The DAO Retrospective Data Assimilation System *

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Abstract

The fixed-lag Kalman smoother of Cohn et al. (1994), or a version of a more computationally feasible approximation of it developed by Todling et al. (1998), is under implementation at the Data Assimilation Office (DAO) to build a retrospective data assimilation system (RDAS). The initial version of the RDAS uses the physical-space statistical analysis system (PSAS; Cohn et al. 1998) and a modification of the incremental analysis update (IAU) procedure of Bloom et al. (1996); a follow up version of the RDAS will require the use of the adjoint of the DAO general circulation model (developed by Y. Yang and I. M. Navon). The retrospective procedure is designed to produce improved analyses as well as improved assimilated fields consequently providing an improved climate representation through data assimilation.

1 The lag-1 RDAS formulation

Let $\mathbf{y}(\sigma)$ represent an n-vector state of the general circulation model (GCM) on its sigma vertical coordinate system. Moreover, let $\mathbf{x}(p)$ represent an m-vector state of the analysis component of the DAS, which in the case of the DAO system is built in pressure coordinates. This way, we can write PSAS analysis increments $\delta\mathbf{x}_{k|k}(p)$, at time t_k, as

$$\delta\mathbf{x}_{k|k}(p) \equiv \mathbf{x}_{k|k}(p) - \mathbf{x}_{k|k-1}(p) = \mathbf{P}^f_{k|k-1}\mathbf{H}^T_k\mathbf{\Gamma}^{-1}_k\mathbf{v}_k \tag{1}$$

where the forecast error covariance matrix $\mathbf{P}^f_{k|k-1}$ is defined on a pressure coordinate system. In the expression above the $r_k \times m$ matrix $\mathbf{H}_k$ converts pressure fields to observation locations, and the $r_k \times r_k$ matrix $\mathbf{\Gamma}_k$ is the error covariance matrix of the r_k-residual vector of observation-minus-forecast $\mathbf{v}_k \equiv \mathbf{x}^o_k - \mathbf{H}_k\mathbf{x}_{k|k-1}(p)$, and is given by

$$\mathbf{\Gamma}_k \equiv \mathbf{H}_k\mathbf{P}^f_{k|k-1}\mathbf{H}^T_k + \mathbf{R}_k \tag{2}$$

where $\mathbf{R}_k$ is the $r_k \times r_k$ observation error covariance matrix.

Assume now the existence of a nonlinear operator Π that transforms GCM prognostic sigma fields on to analysis/first-guess pressure level fields. That is, for a given GCM n-vector background field $\mathbf{y}(\sigma)$ an m-vector first-guess field $\mathbf{x}(p)$ can be derived according to

$$\mathbf{x}(p) \equiv \Pi[\mathbf{y}(\sigma)]\,. \tag{3}$$

The operator Π is nonlinear since it represents more than just simple interpolation procedures: it stands for transformation of variables as well, such as converting potential temperature into heights or deriving sea level pressure and sea level winds through planetary boundary layer effects. In the DAO system, the GCM background state vector is composed of the two zonal and meridional wind components, potential temperature, specific humidity and surface pressure, that is, $y(\sigma) = y(u,v,\theta,q,\pi)(\sigma)$; the analysis state vector is composed of the sea-level pressure, sea-level zonal and meridional wind components, upper-air zonal and meridional wind components, geopotential height and mixing ratio, that is, $x(p) = x(p_{sl}, u_{sl}, v_{sl}, u, v, h, q_m)(p)$.

