INTELLIGENT CONTROL OF FLEXIBLE-JOINT ROBOTIC MANIPULATORS

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Abstract

This paper considers the trajectory tracking problem for uncertain rigid-link, flexible-joint manipulators, and presents a new intelligent controller as a solution to this problem. The prop osed control strategy is simple and computationally efficient, requires lit tle information concerning either the manipulator or actuator/ transmission models, and ensures uniform boundedness of all signals and arbitrarily accurate task-space trajectory tracking.

1. Introduction

The problem of controlling the motion of robotic manipulators in the presence of incomplete information concerning the system model has received considerable attention during the past decade, and much progress has been made in this area. However most of the controllers proposed as solutions to this problem have been designed by neglecting any flexibility associated with the actuator/transmission system and assuming that the actuators are rigidly connected to the manipulator links, As demonstrated in [e.g., 1], joint flexibilities constitute an important component of the complete manipulator dynamic model and thus should be addressed in the controller development process. Recognizing the potential difficulties associated with ignoring the effects of joint flexibility, several researchers have recently considered the problem of controlling rigid-link. flexible-joint (R LFJ) manipulators [e.g., 2-11]. In much of this work. the controller development requires full knowledge of the complex dynamic models for the manipulator and actuator systems [e.g., 2-5]. Research in which controllers are designed with the capability to compensate for uncertainty in the manipulator/actuator system includes adaptive schemes developed using a singular perturbation approach [6,7], which can be used if the joints are sufficiently stiff, and more recent work on robust control strategies and adaptive schemes [8-11] which is valid for arbitrary joint stiffnesses. It is noted that implementation of most of these robust and adaptive controllers requires the calculation of very complex, manipulator-specific quantities, which limits the generality and applicability of these strategies.

This paper introduces a new trajectory tracking controller for uncertain RLFJ manipulators. In contrast with existing schemes, the present strategy is developed using an intelligent control approach which combines ideas from robust control and the recently developed performance- *based adaptive control* methodology [12,13]. This approach effectively exploits the underlying mechanical system structure of the manipulator dynamic model to permit reliance on information regarding this model to be eliminated. and as a consequence overcomes the difficulties associated with previous control methods. Thus the proposed tracking controller possesses a simple and modular structure, is easy to implement. and requires virtually no information regarding either the mechanical or actuat or models. It is shown that the controller ensures uniform boundedness of all signals anc 1 provides arbitrarily accurate tracking control.

2. Preliminaries

Let $\mathbf{p} \in \Re^m$ **define** the position and orientation of the robot encl-effecter relative to a fixed user-dcfkd reference frame and $\theta \in \Re^n$ denote the vector of robot link coordinates. Then the for-ward kinematic and differential kinematic maps between the robot link

coordinates θ and the end-effecter coordinates $\mathbf{p}$ can be written as

$$\mathbf{p} = \mathbf{h}(\theta), \qquad \dot{\mathbf{p}} = J(\theta)\dot{\theta} \tag{1}$$

where $\mathbf{h} : \Re^n \to \Re^m$ is smooth and $J \in \Re^{m \times n}$ is the end-effector Jacobian matrix.

Observe that there are numerous advantages to formulating the manipulator control problem directly in terms of the end-effector coordinates $\mathbf{p}$. **For** example, these coordinates are typically more task-relevant than the link coordinates θ, so that developing the controller in terms of $\mathbf{p}$ can lead to improved performance, efficiency, and implementability. If the manipulator is nonredundant (so that m = n) and is in a region of the workspace where J has full rank, then $\mathbf{p}$ and f? are diffeomorphic and this formulation presents no difficulties. A task-space formulation can also be realized if the manipulator is cinematically redundant (so that m < n) by utilizing, for example, the configuration *control* approach [e.g.. 14, 15]. In what follows, we shall consider nonredundant and redundant robots together and introduce a set of n task-space coordinates x obtained by augmenting p with n — m kinematic functions that define some auxiliary task to be performed by the manipulator [14,15]. To retain generality, we shall require only that the kinematic relationship between θ and x is known and smooth and can be written in a form analogous to (1):

$$\mathbf{x} = \mathbf{h}_a(\theta), \qquad \dot{\mathbf{x}} = J_a(\theta)\dot{\theta} \tag{2}$$

where $\mathbf{h}_a : \Re^n \to \Re^n$ and $J_a \bullet \Re^{n \times n}$. Observe that for x to be a valid task-space coordinate vector the elements of x must be independent in the region of int crest: thus it will be assumed in our development that J_a is of full rank.

