total energy from these inputs is multiplied by the factor G and then transferred to a so-called "power processing" stage, which can actually be quite complex. Generally, most of the power in the form of energetic plasma is passed through this stage directly to some type of magnetic nozzle or "thruster." A portion of the total input power, however, is truly "processed" to run the driver. Unusable energy arising from inefficiencies at this step and others is passed to the thermal management subsystem, which radiates waste heat to space.

An expression for α is obtained by first summing the mass contributions from each of the subsystems represented in Fig. 4 and then dividing by the total power output of the system, that is:

$$\alpha = \frac{\sum m_j}{P_{OUT}} \quad . \tag{10}$$

We assume that the mass of each subsystem can be represented as the product of a distinct mass-power ratio $\hat{\alpha}$ and the power output of that particular subsystem. Noting that the power flows represented in Fig. 4 are expressed as fractions of the power input to the nuclear process P_{IN}, the corresponding subsystem masses are:

$$
\begin{aligned}
m_{\text{SUPPLY (POWER)}} &= m_S = \hat{\alpha}_S[e]P_{IN} \\
m_{\text{DRIVER}} &= m_D = \hat{\alpha}_D[1-e]P_{IN} \\
m_{\text{HEAT DISPOSAL}} &= m_H = \hat{\alpha}_H\left[(1-\eta_D)\frac{1-e}{\eta_D}+(1-\eta_P)G\right]P_{IN} \\
m_{\text{POWER PROCESSOR}} &= m_P = \hat{\alpha}_P[\eta_P G]P_{IN} \\
m_{\text{THRUSTER}} &= m_T = \hat{\alpha}_T \cdot P_{OUT}
\end{aligned}
\qquad (11)
$$

The system output power can be expressed as $P_{OUT} = \left[(1-f)\eta_P\eta_T G\right]P_{IN}$. Solving for the fraction of processor power delivered to the driver yields $f=(1-e)/(\eta_P\eta_D G)$, which upon substitution with all the subsystem masses into Eq. (10) provides the following formulation for α:

$$
\alpha = \frac{\left\{\begin{array}{l}\hat{\alpha}_S\eta_D e+\hat{\alpha}_D(1-e)+\hat{\alpha}_P\eta_D\eta_P G \\ +\hat{\alpha}_H\left[(1-\eta_D)(1-e)+\eta_D(1-\eta_P)G\right] \\ +\hat{\alpha}_T\eta_T\left[\eta_P\eta_D G-(1-e)\right]\end{array}\right\}}{\eta_T\left[\eta_P\eta_D G-(1-e)\right]} \quad . \qquad (12)
$$

The effective mass-power ratio for an electric or power-limited system α_{P-L} is obtained by setting $e = 1$ and $G = 1$ in Eq. (12) to yield:

$$
\alpha_{P-L} = \frac{\hat{\alpha}_S+\hat{\alpha}_P\eta_P+\hat{\alpha}_H(1-\eta_P)}{\eta_T\eta_P}+\hat{\alpha}_T \quad . \qquad (13)
$$

The sum $\hat{\alpha}_S$ + $\hat{\alpha}_P\eta_P$ is often treated as a single term representing the mass-power ratio of the entire power supply subsystem. Apart from the individual subsystem $\hat{\alpha}$'s, α_{P-L} is most sensitive to the efficiency of the power processing subsystem η_P. Lower efficiencies in the power processing subsystem demand a larger power source and greater capacity in the waste heat disposal subsystem. The only way to improve performance of a power-limited system is to reduce the mass-power ratios and increase power processing efficiency.

