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Formation Flying Control of Multiple Spacecraft

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Abstract

The problem of coordination and control of multiple spacecraft (MS) moving in formation is considered. Here, each MS is modeled by a rigid body with fixed center of mass. First, various schemes for generating the desired formation patters are discussed, Then, explicit control laws for formation-keeping and relative attitude alignment based on nearest neighbor-tracking are derived. The necessary data which must be communicated between the MS to achieve effective control are examined. The time-domain behavior of the feedback-controlled MS formation for typical low-Earth orbits is studied both analytically and via computer simulation. The paper concludes with a discussion of the implementation of the derived control laws, and the integration of the MS formation coordination and control system with a proposed inter-spacecraft communication/computing network.

1 Introduction

Future use of multiple micro-spacecraft moving in formation for space exploration, constellation space antennas and interferometers, and space-based global communication systems calls for novel approaches to the design of spacecraft control systems. In particular in recent years, growing emphasis is placed on the concept of separated spacecraft interferometry (SSI). The SSI concept envisioned the collecting apertures to be located on separate spacecraft while central combining instruments to be located on yet another spacecraft. A virtual structure is therefore developed without the real need for maintaining the necessary structural rigidity. The SSI provides measurements unachievable with other techniques and allows long baseline lengths and orient ation changes.

This paper focuses on the development of a control system architecture for the coordination and cent rol of a fleet of micro-spacecraft moving in formation. Here, we are dealing with a collection of systems which interact with each other in a cooperative manner to achieve a common objective. Although control of a single spacecraft is based on well-established control theory concepts and methodologies, the control systems for multiple spacecraft moving in formation require architectures which differ from those of conventional single spacecraft control systems. To provide a desired formation, basic mathematical models for controlled movement of rigid bodies in free space is presented. The control laws for the coordination of spacecraft attitude during motion to achieve a specified objective (e.g. orient each spacecraft along a given direction) is also developed. This is followed by the derivation of control laws for formation keeping and relative attitude alignment, The time-domain behavior of the feedback-controlled formation flying for typical low-Earth orbits is studied both analytically and via computer simulation. Emphasis is placed on determining the information exchange needed for achieving and maintaining a desired formation, and the conditions

for ensuring formation stability in the presence of various types of perturbations. Other important factors such as collision-avoidance, spacecraft failures, loss of communication between spacecraft, and also various constraints imposed by implementation and the physical size of the micro-spacecraft are discussed.

2 Modeling of Multiple Spacecraft in Formation

To simplify the development, we consider only a spacecraft triad modeled by rigid bodies with fixed centers of mass moving in free space under the influence of a gravitational field and external disturbances. We introduce the following coordinate systems in the three-dimensional Euclidean space $\Re^3$: (i) an inertial coordinate system $\mathcal{F}_0$ with orthonormal basis $\mathcal{B}_0 = \{\mathbf{e_X}, \mathbf{e_Y}, \mathbf{e_Z}\}$, and (ii) a set of moving coordinate systems $\mathcal{F}_i, i$ = 1,2,3 whose origins O_i are at the mass centers of the spacecraft. Let $\mathcal{B}_i = \{\boldsymbol{e}_{ix}, \boldsymbol{e}_{iy}, \boldsymbol{e}_{iz}\}$ denote an orthonormal basis associated with the moving coordinate system (abbreviated by "MCS" hereafter) $\mathcal{F}_i$, and [w] $_i$ the representation of the vector w with respect to basis $\mathcal{B}_i$. The the basis vectors in 13$_0$ and $\mathcal{B}_i$ are related by a linear t ransformat ion $\mathbf{C_i}$ defined by

$$\boldsymbol{e}_{ix} = \boldsymbol{C}_i \boldsymbol{e}_x, \quad \boldsymbol{e}_{iy} = \boldsymbol{C}_i \boldsymbol{e}_y, \quad \boldsymbol{e}_{iz} = \boldsymbol{C}_i \boldsymbol{e}_z, \quad i = 1,2,3 \tag{1}$$

whose representation with respect to basis $\mathcal{B}_0$ is given by the direction cosine matrix $C(q_i) = (q_{i4}^2 - \hat{q}_i^T \hat{q}_i)I + 2\hat{q}_i\hat{q}_i^T - 2q_{i4}\boldsymbol{Q}(\hat{q}_i)$, where

$$\boldsymbol{Q}(\hat{q}_i) \triangleq \begin{bmatrix} 0 & -q_{i3} & q_{i2} \\ q_{i3} & 0 & -q_{i1} \\ -q_{i2} & q_{i1} & 0 \end{bmatrix}; \quad \hat{q}_i \triangleq \begin{bmatrix} q_{i1} \\ q_{i2} \\ q_{i3} \end{bmatrix}, \tag{2}$$

and $\boldsymbol{q}_i = [\hat{q}_i^T, q_{i4}]^T$ denotes the unit quaternion with q_{ij} being the Euler symmetric parameters [5] defined by $q_{ij} = \varepsilon_{ij} sin(\phi_i/2), j$ = 1,2,3; $q_{i4} = cos(\phi_i/2)$, satisfying the constraint $\Sigma_{j=1}^4 q_{ij}^2$ = 1, i = 1,2,3 where ϕ_i is the principal angle and the ε_{ij}'s are the components of the principal vector of rotation $\boldsymbol{\ell}_i$ defined by $\boldsymbol{\ell}_i = \varepsilon_{i1}\boldsymbol{e}_x + \varepsilon_{i2}\boldsymbol{e}_y + \varepsilon_{i3}\boldsymbol{e}_z = \varepsilon_{i1}\boldsymbol{e}_{ix} + \varepsilon_{i2}\boldsymbol{e}_{iy} + \varepsilon_{i3}\boldsymbol{e}_{iz}$. The time derivitive of q_{ij} is related to the angular velocity $\omega_i = \omega_{ix}e_{ix} + \omega_{iy}e_{iy} + \omega_{iz}e_{iz}$ of $\mathcal{F}_i$ relative to the inertial coordinate system $\mathcal{F}_0$ by

$$\begin{aligned} \frac{d\hat{q}_i}{dt} &= (q_{i4}\boldsymbol{\omega}_i - \boldsymbol{\omega}_i \times \hat{q}_i)/2, \\ \frac{dq_{i4}}{dt} &= -(\omega_i, \hat{q}_i)/2, \end{aligned} \tag{3}$$

where w x v and w · v denote respectively the cross and scalar products of vectors w and v in $\mathcal{R}^3$.

