

R. Hazoumé\*, J. Chabi Orou\*\*, J. A. Johnson III  
 CeNNAs, Florida A&M University, Tallahassee, FL 32310 (USA)

Abstract

When a simple model for the relationship between the density-temperature fluctuation correlation and mean values is used, we determine that the rate of change of turbulent intensity can influence directly the accretion rate of droplets.

Considerable interest exists in the accretion rate for condensates in nonequilibrium flow with icing' and the potential role which reactant accretion can play in nonequilibrium exothermic reactant processes.<sup>2</sup> Turbulence is thought to play an important role in such flows. It has already been experimentally determined that turbulence influences the sizes of droplets in the heterogeneous nucleation of supersaturated vapors.<sup>3</sup> This paper addresses the issue of the possible influence of turbulence on the accretion rate of droplets.

According to a nonequilibrium model developed earlier<sup>4</sup> the droplet growth rate for flow with changing local pressure is given by:

$$\frac{dr(t)}{dt} = -3BN_n \frac{r_c}{dt} \log \frac{P(t)}{P_\infty} \tag{1}$$

with  $B = (2s)/[r_c T(R/m_n)]$  where  $s$  = surface tension coefficient;  $r_c$  = condensate density; and  $m_n$  = molecular weight of vapor. In this approach, we use the hardcore model for a liquid droplet in which  $dN$ , the number of simple molecules of water in the shell between  $r$  and  $dr$ , is given by  $dN = (k/r^n)(4\pi r^2)dr$ .  $N_n$  is a correction coefficient such that  $0 \leq N_n \leq 4$  and  $N_n = [(n-3)^{n-3}]/[(n-4)^{n-2}]$  for  $n > 4$ .  $P_\infty$  is obtained through the Clausius-Clapeyron equation. For adiabatic flow of a compressible ideal gas the mean value of the square of the local speed of sound is defined as

$$b^2 = \frac{\partial \bar{P}}{(\partial \rho)_{s,g}} = \gamma \frac{\bar{P}}{\rho} \tag{2}$$

where:  $g = C_p/C_v$  is the ratio of the specific heats;  $\bar{g}$  is the mean value of the condensate mass fraction of water;  $\bar{P}$  is the mean pressure function;  $s$  is the mean entropy; and  $\rho$  is the mean density function.

We can readily write from (2) that

$$\frac{\bar{P}}{\rho} = \text{const} \quad \text{and} \quad \frac{\bar{P}}{(b^2)^{(\gamma-1)}} = \text{const} \tag{3}$$

Equations (2) and (3) were written assuming there are fluctuations of the principle physical properties around their mean values as, e.g.,  $g = \bar{g} + g'$  with  $\bar{g}' = 0$ . However  $g'^2 \neq 0$ . This means that we can consider  $P = \bar{P} + P'$ ,  $\rho = \bar{\rho} + \rho'$ ,  $g = \bar{g} + g'$ , and  $T = \bar{T} + T'$ . The equation of state

$$P = \rho \left( \frac{1 - \omega_o}{\mu_i} + \frac{\omega_o - g}{\mu_v} \right) RT \tag{4}$$

becomes

$$\bar{P} + P' = (\bar{\rho} + \rho') \left( \frac{1 - \omega_o}{\mu_i} + \frac{\omega_o - \bar{g} - g'}{\mu_v} \right) R(\bar{T} + T') \tag{5}$$

with  $\omega_o$  = the initial humidity,  $\mu_i$  = the molecular mass of the carrier gas, and  $R$  = the ideal gas constant. Averaging equation (4) yields:

$$\bar{P} = \left( \frac{1 - \omega_o}{\mu_i} + \frac{\omega_o - \bar{g}}{\mu_v} \right) R(\bar{T} \bar{\rho} + \overline{\rho T'}) \tag{6}$$

where a new function,  $\overline{\rho T'}$ , has appeared, this is the density-temperature correlation function involving the fluctuations of density and temperature. We can use this result in eq. (5) above.

