Wave Driven Non-linear Flow Oscillator for the 22-Year Solar Cycle

by

Hans G. Mayr, Charles L. Wolff, and Richard E. Hartle
Goddard Space Flight Center, Greenbelt, MD

August, 2000

Submitted for publication

in

Geophysical Research Letters

Abstract: In the Earth's atmosphere, a zonal flow oscillation is observed with periods between 20 and 32 months, the Quasi Biennial Oscillation. This oscillation does not require external time dependent forcing but is maintained by non-linear wave momentum deposition. It is proposed that such a mechanism also drives long-period oscillations in planetary and stellar interiors. We apply this mechanism to generate a flow oscillation for the 22-year solar cycle. The oscillation would occur just below the convective envelope where waves can propagate. Using scale analysis, we present results from a simplified model that incorporates Hines' gravity wave parameterization. Wave amplitudes <10 m/s can produce reversing zonal flows of 25 m/s that should be sufficient to generate a corresponding oscillation in the poloidal magnetic field. Low buoyancy frequency and the associated increase in turbulence help to produce the desired oscillation period of the flow.

I. Introduction

The Quasi Biennial Oscillation (QBO) dominates the mean zonal circulation in the Earth's stratosphere at low latitudes. The period of this oscillation varies between 20 and 32 months and its amplitude is close to 20 m/s. Employing Kelvin and Rossby-gravity waves, Lindzen and Holton (1968) and Holton and Lindzen (1972) established the important principal that wave-mean-flow interactions can generate the oscillation without external, time dependent forcing; and Plumb (1977) and others further elucidated the dynamical properties of this mechanism. Recent satellite measurements and model results showed that the Kelvin and Rossby-gravity waves are too weak to generate the QBO. Small-scale gravity waves (GW's), invoked by Lindzen (1981) to explain the anomalous temperature and wind reversals in the Earth's upper mesosphere, were therefore introduced as additional or alternative source to drive the QBO (Mengel et al., 1995; Mayr et al., 1997; Dunkerton, 1997).

Analogous to the GW driven terrestrial QBO, we propose that upward propagating GW's interacting with the mean flow may also drive an oscillating zonally symmetric flow with a period of 22 years in the Sun, thus making it a plausible cause for the magnetic cycle (and the 11-year activity cycle). For this mechanism to work, we envision GW's being generated near the solar center. Wolff (1983) showed evidence for rotational beat periods of GW (g-modes in astronomy) although the oscillation periods are difficult to detect convincingly (Palle, 1991).

II. Terrestrial Quasi Biennial Oscillation (QBO) Driven by Gravity Waves

For astrophysicists, we discuss briefly how the terrestrial Quasi Biennial Oscillation (QBO) is generated by GW's. Gravity waves cannot be resolved with global-scale models, and parameterization is required to describe them. Lindzen (1981) first developed such a scheme, and attempts have been made since to provide refinements. In our studies, we adopted Hines' Doppler Spread Parameterization (DSP) that deals with a wave spectrum and accounts for wave-wave and wave-mean-flow interactions. The DSP provides the GW momentum source (MS) and eddy diffusivity K due to wave driven turbulence, but there are uncertainties in the choice of parameters.

We applied the DSP in our Numerical Spectral Model (NSM), introduced by Chan et al. (1994), to study some aspects of the atmosphere dynamics in 2D and 3D settings (Mengel et al., 1995; Mayr et al., 1997, 1998a, 1999). The QBO can be understood as being driven by the MS that accelerates the zonal flow in the presence of K. In Figure 1 we reproduce a numerical simulation of zonal wind oscillations [Figure 7a of Mayr et al. (1998a)] obtained for perpetual equinox with constant solar heating. For this case, using the lowest K, amplitudes of 30 m/s are generated with an oscillation period close to 30 months while the phase progresses downwards, all in qualitative agreement with observations.

Close examination of the GW momentum source reveals (Mayr et al., 1998b) that MS $\propto$ $\partial U/\partial z(U+\sigma|U|/U)^{-2}$, where U is the zonal wind, z is height, and σ is proportional (factor of order one) to the GW horizontal wind variability, σ_h, that increases exponentially with height. A momentum source of this form represents a non-linear function of U that is of third or generally odd order. Given a fundamental frequency of oscillation, ω, such a non-linearity generates the higher harmonics 3ω, 5ω, etc. Importantly, this non-linearity also generates the fundamental frequency itself, so that the oscillation is maintained without external time dependent forcing.

