Spacecraft Attitude Determination Methods

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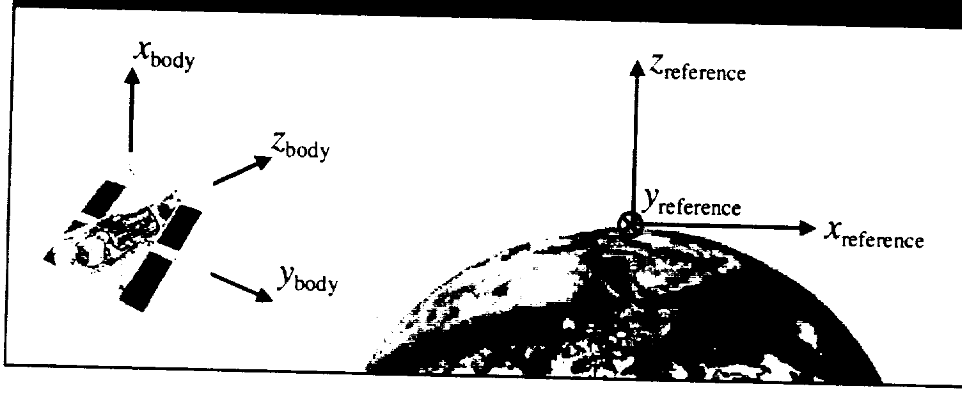
Mechanical and Aerospace Engineering Colloquium
Cornell University
Ithaca, NY November 7, 2000

Outline

- What is spacecraft attitude?
- How do we estimate attitude?
  - References
  - Sensors
  - Mathematical representations of attitude
  - Algorithms
- Representative space missions
- Summary

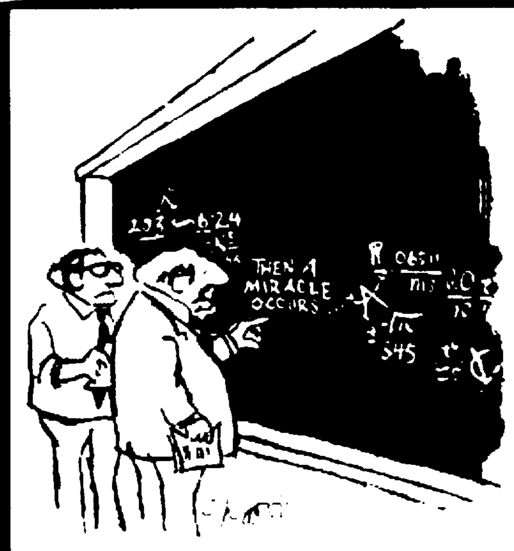
What is attitude?

- 'Attitude' means the orientation of a coordinate frame fixed in the spacecraft body relative to a reference coordinate frame.



What do we observe?

- Angles or vectors to known references
  - Sun (coarse sun sensor or digital sun sensor)
  - Earth (horizon sensor or landmarks)
  - Stars (star trackers or fine guidance sensors)
  - Magnetic field (triaxial magnetometers)
  - RF sources, like GPS (phase interferometry)
  - Motion (inertial sensors — gyros)

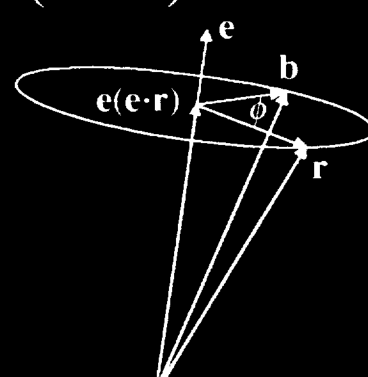


"I think you should be more explicit here in step two."

Euler's Theorem (1775)

- *The general displacement of a rigid body with one point fixed is a rotation about some axis*
- **r** in reference frame, **b** in body frame
- $\mathbf{b} = R(\mathbf{e}, \phi)\,\mathbf{r}$, where

$$R(\mathbf{e}, \phi) = \mathbf{e}\mathbf{e}^T + \cos\phi\,(I - \mathbf{e}\mathbf{e}^T) - \sin\phi\,[\mathbf{e}\times].$$



Cross Product Matrix

- $[\mathbf{e}\times]$ is the cross product matrix

$$[\mathbf{e}\times] \equiv \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix}$$