Let d/dt_o and d/dt_i denote the time derivative operators with respect to $\mathcal{F}_0$ and $\mathcal{F}_i$ respectively; and $\mathcal{D}_i$ the time derivative operator d/dt_o in $\mathcal{F}_i$ defined by $\mathcal{D}_i$ w $\triangleq dw/dt_i + \boldsymbol{\omega}_i$ x w for w ε 73^3. The angular velocities of the i-th spacecraft or $\mathcal{F}_i$ relative to $\mathcal{F}_0$ are given by the following Euler's equations relating that time derivative of the angular momentum $\boldsymbol{I}_i\boldsymbol{\omega}_i$ with respect to $\mathcal{F}_0$ to the control torque τ_{ci}:

$$\mathcal{D}_i(\boldsymbol{I}_i\boldsymbol{\omega}_i) \triangleq d(\boldsymbol{I}_i\boldsymbol{\omega}_i)/dt_i + \boldsymbol{\omega}_i \times (\boldsymbol{I}_i\boldsymbol{\omega}_i) = \boldsymbol{I}_i d\boldsymbol{\omega}_i/dt_i + \boldsymbol{\omega}_i \times (\boldsymbol{I}_i\boldsymbol{\omega}_i) = \tau_{ci} \tag{4}$$

for i = 1,2,3, where $\boldsymbol{I}_i$ is the tensor of inertia associated with the i-th spacecraft. The time derivative of $\boldsymbol{\omega}_i$ may also be taken with respect to $\mathcal{F}_o$.

In formation acquisition and keeping, we are interested in the relative motion between any pair of spacecraft labelled by subscripts i and j. Let the MCS $\mathcal{F}'$ be $\mathcal{F}_i$ as defined earlier (See Fig. 1), and

$\boldsymbol{\rho}_{ji}$ denote the position vector of the j-th spacecraft relative to $\mathcal{F}_i$. The evolution of $\boldsymbol{\rho}_{ji}$ with time is governed by

$$\mathcal{D}_i^2(\boldsymbol{\rho}_{ji}) = (\boldsymbol{f}_{cj} + \boldsymbol{f}_{gj})/M_j - (\boldsymbol{f}_{ci} + \boldsymbol{f}_{gi})/M_i, \tag{5}$$

where $\mathcal{D}_i^2$ denotes the time derivative operator d^2/dt_o^2 in the MCS $\mathcal{F}_i$ given by

$$\mathcal{D}_i^2 \triangleq d^2/dt_i^2 + (d\boldsymbol{\omega}_i/dt_i) \times \cdot + 2\boldsymbol{\omega}_i \times d/dt_i + \boldsymbol{\omega}_i \times (\boldsymbol{\omega}_i \times \cdot). \tag{6}$$

3 Formation-Keeping Control Laws

The movement of spacecraft triad in formation can be achieved in marry ways. Here, we use a simple approach proposed in [6]. Let the first spacecraft be designated as the *leader*, and the remaining spacecraft as *followers*. The leader provides the reference motion $\boldsymbol{r}_1 = \boldsymbol{r}_1(t)$ (relative to $\mathcal{F}_o$) for the followers, and also two nonzero deviation vectors hi(t), $i = 2,3$, such that the point set $\mathcal{P}(t) = \{\boldsymbol{r}_1(t), \boldsymbol{r}_1(t) + \boldsymbol{h}_2(t), \boldsymbol{r}_1(t) + \boldsymbol{h}_3(t)\}$ defines the *desired formation pattern* at time t. It is assumed that the convex hull of P $(t)$is a simplex. We define the positional error for the i-th follower relative to the leader as $\boldsymbol{E}_i(t) \triangleq \boldsymbol{h}_i(t) - \rho_{i1}(t), i = 2,3.$

In what follows, we shall derive formation-keeping control laws assuming that each spacecraft in the triad moves in a low Earth orbit (LEO), and the desired motions for the followers are given by $\boldsymbol{d}_i(t) = \boldsymbol{r}_i(t) + \boldsymbol{h}_i(t)$, $i = 2,3$. Using (5), we obtain the following differential equation for $\boldsymbol{E}_i$:

$$\begin{aligned} \mathcal{D}_i^2(\boldsymbol{E}_i) &\triangleq \ddot{\boldsymbol{E}}_i + \dot{\boldsymbol{\omega}}_i \times \boldsymbol{E}_i + 2\boldsymbol{\omega}_i \times \dot{\boldsymbol{E}}_i + \boldsymbol{\omega}_i \times (\boldsymbol{\omega}_i \times \boldsymbol{E}_i) \\ &= \mathcal{D}_i^2(\boldsymbol{h}_i) + \boldsymbol{f}_{g1}/M_1 - \boldsymbol{f}_{gi}/M_i + \boldsymbol{u}_{c1} - \boldsymbol{u}_{ci}, \end{aligned} \tag{7}$$

where $\boldsymbol{u}_{cj} = \boldsymbol{f}_{cj}/M_j$.