To emulate this non-linear oscillator analytically, a toy model was developed (Mayr et al., 1998b) that will be applied later to make our case for the solar analog. A simplified analytical representation of the MS was applied to the zonal momentum balance at the equator where the Coriolis force vanishes and meridional winds do not come into play. Reduced to a 1D problem, a simplified solution is obtained for the zonal wind oscillation. Given K (the DSP provides) and the vertical wavelength of the QBO, $\lambda \approx 2H$, (H the density scale height), the amplitude and oscillation period can then be estimated.

The MS from the DSP depends on the buoyancy frequency, N, and is proportional to $\sigma_h^3\, k_*$, with the horizontal wave number k_* recommended to be $(100\ \text{km})^{-1} < k_* < (10\ \text{km})^{-1}$. The horizontal wind variability, σ_h, at 20 km is observed to vary from about 2 m/s to 3 m/s. Applying the above toy model at 20 km, we adopt $N = 0.017\ (s^{-1})$, $k_* = (150\ \text{km})^{-1}$, and $K = 0.3\ m^2/s$. The computed amplitude, U, and period, τ, are shown in Figure 2 for varying σ_h and vertical wavelength λ, with respective values around 10 m/s and 24 months that are similar to those from the numerical model. The results reveal that U increases with σ_h due to the increasing MS, and it increases with λ due to the decreasing viscous stress, $(2\pi/\lambda)^2 KU$. And with reduced viscous stress, the time constant and oscillation period, τ, increase. The toy model thus captures the essential features of the QBO and its variations in parameter space, which we take to justify applying it to the Sun.

III. Proposed Wave-Driven 22-Year Oscillation in the Sun

Model

We outline a model for the proposed flow (a Bidecadal Oscillation, BDO) below the convection region of the Sun. Since we are dealing with a high beta plasma, the plasma flow can be derived independently from the magnetic field generated by the flow. At this early stage, we can only suggest that the proposed wave-driven flow oscillation is a plausible driver for the "parasitic" solar poloidal magnetic field. No attempt is made to derive the magnetic field oscillation, which would require a detailed flow model.

Figure 3 illustrates the proposed scenario of wave-driven fluid dynamics we envision to generate the 22-year magnetic cycle. In the radiative regime below the convection region, waves can propagate. The low convective stability there (illustrated in the left panel) represents a dynamical environment that is well suited for wave-mean-flow interaction, wave breaking and turbulence. In a process similar to that responsible for the terrestrial QBO, upward propagating waves deposit momentum and accelerate the mean zonal circulation in the presence of viscous dissipation to produce the oscillation (bottom half of second panel). This oscillation is confined to low latitudes (as illustrated), where the Coriolis force is weak so that momentum cannot be redistributed efficiently by the meridional circulation. Due to the Coriolis force, however, a meridional circulation is set up away from the equator (as illustrated), which is important for the dynamo process. In the presence of a magnetic field, this meridional flow generates a Lorentz force that separates the zonal flow velocities of the electrons and ions to generate oscillating zonal currents. The zonal currents in turn generate a magnetic field whose polarity changes in 11-year intervals, as illustrated in the bottom half of the last panel. Since the meridional flow and Lorentz force go to zero at the equator, the zonal currents also vanish there, as illustrated.

Scale Analysis and Numerical Results

For lack of better knowledge, we extrapolate from our terrestrial experience where applicable. Vertical scale parameters related to the density scale height, H, need to be larger in the Sun by a factor of 10^4. The vertical wavelength of the BDO oscillation is set to λ = 30 Mm, a value of about 0.4H that is proportionately smaller than 2H for Earth. We adopt for the wave amplitude values, σ_h = 5 - 10 m/s, so that the waves carry a negligible fraction of the energy that comes out of the solar interior. In relation to thermal energy, the postulated wave energy is orders of magnitude smaller for the Sun than the Earth. For the horizontal wavenumber of the GW's we adopt a value h = 80, which is 2 times larger than for the Earth and produces $k_* = (6\ \mathrm{Mm})^{-1}$ in the Sun. Unlike the terrestrial case, where the buoyancy frequency is fairly constant throughout the region of the QBO, N varies greatly below the base of the convection region of the Sun. Therefore we present results for a range of N, along with different σ_h values to quantify the wave amplitudes.