- Defined so that $[\mathbf{e}\times]\mathbf{v} = \mathbf{e}\times\mathbf{v}$

Rotation Matrix

- The 3×3 rotation matrix R is
  - orthogonal (transpose = inverse), and
  - proper (determinant = +1)
- Also called the direction cosine matrix (DCM)
- Also called the attitude matrix, denoted by A or D
- Attitude kinematics equation $\mathrm{d}A/\mathrm{d}t = -[\boldsymbol{\omega}\times]A$, where $\boldsymbol{\omega}$ is the angular velocity or body rate vector
- Specification of attitude requires three parameters

Euler Angles

- Parameterization as product of three rotations
  $R_{ijk}(\phi, \theta, \psi) = R(\mathbf{e}_k, \psi)\, R(\mathbf{e}_j, \theta)\, R(\mathbf{e}_i, \phi)$
  - Symmetric, $ijk = 131, 121, 232, 212, 313, 323$
  - Asymmetric, $ijk = 123, 132, 231, 213, 312, 321$
  - 'Gimbal-lock' singularity for some θ
- Generalized Euler Angles (Davenport, 1973)
  - Three rotation axes are not coordinate axes
  - Must have $\mathbf{e}_j$ perpendicular to $\mathbf{e}_i$ and $\mathbf{e}_k$

Other Parameterizations

- It is topologically impossible to have a global nonsingular 3-dimensional parameterization
- Singular 3-dimensional parameterizations
  - Rodrigues parameters, also known as Cayley parameters or the Gibbs vector = $\mathbf{e}\tan(\phi/2)$,
  - Modified Rodrigues parameters, $\mathbf{e}\tan(\phi/4)$
- Nonsingular 4-dimensional parameterization
  - Euler-Rodrigues parameters, or quaternion

Olinde Rodrigues (1795–1850)

- Son of a Jewish accountant in Bordeaux
- Doctorate in math in 1816 from U. of Paris
  - Rodrigues formula for Legendre polynomials
- Banking and utopian socialism for 24 years
  - Important in introducing railroads to France
  - Edited an anthology of workers' poetry
- Published his seminal paper on analysis of rotations in Liouville's Journal in 1840

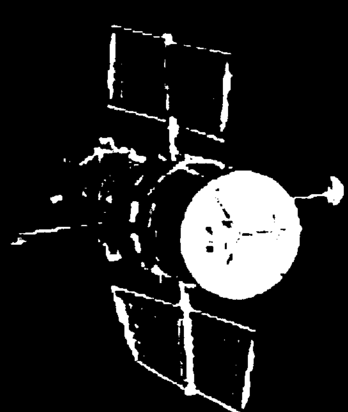
Why Quaternions?

- 4-dimensional non-singular parameterization
  $\mathbf{q} \equiv [q_1 \;\; q_2 \;\; q_3]^T = \mathbf{e}\sin(\phi/2), \quad q_4 = \cos(\phi/2)$
  - One constraint, $|q|^2 = 1$
- Rotation matrix, using half-angle formulas, is
  $R(q) = (q_4^2 - |\mathbf{q}|^2)\, I + 2\mathbf{q}\mathbf{q}^T - 2q_4[\mathbf{q}\times]$
- Simple product rule $R(q)R(q') = R(q \otimes q')$
- Quaternion kinematics $dq/dt = \frac{1}{2}\,\boldsymbol{\omega} \otimes q$

Attitude Determination Methods

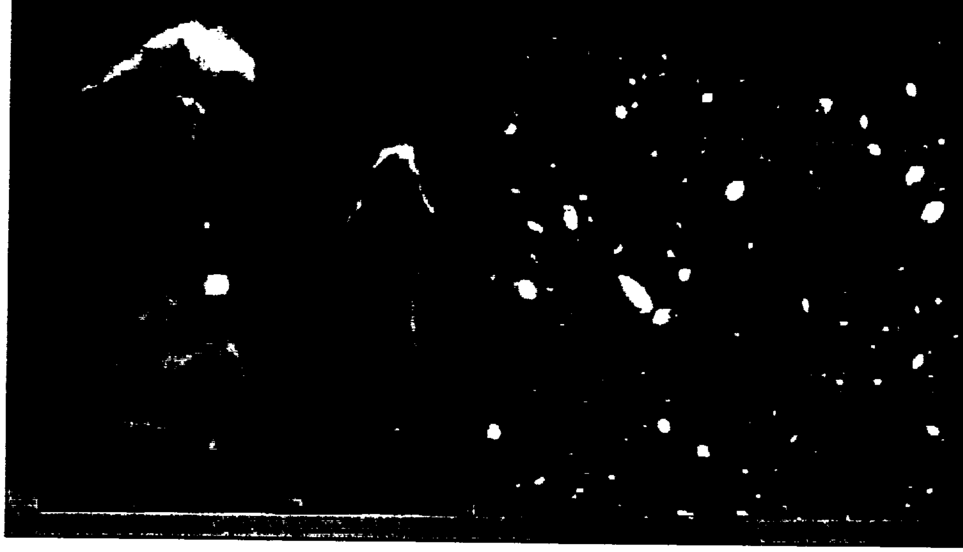
- 'Single Frame' Methods
  - *Ad hoc* methods
  - TRIAD (Black, 1964)
  - Optimal methods (Wahba's problem, 1965)
- Filters
  - Extended Kalman Filter (EKF)
  - H_{∞} filter (Berman, Markley & Shaked, 1993)
  - Predictive filter (Crassidis & Markley, 1997)