Now we make the important simple assumption

$$\overline{\rho'T'} = -\lambda\overline{\rho T} \quad (7)$$

in which eq. (7) is a definition of  $\lambda$ ; with this definition,  $\lambda$  is without consequences for any specific experiment. The equation of state then becomes

$$\frac{\overline{P}}{\overline{\rho}} = l(\overline{g})(1 - \lambda)R\overline{T} = \frac{b^2}{\gamma} \quad (8)$$

where

$$l(\overline{g}) = \frac{1 - \omega_o}{\mu_i} + \frac{\omega_o - \overline{g}}{\mu_v} \quad (9)$$

We must also make it clear that the physical properties of interest are functions of space and time. That is,

$$\overline{T} = \overline{T}(x_i, t), \overline{P} = \overline{P}(x_i, t), \overline{\rho} = \overline{\rho}(x_i, t), \overline{g} = \overline{g}(x_i, t), \lambda = \lambda(x_i, t) \quad (10)$$

where  $i = 1, 2, 3$ ,  $x_i$  is the  $i$  component of the space variable, and  $t$  is the time. To use equation (10), we can use as the simplest adequate physically reasonable model for  $\lambda(x_i, t)$  to be  $-\lambda(x_i, t) = \exp -ft$ . The exponential character of the decay is consistent with the usual long-term behavior of turbulent correlations. With this model, the density temperature correlation function depends only on time and obeys a decay law which is exponential. Finally, it is then usual (with  $U$  and  $L$  as velocity and length parameters respectively) to express all these relations in a non-dimensional form:  $v_i = u_i/U$ ;  $y_i = x_i/L$ ; and  $t = tU/L$ .

We can now include the influence of fluctuations in temperature (as well as the implicit fluctuations in  $P$  and  $\rho$ ) in a turbulence distorted refinement of the local mean square of the local speed of sound  $\overline{a^2}(y_2, \tau)$  by explicitly

requiring  $\frac{\overline{P}(y_2, \tau)}{\overline{\rho}(y_2, \tau)} = \frac{\overline{a^2}}{\gamma}$ . This form is reconciled with equation (8) above by

$$\frac{\overline{b^2}}{\gamma} = \frac{\overline{a^2}}{\gamma} (1 - \lambda(y_2, \tau)) \quad (11)$$

We have shown in equation (3) that  $\overline{P} \propto (\overline{b^2})^{\frac{\gamma}{\gamma-1}}$ . Thus

$$\left. \frac{\partial}{\partial \tau} \text{Log} \overline{P}(y_i, \tau) \right|_{\frac{Y}{\gamma-1}} = \frac{\partial \overline{a^2}(y_2, \tau)}{\partial \tau} + \frac{\partial}{\partial \tau} \text{Log}(1 - \lambda) \quad (12)$$

$$= \frac{\gamma}{\gamma-1} \left\{ \frac{\partial}{\partial \tau} \overline{\text{Log} a^2(y_2, \tau)} - \frac{f}{\exp f\tau + 1} \right\}$$

in which  $f$  is the real positive constant introduced in the definition of 1. We finally obtain from equation (1)

$$\frac{dr(\tau)}{d\tau} = \frac{6\sigma}{\rho_c \left(\frac{R}{\mu_v}\right) \overline{T}} \left\{ \frac{(n-3)^{n-3} \gamma f}{(n-4)^{n-2} (\gamma-1) \exp f\tau + 1} - \frac{\partial \overline{\text{Log} a^2(y_2, \tau)}}{\partial \tau} \right\} \quad (13)$$

When we define the instantaneous turbulent intensity as proportional to  $\overline{a^2}(y_2, \tau)$ , then the interpretation of equation (13) is straightforward. (i) At small values of  $t$  and constant turbulent intensity, ( $\partial \overline{a^2}(y_2, \tau) / \partial \tau = 0$ ), the rate of change in droplet size is determined entirely by the density correlation exponential factor  $f$ . (ii) At large values of  $t$  and constant turbulent intensity, ( $\partial \overline{a^2}(y_2, \tau) / \partial \tau = 0$ ), the rate of change in droplet size has no dependence on turbulence at all. (iii) If the turbulent intensity is not constant, i.e., ( $\partial \overline{a^2}(y_2, \tau) / \partial \tau \neq 0$ ), then the rate of change in droplet size is sensitive to the turbulence at all times. The case  $\partial \overline{a^2}(y_2, \tau) / \partial \tau \neq 0$  requires that a measure of the actual evolution in the strength of turbulent fluctuations be determined and that a model for determining the connection between the mean square local speed of sound and the evolution of turbulence be determined.