For the gain-limited case, the power needed to drive the system comes from the power processing subsystem via a feedback loop. Since no major onboard power supply is required after the process has been initiated, we set $e = 0$ and retain G as a parameter in Eq. (12) to yield the mass-power ratio for a gain-limited system:

$$
\alpha_{G-L} = \frac{\left\{\begin{array}{l}\hat{\alpha}_D\eta_D+\hat{\alpha}_P\eta_D\eta_P G \\ +\hat{\alpha}_H(1-\eta_D)+\eta_D(1-\eta_P)G\end{array}\right\}}{\eta_T(\eta_P\eta_D G-1)}+\hat{\alpha}_T \quad . \qquad (14)
$$

As before, the overall system mass-power ratio is a function of the subsystem $\hat{\alpha}$'s, including the additional parameters of driver mass and efficiency. The most unique feature is the strong dependency on gain, which is not entirely arbitrary. G must be greater than a certain minimum value in order to provide a net positive input power for thrust production and driver operation. This appears as the requirement for a positive value in the denominator of Eq. (14), which can only occur when $G > G_{MIN}$ where $G_{MIN} = (\eta_P\eta_D)^{-1}$. At a minimum, the power gained must overcome losses due to inefficiencies in the driver and power processor subsystems. Values of G above this limit result in lower effective mass-power ratios.

In addition, α does not decrease without limit as gain is raised. Although higher gains diminish the influence of driver mass and efficiency, the power handled by the processor and thruster increases proportionally as well. Consequently, the mass influence of these subsystems becomes dominant with increasing gain, leading to an effective lower bound for α. This can be seen by taking $\lim_{G\to\infty}\alpha_{G-L} = \alpha_{G\infty}$, which yields:

$$
\alpha_{G\infty} = \hat{\alpha}_T + \frac{\hat{\alpha}_P\eta_P+\hat{\alpha}_H(1-\eta_P)}{\eta_T\eta_P} \quad . \qquad (15)
$$

The parameter $\alpha_{G\infty}$ embodies the influence of processor mass and efficiency on the system, and its effect is a strong function of G. The influence of $\alpha_{G\infty}$ on propulsion capability tends to diminish at lower values of G. As G decreases, α becomes more dependent on driver characteristics and increases without limit. Although α approaches an infinite value at the minimum gain condition, the sensitivity to driver characteristics can be ascertained by evaluating α at $G = 0$, thereby removing the influence of gain in Eq. (14). From $\lim_{G \to 0} \alpha_{G-L} = \alpha'_{G0}$, we obtain:

$$\alpha'_{G0} = \hat{\alpha}_T - \frac{\hat{\alpha}_D \eta_D + \hat{\alpha}_H (1 - \eta_D)}{\eta_T} \quad . \qquad (16)$$

α'_{G0} is always a negative quantity, and reflects the impact of driver characteristics on α. Its impact can be explained more clearly by expressing it as a positive value via $\alpha_{G0} = -\alpha'_{G0}$. When α_{G0} is substituted with $\alpha_{G\infty}$ and the definition of G_{MIN} into Eq. (14), the result is an expression for α_{G-L}. This can be expressed in general terms as:

$$\alpha_{G-L} = \Phi \alpha_{G\infty} \quad . \qquad (17)$$

The factor Φ accounts for the relationship between G and G_{MIN}, and the somewhat competing effects of α_{G0} and $\alpha_{G\infty}$, that is:

$$\Phi = \frac{g_G + f_\alpha}{g_G - 1} \quad . \qquad (18)$$

where $g_G = G/G_{MIN}$ and $f_\alpha = \alpha_{G0}/\alpha_{G\infty}$. For the minimum theoretical limit of α_{G-L}, $g_G \to \infty$ and $\Phi \to 1$, thus yielding $\alpha_{G-L} \approx \alpha_{G\infty}$. Similarly, as $g_G \to 1$, $\Phi \to \infty$ and $\alpha_{G-L} \to \infty$. The parameter Φ is always greater than 1, and reveals the degree to which gain overcomes the mass penalties and inefficiencies associated with the driver.

Reference Propulsion Concepts

Evaluating the relationship in Eqs. (17) and (18) requires calculation of G_{MIN}, $\alpha_{G\infty}$ and α_{G0} based on representative mass properties, power flows and efficiencies. Fortunately, a considerable amount of conceptual design information exists from several studies done over the last decade. We refer to the Spherical Torus, VISTA and ICAN-II investigations[3-10] mainly because of their unique (1) extent of design detail, (2) thorough accounting of power flows, and (3) recognition of the significant impact of waste heat handling — an important aspect of all gain-limited concepts.

