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# Local Mesh Refinement in the Space-Time CE/SE Method

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# Local Mesh Refinement in the Space-Time CE/SE Method

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**Abstract.** A local mesh refinement procedure for the CE/SE method which does not use an iterative procedure in the treatments of grid-to-grid communications is described. It is shown that a refinement ratio in the order of 10 can be applied successfully across a single coarse grid/fine grid interface.

## 1 Introduction

To accurately resolve flow properties in high-gradient regions without applying a very fine and costly uniform grid over the entire computational domain, simulation of practical unsteady CFD problems may require implementation of local mesh refinement (LMR) in both spatial and temporal directions. In this paper, we report a new LMR procedure for the CE/SE method [1-3].

It is a must for a LMR procedure to enforce space-time flux conservation across a coarse grid/fine grid interface so that numerical solutions will not be contaminated by spurious reflections originating there [4]. Fortunately, this requirement can be met easily in the CE/SE setting. In particular, an iterative procedure is not needed in the treatments of grid-to-grid communications. Furthermore, it will be shown that a refinement ratio in the order of 10 can be applied successfully across a single interface.

## 2 Preliminaries

To simplify the description of the LMR procedure, consider the PDE

$$\partial u/\partial t + a\partial u/\partial x = 0 \tag{1}$$

and its conservation form

$$\oint_{S(V)} \boldsymbol{h} \cdot d\boldsymbol{s} = 0 \tag{2}$$

where (i) $a > 0$ is a constant; (ii) $S(V)$ is the boundary of any region $V$ in space-time; (iii) $\boldsymbol{h} = (au, u)$; and (iv) $d\boldsymbol{s} = d\sigma\, \boldsymbol{n}$ with $d\sigma$ and $\boldsymbol{n}$, respectively,

being the area and the outward unit normal of a surface element on $S(V)$. Note that $\boldsymbol{h} \cdot d\boldsymbol{s}$ is the flux of $\boldsymbol{h}$ leaving $V$ through the surface element $d\boldsymbol{s}$.

Let $\Omega_1$ denote the set of all mesh points $(j,n)$ (dots in Fig. 1) with $j+n$ being odd integers. For each $(j,n) \in \Omega_1$, let the solution element $\mathrm{SE}(j,n)$ be the *interior* of the *space-time* region bounded by a dashed curve depicted in Fig. 2). For any $(x,t) \in \mathrm{SE}(j,n)$, let $u(x,t)$ and $\boldsymbol{h}(x,t)$, respectively, be approximated by $u^*(x,t\,;j,n)$ and $\boldsymbol{h}^*(x,t\,;j,n)$, where [1]

$$u^*(x,t\,;j,n) = u_j^n + (u_x)_j^n (x - x_j) + (u_t)_j^n (t - t^n) \tag{3}$$

$$\boldsymbol{h}^*(x,t\,;j,n) = \left(au^*(x,t\,;j,n),\ u^*(x,t\,;j,n)\right) \tag{4}$$

It is required that $u = u^*(x,t\,;j,n)$ satisfy Eq. (1). Thus

$$(u_t)_j^n = -a\,(u_x)_j^n \tag{5}$$

Hereafter, $\boldsymbol{q}_j^n$ denotes the column matrix formed by $u_j^n$ and $(u_x)_j^n$, the independent marching variables at $(j,n) \in \Omega_1$.

Let space-time be divided into conservation elements (CEs). Two CEs, i.e., $\mathrm{CE}_-(j,n)$ and $\mathrm{CE}_+(j,n)$ (Figs. 3–4) are assigned to each $(j,n) \in \Omega_1$. Because the boundary of $\mathrm{CE}_-(j,n)$ ($\mathrm{CE}_+(j,n)$) lies within the union of $\mathrm{SE}(j,n)$ and $\mathrm{SE}(j-1,n-1)$ ($\mathrm{SE}(j+1,n-1)$), $\boldsymbol{q}_j^n$ can be solved for in terms of $\boldsymbol{q}_{j\pm1}^{n-1}$ by substituting $V = CE_\pm(j,n)$ and $\boldsymbol{h} = \boldsymbol{h}^*$ in Eq. (2). The resulting scheme is the $a$ scheme which has the property that the flux of $\boldsymbol{h}^*$ is conserved over the union of any combination of CEs [1].