Assuming a central Newtonian gravitational force field, the gravitational force acting on the i-th follower has the form: $\boldsymbol{f}_{gi} = -\mu M_i \boldsymbol{r}_i/\|\boldsymbol{r}_i\|^3$, where μ is the geocentric gravitational constant; $\boldsymbol{r}_i$ is the vector specifying the position of the mass center of the i-th spacecraft relative to the inertial frame $\mathcal{F}_o$; and $\|\boldsymbol{r}_i\|$ the Euclidean norm of $\boldsymbol{r}_i$. Let $\boldsymbol{e}_{ri} = \boldsymbol{r}_i/\|\boldsymbol{r}_i\|$. We can write

$$\begin{aligned} \boldsymbol{r}_i &= \|\boldsymbol{r}_i\|\boldsymbol{e}_{ri} \\ &= \|\boldsymbol{r}_i\|\{(\boldsymbol{e}_{ri} \cdot \boldsymbol{e}_{ix})\boldsymbol{e}_{ix} + (\boldsymbol{e}_{ri} \cdot \boldsymbol{e}_{iy})\boldsymbol{e}_{iy} + (\boldsymbol{e}_{ri} \cdot \boldsymbol{e}_{iz})\boldsymbol{e}_{iz}\}, \end{aligned} \tag{8}$$

where $\boldsymbol{r}_j = \boldsymbol{r}_i + \boldsymbol{\rho}_{ji}$. For LEO spacecraft triad moving in formation at approximately the same altitude, we have $\|\boldsymbol{r}_i\|/\|\boldsymbol{r}_1\| \cong 1$. Thus, the components of $(\boldsymbol{f}_{g1}/M_1 - \boldsymbol{f}_{gi}/M_i)$ in (7) with respect to basis $\mathcal{B}_i$ can be approximated by

$$\begin{aligned} (\boldsymbol{f}_{g1}/M_1 - \boldsymbol{f}_{g1}/M_i)_{ik} &= \omega_{io}^2\{\|\boldsymbol{r}_i\|(\boldsymbol{e}_{ri} \cdot \boldsymbol{e}_{ik}) - (\|\boldsymbol{r}_i\|/\|\boldsymbol{r}_i\|)^3(\rho_{1ik} + \|\boldsymbol{r}_i\|(\boldsymbol{e}_{ri} \cdot \boldsymbol{e}_{ik}))\} \\ &\cong -\omega_{io}^2\rho_{1ik}, \quad k = x, y, z, \end{aligned} \tag{9}$$

where $\omega_{io}^2(t) = \mu/\|\boldsymbol{r}_i(t)\|^3$ is the orbital angular speed of the i-th spacecraft about the origin of $\mathcal{F}_o$ at the time t. Substituting (9) into (7), making use of the identity $\boldsymbol{\omega}_i \times (\boldsymbol{\omega}_i \times \boldsymbol{E}_i) = (\boldsymbol{\omega}_i \cdot \boldsymbol{E}_i)\boldsymbol{\omega}_i - \|\boldsymbol{\omega}_i\|^2\boldsymbol{E}_i$, and assuming $\omega_{io} \cong \omega_o$ (a positive constant) lead to

$$\ddot{\boldsymbol{E}}_i + \dot{\boldsymbol{\omega}}_i \times \boldsymbol{E}_i + 2\boldsymbol{\omega}_i \times \dot{\boldsymbol{E}}_i + (\boldsymbol{\omega}_i \cdot \boldsymbol{E}_i)\boldsymbol{\omega}_i + (\omega_{io}^2 - \|\boldsymbol{\omega}_i\|^2)\boldsymbol{E}_i = \mathcal{D}_i^2(\boldsymbol{h}_i) + \omega_o^2\boldsymbol{h}_i + \boldsymbol{u}_{c1} - \boldsymbol{u}_{ci}. \tag{10}$$

To derive a formation-keeping control law for the i-th follower, we consider the time rate-of-change of the positive definite function $V_i = (K_{1i}\boldsymbol{E}_i \cdot \boldsymbol{E}_i + \dot{\boldsymbol{E}}_i \cdot \dot{\boldsymbol{E}}_i)/2$, $K_{1i} > 0$. It can be verified [8] that if we set

$$\begin{aligned} \boldsymbol{u}_{ci} &= \boldsymbol{I}_i^{-1}(-\boldsymbol{\omega}_i \times (\boldsymbol{I}_i\boldsymbol{\omega}_i) + \boldsymbol{\tau}_{ci}) \times (\boldsymbol{h}_i - \boldsymbol{E}_i) + (\boldsymbol{\omega}_i \cdot (\boldsymbol{h}_i - \boldsymbol{E}_i))\boldsymbol{\omega}_i \\ &+ (\omega_{io}^2 - \|\boldsymbol{\omega}_i\|^2)(\boldsymbol{h}_i - \boldsymbol{E}_i) + K_{1i}\boldsymbol{E}_i + K_{2i}\dot{\boldsymbol{E}}_i + 2\boldsymbol{\omega}_i \times \dot{\boldsymbol{h}}_i + \ddot{\boldsymbol{h}}_i + \boldsymbol{u}_{c1}, \end{aligned} \tag{11}$$

where $\dot{\boldsymbol{h}}_i = d\boldsymbol{h}_i/dt_i, \ddot{\boldsymbol{h}}_i = d^2\boldsymbol{h}_i/dt_i^2$, and K_{2i} is a positive constant, then $dV_{i1}/dt = -K_{2i}\|\dot{\boldsymbol{E}}_i\|^2 < 0$.

The equation for $[\boldsymbol{E}_i]_i$ corresponding to the feedback-controlled system is given by

$$[\ddot{\boldsymbol{E}}_i]_i + (K_{2i}[\boldsymbol{I}] + 2\boldsymbol{Q}([\boldsymbol{\omega}_i]_i(t)))[\dot{\boldsymbol{E}}_i]_i + K_{1i}[\boldsymbol{E}_i]_i = 0 \tag{12}$$

Since $\boldsymbol{Q}([\boldsymbol{\omega}_i]_i)$ is slew-symmetric, all solutions $[\dot{\boldsymbol{E}}_i, \boldsymbol{E}_i]_i(t) \longrightarrow (\boldsymbol{O}, 0) \epsilon \mathcal{R}^6$ as $t + \infty$ for any $K_{1i}, K_{2i} > 0$.