Consider an n link RLFJ manipulator with actuator coordinates $\phi \in \Re^n$ and actuator torques $\mathbf{u} \in \mathbf{W}$. The task-space dynamic model for this manipulator system is a $4n$th order differential **equation** relating the end-effector coordinates x and the system control input $\mathbf{u}$:

$$\mathbf{F} = H\ddot{\mathbf{x}} + V_{cc}\dot{\mathbf{x}} + \mathbf{G}, \qquad \mathbf{T} = J_a^T \mathbf{F} \tag{3a}$$

$$\mathbf{u} = J_m \ddot{\phi} + K_m \mathbf{T} \tag{3b}$$

where $\mathbf{F} \in \Re^n$ is the generalized force associated with x, $H(x) \in \Re^{n \times n}$ is the manipulator inertia matrix, V&(x, $\dot{\mathbf{x}}$)$\in \Re^{n \times n}$ quantifies Coriolis and centripetal acceleration effects, $G(x) \in \Re^n$ is the vector of gravity forces, $\mathbf{P} \in \Re^n$ is the vector of forces and moments exert ed by the end-effector on the environment, and J_m, $K_m \in \Re^{n \times n}$ are positive, constant, diagonal matrices which characterize the actuator dynamics. Note that in obtaining the RLFJ manipulator model (3) we have scaled H, V_{cc}, and G by the joint stiffness, introduced the definition T = $\phi - \theta$, and assumed that actuator rotor motion is a pure rotation relative to an inertial frame. It is well known that the rigid-link manipulator dynamics (3a) possesses considerable structure. For example, for any set of generalized coordinates x, the dynamic model terms H, G are bounded functions of x whose time derivatives $\dot{H}$. G are also bounded in x and depend linearly on $\dot{\mathbf{x}}$, the matrix H is symmetric and positive-definite, the matrix V_{cc} *is* bounded in x and depends linearly on x, and the matrices H and V_{cc} are related according to H = V_{cc} + V_{cc}^T. Additionally, V&(x: x) y = V&(x? y)$\dot{\mathbf{x}}$ $\forall$y, and if y and y are bounded then V&(x; y) is bounded and V_{cc} (x, y) grows linearly with x.

In this paper we shall address the trajectory tracking problem. The control objective for tracking is to ensure that the manipulator/actuator system (3) evolves from its initial state to the desired final state along some specified task-space trajectory $\mathbf{x}_d(t) \in \Re^n$ (where $\mathbf{x}_d$ is bounded with bounded derivatives). In what follows, it is assumed that the manipulator/actuator system state $\theta, \dot{\theta}, \phi$, and $\dot{\phi}$ is measurable. Observe that the dynamic

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model (3) consists of two cascaded dynamical systems. One consequence of this structure is that the rig-id-link manipulator input **T** cannot be commanded directly, as is assumed in the design of controllers at the link torque input level, and instead must be realized as the output of the actuator' dynamics (3b) through proper specification of the actuator control input **u**. The structure of the RLFJ manipulator dynamics (3) suggests partitioning the control system design problem into two subproblems: regard **T** as the control input for the subsystem (3a) and specify the desired evolution of this variable $\mathbf{T}_d(t)$ in such a way that if $\mathbf{T} = \mathbf{T}_d$ then accurate tracking would be achieved, and then specify the actual control input **u so** that **T** closely tracks $\mathbf{T}_d$. This approach to controller design is adopted in this paper, so that the proposed control system consists of two subsystems: an adaptive strategy that provides the (fictitious) control input $\mathbf{T}_d$ required to ensure that the system (3a) pm-forms as desired, and a robust control scheme that determines the (actual) control input **u** which guarantees that the system (3b) evolves with **T** closely tracking $\mathbf{T}_d$.