The Spherical Torus (ST) vehicle concept[4] employs a magnetic confinement fusion (MCF) device that is geometrically similar to tokamaks and other toroidal containment systems. The MCF regime occupies the low end of plasma density, and requires long confinement times to sustain ignition conditions. This device's low aspect ratio could mitigate many of the instabilities previously encountered with tokamaks. However, it requires external methods of heating to raise the plasma to ignition temperatures, and large magnetic fields to compress the nuclear fuel to fusion densities. The most recent assessments of the ST employ D^3He fuel within the reactor to heat a hydrogen propellant, and a magnetic nozzle to expand it for thrust. The vehicle is sized for a Saturn rendezvous, with a thrust and I_{sp} of 26,000 N and 38,612 sec, respectively.

Representative of the opposite end of the operational spectrum is the VISTA concept,[5,6] which is based on inertial confinement fusion (ICF). ICF involves laser or particle beam bombardment of fusionable pellets to generate a much higher density plasma — ten or more orders of magnitude higher than MCF plasmas. ICF drivers typically employ very high energies. The most recent VISTA studies assume a conical configuration, using arrayed lasers to initiate fusion in targets ejected at the cone's vertex. A superconducting magnetic coil directs the resulting plasma for thrust. The quoted performance numbers are based on DT fusion, but the references also mention eventual use of DD and

D^{3}He fusion. The VISTA studies focus on roundtrip human missions to Mars, although the capabilities would enable missions into the outer solar system. The thrust and I_{sp} are 185,000 N and 12,562 sec, respectively.

The third concept, ICAN–II,[7,8,9] is a hybrid approach that combines ICF with use of antimatter to initiate combined fission-fusion in a compressed nuclear target. A pellet of DT and U-235 is compressed with light ion beams and irradiated with a low-intensity beam of antiprotons. The antiprotons initiate a hyper-neutronic fission process in the U-235 that rapidly heats and ignites the DT core. The resulting radiation from fission and fusion is transformed via wavelength shifter material before generating thrust through ablation of material from a spherical thrust shell. Since the antimatter actually initiates the reactions, this approach requires lower driver energies than other ICF concepts. However, the driver power is still significant, and this approach depends on dramatic improvements in technologies for antimatter production and storage.[7,8,9] Like VISTA, the reference mission is a roundtrip mission to Mars, but with a thrust of 100,000 N and I_{sp} of 13,500 sec.

The mass distributions and subsystem power levels for these three concepts vary considerably due in large part to their being sized to different mission requirements. However, the basic processes are similar enough that the subsystems and power flows can be defined in accordance with the model shown in Fig. 4. This yields the terms shown in Table 1, which highlight some noteworthy differences in the systems' power flows.

In all cases, the "effective" power output discounts unrecoverable losses of radiation to space in the form of high-energy photons and neutrons. This has a relatively small effect on the fusion power multiplication of ST, in which only 8.5% of the power is presumed unrecoverable and lost to space. By contrast, it has a significant effect on the VISTA and ICAN–II concepts, where 72% and 84% of the fusion power generated is considered lost, respectively.

A few cases include an interval of onboard power utilization as part of the driver requirement to simplify power flows. For example, ST specifies a 2 MW fission reactor to assist in initial ignition and to provide onboard power. This 2 MW is added to the power required to drive the fusion process, thus yielding a total value of 156 MW input power for the driver subsystem.

The concepts also differ in their accounting of jet power. The MCF-based ST converts 67% of its effective fusion power into jet power. However, the ICF systems have much lower percentages – 27% and 15% for VISTA and ICAN–II, respectively. The ICF concepts redirect isotropically expanding plasmas rearward of the vehicle, while the ST adds fusion energy to injected hydrogen propellant, which is ionized and expanded in a magnetic nozzle.