Hubble Space Telescope (HST)

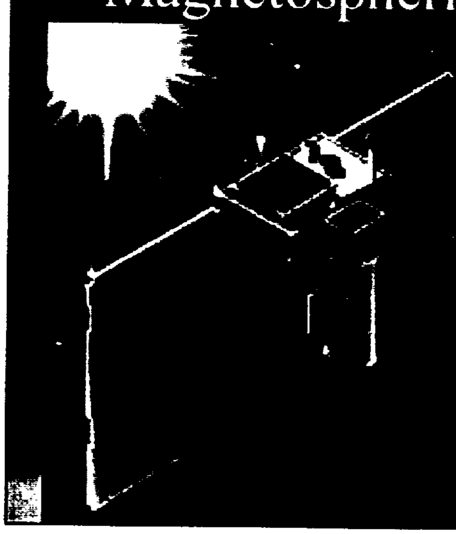


- Launch April 1990
- Mass = 11000 kg
- Pointing = 0.007 arc seconds
- Attitude sensors:
  - Fine Guidance Sensors
  - Gyros
  - Star trackers
  - Sun sensors
  - Magnetometers

HST Science

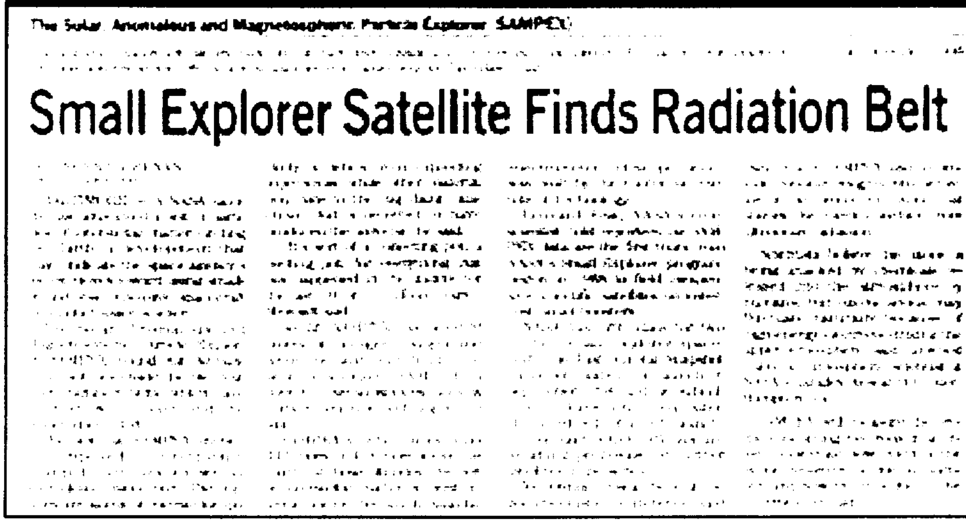


Solar, Anomalous, and Magnetospheric Particle Explorer



- Launch July 1992
- The first Small Explorer (SMEX) mission
- Mass = 160 kg
- Pointing = 2°
- Attitude sensors
  - Sun Sensors (0.25°)
  - Magnetometers

SAMPEX Science


The Solar, Anomalous and Magnetospheric Particle Explorer (SAMPEX)

Small Explorer Satellite Finds Radiation Belt

Two Contrasting Missions

- Differences
  - HST pointing six orders of magnitude finer
  - HST more complex, uses more sensors
- Similarities
  - Onboard attitude determination
  - Programmable digital computer
  - HST gyroless safemode uses the same sensor information as SAMPEX