Remarks. RI: The model-dependent Control law (11) corresponds to state-feedback linearization controls which involve partial cancellation of the terms in (10). Assuming perfect cancellation, there is no coupling between the equations for the tracking errors of followers given by (12). Since perfect cancellation is not achievable physically due to inaccurate knowledge of the model parameter values, sensor errors, act uator saturation, and unmodelled external dist urbances, it is of import ante to determine the effect of imperfect cancellation on the behavior of the feedback-controlled system. Here, we model this imperfection by introducing a persistent disturbance N in (12) as follows:

$$\begin{aligned} &[\ddot{\boldsymbol{E}}_i]_i + (K_{2i}[\boldsymbol{I}] + 2\boldsymbol{Q}([\boldsymbol{\omega}_i]_i(t)))[\dot{\boldsymbol{E}}_i]_i + K_{1i}[\boldsymbol{E}_i]_i \\ &= \boldsymbol{N}(t, [\boldsymbol{\omega}_i(t)]_i \cdot [\rho_{ji}(t)]_i, [\boldsymbol{u}_{cj}(t)]_i, [\boldsymbol{E}_i]_i, [\dot{\boldsymbol{E}}_i]_i), \end{aligned} \tag{13}$$

and require that the zero state is totally stable [7]. Since $\boldsymbol{B}_1$ is a skew-symmetric matrix, the zero state of (12) is uniformly asymptotically stable for any $K_{1i}, K_{2i} > 0$. Then, total stability of the zero state follows from a well-known theorem of Malkin [7].

R2: The control laws (11) for the followers require the knowledge of its own attitude control law $\boldsymbol{\tau}_{ci}$ and the control law $\boldsymbol{u}_{c1}$ of the leader. The latter information must be transmitted to the follower spacecraft. Note also that control laws (11) can be rewritten as

$$\begin{aligned} \boldsymbol{u}_{ci} = \boldsymbol{I}_i^{-1}(-\boldsymbol{\omega}_i \times (\boldsymbol{I}_i\boldsymbol{\omega}_i) + \boldsymbol{\tau}_{ci}) \times \boldsymbol{E}_i - (\boldsymbol{\omega}_i \cdot \boldsymbol{E}_i)\boldsymbol{\omega}_i - (\omega_{io}^2 - \|\boldsymbol{\omega}_i\|^2)\boldsymbol{E}_i \\ + K_{1i}\boldsymbol{E}_i + K_{2i}\dot{\boldsymbol{E}}_i + \boldsymbol{u}_{ci} + \mathcal{D}_i^2(\boldsymbol{h}_i) + \omega_{io}^2\boldsymbol{h}_i. \end{aligned} \tag{14}$$

The terms $\boldsymbol{u}_{ci}$ and $\mathcal{D}_i^2(\boldsymbol{h}_i) + \omega_{io}^2\boldsymbol{h}_i$ in (11') correspond to a feed-forward control. When the norms of these terms are large, the norm of $\boldsymbol{u}_{ci}$ is also large. This situation may be alleviated by replacing the term by a suitable scaling depending on the norm of $\mathcal{D}_i^2(\boldsymbol{h}_i) + \omega_{io}^2\boldsymbol{h}_i$. In the important special case where the spacecraft move in a nearly circular LEO and the deviation vector $\boldsymbol{h}_i$ rotates about the Earth's center with angular velocity $\omega_i \cong \omega_{io}$, then $\mathcal{D}_i^2(\boldsymbol{h}_i) + \omega_{io}^2\boldsymbol{h}_i \cong \boldsymbol{O}$ or $\boldsymbol{h}_i$ is close to a solution of the simple harmonic oscillator equation $d^2\boldsymbol{h}_i/dt_o^2 + \omega_{io}^2\boldsymbol{h}_i = O$.

R3: It is evident that if each follower applies control law (11), then the desired formation pattern $\mathcal{P}$ = P(i), $t \geq 0$, is asymptotically stable, i.e. given any real number $\epsilon > 0$, there exists a $\delta > 0$ such that $\triangle(t) < 6 \Rightarrow \triangle(t) < \epsilon$ for all $t \geq 0$. Moreover, $\triangle(t) \rightarrow O$ as $t \rightarrow \infty$, where

$$\triangle(t) = \left(\sum_{i=2}^{3} \{\sigma_{1i} \|\boldsymbol{E}_i(t)\|^2 + \sigma_{2i} \|\dot{\boldsymbol{E}}_i(t)\|^2\} \right)^{1/2}, \tag{15}$$

and σ_{1i}, σ_{2i} are specified positive weighting coefficients. Here, asymptotic stability is only local in the sense that the convergence of $\triangle(t)$ to O as $t \to 00$ is attained if the deviation of the initial formation pattern at t = O from the desired one is sufficiently small. In physical situations, the possibility of collision between spacecraft must also be considered.

R4: It has been shown recently [9] that under certain mild conditions, asymptotic stability of the formation pattern $\mathcal{P} = \mathcal{P}(t)$ can be achieved using simplified versions of control laws (11') given by

$$\boldsymbol{u}_{ci} = K_{1i}\boldsymbol{E}_i + K_{2i}\dot{\boldsymbol{E}}_i + \{\boldsymbol{u}_{ci} + \mathcal{D}_i^2(\boldsymbol{h}_i) + \omega_{io}^2 \boldsymbol{h}_i\}, i = 2,3, \tag{16}$$

which correspond to proportional-plus-rate feedback plus feed-forward controls. The inclusion of the feed-forward control is essential for asymptotic stability.