It is now convenient to introduce an operator Π^+ which takes an analysis state vector $\mathbf{x}(p)$ in to a model state vector $\mathbf{y}(\sigma)$ according to

$$\mathbf{y}(\sigma) \equiv \Pi^+[\mathbf{x}(p)]\,. \tag{4}$$

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Once these two transformations Π and Π^+ have been defined we can introduce there corresponding Jacobian operators as

$$\boldsymbol{\Pi} \equiv \left. \frac{\partial \Pi[\mathbf{y}]}{\partial \mathbf{y}} \right|_{\mathbf{y}=\mathbf{y}(\sigma)}, \tag{5}$$

$$\boldsymbol{\Pi}^+ \equiv \left. \frac{\partial \Pi^+[\mathbf{x}]}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}(p)}. \tag{6}$$

respectively. The actual implementation of Π^+ is such that it renders minimal the difference $||\mathbf{x}(p) - \Pi[\Pi^+[\mathbf{x}(p)]]||$. In other words, the error is minimal when transforming an analysis state vector in to a model state vector and then back to an analysis state vector. In a nonlinear sense, Π^+ is a pseudo-inverse of Π. Analogously, $\boldsymbol{\Pi}^+$ must be a pseudo-inverse of $\boldsymbol{\Pi}$; in practice, numerical inaccuracies and the character of the original nonlinearities are such that this can only be achieved to a certain extent.

With the definitions above, the increment (1) can be converted in to a model increment vector as

$$\delta \mathbf{y}_{k|k}(\sigma) = \Pi^+[\mathbf{x}_{k|k}(p)] - \mathbf{y}_{k|k-1}(\sigma). \tag{7}$$

which is then be used, after proper rescaling, as the IAU forcing term (see Bloom et al. 1996).

From Todling and Cohn (1996) the lag-1 retrospective analysis increment is given by

$$\delta \mathbf{x}_{k-1|k,k-1}(p) \equiv \mathbf{x}_{k-1|k}(p) - \mathbf{x}_{k-1|k-1}(p) = \mathbf{P}^a_{k-1|k-1}\mathbf{A}^T_{k,k-1}\mathbf{H}^T_k\boldsymbol{\Gamma}^{-1}_k\mathbf{v}_k, \tag{8}$$

which as in the regular (filter) analysis is given in pressure coordinates. The operator $\mathbf{A}^T_{k,k-1}$ is the adjoint of the tangent linear model of the GCM and of all transformations involved in taking state vectors from the model to the analysis grid and vice-versa; the operator $\mathbf{P}^a_{k-1|k-1}$ is the analysis error covariance obtained from

$$\mathbf{P}^a_{k-1|k-1} = (\mathbf{I} - \mathbf{K}_{k-1|k-1}\mathbf{H}_{k-1})\mathbf{P}^f_{k-1|k-2}. \tag{9}$$

To derive the proper equation for the operator $\mathbf{A}_{k,k-1}$ we introduce the GCM evolution equation through an operator $\mathcal{M}$ that acts on model fields as follows

$$\mathbf{y}_{k|k-1}(\sigma) = \mathcal{M}[\mathbf{y}_{k-1|k-1}(\sigma)]. \tag{10}$$

The nonlinear GCM evolution operator $\mathcal{M}$ has the Jacobian operator defined as

$$\mathbf{M}(\sigma) \equiv \left. \frac{\partial \mathcal{M}[\mathbf{y}]}{\partial \mathbf{y}} \right|_{\mathbf{y}=\mathbf{y}(\sigma)} \tag{11}$$

which also acts on model (perturbation) fields. Together with the Jacobians introduced previously, the operator $\mathbf{A}_{k,k-1}$ can be shown to have the following explicit form

$$\mathbf{A}_{k,k-1}(p) \equiv \boldsymbol{\Pi}_k\mathbf{M}_{k,k-1}(\sigma)\boldsymbol{\Pi}^+_{k-1}. \tag{12}$$

The need for the adjoint of $\mathbf{A}_{k,k-1}(p)$ in (8) indicates the requirement for constructing not only the adjoint, $\mathbf{M}^T_{k,k-1}$, of the model's tangent linear operator $\mathbf{M}_{k,k-1}$, but also the adjoint of both $\boldsymbol{\Pi}_k$ and $\boldsymbol{\Pi}^+_{k-1}$. The final lag-1 retrospective analysis increment on the model variables is derived as

$$\begin{aligned} \delta \mathbf{y}_{k-1|k,k-1}(\sigma) &= \mathbf{y}_{k-1|k}(\sigma) - \mathbf{y}_{k-1|k-1}(\sigma) \\ &= \Pi^+\left(\mathbf{x}_{k-1|k-1}(p) + \delta\mathbf{x}_{k-1|k,k-1}(p)\right) - \mathbf{y}_{k-1|k-1}(\sigma). \end{aligned} \tag{13}$$

This increment can be rescaled properly and used in the IAU procedure to obtain the retrospective IAU assimilation. The motivation for using the retrospective analysis increments $\delta\mathbf{x}_{k-1|k-1}(p)$ to produce an IAU increment that is then used to force the model and perform another GCM integration is similar to that of the original IAU procedure, i.e., to produce a consistent dynamically balanced and continuous history of the model fields.