3. **Tracking** Control **Scheme**

Let $e = \mathbf{x}_d - \mathbf{x}$ denote the task-space trajectory tracking error and $E = \mathbf{T}_d - \mathbf{T}$ represent the link torque tracking error. Consider the following tracking controller for RLFJ manipulators:

$$\mathbf{F}_d = A(t)\ddot{\mathbf{x}}_d + B(t)\dot{\mathbf{x}}_d + \mathbf{f}(t) + k_1\gamma^2\mathbf{w} + k_2\gamma^2\mathbf{e}$$
$$w = -2\gamma\mathbf{w} + \gamma^2\dot{\mathbf{e}}$$
$$\mathbf{T}_d = J_a^T\mathbf{F}_d \qquad (4)$$
$$\mathbf{u} = \mathbf{f}_0(t) + [\mathbf{b}(t)\operatorname{sat}(\frac{\mathbf{s}}{\epsilon})] + k_a\mathbf{s}$$

where the notation $[\mathbf{gh}] = [g_1 h_1, g_2 h_2, \ldots g_n h_n]^\top \in \Re^n$ (for any two n-vectors **g**, **h**) and $\operatorname{sat}(\mathbf{g}) = [\operatorname{sat}(g_1), \operatorname{sat}(g_2), \ldots \operatorname{sat}(g_n)]^T \in \Re^n$ (with $\operatorname{sat}(\cdot)$ the standard saturation function) is introduced, $\mathbf{s} = \dot{\mathbf{E}} + \lambda\mathbf{E}$ is the weighted torque-torque rate tracking error, $\mathbf{f}_0$ **(t)**, $\mathbf{b}(t) \in \Re^n$ **are** robust control terms, $\mathbf{f(f)} \in \Re^n$ **and** $A(t)$, $B(t) \in \Re^{n \times n}$ are adaptive gains! and k_1, k_2, $\gamma, k_a, \epsilon, \lambda$ are positive scalar constants. The robust control terms f., b are smooth vector functions which are defined in the proof of the Theorem below, and the adaptive gains **f**, A, B are adjusted according to the following simple update laws:

$$\dot{\mathbf{f}} = -\sigma_1\mathbf{f} + \beta_1\mathbf{q}$$
$$\dot{A} = -\sigma_2 A + \beta_2\mathbf{q}\ddot{\mathbf{x}}_d^T \qquad (5)$$
$$\dot{B} = -\sigma_3 B + \beta_3\mathbf{q}\dot{\mathbf{x}}_d^T$$

where $\mathbf{q} = \mathbf{e} + k_2\mathbf{e}/k_1\gamma - \mathbf{w}/\gamma$ is the weighted and filtered position-velocity tracking error and the σ_i and β_i are positive scalar adaptation gains.

The stability properties of the proposed tracking strategy (4),(5) are summarized in the following theorem.

Theorem: The control scheme (4),(5) ensures that (3) evolves with all signals (semiglobally) uniformly bounded provided γ is chosen sufficiently large and **b is** properly defined. Moreover, the trajectory tracking error e,& is guaranteed to converge exponentially to a compact set which can be made arbitrarily small.