4 Attitude Control

Let the desired attitude and angular velocity of the i-th follower at time t relative to the inertial coordinate system $\mathcal{F}_o$ be specified respectively by the MCS $\mathcal{F}_i^d(t)$and @(t), which may depend on the attitude and angular velocity of the leader, eg. the desired attitude and angular velocity of the i-th follower correspond exactly to $\mathcal{F}_1(t)$ and $\boldsymbol{\omega}_1(t)$ respectively. It is of interest to control the *relative* attitudes and angular velocities between the spacecraft. Here, we shall derive control laws for the followers which are expressed in terms of their instantaneous attitudes and angular velocities of relative to the inertial coordinate system $\mathcal{F}_o$ or to the MCS $\mathcal{F}_1$.

Let the unit quaternion corresponding to $\mathcal{F}_i^d(t)$ relative to the inertial coordinate system $\mathcal{F}_o$ be denoted by $\boldsymbol{q}_i^d(t) = [\widehat{\boldsymbol{q}}_i^d(t), \boldsymbol{q}_{i4}^d(t)]^T$. We assume that $\boldsymbol{q}_i^d$ and $\boldsymbol{\omega}_i^d$ are consistent in the sense that they satisfy (3) and (4) with" control torque τ_i^d. We introduce the deviations $\delta\boldsymbol{q}_i = \boldsymbol{q}_i^d - \boldsymbol{q}_i = [\widehat{\boldsymbol{q}}_i^d - \widehat{\boldsymbol{q}}_i, q_{i4}^d - q_{i4}]^T$ and @ $\triangleq \boldsymbol{\omega}_i^d - \boldsymbol{\omega}_i$. It can be verified that $\delta\boldsymbol{q}_i$ satisfies

$$\left.\begin{aligned} \frac{d\delta\widehat{\boldsymbol{q}}_i}{dt_o} &= (q_{i4}\boldsymbol{\omega}_i^d - q_{i4}\boldsymbol{\omega}_i - \boldsymbol{\omega}_i^d \times \widehat{\boldsymbol{q}}_i^d + \boldsymbol{\omega}_i \times \widehat{\boldsymbol{q}}_i)/2, \\ \frac{d\delta q_{i4}}{dt} &= -(\boldsymbol{\omega}_i^d \cdot \widehat{\boldsymbol{q}}_i^d - \boldsymbol{\omega}_i, \widehat{\boldsymbol{q}}_i)/2. \end{aligned}\right\} \tag{17}$$

To derive an attitude control law, we consider the following positive definite function $V_{1i} = K_{qi}\dot{V}_{1i} + \ddot{V}_{1i}$ defined on R^7, where K_{qi} is a given positive constant and $\dot{V}_{1i} = \delta q_{i4}^2 + \delta\widehat{\boldsymbol{q}}_i \cdot \delta\widehat{\boldsymbol{q}}_i$; $\ddot{V}_{1i} = (\delta\boldsymbol{\omega}_i \cdot I_i\delta\boldsymbol{\omega}_i)/2$. The time derivative of V_{1i} along the solutions of the equations for $\delta q_i, \delta\boldsymbol{\omega}_i$ is given by

$$\frac{dV_{1i}}{dt} = \{K_{qi}(q_{i4}^d\delta\widehat{\boldsymbol{q}}_i - \delta q_{i4}\widehat{\boldsymbol{q}}_i^d - \widehat{\boldsymbol{q}}_i^d \times \delta\widehat{\boldsymbol{q}}_i) + \tau_{ci}^d - \tau_{ci} - \boldsymbol{\omega}_i^d \times (\boldsymbol{I}_i\delta\boldsymbol{\omega}_i)/2\} \cdot \delta\boldsymbol{\omega}_i. \tag{18}$$

Thus, if we set

$$\tau_{ci} = K_{qi}(q_{i4}^d\delta\widehat{\boldsymbol{q}}_i - \delta q_{i4}\widehat{\boldsymbol{q}}_i^d - \widehat{\boldsymbol{q}}_i^d \times \delta\widehat{q}_i) + \tau_{ci}^d - \boldsymbol{\omega}_i^d \times (\boldsymbol{I}_i\delta\boldsymbol{\omega}_i)/2 + K_{\omega i}I_i\delta\boldsymbol{\omega}_i, \tag{19}$$

where $K_{\omega i}$ is a positive constant, then $dV_{1i}/dt = -K_{\omega i}\delta\boldsymbol{\omega}_i \cdot \boldsymbol{I}_i\delta\boldsymbol{\omega}_i \leq 0$ and $Vii(t) \leq V_{1i}(0)$ for all $t \geq 0$ implying uniform boundedness of $\|\delta\omega_i(t)\|$ for all $t \geq 0$, By considering d^2V_{1i}/dt^2, and making use of Barbalat's Lemma [10], we can deduce that $(dV_{1i}/dt)(t) \to O$ *as* $t \to 00$, or $\omega_i(t) \to \omega_i^d(t)$ as $t \to \infty$. But it does not follow that $\delta q_{i4}(t) \to \boldsymbol{O}$ and $\delta\widehat{\boldsymbol{q}}_i(t) \to$ O as $t \to 00$.

