

## **Estimates of internal tide energy fluxes from Topex/Poseidon altimetry: Central North Pacific**

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Short title: INTERNAL TIDE ENERGY FLUXES

## Introduction

The global ocean tide dissipates 25–30% of its total power loss ( $\sim 1$  TW) near large bathymetric features in the open sea. This is the conclusion of a recent study of global cotidal charts deduced from Topex/Poseidon satellite altimeter data [Egbert and Ray, 2000]. The physical mechanism responsible for this dissipation is apparently scattering of energy into internal tides and other baroclinic motions and turbulence. Such scattering complements dissipation by turbulent friction in the bottom boundary layer of shallow seas, which is the traditional tidal energy sink.

While the Egbert-Ray conclusion rests on analysis of the barotropic tide, internal tide signals in satellite altimetry open a second, independent approach to quantifying via altimetry the barotropic-to-baroclinic conversion rate. Under certain conditions internal tides are detectable in satellite altimetry as small (several cm) oscillations in the tide estimated at points along repeating satellite tracks [Ray and Mitchum, 1996, 1997]. The wavelengths are of order 100–200 km, consistent with typical baroclinic phase speeds of  $2\text{--}3\text{ m s}^{-1}$ . We show here that such signals enable direct mapping of internal-tide sources, sinks, and power transmission routes throughout the ocean. We use a combination of two-dimensional wave fitting to the altimetry with realistic numerical solutions for the relation between the surface elevations and the current and pressure at depth (based on climatological hydrographic data). Both progressive and standing-wave elements are allowed for. Energy fluxes follow by integration of pressure  $\times$  current velocity from surface to seafloor. Successive employment of this procedure over small oceanic regions allows maps of internal-tide energy fluxes to be produced over arbitrarily wide stretches of the ocean.

Before proceeding, however, we should note certain limitations to the altimetric analysis of internal tides. These limitations suggest that internal-tide energy fluxes represent in many cases only lower bounds. The limitations are: (1) Only temporally coherent internal tides (i.e., those that maintain phase lock with the astronomical potential over long timespans) are detectable with altimetry. Such coherent tides may be exceptional, since temporal variability is generally considered the norm [e.g., Wunsch, 1975]. (2) The satellite tracks are widely spaced relative to internal-tide wavelengths, causing possible loss of spatial resolution and smearing of signal in our wave fitting. (3) We assume that a plane wave adequately describes the internal tide over distances of several hundred km. In regions of complex topography and multiple sources, this assumption may be inadequate. (4) Only the first baroclinic mode is considered. There is some evidence for higher modes in the altimetry [e.g., Ray and Mitchum, 1996], but the signals are significantly smaller and more difficult to extract.

they commonly exceed  $10^3 \zeta_0$  at mid-depths. The power transmission, however, is in the wave direction  $\phi$  at all depths. The total transmission is given by

$$P = \frac{1}{2} \int_{-D}^0 u(z) p(z) dz. \quad (3)$$

Finally, since  $u(z)$  and  $p(z)$  in (3) are linearly proportional to the assumed surface amplitude  $|\zeta_0|$ , the energy flux  $P$  in direction  $\phi$  is scaled by

$$(a^+)^2 - (a^-)^2.$$

(It is assumed that  $a^+ \geq a^-$ ; if not,  $a^+, a^-$  are interchanged with  $(\phi + \pi)$  replacing  $\phi$ . In the special case of a purely standing wave,  $a^+ = a^-$ , the energy flux is zero.)

The above procedure is applied to the altimetric tide estimates falling within a small oceanic region. The size of this region is a matter of some delicacy. It should be large enough to ensure robust fits to Eq. (1), which calls for a spread of data in  $(x, y)$ , necessitating 2 to 4 altimeter tracks. But it should also be small enough that the observed internal tide can be expressed by the simple plane wave, and especially it should avoid combining ocean regions with markedly different internal tide characteristics. The choice is clearly a tradeoff, and the widely spaced tracks of Topex/Poseidon ( $2.835^\circ$  in longitude) are probably only marginally adequate to the task. In the work shown here we have chosen ocean regions of size  $4^\circ$  (longitude)  $\times$   $3^\circ$  (latitude), overlapping on a  $1^\circ \times 1^\circ$  grid. The size ensures at least 3 or 4 different altimeter tracks in each region and at least 2 full wavelengths of the internal tide (of typical wavelength 150 km).