Proof: Observe first that the actuator dynamics (3b) can be written

$$J_m\dot{\mathbf{s}} = \mathbf{f}_m(\theta, \dot{\theta}, \phi, \dot{\phi}) - \mathbf{u} \qquad (6)$$

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where $\mathbf{f}_m(\theta, \theta, \phi, \dot{\phi})$ is a smooth function obtained through routine manipulation. Applying the control law (4) to the manipulator dynamics (3) yields the closed-loop error dynamics

$$H\ddot{\mathbf{e}} + V_{cc}\dot{\mathbf{e}} + k_1\gamma^2\mathbf{w} + k_2\gamma^2\mathbf{e} + \Phi_f + \Phi_A\ddot{\mathbf{x}}_d + \Phi_B\dot{\mathbf{x}}_d + V_{cd}\dot{\mathbf{e}} - J_a^{-T}\mathbf{E} = O$$

$$J_m\dot{\mathbf{s}} + k_a\mathbf{s} + [\mathbf{b}\mathrm{sat}\,(\frac{\mathbf{s}}{\epsilon})] + \mathbf{f}_0 - \mathbf{f}_m = \mathbf{0} \tag{7}$$

where $\Phi_f = \mathbf{f} - \mathbf{G}$, $\Phi_A = A - H$, $\Phi_B = B - V_{cd}$, and the notation $V_{cd} = V_{cc}(\mathbf{x}, \dot{\mathbf{x}}_d)$ is introduced.

Consider the Lyapunov function candidate

$$V = \frac{1}{2}\dot{\mathbf{e}}^T H\dot{\mathbf{e}} + \frac{1}{2}k_2\gamma^2\mathbf{e}^T\mathbf{e} + \frac{1}{2}k_1\mathbf{w}^T\mathbf{w} + \frac{k_2}{k_1\gamma}\mathbf{e}^T H\dot{\mathbf{e}} - \frac{1}{\gamma}\mathbf{w}^T H\dot{\mathbf{e}}$$

$$+ \frac{1}{2}\mathbf{s}^T J_m\mathbf{s} + k_a\lambda\mathbf{E}^T\mathbf{E} + \frac{1}{2\beta_1}\Phi_f^T\Phi_f + \frac{1}{2}\mathrm{tr}[\frac{1}{\beta_2}\Phi_A\Phi_A^T + \frac{1}{\beta_3}\Phi_B\Phi_B^T] \tag{8}$$

and note that V is a positive-definite and proper function of the closed-loop system state if γ is chosen sufficiently large. Computing the derivative of (8) along (7) and simplifying permits the following upper bound on $\dot{V}$ to be established:

$$\dot{V} \le -\lambda_m(Q^*)\|\mathbf{z}_1\|^2 - k_a\|\dot{\mathbf{E}}\|^2 - k_a\lambda^2\|\mathbf{E}\|^2 + \frac{3}{\sigma_{min}}\|\mathbf{z}_1\|\|\mathbf{E}\| - \frac{\eta_1}{\beta_{min}}\|\Psi\|^2$$

$$+ \frac{k_2 k_{cc}}{k_1\gamma}\|\mathbf{e}\|\|\dot{\mathbf{e}}\|^2 + \frac{k_{cc}}{\gamma}\|\mathbf{w}\|\|\dot{\mathbf{e}}\|^2 + \eta_2\epsilon + \frac{\eta_3}{\beta_{min}} \tag{9}$$

where $\lambda_m(\cdot), \lambda_M(\cdot)$ denote the minimum and maximum eigenvalue of the matrix argument, respectively, k_{cc} satisfies $\|V_{cc}\|_F \le k_{cc}\|\dot{\mathbf{x}}\| \, \forall\mathbf{x}$, k_{cd} is an upper bound on V_{cd}, v_M is an upper bound on $\|\dot{\mathbf{x}}_d\|$, the η_i are positive scalar constants which do not increase as ϵ is decreased and the β_i are increased, $\mathbf{f}_0$ is any (nominal) estimate for $\mathbf{f}_m$ (for example, $\mathbf{f}_0 = O$ can be used), $\mathbf{b}$ is chosen so that $b_i \ge \max[1, (f_{mi} - f_{0i})^2]$ for $i = 1, 2,..., n$, γ is chosen so that $\gamma \ge \max[1, k_2/k_1]$, $\mathbf{z}_1 = [\|\mathbf{e}\|\|\dot{\mathbf{e}}\|\|\mathbf{w}\|]^T$, $\Psi = [\|\Phi_f\|\|\Phi_A\|_F\|\Phi_B\|_F]^T$, $\beta_{min} = \min(\beta_i)$, $\beta_{max} = \max(\beta_i)$, σ_{min} is the minimum singular value of the matrix J_a (recall that J_a is assumed to be nonsingular in the region of interest, so that σ_{min} is nonzero), and