Individual $\hat{\alpha}$'s can be calculated by dividing each subsystem's mass by its power output. These $\hat{\alpha}$'s are shown in Table 2, along with their associated efficiencies, which are nothing more than the power output divided by the power input. The differences between input and output power for the driver, processor and thruster are treated as inputs to the heat disposal subsystem.

These values of $\hat{\alpha}$ generally represent foreseeable technological limits of subsystem performance. All three concepts feature power processing and thruster mass-power ratios in the range of 10^{-3} to 10^{-2} kg/kW, which individually approximate the α required for rapid interplanetary flight. The differences in heat disposal $\hat{\alpha}$ are greater — ranging from 10^{-3} to 10^{-1} kg/kW. This is primarily because all three concepts employ different types of designs. The ST uses four sets of rectangular radiator panels in a "cross" geometry; VISTA assumes a conical heat pipe radiator system; and ICAN–II uses a liquid droplet radiator system — a promising but less developed technology.

The resulting values of $\alpha_{G\infty}$, α_{G0}, G_{MIN} and α_{G-L} for all three concepts are shown in Table 3. A notable feature is the relatively high value of G_{MIN} for VISTA. This is due to the conservative value of 6% assumed for laser driver efficiency. As a result, the gain ratio g_G for VISTA is actually lower than the ST, despite the fact that VISTA has a gain that is over four times greater than that concept.

Both of the ICF concepts have lower α_{G-L} than the ST due to their intrinsically higher gain. In fact, the α_{G-L} for the ICF concepts are relatively close to their respective values of $\alpha_{G\infty}$. Therefore, further increasing the gain for these systems will have only modest effects on improving α_{G-L}, even if all subsystem masses remain the same. There is more margin for improvement with the ST should research indicate higher possible gains for an MCF system.

Figure 5 shows the effect of varying gain on α_{G-L}. Each curve exhibits asymptotes at $g_G \to 1$ and $g_G \to \infty$. VISTA is distinct in that its α_{G-L} is more sensitive to gain than the other two concepts. This is because of the comparable influence between its driver and processor characteristics ($\alpha_{G0} \approx \alpha_{G\infty}$ to an order of magnitude), which causes α_{G-L} to approach its minimum theoretical value over a smaller magnitude variation in gain. However, VISTA's low driver efficiency of 6% yields a higher minimum gain condition than either ST or ICAN–II. Its relatively high processor mass also yields a $\alpha_{G\infty}$ = 0.075 kg/kW, which is higher than ST. Even with these more conservative masses and efficiencies, VISTA ultimately achieves a lower α_{G-L} than the ST by operating at a higher gain $G = 279$.

Compared to VISTA, the ST has lower values of G_{MIN} and $\alpha_{G\infty}$, but has a higher value of f_α. This results in a lower sensitivity to gain, as noted by the more gradual variation in Fig. 5. The ST's design point G of 68 yields a higher α_{G-L} than that of VISTA. The values of α_{G-L} for ICAN–II are substantially higher than the other concepts for G < 2000. This is primarily due to its large driver mass and value of f_α. Above this point, ICAN–II's lower value of $\alpha_{G\infty}$ enables it to achieve the lowest α_{G-L} of all, albeit at very high gain.

