

Jerk Minimization Method for Vibration Control in Buildings

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Abstract

In many vibration minimization control problems for high rise buildings subject to strong earthquake loads, the emphasis has been on a combination of minimizing the displacement, the velocity and the acceleration of the motion of the building. In most cases, the accelerations that are involved are not necessarily large but the change in them (jerk) are abrupt. These changes in magnitude or direction are responsible for most building damage and also create discomfort like motion sickness for inhabitants of these structures because of the element of surprise. We propose a method of minimizing also the jerk which is the sudden change in acceleration or the derivative of the acceleration using classical linear quadratic optimal controls. This was done through the introduction of a quadratic performance index involving the cost due to the jerk; a special change of variable; and using the jerk as a control variable. The values of the optimal control are obtained using the Riccati equation.

1. Introduction

Classical optimal controls in buildings typically consider the control of displacements and velocities [Loh and Ma, 1994, Soong and Yang, 1988]. There is minimum consideration for the control of acceleration [Yang and Li, 1991] or the rate of change of acceleration, otherwise known as **jerk** [Finney and Thomas, 1994]. This imposes limitation on the effective control of motion sickness resulting from jerks. There is therefore a need for more precision about the modeling of control forces in buildings since seismic excitation is jerky. For a building structure subjected to earthquake excitations, a jerk minimization method for vibration control is proposed.

2. Equations of motion

We consider a MDOF system of the form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{D}\mathbf{u} + \mathbf{E}\mathbf{f} \quad (1)$$

where \mathbf{x} is the n -dimensional displacement vector, \mathbf{M} , \mathbf{C} , and \mathbf{K} are respectively the $n \times n$ mass, damping and stiffness matrices. \mathbf{u} is the m -dimensional control vector forces and \mathbf{f} is

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the r -dimensional external excitation vector forces, whereas, \mathbf{D} and \mathbf{E} are respectively the $n \times m$ and $n \times r$ location matrices of the controls and external forces.

3. Control algorithm

We seek to minimize the performance index of the form

$$\mathbf{J} = \int_0^T (\mathbf{x}^T \mathbf{Q}_1 \mathbf{x} + \dot{\mathbf{x}}^T \mathbf{Q}_2 \dot{\mathbf{x}} + \ddot{\mathbf{x}}^T \mathbf{Q}_3 \ddot{\mathbf{x}} + \ddot{\mathbf{x}}^T \mathbf{Q}_4 \ddot{\mathbf{x}} + \mathbf{u}^T \mathbf{R} \mathbf{u}) dt \quad (2)$$

which is the analogous of the usual linear quadratic cost, where $\mathbf{Q}_i, i = 1, \dots, 3$, are symmetric positive semi-definite matrices and \mathbf{Q}_4 and \mathbf{R} are symmetric positive definite matrices.

Introducing the change of variable

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} \quad (3)$$

and using the jerk as a control variable via

$$\tilde{\mathbf{u}} = \begin{bmatrix} \mathbf{u} \\ \mathbf{w} \end{bmatrix}, \text{ with } \mathbf{x} = \mathbf{w}, \text{ system (1) can be transformed into}$$

$$\dot{\mathbf{z}} = \mathbf{A} \mathbf{z} + \mathbf{B} \tilde{\mathbf{u}} + \mathbf{H} \mathbf{f}, \quad \mathbf{z}(0) = \mathbf{z}_0 \quad (4)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} & \mathbf{0} \\ -\mathbf{M}^{-1} \mathbf{K} & -\mathbf{M}^{-1} \mathbf{C} & \mathbf{0} \\ 0 & 0 & 0 \end{bmatrix} \quad (5)$$

is the $3n \times 3n$ system matrix,

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{M} \mathbf{M}^{-1} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \text{ and } \mathbf{H} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \mathbf{E} \\ 0 \end{bmatrix} \quad (6)$$

are $3n \times (n+m)$ and $3n \times r$ location matrices respectively for the controls and external forces. Here \mathbf{O} and \mathbf{I} denote respectively the zero matrices and the identity matrix of appropriate dimensions. The corresponding performance index is given by

$$\mathbf{J} = \int_0^T (\mathbf{z}^T \mathbf{Q} \mathbf{z} + \tilde{\mathbf{u}}^T \tilde{\mathbf{R}} \tilde{\mathbf{u}}) dt \quad (7)$$

where

$$\mathbf{Q} = \begin{bmatrix} \hat{\mathbf{Q}}_1 & \hat{\mathbf{0}} & \hat{\mathbf{0}} \\ \mathbf{0} & \mathbf{Q}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q}_3 \end{bmatrix} \text{ is a symmetric positive semi-definite matrix and } \tilde{\mathbf{R}} = \begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_4 \end{bmatrix} \text{ is a}$$

symmetric positive definite matrix.