Plotted versus N and σ_h, we present in Figure 4 the amplitude of zonal flow oscillation, U, and period of oscillation, τ. We also present the variations in eddy diffusivity, K, provided by the DSP for the chosen parameters. The results show that with increasing wave amplitude, σ_h, both K and U increase as one expects. With decreasing N (buoyancy frequency), K increases as expected; and if K were constant, U would also increase. But the increase in K reverses this trend and causes the small decrease in U as Figure 4 shows. The oscillation period, τ, decreases with increasing σ_h, because of the concomitant increase in acceleration. As seen from Figure 4, τ also decreases with decreasing N. This latter trend is understandable and shows the important role that low N plays in producing reasonable τ values. Given the large length-scales in the Sun due to its large temperatures, the flow cycle period would be exceedingly long, hundreds of years, if it were not for the low buoyancy frequency (marginal convective stability) just below the base of the convection region.

IV. Summary, Qualification, and Possible Observational Tests

Since the amplitudes of long-period waves are unknown in the solar interior, our analysis must be speculative. To make the case for an astrophysical metronome, we carried out a scale analysis by extrapolating our terrestrial experience to the Sun. Using the terrestrial QBO as a fluid dynamical laboratory for insight, we applied a simple heuristic model to the radiative regime in the Sun where waves can propagate. The estimates obtained from this model lead us to conclude that the radiative regime near the base of the solar convection region where low convective stability resides, is favorable to produce a wave-driven flow, oscillating with period on the order of 22 years. With reasonable wave amplitudes, the estimated amplitude of the zonal flow oscillation can be made to be of order 25 m/s, which is comparable to the differential flows that drive well-known solar dynamo models. In the region allocated for this flow oscillation, the eddy diffusivity generated by wave driven turbulence is enhanced due to low convective stability. This reduces the turbulent momentum transfer between ions and electrons and would aid in generating the zonally symmetric currents that produce the oscillating magnetic field.

It is important to qualify our model. We have only shown that it is possible to generate reasonable flow amplitudes and oscillation periods by choosing what we consider to be reasonable GW parameters. While Hines' GW parameterization, DSP, has a strong physical basis and has been applied with some success in the terrestrial setting, our understanding of GW processes is still rudimentary. As pointed out earlier, the DSP comes with several free parameters. For a given GW wind variability, σ_h, we could have chosen parameters, within the range recommended by the DSP, that do produce not enough of a momentum source and/or too large an eddy diffusivity so that the desired BDO does not develop in the model. And almost the same could be said about our inability to produce uniquely a QBO in the terrestrial setting for which we know so much more about GW's. Thus, planetary waves may also be involved in driving the oscillating flow in the Sun, as discussed for the QBO by Lindzen and Holton.

Helioseismology, solar surface temperature, and eventually oblateness might show evidence of the BDO. As emphasized above, the range of free parameters prevents any firm prediction, but as an example, we discuss a BDO of peak strength 20 m/s and a thickness α times the density scale height. If $\alpha = 0.2$ or larger, helioseismology could probably measure such a flow now (Thompson, et al., 1996). The solar oblateness and the extra brightness of equatorial latitudes have been measured by Kuhn, et al. (1998, 1987). They find low latitudes have a surface temperature about 1.5 K larger than mid-latitudes and see a total oblateness of about 8 milliarcsec. If the current 10% precision in oblateness were improved by two orders of magnitude, one should be able to see the effect of the BDO on the Sun's shape. As the BDO flow dissipates into heat each 11 year cycle, the resulting temperature difference between low and mid-latitudes might already be detectible in data now accumulating. Let the BDO occupy all low latitudes up to 30°. Then its kinetic energy is about $0.5\rho U^2(2\pi r_c^2 \alpha H)$, where r_c is its radial location and ρ is the mean density. Converting most of this to heat during 4 years of rapid decay represents a fractional change in the solar irradiance of $\Delta I/I = 5\times10^{-7}\alpha U^2$ (U in m/s) at these latitudes. The linearized Stefan Boltzmann law, $4\Delta T/T \sim \Delta I/I$, gives a surface temperature change $\Delta T \sim 0.06$ K at low latitudes for $\alpha = 0.2$, which should be measurable with today's techniques. (For brevity, we ignored diffusion in latitude and thus have only an upper limit to ΔT.) The mean power dissipated by the above flow is $\sim 10^{-5}$ times the solar luminosity. We note that this is $\sim 10^{3}$ times as large as the power (averaged over 11 years) released by solar flares (deJager, 1970). If large scale convection can carry much of this energy near to the surface, this example-BDO may have enough power to supply all forms of solar activity. Wolff (1984) has suggested several times that large-scale convection and its resulting small-scale vortices, not magnetism, are a more basic cause of solar activity and its distribution over the solar surface.