$$Q^* = \begin{bmatrix} \frac{k_2^2\gamma}{k_1} & -\frac{k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) & 0 \\ -\frac{k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) & (\frac{\gamma}{2} - \frac{k_2}{k_1\gamma})\lambda_m(H) - k_{cd} & -\lambda_M(H) - \frac{k_{cd}}{2\gamma} - \frac{k_{cc}v_M}{2\gamma} \\ 0 & -\lambda_M(H) - \frac{k_{cd}}{2\gamma} - \frac{k_{cc}v_M}{2\gamma} & k_1\gamma \end{bmatrix}$$

(note that Q^* is positive-definite if γ is chosen large enough). Next let $\mathbf{z}_2 = [\|\mathbf{z}_1\|\|\mathbf{E}\|\|\dot{\mathbf{E}}\|]^T$ and

$$Q = \begin{bmatrix} \lambda_m(Q^*) & -3/(2\sigma_{min}) & 0 \\ -3/(2\sigma_{min}) & k_a\lambda^2 & 0 \\ & & k_a \end{bmatrix}$$

and notice that Q is positive-definite if k_a is chosen large enough. If ϵ is chosen to be inversely proportional to β_{min} and β_{max}/β_{min} is fixed, then there exist positive scalar

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constants η_4, η_5 that does not increase as γ and β_{min} increase, and positive scalar constants λ_i independent of γ and β_{min}, such that V and V in *(8)* and (9) can be bounded as

$$\lambda_1 \parallel z' \parallel^2 + \frac{\lambda_2}{\beta_{min}} \parallel \Psi \parallel^2 \le V \le \lambda_3 \parallel z_2 \parallel^2 + \frac{\lambda_4}{\beta_{min}} \parallel \Psi \parallel^2$$

$$v \le -(\lambda_m(Q) - \frac{\eta_4}{\gamma}V^{1/2}) \parallel z' \parallel^2 - \frac{\lambda_5}{\beta_{min}} \parallel \Psi \; II'+ *$$

Now choose two scalar constants V_M, V_m *so* that $V_M > V_m \ge$ *V(0)*, and define $c_M = \lambda_m$ **(Q)** $- \eta_4 V_M{}^{1/2}/\gamma$; then choose γ large enough so that $c_M > 0$ (this is always possible). Let $\delta = \max(\lambda_3/c_M, \lambda_4/\lambda_5)$ and choose β_0 so that $\eta_5\delta/\beta_0 < V_m$ (this is always possible). Then selecting $\beta_{min} \ge \beta_0$ ensures that if $V_m \le V < V_M$ then V < 0. This condition together with $V_M > V_m \ge V(0)$ implies that $V(t) \le V_M \forall t$, *so* that $c(t) = \lambda_m(Q) - \eta_4 V^{1/2}(t)/\gamma > c_M > 0 \forall t$ and

$$\dot{V} \le -c_M \mid z_2 \parallel^2 - \frac{\lambda_5}{(\beta_0 + \Delta\beta)} \parallel \Psi \parallel^2 + \frac{\eta_5}{(/\sim. + \Delta\beta)}$$

where $\Delta\beta = \beta_{min} - \beta_0$ and it is assumed that β_{min} is chosen so that $\Delta\beta > 0$. The ultimate boundedness results developed in [13,14] are now directly applicable and permit the conclusion that $\parallel z_2 \parallel, \mid \Psi \parallel$ are uniformly bounded and that $\parallel z_2 \parallel, \parallel \Psi \parallel$ converge exponent ially to the" closed balls B_{r_1}, B_{r_2}, respective] y, where "

$$r_1 = \left(\frac{\delta\eta_{14}}{\lambda_1(\beta_0 + \Delta\beta)} \right)^{1/2}$$

$$r_2 = \left(\frac{\delta\eta_{14}}{\lambda_2} \right)^{1/2}$$

Observe that the radius of the ball to which $\parallel z_2 \parallel^2$ is guaranteed to converge can be decreased as desired simply by increasing $\Delta\beta$. ∎