2 The approximate RDAS

Let us consider the expression for the lag-1 retrospective increment when we take the approximation of replacing the adjoint of the dynamics operator $\mathbf{A}_{k,k-1}^T$ with the identity matrix. In this case, we can write the lag-1 retrospective analysis increment as

$$
\begin{aligned}
\delta\tilde{\mathbf{x}}_{k-1|k,k-1} &= \mathbf{P}_{k-1|k-1}^a \mathbf{H}_k^T \mathbf{\Gamma}_k^{-1} \mathbf{v}_k \\
&= (\mathbf{I} - \mathbf{K}_{k-1|k-1}\mathbf{H}_{k-1})\mathbf{P}_{k-1|k-2}^f \mathbf{H}_k^T \mathbf{\Gamma}_k^{-1} \mathbf{v}_k
\end{aligned}
\tag{14}
$$

where the last equality is obtained after substituting the analysis error covariance at t_{k-1} with its explicit form, and a tilde is used to indicate this is an approximate quantity.

In practice, in many three-dimensional variational systems, the forecast error covariance matrix $\mathbf{P}_{k-1|k-2}^f$ is constant in time[1]. If we take this matrix to be constant we can write

$$
\delta\tilde{\mathbf{x}}_{k-1|k,k-1} = (\mathbf{I} - \mathbf{K}_{k-1|k-1}\mathbf{H}_{k-1})\delta\mathbf{x}_{k|k} \tag{15}
$$

where $\delta\mathbf{x}_{k|k}$ is the regular (filter) analysis increment. Thus, the expression above provides an extremely simple way of calculating the lag-1 retrospective increment given the analysis increment. As a matter of fact, the last term in the expression above corresponds to an application of PSAS to the vector $\mathbf{H}_{k-1}\delta\mathbf{x}_{k|k}$, which replaces the innovation vector needed when calculating the filter analysis. This quantity represents the projection of the current (filter) analysis increments on the observation grid of the previous analysis time.

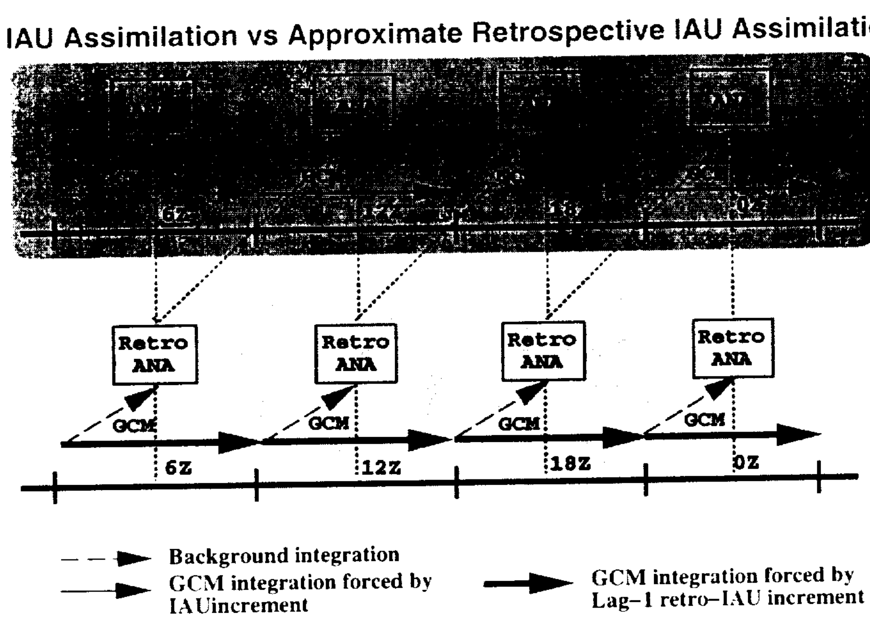

Figure 1: Schematic representation of IAU (within heavy gray rectangle) and retrospective IAU (within light gray rectangle). Dashed arrows represent the GCM integration; solid arrows represent GCM integration forced by IAU (thin arrows) and retrospective IAU increment (thick arrows).