## Results for Central North Pacific

Figure 1 shows energy flux vectors of the  $M_2$  first-mode internal tide throughout the central North Pacific Ocean, derived by the above method. The area includes the Hawaiian Ridge and part of the Aleutian Trench. Altimeter estimates of the internal tide across the Hawaiian Ridge were previously discussed by *Ray and Mitchum* [1996, 1997], and along-track estimates of the phase were observed to show clear wave propagation away from the ridge, both northwards and southwards. The flux vectors here give a more detailed picture of the propagation and depict more clearly the regions of intense generation. The reader is urged to compare this picture with similar diagrams in *Kang et al.* [2000] and *Merrifield and Holloway* [2000]; the former use a simple two-layer dynamic model, the latter a series of three-dimensional models, to deduce internal-tide energy fluxes (although in somewhat more limited domains than Fig. 1). All three pictures are remarkably consistent in their depiction of the generation regions and the strong propagation somewhat localized to a series of fairly narrow beams.

## Prospects

Space precludes showing regions complementing that of Fig. 1. We may briefly mention one of the clearest examples in the altimetry: the region surrounding the Tuamotu Archipelago in the South Pacific (amplitudes were shown by *Ray and Mitchum* [1997]). In that region the integrated energy flux amounts to 11 GW.

Further application of the methods of this paper to global scales is straightforward to the extent that robust tidal estimates can be computed along altimeter tracks. We find that this is somewhat problematic in regions of high mesoscale activity, even after 8 years of data. In those regions more sophisticated signal extraction techniques may be called for.

**Figure 1.** Energy flux vectors of the first baroclinic mode of the  $M_2$  internal tide, computed on a  $1^\circ$  grid of overlapping  $4^\circ \times 3^\circ$  bins. Flux vectors smaller than 100 W/m are not shown. Bathymetry contours are given every 1000 meters. The large triangle near  $35^\circ\text{N}$ ,  $160^\circ\text{W}$  is the RTE87 acoustic array described by *Dushaw et al.* [1995].

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**Abstract.** Energy fluxes for first-mode  $M_2$  internal tides are deduced throughout the central North Pacific Ocean from Topex/Poseidon satellite altimeter data. Temporally coherent internal tide signals in the altimetry, combined with climatological hydrographic data, determine the tidal displacements, pressures, and currents at depth, which yield power transmission rates. For a variety of reasons the deduced rates should be considered lower bounds. Internal tides are found to emanate from several large bathymetric structures, especially the Hawaiian Ridge, where the integrated flux amounts to about 6 gigawatts. Internal tides are generated at the Aleutian Trench near  $172^\circ\text{W}$  and propagate southwards nearly 2000 km.

## Introduction

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Before proceeding, however, we should note certain limitations to the altimetric analysis of internal tides. These limitations suggest that internal-tide energy fluxes represent in many cases only lower bounds. The limitations are: (1) Only temporally coherent internal tides (i.e., those that maintain phase lock with the astronomical potential over long timespans) are detectable with altimetry. Such coherent tides may be exceptional, since temporal variability is generally considered the norm [e.g., Wunsch, 1975]. (2) The satellite tracks are widely spaced relative to internal-tide wavelengths, causing possible loss of spatial resolution and smearing of signal in our wave fitting. (3) We assume that a plane wave adequately describes the internal tide over distances of several hundred km. In regions of complex topography and multiple sources, this assumption may be inadequate. (4) Only the first baroclinic mode is considered. There is some evidence for higher modes in the altimetry [e.g., Ray and Mitchum, 1996], but the signals are significantly smaller and more difficult to extract.



## Method of analysis

Eight years of Topex/Poseidon altimeter data (286 repeat cycles) are used to estimate the  $M_2$  tide at successive points (5.75 km separation) along each satellite track. Simple high-pass filtering (cutoff approximately 400 km) adequately removes the barotropic tide over the open ocean.

Our analysis of these data proceeds in two stages: (A) resolution of wave directions and amplitudes of the internal tide from the altimetry; (B) integration of power transmission vs. depth by combining (A) with climatological hydrography. We must allow for a standing wave component in stage (A), but our method accomodates only two amplitudes and phases in a single arbitrary direction  $\phi$  and its converse ( $\phi + \pi$ ). Thus, it is assumed that within a small element of area the wave motion is represented in local cartesian coordinates ( $x$  east,  $y$  north) by

$$\zeta(x, y, t) = a^+ \cos(\sigma t - \Theta - p^+) + a^- \cos(\sigma t + \Theta - p^-) \quad (1)$$

where  $\Theta = \kappa(x \sin \phi + y \cos \phi)$  and  $\sigma$  is tidal frequency.

The parameters of (1) are selected by least squares from the profiles of high-passed tidal signal, in phase and in quadrature with the tide potential, along ascending and descending tracks within the chosen area. The model has six free parameters [ $a^+(\cos, \sin)p^+, a^-(\cos, \sin)p^-, \kappa, \phi$ ] of which the first four are estimated from elementary normal equations for an array of values ( $\kappa, \phi$ ). The wavenumber  $\kappa$  and direction of propagation  $\phi$  are selected for overall minimum residual variance within the array.