For the $a$-$\epsilon$ scheme [1], which reduces to the $a$ scheme if $\epsilon = 0$, again $\boldsymbol{q}_j^n$ is a function of $\boldsymbol{q}_{j\pm1}^{n-1}$. However, the flux of $\boldsymbol{h}^*$ generally is not conserved over the CEs referred to above. Rather, for each $(j,n) \in \Omega_1$, it is conserved over $\mathrm{CE}(j,n)$ (the union of $\mathrm{CE}_\pm(j,n)$—see Fig. 5). To proceed further, in the following, the concept of modified flux will be introduced.

By definition, the modified flux at each of the line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ $\overline{DE}$, $\overline{EF}$, and $\overline{FA}$, which form the boundary of $\mathrm{CE}(j,n)$, is the corresponding flux of $\boldsymbol{h}^*$. On the other hand, the modified flux at $\overline{AD}$, i.e., the interface that divides $\mathrm{CE}_\pm(j,n)$, is defined such that the total modified flux leaving the boundary of $\mathrm{CE}_-(j,n)$ vanishes. Note that the modified flux at $\overline{AD}$, except in the case of the $a$ scheme, is not equal to the corresponding flux of $\boldsymbol{h}^*$. Because (i) the total modified flux leaving the boundary of $\mathrm{CE}(j,n)$ vanishes; and (ii) the modified flux leaving $\mathrm{CE}_-(j,n)$ through $\overline{AD}$ is the negative of that leaving $\mathrm{CE}_+(j,n)$ through $\overline{AD}$, one concludes that the total modified flux leaving the boundary of $\mathrm{CE}_+(j,n)$ also vanishes. In other words, the modified flux is conserved over $\mathrm{CE}_\pm(j,n)$ at each $(j,n) \in \Omega_1$.

The $a$-$\epsilon$ scheme obviously can also be defined over $\Omega_2$, the set of $(j,n)$ (not marked in Fig. 1) with $j+n$ being even integers. Moreover, two schemes defined over $\Omega_1$ and $\Omega_2$ can be fused into a "dual" scheme which is defined over $\Omega \stackrel{\text{def}}{=} \Omega_1 \cup \Omega_2$. Note that, in practical applications, a solution of a dual scheme may not be decoupled into two independent solutions.

Let $(j,n) \in \Omega_1$, then $(j+1,n) \in \Omega_2$. By definition, $\mathrm{CE}_+(j,n)$ coincides with $\mathrm{CE}_-(j+1,n)$. However, two different flux conservation conditions are defined over their co-boundary. In fact, two different modified fluxes tied to, respectively, these two coincident CEs, are defined at any boundary line segment. Obviously, the same complication arises for any CE. As a result, in case that the solution of a dual scheme becomes coupled, the concept of global flux conservation is not well defined in terms of these individual modified fluxes. However, *if the generalized flux at any line segment that forms the boundary of a CE is defined as the average of the two local modified fluxes, then the generalized flux is conserved over the union of any combination of CEs.* The proof follows from (i) the generalized flux is conserved over each CE—a fact that can be established by summing over the two local modified flux conservation conditions tied to this CE; and (ii) the generalized flux leaving one CE through an interface is the negative of that leaving the neighboring CE through the same interface.

## 3 Time Marching

As an example, a step-by-step description of the marching from the 0th to 1st time level (Fig. 6) will be described. Note that (i) the simple grid structure used here does not limit the generality of the following discussions; and (ii) *two $\boldsymbol{q}$ s and thus two symbols (e.g., $A_3^1$ and $B_1^1$) are assigned to any common mesh point of a coarse grid and a fine grid.*

I. Because $a > 0$, $\boldsymbol{q}(A_1^1)$ is specified as a part of the left boundary condition while $\boldsymbol{q}(C_3^1)$ is extrapolated from $\boldsymbol{q}(C_2^0)$, i.e., $\boldsymbol{q}(C_3^1) = \boldsymbol{q}(C_2^0)$.

II. $\boldsymbol{q}(A_2^1)$ is determined in terms of $\boldsymbol{q}(A_1^0)$ and $\boldsymbol{q}(A_3^0)$ using the dual $a$-$\epsilon$ scheme with $\epsilon = 0.5$ [1]. Similarly, $\boldsymbol{q}(B_2^{1/2})$ ($\boldsymbol{q}(C_2^1)$) is determined in terms of $\boldsymbol{q}(B_1^0)$ and $\boldsymbol{q}(B_3^0)$ ($\boldsymbol{q}(C_1^0)$ and $\boldsymbol{q}(C_3^0)$).