Acknowledgement: One of us (HGM) is greatly indebted to Prof. Colin O. Hines for his earliest and active support of the idea to propose the QBO mechanism for the 22-year solar cycle.

References

Chan, K. L., H. G. Mayr, J. G. Mengel, and I. Harris, A 'stratified' spectral model for stable and convective atmospheres, *J. Comp. Phys.*, 113, 165, 1994a

De Jager, C., *Solar Terrestrial Physics: Part I*, ed E. Dyer (Dordrecht: Reidel), 1, 1972

Dunkerton, T., J., The role of gravity waves in maintaining the quasi-biennial oscillation, *J. Geophys. Res.*, 102, 26053, 1997

Hines, C. O., Doppler-spread parameterization of gravity-wave momentum deposition in the middle atmosphere, 1, Basic formulation, *J. Atmos. Solar-Terr. Phys.*, 59, 371, 1997a

Hines, C. O., Doppler-spread parameterization of gravity-wave momentum deposition in the middle atmosphere, 2, Broad and quasi monochromatic spectra, and implementation, *J. Atmos. Solar-Terr. Phys.*, 59, 387, 1997b

Holton, J. R., and R. S. Lindzen, An updated theory for the quasi-biennial cycle of the tropical stratosphere, *J Atmos. Sci.* 29, 1076-1080, 1972

Kuhn, J. R., Libbrecht, K. G., and Dicke, R. H., Evidence of global circulation currents from solar-limb temperature variations, *Nature*, 328, 326, 1987

Kuhn, J. R., Bush, R. I., Scheick, X., and Scherrer, P., The Sun's shape and brightness, *Nature*, 392, 155, 1998

Lindzen, R. S., and J. R. Holton, A theory of the quasi-biennial oscillation, *J. Atmos. Sci.*, 25, 1095, 1968

Lindzen R. S., Turbulence and stress due to gravity wave and tidal breakdown, *J. Geophys., Res.*, 86, 9707-9714, 1981

Mayr, H. G. et al., The gravity wave Doppler spread theory applied in a numerical Model of the middle atmosphere, 2, Equatorial oscillations, *J. Geophys. Res.*, 102, 26093, 1997

Mayr, H. G., J. G. Mengel, and K. L. Chan, Equatorial oscillations maintained by gravity waves as described with the Doppler spread parameterization: I. Numerical experiments, *J. Atmos. Solar-Terr. Phys.*, 60, 181, 1998a

Mayr, H. G., R. E. Hartle, and K. L. Chan, Equatorial oscillations maintained by gravity waves as described with the Doppler spread parameterization: II. Heuristic analysis, *J. Atmos. Solar-Terr. Phys.*, 60, 201, 1998b

Mayr, H. G. et al., The role of gravity waves in maintaining the QBO and SAO at equatorial latitudes, *Adv. Space Res.*, 24 (11), 1531, 1999

Mengel, J. G. et al., Equatorial oscillations in the middle atmosphere generated by small scale gravity waves, *Geophys. Res. Lett.*, 22 (22), 3027, 1995

Palle, P. L., The search for solar gravity waves, *Adv. Space Res.*, 11 (4), 29, 1991

Plumb, R. A., The interaction of two internal waves with the mean flow: Implications for the theory of the quasi-biennial oscillation, *J. Atmos. Sci.*, 34, 1847, 1977

Thompson, M. J., et al, Differential rotation and dynamics of the solar interior, *Science*, 272, 1300, 1996

Wolff, C. L., The rotational spectrum of g-modes in the Sun, *Astrophys. J.*, 264, 667, 1983

Wolff, C. L., Solar irradiance changes caused by g-modes and large scale convection, *Solar Physics*, 93, 1, 1984

Figure Captions

Figure 1: Contour plots of zonal winds at 4°N computed for perpetual equinox, i.e., without external time dependent forcing. A steady flow of gravity waves -- whose interaction with the background flow is described with Hines' Doppler Spread Parameterization -- drives in a 2D model an oscillation with a period of about 30 months. (Figure taken from Mayr et al., 1998a.)