In stage (B) we use the climatological average of potential density profile for the entire water column close to the ocean area covered by (A). The hydrographic data [Levitus *et al.*, 1994] are presented in typically 30 layers; we use all these layers to integrate the equation for oscillations of vertical velocity  $w(z)$  from surface to bottom:

$$\frac{\partial^2 w}{\partial z^2} + \frac{\kappa'^2 N^2(z)}{\sigma^2 - f^2} w = 0, \quad w(0) = w(-D) = 0, \quad (2)$$

where  $f$  is the inertial frequency and  $N(z)$  the buoyancy frequency in each layer. Solution of (2) involves the extraction of eigenvalues for the wavenumber  $\kappa' = \kappa'_1, \kappa'_2, \dots$ , to satisfy the boundary conditions  $w = 0$  at surface and bottom  $z = -D$ . We find that, in the large majority of cases, the first mode  $\kappa'_1$  corresponds fairly closely with the wavenumber  $\kappa$  derived from the altimetry in stage (A). We have therefore restricted so far our solutions to the first mode. The oscillatory variables for internal vertical displacement  $\zeta(z)$ , horizontal current  $u(z)$ , and pressure  $p(z)$  are all proportional to  $w(z)$  or to  $\partial w / \partial z$  with the same phase. By the usual approximation the surface elevation (the only signal seen by the altimetry) is given by  $\zeta_0 = p(0) / \rho_0 g$ , where  $\rho_0$  is surface water density and  $g$  is gravity.

As is well known [e.g., Apel, 1987], the internal elevations  $\zeta(z)$  are inverted with respect to  $\zeta_0$  and

they commonly exceed  $10^3 \zeta_0$  at mid-depths. The power transmission, however, is in the wave direction  $\phi$  at all depths. The total transmission is given by

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The strongest generation region along the Hawaiian Ridge appears between longitudes  $169^{\circ}$ – $165^{\circ}$ W. In that region fluxes toward the south exceed  $2400 \text{ W m}^{-1}$ , averaged over the rectangular ocean bin. (Note that, in addition to reasons listed in the Introduction for altimeter estimates yielding lower bounds, vectors in the immediate vicinity of generation points are slightly too small: the bins used there for wave fitting will overlap both sides of the ridge, resulting in reduced amplitudes or apparent standing-wave situations.)

The total integrated energy flux into internal tides along the ridge can be determined by integrating regions of positive flux divergence near the ridge. The result (which includes the ridge system at  $20^{\circ}$ N, west of Johnston Atoll) is 6 gigawatts. *Kang et al.* [2000] arrive at a very similar total of 5 GW. *Merrifield and Holloway* [2000] compute 9.7 GW from their 3-D model, with 62% (6.0 GW) arising from the first internal mode. The rough “ballpark” estimate of 15 GW by *Ray and Mitchum* [1997] was based simply on altimeter amplitudes averaged along the ridge and could easily be off by a factor of 2 (they noted “not unreasonable” agreement with the figure of 8 GW obtained by *Morozov* [1995].) *Egbert and Ray* [2000] estimate that 20 GW is extracted from the barotropic tide over the Hawaiian Ridge. Whether the difference of 14 GW (10 GW for all modes) is expended directly on mechanical mixing in the region is a question of ongoing investigation and measurements (Hawaii Ocean Mixing Experiment).

According to Fig. 1, generation along the Aleutian Trench is primarily limited to a small region in the vicinity of  $170^{\circ}$ – $176^{\circ}$ W. The waves follow a remarkably narrow beam southwards, spreading slightly, and reaching at least latitude  $35^{\circ}$ N. The integrated flux along the  $51^{\circ}$ N parallel is only 0.6 GW.

Also shown in Fig. 1 is the RTE87 triangular acoustic array from which *Dushaw et al.* [1995] first deduced the presence of a coherent internal-tide signal emanating from the Hawaiian Ridge. Our fluxes reconfirm that conclusion, although they show a picture more complicated than a simple series of plane-wave crests parallel to the ridge. They also show that the northwest corner of the acoustic array must have observed signals emanating from the north. *Dushaw et al.* reckoned a northward energy flux of  $180 \text{ W m}^{-1}$ .

Decay scales for internal tides are clearly dependent on locale. For example, at longitude  $170^{\circ}$ W, the decay scale northwards from the Hawaiian Ridge is relatively short, while the decay scale southwards from the Aleutian Trench is many times longer.

## Prospects

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Received \_\_\_\_\_

Submitted to *Geophys. Res. Lett.*, October 2, 2000

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