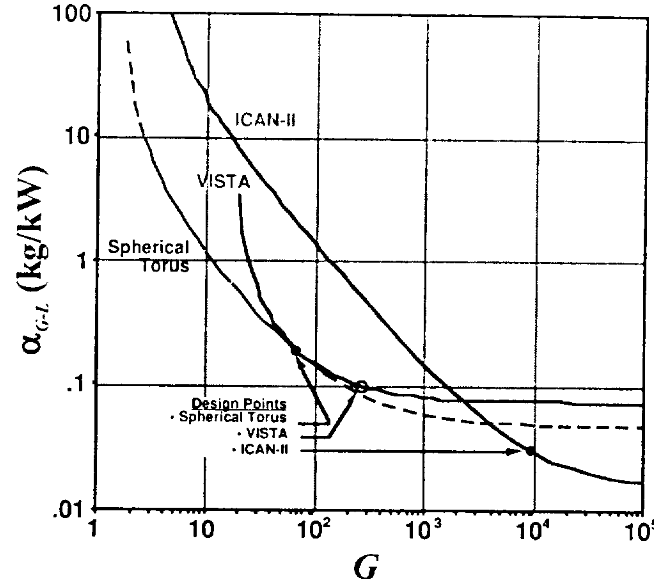

Figure 5: Sensitivity of α_{G-L} to Gain for Fusion Propulsion Concepts

In summary, the calculation of the parameters G_{MIN}, $\alpha_{G\infty}$ and α_{G0} together with examination of Eq. (17) through Fig. 5 demonstrates the differences in performance capabilities of fusion propulsion systems. Nevertheless, these parameters also serve to "normalize" different concepts to ensure a more consistent comparison of performance. All gain-limited systems exhibit the same functional behavior, as shown by the asymptotic nature of the curves in Fig. 5. Nevertheless, G_{MIN}, $\alpha_{G\infty}$ and α_{G0} for a given concept define the shape of both asymptotes and the "elbow" of the α_{G-L} versus G curve. The effects of these parameters are examined further in the next section.

It is useful to note that it is difficult to improve performance at both ends of this curve by evolving a given subsystem to simultaneously lower both asymptotes—that is, enabling a lower α_{G-L} by decreasing both G_{MIN} and $\alpha_{G\infty}$. Only one parameter is common between these two asymptotic limits: the power processing efficiency η_P. It is somewhat surprising that this parameter if of central importance for fusion-based spacecraft; conventional wisdom has it that improvement of the driver provides the greatest payoff. While the foregoing is indeed important.

Fig. 5 illustrates that improving fusion power conversion holds the key to lowering both the required gain and mass-power limitations on spacecraft performance.

Discussion

The data derived for the three reference propulsion concepts can be used to further examine the nature of Φ. The functional dependence of Φ on g_G and f_α is illustrated in Fig. 6. Earlier we noted that $\tau_{RT} \propto \alpha_{G-L}^{1/3}$, which stresses the need to achieve as low of α_{G-L} as possible. From Eqs. (17) and (18), it is obvious that increasing g_G through operation at higher gain has this effect. Although the impact of α_{G0}, $\alpha_{G\infty}$ and G_{MIN} is not as obvious, the relative influence of these three parameters can be assessed through the following ratios of partial derivatives:

$$\frac{\left(\partial\alpha_{G-L}/\partial\alpha_{G\infty}\right)}{\left(\partial\alpha_{G-L}/\partial\alpha_{G0}\right)} = g_G \ , \tag{19}$$

$$\frac{\left(\partial\alpha_{G-L}/\partial\alpha_{G\infty}\right)}{\left(\partial\alpha_{G-L}/\partial G_{MIN}\right)} = \frac{(g_G - 1)G_{MIN}}{\alpha_{G\infty} + \alpha_{G0}} \ . \tag{20}$$

Since $g_G > 1$, Eq. (19) implies that α_{G-L} is *always* more sensitive to changes in $\alpha_{G\infty}$ than in α_{G0}. Improving processor mass and efficiency via reduction in $\alpha_{G\infty}$ contributes more to lowering α_{G-L} than decreasing α_{G0} through improvements in driver characteristics. The relative sensitivity of α_{G-L} to $\alpha_{G\infty}$ and G_{MIN} embodied by Eq. (20) is not as obvious, since it depends more on propulsion system design. However based on the data in Table 3, it appears that Eq. (20) is generally greater than 1. As in the case of α_{G0}, $\alpha_{G\infty}$ transcends G_{MIN} in its influence on α_{G-L}.

The parameter $\alpha_{G\infty}$ is the fundamental delimiter of gain-limited propulsion performance, and Φ should be treated as a factor that causes α_{G-L} to deviate from this limit. Table 3 shows that the values of $\alpha_{G\infty}$ that can be obtained with foreseeable technology are very close to the α_{G-L} required for rapid interplanetary space flight. Therefore, the goal is to achieve a Φ that is as close to 1 as possible.