The expression for retrospective analysis increments can be extended to lags higher than one, with the general lag ℓ result being

$$
\delta\tilde{\mathbf{x}}_{k-\ell|k,k-1} = (\mathbf{I} - \mathbf{K}_{k-\ell|k-\ell}\mathbf{H}_{k-\ell})\delta\tilde{\mathbf{x}}_{k-\ell+1|k,k-1} \tag{16}
$$

where $\delta\tilde{\mathbf{x}}_{k-\ell|k,k-1} \equiv \tilde{\mathbf{x}}_{k-\ell|k} - \tilde{\mathbf{x}}_{k-\ell|k-1}$. The equation above states that each lag of the approximate retrospective procedure can be obtained from the retrospective increment of the previously calculated

[1] This is not the case in the DAO's System, however, for all practical purposes the forecast error covariance matrix can be considered to be constant since it varies very slowly in time.

retrospective increment with one additional PSAS application. With the assumption of constant forecast error covariances, the same can be shown to be true for the algorithm involving the adjoint operator(s) of the previous section. We remark further that:

- When the forecast error covariance matrix $\mathbf{P}^f$ is kept constant, the only time dependence in the gain matrix $\mathbf{K}_{k-1|k-1}$ comes from the time dependence of the observing network.
- It can be shown that, when the forecast errors are indeed independent of the dynamics, expression (16) corresponds to the optimal solution for the lag-ℓ retrospective increments, in the linear, minimum variance sense.
- Furthermore, when the observing system is constant in time, the lag-1 retrospective analysis algorithm corresponds to a single step of the bias estimation procedure of Dee and da Silva (1999), when the bias first-guess is null. In other words, each lag-1 retrospective analysis corresponds to a less biased analysis than that of the filter.

A schematic representation of the approximate lag-1 RDAS is shown in Fig. 1, where the thick solid arrows in the lower, light shaded, rectangle indicate the retrospective IAU assimilation trajectory.

3 Preliminary results

In this section we briefly show the performance of the RDAS versus that of the regular DAS using an identical-twin experiment set up. We construct observations from a control run of the DAS. The control run is a complete DAS run with the full observing network of rawinsondes, TOVS height retrievals, conventional surface data and Wentz total precipitable water. This experiment serves as the truth, from which we construct observations, for the following experiments, at exactly the same locations as the original observations.