Figure 2: Zonal wind amplitudes, U, and corresponding oscillation periods computed for Earth from an heuristic analytical model. With fixed eddy viscosity of 0.3 m^2/s, the oscillations are derived at 20 km for different values of σ_h and vertical wavelengths of oscillation, λ. The amplitudes and periods are in qualitative agreement with the numerical result shown in Figure 1.

Figure 3: Schematic illustration of the wave driven oscillator referred to as Bidecadal Oscillation (BDO), proposed to generate the dynamo for the 22-year solar magnetic cycle. Near the base of the convection region where the convective stability (N^2) approaches zero, the dynamical conditions are well suited for waves to generate turbulence and deposit momentum in the background flow. The zonal flow oscillation (BDO) thus generated, analogous to the terrestrial QBO, is accompanied by a meridional circulation. The meridional flow produces a Lorentz force in the fully ionized plasma of the Sun. In the presence of a magnetic field, this force drives oscillating zonal flow currents. And the zonal currents can produce a poloidal magnetic field that changes polarity in 11-year intervals.

Figure 4: Eddy diffusivities, K, zonal flow amplitudes, U, and oscillation periods, τ, derived with the heuristic model applied to the Sun. Using scale analysis, the adopted parameters were chosen in part by extrapolating from our terrestrial experience. The amplitudes of long-period

waves in the solar interior are unknown, and so our analysis must be speculative. The results suggest that the proposed wave-driven oscillator needs to be placed near the base of the convection region where the buoyancy frequency is sufficiently low to produce a period of 22 years that otherwise would be much longer considering the large scale height of the solar atmosphere.