III. $\boldsymbol{q}(B_1^{1/2})$ is determined in terms of $\boldsymbol{q}(A_2^0)$, $\boldsymbol{q}(A_3^0)$ and $\boldsymbol{q}(B_2^0)$ assuming

$$u_x(B_1^{1/2}) = \left[u(B_1^{1/2}) - \left(u(A_2^0) + (\Delta t/2)u_t(A_2^0)\right)\right]/\Delta x \tag{6}$$

$$F(\overrightarrow{A_3^0 B_1^{1/2}}) + F(\overrightarrow{B_1^{1/2} B_2^{1/2}}) + F(\overrightarrow{B_2^0 B_2^{1/2}}) + F(\overrightarrow{B_2^0 B_1^0}) = 0 \tag{7}$$

Here (i) $u_t(A_2^0) = -a\, u_x(A_2^0)$ (see Eq. (5)); and (ii) $F(\overrightarrow{A_3^0 B_1^{1/2}})$, as an example, denotes the flux of $\boldsymbol{h}^*$ leaving the rectangle $A_3^0 B_1^{1/2} B_2^{1/2} B_2^0$ through $\overrightarrow{A_3^0 B_1^{1/2}}$ as evaluated using $\boldsymbol{q}(A_3^0)$. Similarly, $\boldsymbol{q}(B_3^{1/2})$ is determined in terms of $\boldsymbol{q}(B_2^0)$, $\boldsymbol{q}(C_1^0)$ and $\boldsymbol{q}(C_2^0)$. $\boldsymbol{q}(B_2^1)$ is then determined in terms of $\boldsymbol{q}(B_1^{1/2})$ and $\boldsymbol{q}(B_3^{1/2})$ using the special dual $a$-$\epsilon$ scheme referred to above.

IV. $\boldsymbol{q}(B_1^1)$ is determined in terms of $\boldsymbol{q}(B_2^{1/2})$, $\boldsymbol{q}(A_3^0)$ and $\boldsymbol{q}(A_2^0)$ assuming

$$u_x(B_1^1) = \left[u(B_1^1) - \left(u(A_2^0) + \Delta t\, u_t(A_2^0)\right)\right]/\Delta x \tag{8}$$

$$F(A_3^0, \overrightarrow{B_1^{1/2} B_1^1}) + F(\overrightarrow{B_1^1 B_2^1}) + F(\overrightarrow{B_2^{1/2} B_2^1}) + F(\overrightarrow{B_2^{1/2} B_1^{1/2}}) = 0 \tag{9}$$

Here $F(A_3^0, \overline{B_1^{1/2}B_1^1})$ denotes the flux of $\boldsymbol{h}^*$ at $\overline{B_1^{1/2}B_1^1}$ evaluated using $\boldsymbol{q}(A_3^0)$. Similarly, $\boldsymbol{q}(B_3^1)$ is determined in terms of $\boldsymbol{q}(B_2^{1/2})$, $\boldsymbol{q}(C_1^0)$ and $\boldsymbol{q}(C_2^0)$.

V. $\boldsymbol{q}(A_3^1)$ is determined in terms of $\boldsymbol{q}(B_1^0)$, $\boldsymbol{q}(B_1^{1/2})$, $\boldsymbol{q}(B_2^{1/2})$ and $\boldsymbol{q}(A_2^0)$ assuming

$$u_x(A_3^1) = \left[\left(u(B_2^{1/2}) + (\Delta t/2)u_t(B_2^{1/2})\right) - u(A_3^1)\right]/(\Delta x/2) \tag{10}$$

$$F(\overrightarrow{B_1^0B_1^{1/2}}) + F(\overrightarrow{B_1^{1/2}B_1^1}) + F(\overrightarrow{A_3^1A_2^1}) + F(\overrightarrow{A_2^0A_2^1}) + F(\overrightarrow{A_2^0A_3^0}) = 0 \tag{11}$$

Similarly, $\boldsymbol{q}(C_1^1)$ is determined in terms of $\boldsymbol{q}(B_3^0)$, $\boldsymbol{q}(B_3^{1/2})$, $\boldsymbol{q}(B_2^{1/2})$ and $\boldsymbol{q}(C_2^0)$.