SAMPEX Attitude Determination

- TRIAD is used for attitude determination
  - A 'single frame' method
  - Measurements at one instant of time
- Measurements represented as (two) unit vectors
  - Direction from the spacecraft to the Sun
  - Direction of the Earth's magnetic field
  - Coordinates in reference and body frames
  - **Minimum number required for a solution**

TRIAD (Black, 1964)

- Orthogonal triad of body frame unit vectors:
  $\mathbf{b}_1$ = spacecraft-to-Sun unit vector
  $\mathbf{b}_2$ = perpendicular to Sun and magnetic field
  $\mathbf{b}_3 = \mathbf{b}_1 \times \mathbf{b}_2$
- Corresponding triad of reference frame vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ from ephemerides and field models
- $A = [\,\mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\,][\,\mathbf{r}_1\ \mathbf{r}_2\ \mathbf{r}_3\,]^T = \Sigma_i\, \mathbf{b}_i\, \mathbf{r}_i^T$

Wahba's Problem (1965)

- Optimal estimate from n vector observations
- Find the proper orthogonal A that minimizes the loss function, with positive weights a_i,

  $$L(A) = \tfrac{1}{2}\, \Sigma_i\, a_i\, |\mathbf{b}_i - A\mathbf{r}_i|^2,$$

  where $\{\mathbf{b}_i\}$ are the body frame unit vectors, and $\{\mathbf{r}_i\}$ are the same vectors in the reference frame.
- Can write $L(A) = \Sigma_i\, a_i - \mathrm{trace}(AB^T)$, where

  $$B \equiv \Sigma_i\, a_i\, \mathbf{b}_i\, \mathbf{r}_i^T$$

Procrustes Problem

- Wahba's problem is equivalent to the Procrustes problem: to find the orthogonal matrix A that is closest to B in the Frobenius norm, defined by

  $$||M||^2 \equiv \Sigma_{ij} M_{ij}^2 = \mathrm{trace}(MM^T)$$

- $||A - B||^2 = \mathrm{trace}[(A - B)(A - B)^T]$

  $$= \mathrm{trace}(AA^T) - 2\,\mathrm{trace}(AB^T) + \mathrm{trace}(BB^T)$$

- Since $\mathrm{trace}(AA^T) = \mathrm{trace}(I) = 3$,

  this is minimized by maximizing $\mathrm{trace}(AB^T)$

Davenport's q Method (1977)

- $A(q) = (q_4^2 - |\mathbf{q}|^2)\, I + 2\mathbf{q}\mathbf{q}^T - 2q_4[\mathbf{q}\times]$,

  is a homogeneous quadratic function of q, so

  $L(A) = \Sigma_i a_i - \text{trace}(AB^T) = \Sigma_i a_i - q^T K q$,

  where K is a symmetric, traceless 4×4 matrix whose elements are linear in the elements of B.
- The optimal quaternion q_{opt} is the eigenvector of K with the maximum eigenvalue λ_{max}.

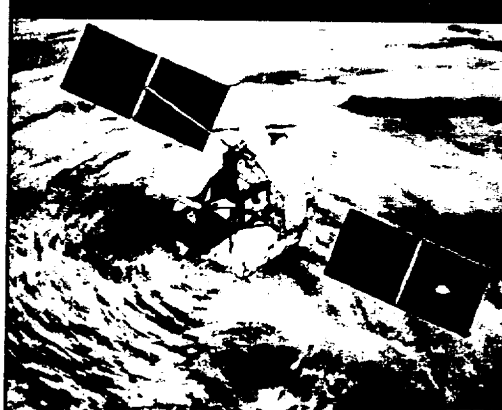
Faster (but less robust) Solutions of Wahba's Problem

- Shuster's QUaternion ESTimator (1978)
  - Solve the equation det $(\lambda I - K) = 0$ iteratively, starting from $\lambda_0 = \Sigma_i a_i = \lambda_{max} + L(A_{opt})$
  - Then simple matrix algebra gives q_{opt}
  - Error covariance is $P = [\, \Sigma_i a_i (I - \mathbf{b}_i \mathbf{b}_i^T)]^{-1}$, if weights are inverse measurement variances.
- Mortari's ESOQ (1996) and ESOQ2 (1997)