The usual necessary conditions for optimal control are given by

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \tilde{\mathbf{u}}} &= \mathbf{0} \\ \frac{\partial \mathcal{L}}{\partial \mathbf{z}} &= \boldsymbol{\lambda}^T \end{aligned} \quad (8)$$

with

$$\boldsymbol{\lambda}^T(\mathbf{T}) = \mathbf{0} \quad (9)$$

and Hamiltonian

$$\mathbf{L}(\mathbf{z}, \tilde{\mathbf{u}}, t) = \mathbf{z}^T \mathbf{Q} \mathbf{z} + \tilde{\mathbf{u}}^T \tilde{\mathbf{R}} \tilde{\mathbf{u}} + \boldsymbol{\lambda}^T (\mathbf{A} \mathbf{z} + \mathbf{B} \tilde{\mathbf{u}} + \mathbf{H} \mathbf{f}) \quad (10)$$

The above system yields

$$\tilde{\mathbf{u}} = -\frac{1}{2} \tilde{\mathbf{R}}^{-1} \mathbf{B}^T \boldsymbol{\lambda} \quad (11)$$

$$\dot{\boldsymbol{\lambda}} = -\mathbf{A}^T \boldsymbol{\lambda} - 2\mathbf{Q} \mathbf{z}, \quad \mathbf{A}^T(\mathbf{T}) = \mathbf{0} \quad (12)$$

This problem can be solved by the **Riccati** equation approach. We assume that the control is regulated by the generalized state vector, i.e., we seek a solution of the form

$$\mathbf{a}(t) = \mathbf{P}(t) \mathbf{z}(t). \quad (13)$$

Here $\mathbf{P}(t)$ is a symmetric and differentiable matrix. When the external excitation vector is neglected, $\mathbf{P}(t)$ satisfies the so called **Riccati** differential equation

$$\mathbf{P}(t) + \mathbf{P}(t)\mathbf{A} - \frac{1}{2}\mathbf{P}(t)\mathbf{B}\tilde{\mathbf{R}}^{-1}\mathbf{B}^T\mathbf{P}(t) + \mathbf{A}^T\mathbf{P}(t) + 2\mathbf{Q} = \mathbf{O}, \quad \mathbf{P}(T) = \mathbf{O} \quad (14)$$

The solution is obtained through backward integration in time. Since in most structural engineering applications the matrix $\mathbf{P}(t)$ remains constant throughout and drops to zero near T , \mathbf{P} can be assumed constant. It follows that \mathbf{P} satisfies the algebraic Riccati equation

$$\mathbf{P}\mathbf{A} - \frac{1}{2}\mathbf{P}\mathbf{B}\tilde{\mathbf{R}}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{A}^T\mathbf{P} + 2\mathbf{Q} = \mathbf{O} \quad (15)$$

$$\tilde{\mathbf{u}}(t) = -\frac{1}{2}\tilde{\mathbf{R}}^{-1}\mathbf{B}^T\mathbf{P}\mathbf{z}(t) \quad (16)$$

If the solution of the Riccati equation obtained is written the form

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{P}_{13} \\ \mathbf{P}_{21} & \mathbf{P}_{22} & \mathbf{P}_{23} \\ \mathbf{P}_{31} & \mathbf{P}_{32} & \mathbf{P}_{33} \end{bmatrix} \quad (17)$$

where each \mathbf{P}_i is an $n \times n$ matrix, then

$$\mathbf{u}(t) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{D}^T\mathbf{M}^{-1T}\mathbf{P}_2\mathbf{z}(t) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{D}^T\mathbf{M}^{-1T}(\mathbf{P}_{21}\mathbf{x}_1(t) + \mathbf{P}_{22}\mathbf{x}_2(t) + \mathbf{P}_{23}\mathbf{x}_3(t)) \quad (18)$$

and

$$\mathbf{w}(t) = -\frac{1}{2}\mathbf{Q}_4^{-1}\mathbf{P}_3\mathbf{z}(t) = -\frac{1}{2}\mathbf{Q}_4^{-1}(\mathbf{P}_{31}\mathbf{x}_1(t) + \mathbf{P}_{32}\mathbf{x}_2(t) + \mathbf{P}_{33}\mathbf{x}_3(t)) \quad (19)$$

hence from (3), the expressions (18) and (19) can be rewritten as

$$\mathbf{u}(t) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{D}^T\mathbf{M}^{-1T}(\mathbf{P}_{21}\mathbf{x}(t) + \mathbf{P}_{22}\dot{\mathbf{x}}(t) + \mathbf{P}_{23}\ddot{\mathbf{x}}(t)) \quad (20)$$

and likewise

$$\mathbf{w}(t) = \mathbf{x}(t) = -\frac{1}{2}\mathbf{Q}_4^{-1}(\mathbf{P}_{31}\mathbf{x}(t) + \mathbf{P}_{32}\dot{\mathbf{x}}(t) + \mathbf{P}_{33}\ddot{\mathbf{x}}(t)). \quad (21)$$

Once the matrix \mathbf{P} is obtained and the case the generalized state vector is available for measurement the control can be designed by appropriately choosing the weighting matrices \mathbf{Q} and \mathbf{R}

4. Applications

The analysis proposed will lead to the design of more efficient controls for jerky excitation like earthquake loads and the design of better controllers to prevent motion sickness.

5. Conclusion

The concept of jerk minimization was introduced in this paper with a detail of how this promising new concept can be applied to linear systems via quadratic performance index involving the cost due to the jerk. This was done through a special change of variable and using the jerk as a control variable. The values of the optimal control are obtained using the so called of Riccati equation. This result is a fruitful step toward designing efficient controllers for the minimization of vibration in structures.

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