Note that, *at each line segment on a coarse grid/fine grid interface, two fluxes of* $\boldsymbol{h}^*$ *are evaluated using a coarse-grid* $\boldsymbol{q}$ *and a fine-grid* $\boldsymbol{q}$*, respectively. Moreover, each of these fluxes is involved in a coarse-grid conservation condition and a fine-grid conservation condition. As a result, for the current marching procedure, it can be shown that the generalized flux is conserved over the union of any combination of interior CEs.*

## 4 Mesh Reconstruction

For time-dependent problems, spatial mesh reconstruction may be implemented at any designated time level such that high-gradient regions are always covered by fine grids. In the following, a step-by-step description of a reconstruction procedure that preserves generalized-flux conservation will be explained using Fig. 7 (Note: $C_{3/2}$ is the midpoint of $\overline{C_1C_2}$).

I. No reconstruction is required for the mesh points to the left of $A_2$ or to the right of $C_2$. Thus $\boldsymbol{q}(A_1') = \boldsymbol{q}(A_1)$ and $\boldsymbol{q}(C_2') = \boldsymbol{q}(C_3)$.

II. $\boldsymbol{q}(A_2')$ is determined in term of $\boldsymbol{q}(A_2)$, $\boldsymbol{q}(B_1)$ and $\boldsymbol{q}(B_2)$ assuming

$$F(\overrightarrow{A_2'A_1'}) = F(\overrightarrow{A_2A_1}), \quad F(\overrightarrow{A_2'A_3'}) = F(\overrightarrow{B_1B_2}) + F(\overrightarrow{B_2B_3}) \tag{12}$$

III. $\boldsymbol{q}(B_2')$ is determined in terms of $\boldsymbol{q}(C_1)$ and $\boldsymbol{q}(C_2)$ assuming

$$F(\overrightarrow{B_2'B_1'}) = F(C_2, \overline{C_{3/2}C_1}), \quad F(\overrightarrow{B_2'B_3'}) = F(C_1, \overline{C_{3/2}C_2}) \tag{13}$$

IV. $\boldsymbol{q}(A_3')$ is determined in terms of $\boldsymbol{q}(B_2)$, $\boldsymbol{q}(B_3)$ and $\boldsymbol{q}(B_2')$ assuming

$$u_x(A_3') = (u(B_2') - u(A_3'))/(\Delta x/2), \quad F(\overrightarrow{A_3'A_2'}) = F(\overrightarrow{B_3B_2}) + F(\overrightarrow{B_2B_1'}) \tag{14}$$

V. $\boldsymbol{q}(B_1')$ is determined in terms of $\boldsymbol{q}(C_1)$ and $\boldsymbol{q}(A_2')$ assuming

$$u_x(B_1') = (u(B_1') - u(A_2'))/\Delta x, \quad F(\overrightarrow{B_1'B_2'}) = F(\overrightarrow{C_1C_{3/2}}) \tag{15}$$

VI. $\boldsymbol{q}(B_3')$ is determined in terms of $\boldsymbol{q}(C_2)$ and $\boldsymbol{q}(C_2')$ assuming

$$u_x(B_3') = (u(C_2') - u(B_3'))/\Delta x, \quad F(\overrightarrow{B_3'B_2'}) = F(\overrightarrow{C_2C_{3/2}}) \tag{16}$$

VII. $\boldsymbol{q}(C_1')$ is determined in terms of $\boldsymbol{q}(C_2)$ and $\boldsymbol{q}(B_2')$ assuming

$$u_x(C_1') = (u(C_1') - u(B_2'))/(\Delta x/2), \quad F(\overrightarrow{C_1'C_2'}) = F(\overrightarrow{C_2C_3}) \tag{17}$$

## 5 Numerical Results

Assuming $a = 1$, $\Delta x = 0.01$ and $\Delta t = 0.009$, an exact solution to Eq. (1), i.e.,

$$u = (10/\sqrt{\pi}) \exp\left[-100(x - t - 0.3)^2\right] \tag{18}$$

is solved numerically using the current LMR procedure with a grid refinement ratio $R = 10$ in both space and time. The initial condition ($t = 0$) and mesh distribution are shown in Fig. 8 while the exact solution (a solid line) and numerical solution (dots) along with the mesh distribution at $t = 0.9$ are depicted in Fig. 9 (Note: due to a consideration related to generalized flux, here each dot represents a midpoint solution, i.e., the average mesh value of two neighboring mesh points). It is seen that the numerical solution match very smoothly across the two coarse grid/fine grid interfaces.