My Algorithms for Solving Wahba's Problem

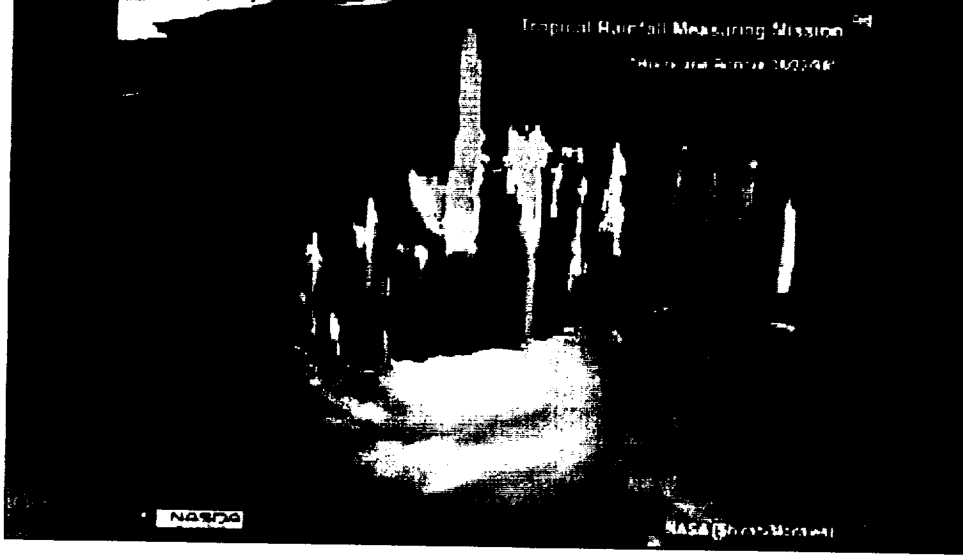
- Singular Value Decomposition Method (1987)
  $B = U\,\mathrm{diag}([s_1, s_2, s_3])\,V^T$, then $A_{opt} = UV^T$
  with U, V proper orthogonal and $s_1 \geq s_2 \geq |s_3|$.
  $P = U\,\mathrm{diag}([(s_2+s_3)^{-1}, (s_3+s_1)^{-1}, (s_1+s_2)^{-1}])U^T$
- Fast Optimal Attitude Matrix (FOAM, 1992)
  $A_{opt} = \zeta^{-1}[(\kappa + \|B\|^2)B + \lambda_{max}\,\mathrm{adj}B + BB^TB]$,
  with $\zeta \equiv \kappa\lambda_{max} - \det B$ and $\kappa \equiv \frac{1}{2}(\lambda_{max}{}^2 - \|B\|^2)$
  $P = \zeta^{-1}(\kappa I + BB^T)$

Tropical Rainfall Measuring Mission (TRMM)



- Launch November 1997
- Mass = 3500 kg (largest ever built at GSFC)
- Pointing = 0.2°
- Attitude sensors:
  - Static Earth sensor
  - Gyros
  - Sun sensors
  - Magnetometers

TRMM Science



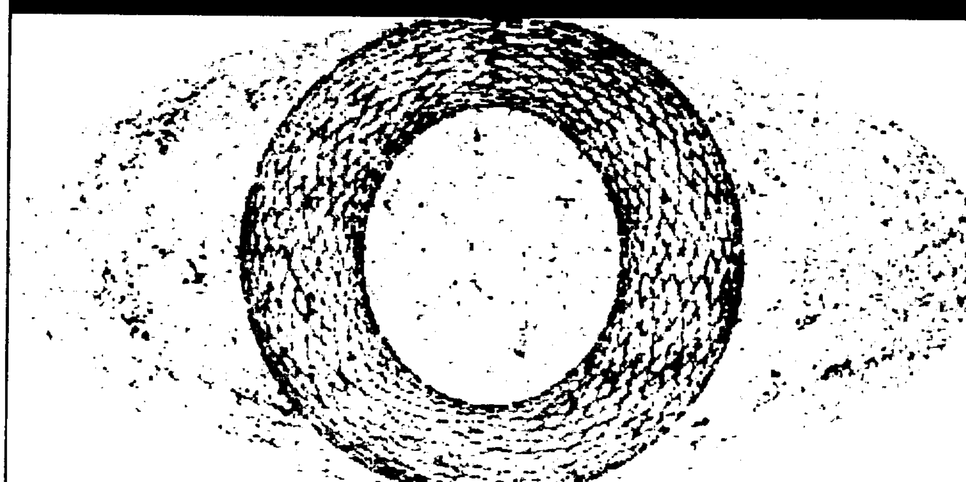
TRMM Attitude Determination

- Science mode uses $R_{321}(yaw, pitch, roll)$
  - Relative to an Earth-pointing reference
  - Earth sensor gives *pitch* and *roll* directly
  - *yaw* determined from gyro and sun sensors
  - Gimbal lock would be at *pitch* $= 90°$
- Independent contingency mode
  - Uses gyros, sun sensors, and magnetometer
  - Quaternion EKF