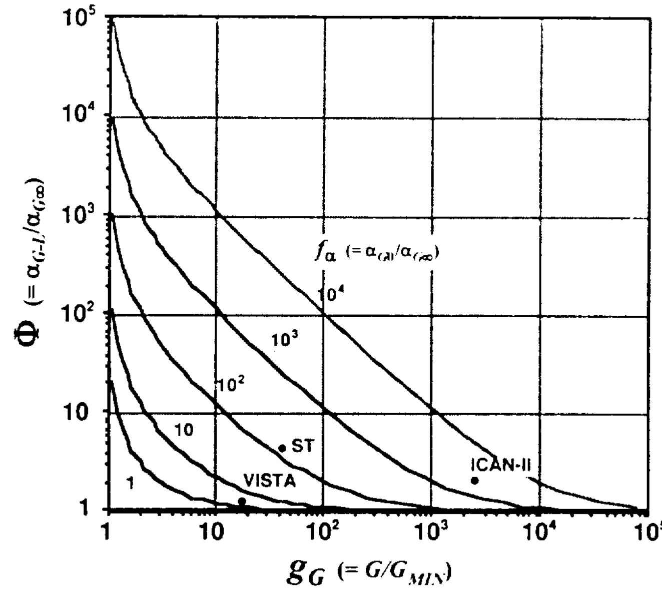

Figure 6: Sensitivity of Φ to Gain and Propulsion Subsystem Characteristics

One way is through a high value of g_G, which offsets the negative impacts of driver mass and inefficiency — both of which tend to increase α_{G0} and Φ. The ability to reduce driver effects depends on the value assumed for f_α. For instance, operating at a low g_G of 10 with a roughly intermediate f_α of 100 yields a Φ of 12.2, which translates to an α_{G-L} that is 12.2 times greater than $\alpha_{G\infty}$. Raising g_G to 100, however, drops Φ to 2.02, while operating at g_G = 1000 yields a Φ of 1.10 – only 10% greater than the minimum limit of $\alpha_{G\infty}$.

Again, $\alpha_{G\infty}$ sets the basic limit, regardless of the values of g_G and f_α. Unless $\alpha_{G\infty}$ is several orders of magnitude less than α_{G-L}, operation at g_G between 1 and 10 provides no benefit from a performance standpoint. For example in the case where f_α = 100, g_G could be as low as 2.02 only if $\alpha_{G\infty}$ were two orders of magnitude less than the required α_{G-L}. However, a g_G of 12.2 would decrease Φ by one order of magnitude and yield a α_{G-L} only 10 times $\alpha_{G\infty}$.

The asymptotic relationships in Fig. 6 clearly indicate a tradeoff between gain and driver characteristics. That is, it appears that g_G and f_α can be mutually varied to yield a specific value of Φ. At large values of Φ, sizable variations in f_α translate to comparatively smaller changes in g_G. This could be viewed as advantageous if the goal is to relax driver requirements while keeping increases in gain low. However, it could also be regarded negatively if the object is to reduce gain requirements by improving driver characteristics.

At the low Φ of interest here, the relationship between variation of f_α and g_G is not as obvious. The relative sensitivity about a constant value of Φ can be shown through the ratio of df_α/f_α to dg_G/g_G, that is:

$$\frac{df_\alpha}{f_\alpha} \cdot \frac{g_G}{dg_G} = \frac{\Phi + f_\alpha}{f_\alpha} \quad . \tag{21}$$

Equation (21) confirms the potentially significant relaxation of driver requirements for small increases in g_G when $\Phi >> f_\alpha$. However at low values of Φ, especially if $\Phi << f_\alpha$, the relative sensitivity becomes approximately equivalent. In other words, an order of magnitude variation in f_α requires a comparable order of magnitude change in g_G to maintain the same value of Φ and α_{G-L}.

Although we have defined $\alpha_{G\infty}$ and α_{G0} according to the subsystems and power flows in Fig. 4, it is important to note that the expression for Φ in Eq. (18) is applicable to any power-balance involving "feedback" to sustain energy release. Even though the equations for calculating $\alpha_{G\infty}$ and α_{G0} may vary significantly from concept to concept, Eq. (10) always reduces to the form of Eqs. (17) and (18), which represents the general relationship between α_{G-L}, G and subsystem characteristics.