Assuming $\Delta x = 4 \times 10^{-3}$, $\Delta t = 1.5 \times 10^{-3}$ and $R = 4$, the Sod's shock tube problem [1] is also solved using a similar Euler LMR procedure. The results at $t = 0.195$ are shown in Fig. 10(a) while a close-up view covering only the neighborhoods of grid interfaces is depicted in Fig. 10(b). Note that a simulation using $R = 64$ also has been carried out successfully.

## 6 Conclusions

A simple CE/SE LMR procedure has been described. Its robustness is demonstrated by numerical examples with grid refinement in the order of 10.

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[3] Other CE/SE references: www.grc.nasa.gov/www/microbus.

[4] M.J. Berger: On conservation at grid interfaces. *SIAM J. Numer. Anal.*, **24(5)**, 1986, pp. 967-984.

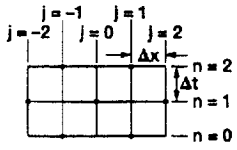



Figure 1.—A staggered space-time mesh.

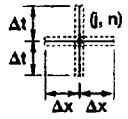



Figure 2.—SE (j, n).

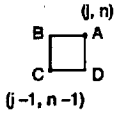



Figure 3.—CE_(j, n).

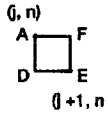



Figure 4.—CE$_+$ (j, n).

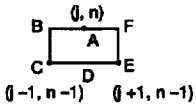



Figure 5.—CE (j, n).

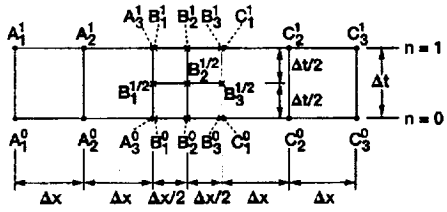



Figure 6.—A coarse-grid/fine-grid space-time mesh.

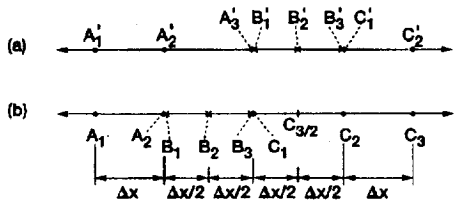



Figure 7.—Mesh reconstruction. (a) Reconstructed mesh. (b) Original mesh.

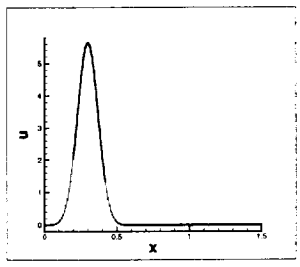



**Fig. 8.** Initial condition (Eq. (18)).

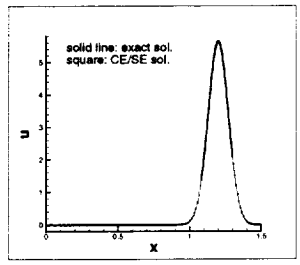
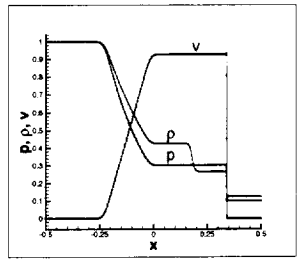
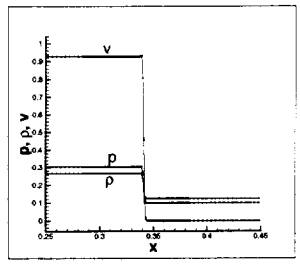



**Fig. 9.** Exact and numerical solutions at $t = 0.9$.




(a) Overall solution.




(b) Close-up view.

**Fig. 10.** Solution for Sod's problem at $t = 0.195$.

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**13. ABSTRACT** ***(Maximum 200 words)***

A local mesh refinement procedure for the CE/SE method which does not use an iterative procedure in the treatments of grid-to-grid communications is described. It is shown that a refinement ratio higher than 10 can be applied successfully across a single coarse grid/fine grid interface.

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