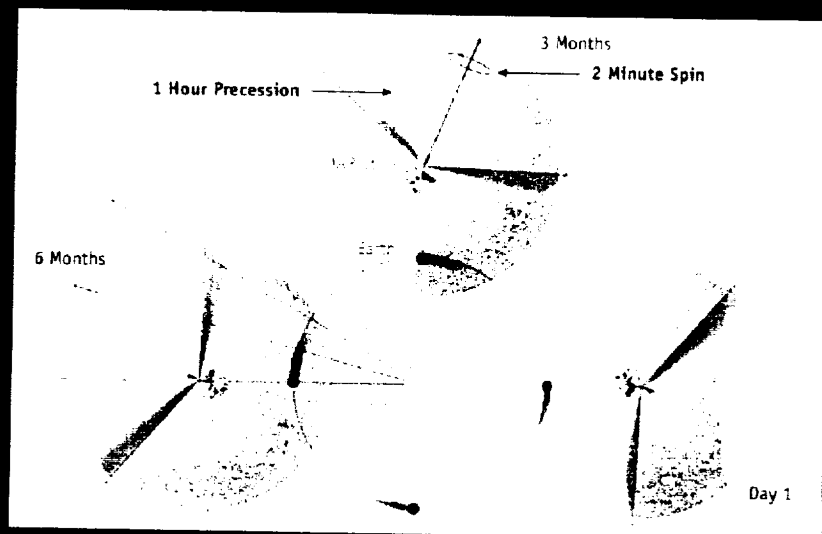
Microwave Anisotropy Probe (MAP), a MIDEX

- Launch November 2000
- L2 orbit, 1.5 million km behind the Earth
- Mass = 830 kg
- Pointing = 0.03°
- Attitude sensors:
  - Star trackers
  - Gyros
  - Sun sensors

MAP Science
2.728 K ± 200 μK



MAP Scan Pattern



MAP Attitude Determination

- Target attitude is $R_{313}(\omega_{slow}\, t,\ 22.5^\circ,\ \omega_{fast}\, t)$ relative to a Sun-pointing reference
  - ω_{slow} 1 revolution/hour about Sun line
  - ω_{fast} 0.5 revs/minute about spacecraft z axis
- Attitude determination is a quaternion EKF, using gyro, star tracker, and sun sensor data

Quaternion EKF

- Straightforward 'additive' EKF has $q = q_{est} + \Delta q$
- EKF uses error covariance $P_{4\times 4} \equiv E\{(\Delta q)(\Delta q)^T\}$
- The normalization constraint is, to first order, $1 = |q|^2 = |q_{est}|^2 + 2(\Delta q)^T q_{est} = 1 + 2(\Delta q)^T q_{est}$.
- So $(\Delta q)^T q_{est} = 0$ to first order in the error.
- Thus $P_{4\times 4} q_{est} = E\{(\Delta q)(\Delta q)^T\}\, q_{est} = 0$.
- The covariance is not positive definite.
- What to do?

Multiplicative Quaternion EKF

- $q = \delta q \otimes q_{est}$, with δq and q_{est} unit quaternions.
- Use a *local* nonsingular 3-dimensional parameterization $\mathbf{x}$ of δq (*e.g.* the Gibbs vector).
- The multiplicative EKF filters $\mathbf{x}$ instead of q.
- $P_{3\times 3} \equiv E\{\mathbf{x}\mathbf{x}^T\}$ is nonsingular.
- After an update, q_{est} is *reset* with $\mathbf{x}$ information.
- This method was invented in 1969 (SPARS), and has been used in NASA spacecraft since 1978.

Swift Gamma Ray Burst Explorer, another MIDEX

- Launch 2003
- Mass – 1300 kg
- Pointing = 3 arc seconds
- Attitude sensors:
  - Star trackers
  - Ring Laser Gyros
  - Sun sensors
  - Magnetometer

Summary

- Attitude requires 3 parameters, but there is no global nonsingular 3-d parameterization
- Onboard attitude determination is the norm, using either 'single frame' or filtering methods
- Several mathematical representations of attitude are used, often on the same spacecraft; quaternions are used in the EKF and QUEST
- Attitude determination precision requirements vary over at least six orders of magnitude