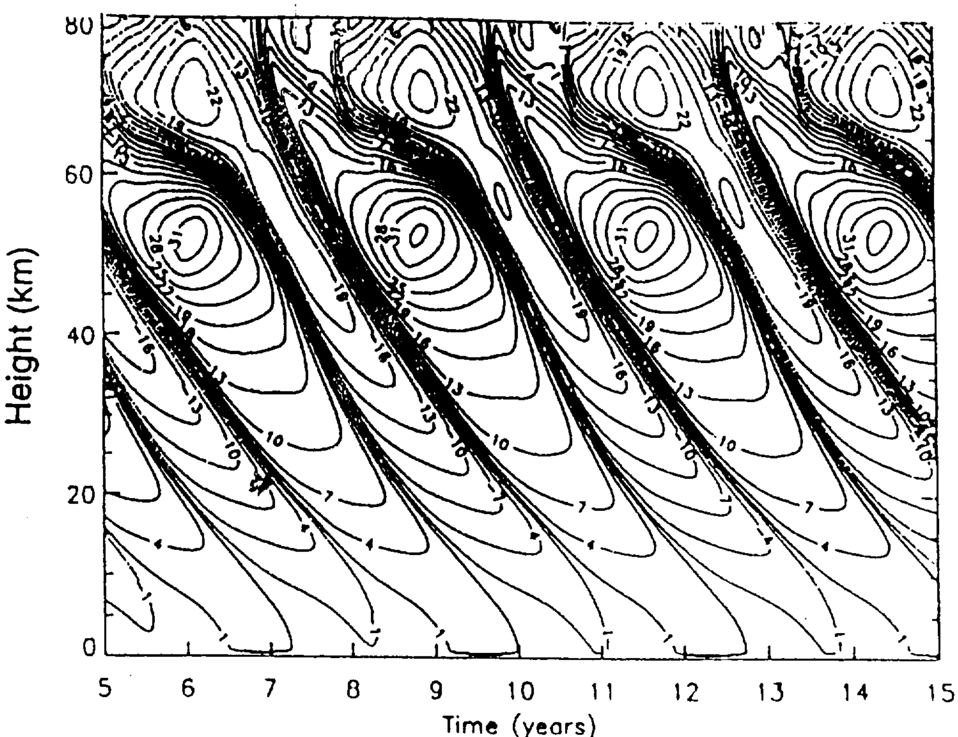

Figure 1

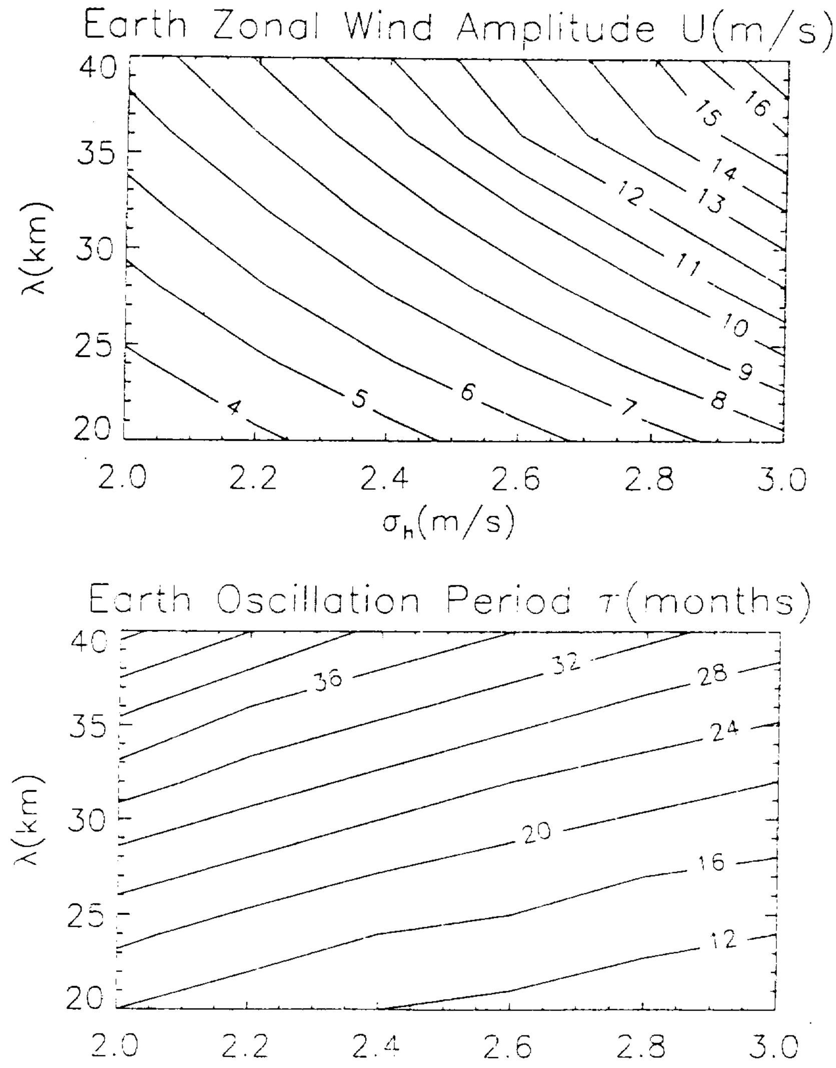

Figure 2