4. Conclusions

This paper presents a new solution to the motion control problem for uncertain RLFJ manipulators. The proposed control strategy is simple and computationally efficient, requires little information concerning either the manipulator or actuator/transmission models, and ensures uniform b oundedness of all signals and arbitrarily accurate task-space trajectory tracking. Future research will involve the implementation of the controllers for robotic applications in hazardous and unstructured environments.

5. Acknowledgments

6. References

1. Good, M., L. Sweet, and K. Strobel, "Dynamic Models for Control System Design of Integrated Robot and Drive Systems", *ASME Journal of Dynamic Systems, Measurement, and Control,* Vol. 107, 1985, No. 1. pp. 53-59

2. Spong, M., "Modeling and Control of Elastic Joint Robots", *ASME Journal of Dynamic Systems, Measurement, and Control,* Vol. 109, No. 4, 1987, pp. 310-319

3. Khorasani, K., "Nonlinear Feedback Control of Flexible Joint Manipulators". *IEEE Transactions on Automatic Control,* Vol. 35, No. 10.1990, pp. 1145-1149

4. Dawson, D., Z. Qu, J. Carroll, and M. Bridges, "Control of Robot Manipulators in the Presence of Actuator Dynamics", *International Journal of Robotics and Automation.* Vol. 8, *No.* 1, 1993, pp. *13-21*

5. Nicosia, S. and P. Tomei, "Design of Global Tracking Controllers for Flexible-Joint Robots", *Journal of Robotic Systems,* Vol. 10, No. 6, 1993, pp. 835-846

6. Spong, M., "Adaptive Control of Flexible Joint Manipulators", *Systems and Control Letters,* Vol. 13, 1989, pp. 15-21

7. Khorasani, K., "Adaptive Control of Flexible-Joint Robots", *IEEE Transactions on Robotics and Automation,* Vol. 8, No. 2, 1992, pp. 250-267

8. Lozano, R. and B. Brogliato, "Adaptive Control of Robot Manipulators with Flexible Joints", *IEEE Transactions on Automatic Control,* Vol. 37, 'No. 2, 1992, pp. 174-181

9. Dawson, D., Z. Qu, and M. Bridges, "Hybrid Adaptive Control for Tracking of Rigid-Link Flexible-Joint Robots", *IEE Proceedings D,* Vol. 140, No. 3, 1993, pp. 155-1.59

10. Tomei, P., "Tracking Control of Flexible Joint Robots with Uncertain Parameters and Disturbances", *IEEE Transactions on Automatic Control,* Vol. 39, No, 5. 1994, pp. 1067-1072

11. Qu, z., "Input-Output Robust Tracking Control Design for Flexible Joint Robots", *IEEE Transactions on Automatic Control,* Vol. 40, No. 1, 1995, pp. 78-83

12. Colbaugh, R., K. Glass, and H, Seraji, "Performance-Based Adaptive Tracking Control of Robot Manipulators", *Journal of Robotic Systems,* Vol. 12, No. 8, 1995, pp. 517-530

13. Colbaugh, R., H. Seraji, and K. Glass, "Adaptive Compliant Motion Control for Dexterous Manipulators", *International Journal of Robotics Research,* Vol. 1.4, No. 3, 1995, pp. 270-280

14. Seraji, H., "Configuration Control of Redundant Manipulators: Theory and Implementation", *IEEE* Transactions on *Robotics and Automation,* Vol. 5, No. 4.1989, pp. 472-490

15. Glass, K., R,. Colbaugh, D. Lim, and H. Seraji, "Real-time Collision Avoidance for Redundant Manipulators", *IEEE Transactions on Robotics and Automation,* Vol. 11, NO. 3, 1995, pp. 448-457