To proceed further, we make use of the fact that the quaternion $(\Delta\hat{q}_i, \Delta q_{i4})$ of the desired attitude or $\mathcal{F}_i^d$ *relative to* $\mathcal{F}_i$ is related to the quaternions $(\hat{q}_i^d, q_{i4}^d)$ for $\mathcal{F}_i^d$, and $(\hat{q}_i, q_{i4})$ for $\mathcal{F}_i$ relative to the inertial coordinate system $\mathcal{F}_o$ by

$$\Delta\hat{q}_i = q_{i4}\hat{q}_i^d - q_{i4}^d\hat{q}_i - \hat{q}_i \times \hat{q}_i^d, \quad \Delta q_{i4} = q_{i4}q_{i4}^d + \hat{q}_i \cdot \hat{q}_i^d. \tag{20}$$

When $\mathcal{F}_i$ coincides with $\mathcal{F}_i^d$ (i.e. $\delta\hat{q}_i$ = O and δq_{i4} = O), we have $\Delta\hat{q}_i$ = O and $\Delta q_{i4} = 1$. Using (18), control law (17) can be rewritten as

$$\tau_{ci} = -K_{qi}\,\Delta\,q_i + \tau_{ci} - \omega_i^d \times (I_i\delta\omega_i)/2 + K_{\omega i}I_i\delta\omega_i. \tag{21}$$

Following an analysis similar to that given in [II], conclude that $\Delta\hat{q}_i$ and $\delta\omega_i(t) \to 0$ as $t \to \infty$ for any positive K_{qi} and $K_{\omega i}$.

5 Implementation of Control Laws

We observe that the implementation of control laws (11) and (19) for formation keeping requires a knowledge of $[E_i]_i, [\dot{E}_i]_i, [\omega_i]_i; [\tau_{ci}]_i$ and $[u_{ci}]_i$ at any time t. The quantities $[E_i]_i$ and $[\dot{E}_i]_i$ can be determined from $[\rho_{1i}]_i$ and $[\dot{\rho}_{1i}]_i$ which require measurement of the position and velocity of the leader relative to the i-th follower spacecraft. These quantities can also be obtained by transmitting the position and velocity of leader to the i-th follower. Also, the control of the leader at any time must also be transmitted to the i-t h follower spacecraft. When one or more spacecraft failure occurs, one may adopt the following backup schemes for control law implement aiton depending on the nature of failure:

(i) Inter-spacecraft Communicaiton System Failure: One may obtain estimates of $[\rho_{1i}]_i$ and $[\dot{\rho}_{1i}]_i$ by using on-board optical range sensors, or by setting the relative position and velocity between the failed and active spacecraft at their nominal values temporarily until the failure is recovered.

(ii) Overall Spacecraft Failure: Here the failure is sufficiently severe such that the failed spacecraft is no longer useful. In this case, it should be removed from the formation by deorbiting or by manual retrieval, If the failed spacecraft is replaced by a backup spacecraft, then it is necessary to reconfigure the formation, The control laws for steering the remaining active spacecraft from the old to the new formation requires separate consideration. This aspect will be discussed elsewhere.

We note also that in the derivation of foregoing control laws, no constraints have been imposed on the magnitude of the control variables. In the presence of bounded controls, one expects that the rate of decay of $\|([E_i]_i, [\dot{E}_i]_i)(t)\|$ and $\|(\delta\omega_i, \delta q_i)(t)\|$ to zero would be reduced by when one or more of the control variables takes on its extreme values.

Finally, for a real mission, it is necessary to consider discrete-time versions of the proposed control laws. In view of the limited fuel on-board, it is generally undesirable to have continuously acting controls. Therefore, the system response corresponding to the cent rol laws derived here serves as a basis for comparison between the idealized and the actual responses.

6 Fleet Coordination

For a fleet of spacecraft, one may require complete autonomy in each spacecraft in the sense that all the decisions for determining its future behavior are made on-board without the assistance of external agents. Although this approach provides enhanced operational reliability, it may not be cost effective since each spacecraft must contain all the essential hardware and software for

coordination and control. An alternative approach is to require each spacecraft to have only the basic hardware and software for attitude control and orbital nameuvering. The more complex tasks in fleet coordination and control are shared by all the spacecraft in the fleet. Moreover, some of the spacecraft may be equipped with special hardware and software to perform particular tasks for the entire fleet.

The fleet coordination is achieved with the aid of an inter-spacecraft communication network (eg. radio or optical links). This network has the following basic functions:

(i) Communicating the necessary data for fleet formation-keeping and relative attitude control;

(ii) Linking the computers in the spacecraft to form a distributed computing network thereby increasing the computational capability of the fleet for more computational intensive tasks such as on-board interferometer data processing,

In the realization of the first function, each fleet leader broadcasts its position and attitude with respect to a specified inertial frame, and the follower spacecraft broadcast their positions and velocities relative to their leader to achieve formation alignment.

7 Simulation Studies

Extensive simulation studies have been made to determine the performance of the proposed control laws for formation keeping and attitude regulation in the presence of actuator saturation, variations in spacecraft parameters, and loss of communication between spacecraft. Only typical results will be presented here.

We assume that the leader of the spacecraft triad moves along an inclined circular orbit $\mathcal{O}_1$ about the Earth with inclination angle $(\pi/2 - \varphi_{inc})$ and ascending node along the Y-axis. For convenience, we introduce a geocentric fixed cartesian coordinate frame $\mathcal{F}_o$ with origin O at the Eart h's center along with a spherical coordinate system (r, θ, ϕ) wit h orthonormal basis $\{e_r, e_\theta, e_\phi\}$. Its motion in spherical coordinates is given by

$$\begin{aligned} r_1(t) &= r_o, \quad \theta_1(t) = cos^{-1}\{cos(\varphi_{inc})\, cos(\theta_o - \omega_o t)\}, \\ \phi_1(t) &= tan^{-1}\{-tan(\theta_o - \omega_o t)/sin(\varphi_{inc})\}, \end{aligned} \tag{22}$$

where r_o is a given orbital radius, and $\omega_o = \sqrt{\mu/r_o^3}$. Here, for simplicity, we have set the desired orbit al radius for all spacecraft to r_o.