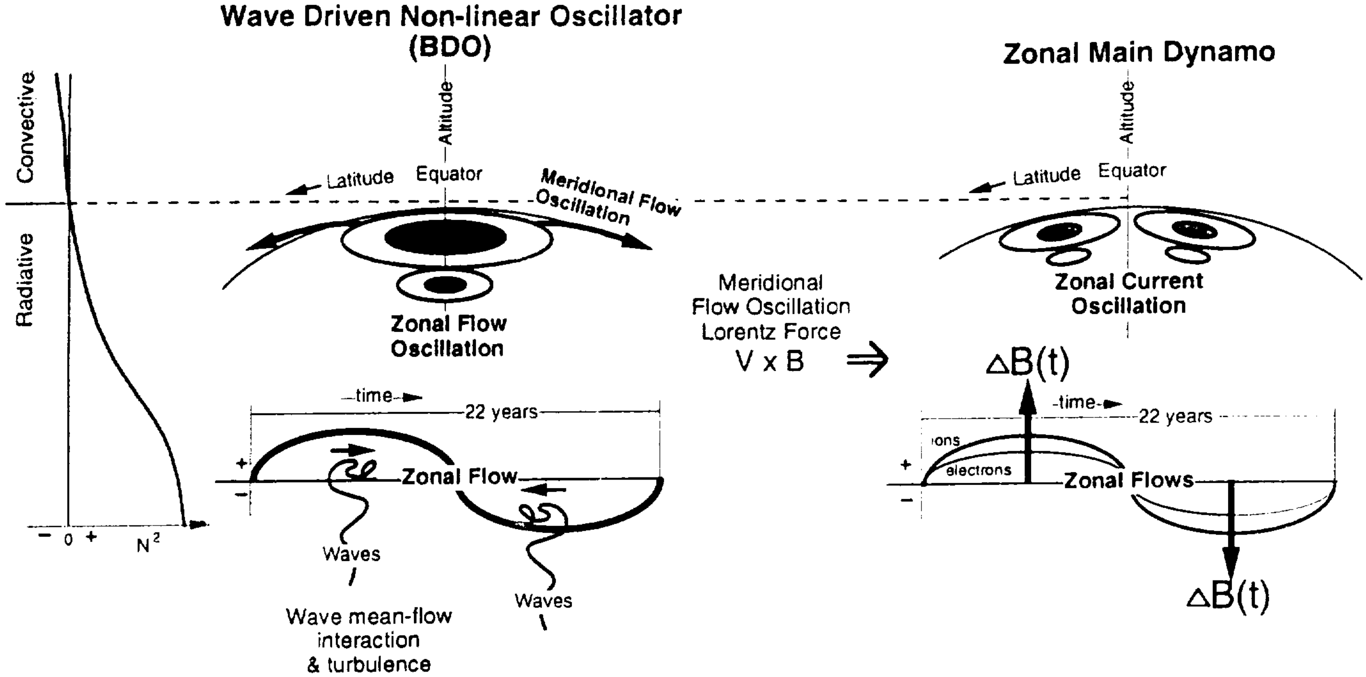

Figure 3

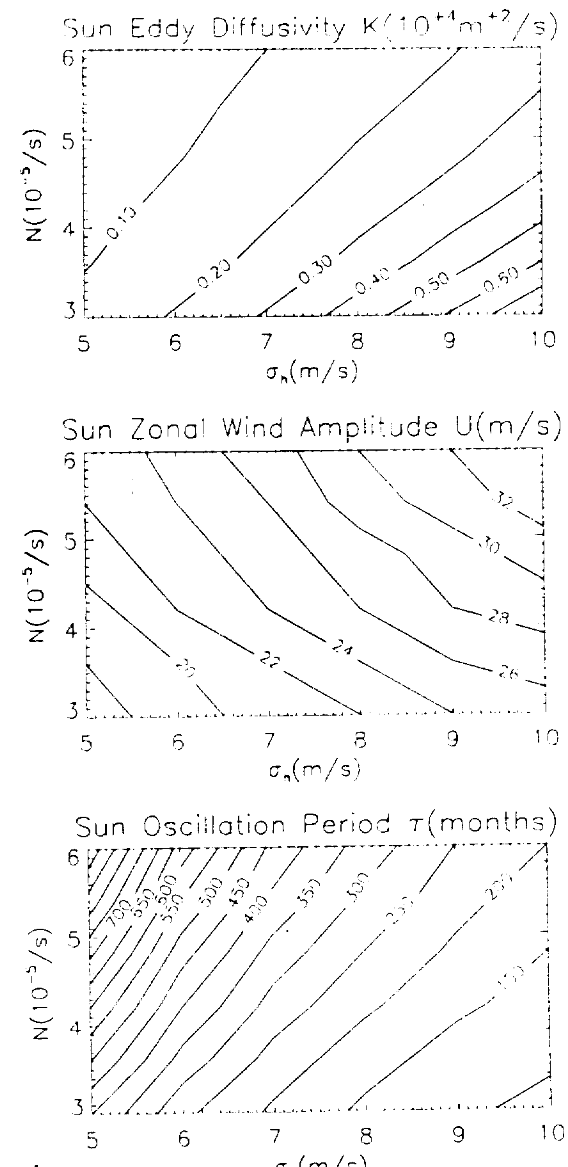

Figure 4