Conclusions

Fusion propulsion offers the promise of achieving the high I_{sp} and low mass-power ratios required for rapid interplanetary space flight. However, practical implementation of such systems will demand extremely low $\hat{\alpha}$ and high efficiencies in several key propulsion subsystems. Even with the technology projections from several design studies, the required energy gains for ambitious interplanetary missions (α ~ 0.001 to 0.1 kg/kW) range upwards of 100 to possibly 1,000 or greater.

This result stems from the fact that the $\hat{\alpha}$ and η of the subsystems downstream of the nuclear process set the minimum limit on attainable α. Energy gain has no influence on this limit, but it can substantially offset the burden associated with the driver. Improving driver characteristics through reduced mass and increased efficiency can reduce gain requirements. However even with the most optimistic projections of driver performance, the energy gains will be substantial — on the order of 100 or more.

We can conclude that the challenge of applying fusion to propulsion applications, at least in the manner envisioned in these studies, is as great or even greater than ground applications for commercial power. It is appropriate to question whether systems requiring complex drivers and power processing subsystems will ever achieve the mass characteristics necessary for interplanetary flight. These complexities, which generally result in massive and/or inefficient systems, represent the main obstacles to practical gain-limited systems.

References

1. Frisbee, R. H. and Leifer, S. D., "Evaluation of Propulsion Options for Interstellar Missions," AIAA 98-3403, July 1998.

2. Borowski, S. K., "Comparison of Fusion/ Antiproton Propulsion Systems for Interplanetary Travel," *Fusion Energy in Space Propulsion: AIAA Progress in Astronautics and Aeronautics*, Vol 167, Kammash, T. ed., 1995.

3. Frisbee, R. H. and Leifer, S. D., "Evaluation of Propulsion Options for Interstellar Missions," AIAA 98-3403, July 1998.

4. Borowski, S. K., "Comparison of Fusion/ Antiproton Propulsion Systems for Interplanetary Travel," *Fusion Energy in Space Propulsion: AIAA Progress in Astronautics and Aeronautics*, Vol 167, Kammash, T. ed., 1995

5. Williams, C. H., "An Analytic Approximation to Very High Specific Impulse and Specific Power Interplanetary Space Mission Analysis," *NASA Technical Memorandum (NASA-TM-107058)*, September 1995.

6. Williams, C. H., Borowski, S. K., Dudzinski, L. A., and Juhasz, A. J., "A Spherical Torus Nuclear Fusion Reactor Space Propulsion Vehicle Concept for Fast Interplanetary Piloted and Robotic Missions," AIAA 99-2704, June 1999.

7. Orth, C. D., "VISTA project" presentation, NASA MSFC/JPL Visit to LLNL, September 22-23 1998.

8. Orth, C. D., Klein, G., Sercel, J., Hoffman, N., Murray, K., and Chang-Diaz, F., "VISTA: A Vehicle for Interplanetary Space transport Applications Powered By Inertial Confinement Fusion." Report UCRL-LR-110500, University of California, Lawrence Livermore National Laboratory, Livermore, CA 94550 (1998).

9. Gaidos, G., Lewis, R. A., Smith, G. A., Dundore, B. and Chakrabarti, S., "Antiproton-Catalyzed Microfission/Fusion Propulsion Systems for Exploration of the Outer Solar System and Beyond," *Space Technology and Applications International Forum*, El-Genk, M. S. ed., 1998.

10. Fulmer, J. R., "Radiator Design and Mars Payload for an Antiproton-Catalyzed Microfission/fusion Propelled Spacecraft," Baccalaureate Honors Thesis, Pennsylvania State University, 1997.

11. Lewis, R. A., personal communication.

12. Werthman, W. L., "Antiproton-Catalyzed Microfission/fusion Space Propulsion," Masters Thesis, Pennsylvania State University, 1995.