The desired motions for the second and third spacecraft correspond to two circular orbits with the same inclination angle $(\mu/2 - \varphi_{inc} = 8.2\mu/180\,\text{rad.}$, but with ascending nodes at (r, θ, ϕ) = $(r_o, \mu/2, \triangle\phi)$ and $(r_o, \mu/2, -\triangle\phi)$ respectively, where $\triangle\phi = 9.5\mu/180\,\text{rad.}$ We adopt the simplified control law (11") for formation keeping, where the deviation vector $h_2(t)$ is given by

$$\begin{aligned} h_2(t) = r_o\{&(-cos(\triangle\phi)sin\,(\varphi_{inc})cos(\bar{\theta}(t)) + sin(\triangle\phi)sin(\bar{\theta}(t))e_X \\ &+ (sin(\triangle\phi)sin(\varphi_{inc})\, cos(\bar{\theta}(t)) + cos(\triangle\theta)sin(\bar{\theta}(t)) - sin(\theta_o - \omega_o t))e_Y \\ &+ (cos(\varphi_{inc})cos(\bar{\theta}(t) - \triangle\theta) - cos(\theta_o - \omega_o t))e_Z. \end{aligned} \tag{23}$$

where $\bar{\theta}(t) = 19_0 - \omega_o t - \triangle\theta$. The deviation vector $h_3(t)$ has the same form as (21) except with $\triangle\phi$ replaced by $-\triangle\phi$. Evidently, hi(t) satisfies $d^2h_i\,(t)/dt_o^2 + \omega_o^2 h_i(t) = O$ for all t and i = 2,3. To specify the desired attitude of the i-th follower, we introduce the (1-2-3) Euler angles $(\Theta_i, \Psi_i, \Phi_i)$ corresponding to a rotation of Θ_i about the X-axis followed by a rotation of Ψ_i about the rotated

Y-axis, and a rotation of Φ_i about the rotated Z-axis. The desired Euler angles for the followers are given by:

$$\Theta_2^d(t) = \Theta_3^d(t) = \bar{\theta}(t),\ \Psi_2^d(t) = \Psi_3^d(t) = \varphi_{inc},$$
$$\Phi_2^d(t) = \triangle\phi,\ \Phi_3^d(t) = -\triangle\phi. \tag{24}$$

Thus, the desired attitude of the i-th spacecraft can be expressed in terms of the following quaternions:

$$q_{24}^d(t) = \{1 + cos(\triangle\phi)cos(\varphi_{inc}) - sin(\triangle\psi)sin(\varphi_{inc})\ sin(\bar{\theta}(t)) + cos(\bar{\theta}(t))(cos(\triangle\psi) + cos(\varphi_{inc}))\}^{\frac{1}{2}}/2; \tag{25}$$

$$q_{21}^d(t) = \{sin(\triangle\phi)sin(\varphi_{inc})cos(\bar{\theta}(t)) + sin(\bar{\theta}(t))(cos(\triangle\phi) + cos(\varphi_{inc}))\}/(4q_{24}^d(t)); \tag{26}$$

$$q_{22}^d(t) = \{sin(\varphi_{inc})(1 + cos(\triangle\phi)cos(\bar{\theta}(t))) - sin(\triangle\phi)sin(\bar{\theta}(t))\}/(4q_{24}^d(t)); \tag{27}$$

$$q_{23}^d(t) = \{cos(\triangle\phi)\ sin(\varphi_{inc})sin(\bar{\theta}(t)) + sin(\triangle\phi)cos(\bar{\theta}(t)) + cos(\varphi_{inc}))\}/(4q_{24}^d(t)). \tag{28}$$

The quaternions corresponding to the desired attitude of the third follower have the same form as (23) except with $\triangle\phi$ replaced by – A+. We require every spacecraft to spin about its z-axis with constant angular speed ω_s. Thus, the desired angular velocity for the i-th follower is given by

$$\boldsymbol{\omega}_i^d = \omega_o cos(\varphi_{inc})\boldsymbol{e}_X + \omega_o sin(\varphi_{inc})\boldsymbol{e}_Z + \omega_s \boldsymbol{e}_{iz}^d = \omega_s \boldsymbol{e}_{ix}^d + \omega_s \boldsymbol{e}_{iz}^d, \tag{29}$$

where $\{\boldsymbol{e}_{ix}^d, \boldsymbol{e}_{iy}^d, \boldsymbol{e}_{iz}^d\}$ corresponds to the basis of the body coordinate system $\mathcal{F}_i^d$ associated with the i-th follower with the desired attitude.

Figure 4 shows a typical time-domain response of the MS fleet with the simplified formation-keeping control law(11") and attitude control law (19) in the presence of actuator saturation, The spacecraft parameter values used in the simulation study are given in Table 1. The corresponding time-domain response of the MS fleet with the $[u_{c1}]_i$ term in (11") set to zero (to simulate the loss of communication between the MS) was also determined. The results do no differ significantly from those shown in Fig. 4. Next, the effect of inertia pertrubations on the time-domain response of the spacecraft triad was studied. It was found that the qualitative behavior of the response is essentially identical to that of the unperturbed case.

8 Concluding Remarks

In this paper, control laws for a spacecraft triad moving in formation have been derived using a simplified model for a rigid spacecraft. These control laws require the knowledge of the relative displacements and attitudes of the spacecraft and its neighbors. Simulation results based on a generic spacecraft model showed that the derived control law are effective in formation and relative attitude alignment provided that the magnitude of the initial deviation from the desired state is sufficiently small so that collisions between the spacecraft do not occur. Finally, in this work, important factors such as data processing time-delay and time discretization arising in physical implementation have not been taken into consideration. Nevertheless, the results reveal the basic structure of the control laws and the required inter-spacecraft data required for their implementation. Finally, the problems associated with the physical implementation of the control laws in terms of the state-of-the-art hardware and fuel consumption for control are not considered here, and they require further study.