The overall features just described are summarized in Fig. 1. It has already been shown<sup>2</sup> that, at fixed thermodynamic conditions, the size of a droplet in condensing flow is influenced by the strength of the local turbulence and, by inference, the value of the Reynolds. Equation (13) above sets conditions for the sensitivity of the condensate accretion to turbulence, based on basic physics principles, which can now be tested by experiments.

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#### References

\*Visiting Research Scientist. Permanent Address: **Université Nationale du Bénin, Cotonou, Bénin**  
\*\*Graduate Research Assistant. Current Address: **Université Nationale du Bénin, Cotonou, Bénin**

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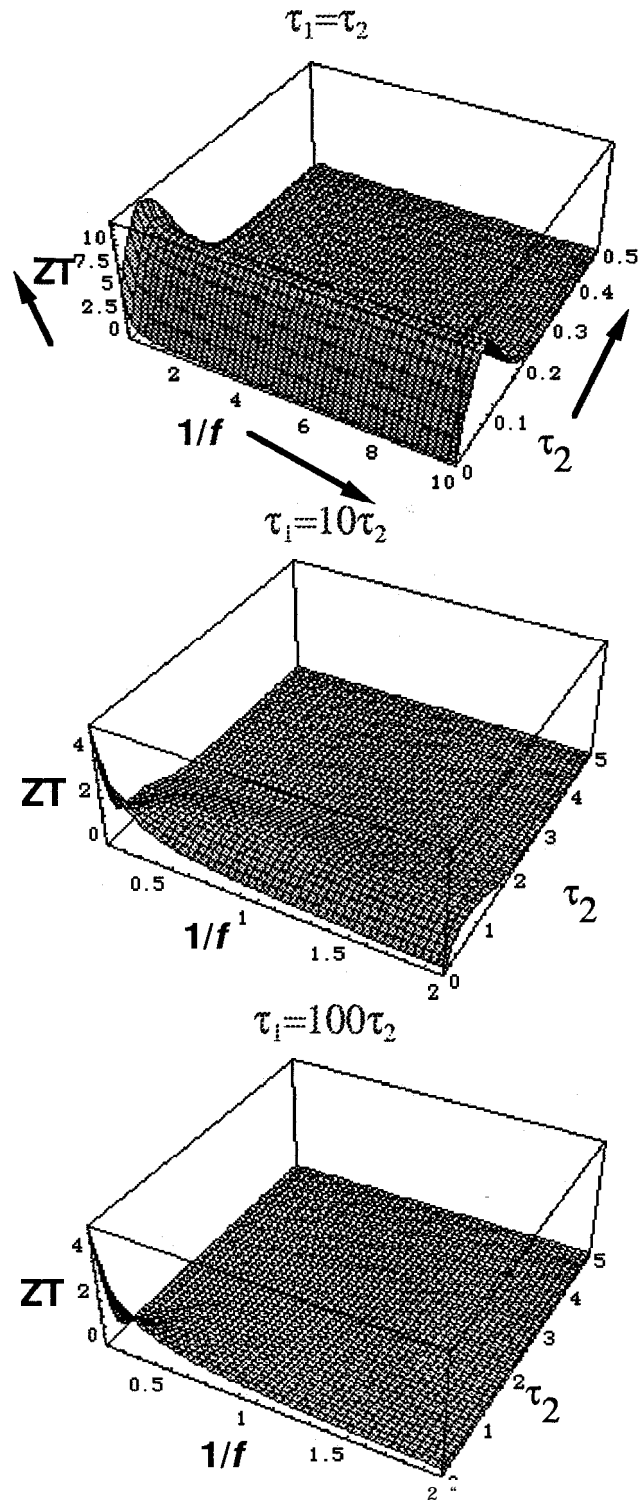


Figure 1. Sensitivity in the Evolution of Turbulent Distortion of Droplet Growth Rate with the Persistence of the Turbulence Pulse. The symbol  $f$  is defined in the text;  $\tau_2$  is the laboratory time;  $\tau_1$  is the persistence time of adistortion in the turbulence; and  $ZT$  is the product of the left hand side of eq. (13) with the prefactors of the right hand side's bracketed expression. The plots show the differences between a short puke, in the top display, and a long and relatively unchanging turbulence in the lower display.