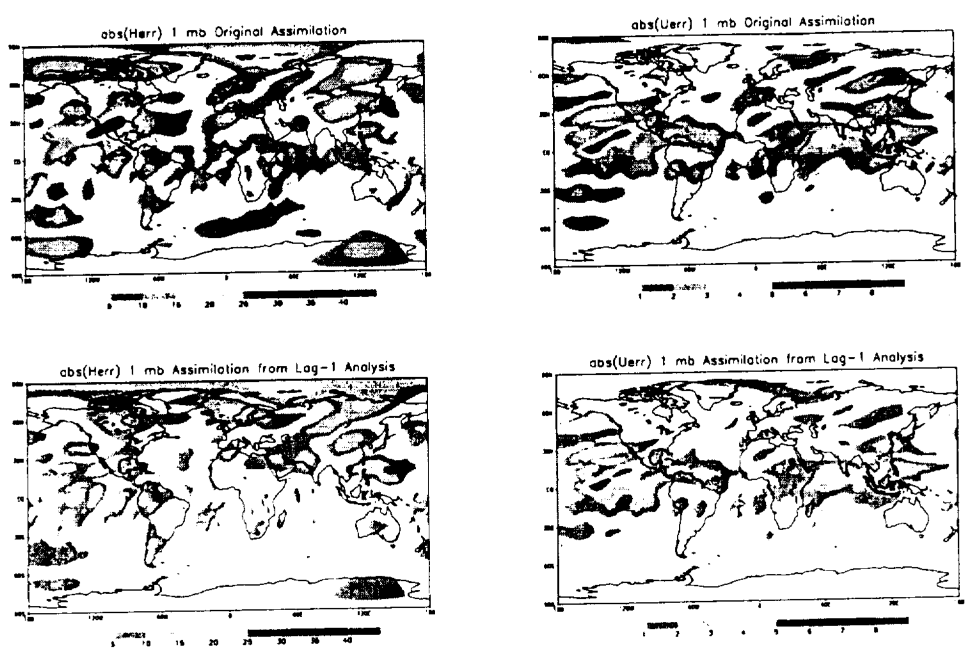

Figure 2: Assimilation versus lag-1 retrospective assimilation. Panels show 10-day mean absolute error for heights (left) and zonal winds (right), at 1 mb. Top panels are for assimilation, bottom panels are for retrospective assimilation.

Two experiments are ran for a 10-day period with the DAS and the RDAS. In Fig. 2 we show the (10-day) mean absolute error in both these experiments for the the height and zonal wind fields at 1 mb. High in the stratosphere the main (only) source of data are the TOVS height retrievals. We see from the figure a considerable improvement in the assimilation when using the retrospective procedure (bottom panels). The reduction in errors in the height field is more pronounced than for the winds since the only source of wind correction this high up comes from the multivariate aspect of PSAS. Since the error covariance formulation in PSAS is nearly geostrophic, with a weak relaxation from geostrophy near the equator, wind error reduction is not as significant along the tropics.

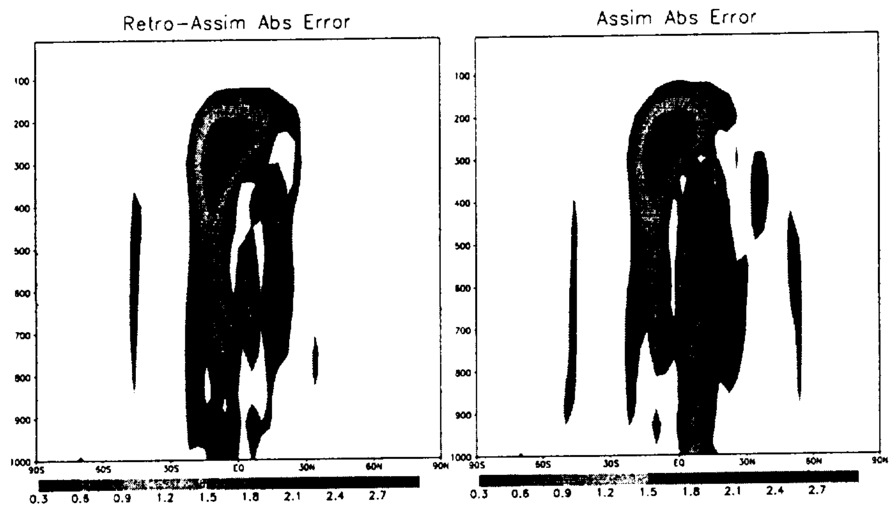

Figure 3: Absolute mean error in mass stream function for lag-1 retrospective assimilation (left) and assimilation (right).

Another indication of the overall improvement obtained in the assimilation through the RDAS is seen in Fig. 3 where the absolute error in the mass-stream function, averaged over the 10-day period of the experiments, is shown. We see a slight improvement in the Hadley circulation during this period when comparing the RDAS (left panel) against the DAS (right panel). Other climatologically significant quantities show similar marginal improvement.

4 Conclusions

Preliminary results indicate the lag-1 RDAS to provide improvements over the DAS. Experiments for time scales of a month to a season are being conducted to assess more accurately the impact on the assimilation climatology. Examination and experimentation of the complete RDAS procedure involving the adjoint operators will follow.