M_i	mass of MS	$10kg.$
I_{ix}	moment of inertia about x-axis	$0.3646\,kgm^2.$
I_{iy}	moment of inertia about y-axis	$0.2734kgm^2.$
I_{iz}	moment of inertia about z-axis	$0.3125\,kgm^2.$
r_o	desired orbital radius of MS	$7.13814\ x10^6m.$
$\omega_o = \sqrt{\mu/r_o^3}$	orbital angular speed of fleet leader	$0.001\,rad./sec.$
ω_s	desired spin speed about z-axis	$0.01rad./sec.$
$\varphi_{inc} = \pi/2$	inclination angle of reference orbits	$8.2\pi/180rad.$
$\triangle\phi$	azimuthal angle associated with the ascending node of reference orbits	$0.2rad.$
$\triangle\theta$	MS separation angle	$\pi/120rad.$

Table 1: Values of microspacecraft and orbital parameters for simulation study,

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$[\boldsymbol{E}_2(0), \dot{\boldsymbol{E}}_2(0)]_2$ = [5 2 -5, 00 O]; $[\omega_2(0)]_2$ = [0.02 0,02 0.02];
$\boldsymbol{q}_2(0)$ = [0.3 0.1 0.2 0.9272618];
$[\boldsymbol{E}_3(0), \dot{\boldsymbol{E}}_3(0)]_3$ = [-1 1 -1, 00 O]; $[\omega_3(0)]_3$ = [−0.01 0.015 - 0.01];
$\boldsymbol{q}_3(0)$ = [–0.2 0.2 0.3 0.910433];

follower spacecraft 2 (solid curves); follower spacecraft 3 (dashed curves); Saturation levels: $\|f_{cij}\| \leq 1\ N$; $\|\tau_{cij} \leq 0.05 N.m.$, i = 2,3; j = x, y, z.

Fig. 3a: Positional tracking errors (m) vs.time.
Fig. 3b: Angular velocities (m/see) vs. time.
Fig. 3c: Quaternions vs. time.
Fig. 3d: Control forces (N) vs. time.
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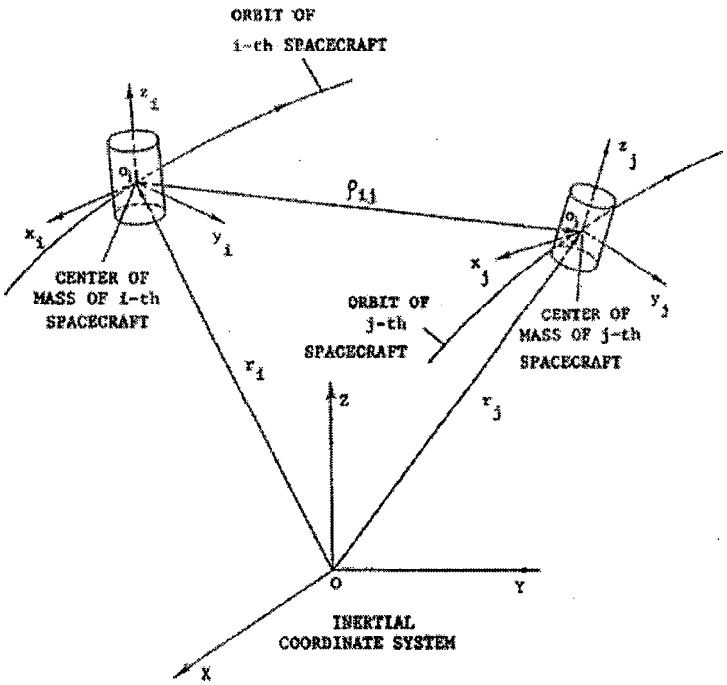

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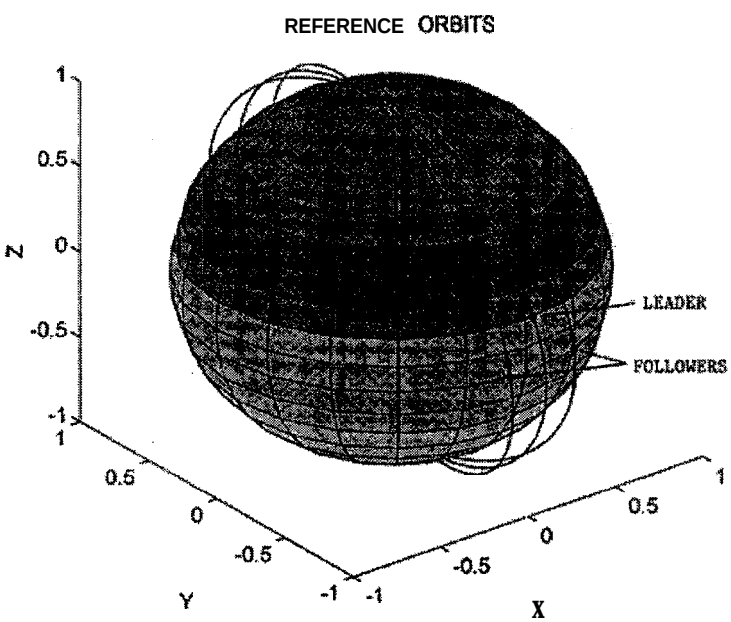

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$[E_2(0), \dot{E}_2(0)]_2$ =[52 -5, 00 O];

[CO,(o)], = [0.02 0.02 0.02];

$q_2(0)$= [0.3 0.10.20.927261 8];

$[E_3(0), \dot{E}_3(0)]_3$ = [−1 2-1, 0 0 0];

$[\omega_3(0)]_3$ = [-0.01 0.015- 0.01];

$q_3(0)$ = [−0.2 0.20.3 0.910433];

The Follower spacecraft 2 is shown with solid lines and the follower spacecraft 3 is shown with dashed lines; Saturation levels:

$|f_{cij}| \leq 1$ N; $|\tau_{cij}| \leq 0.05$ N.m, 1=2,3; j=x,y,z.

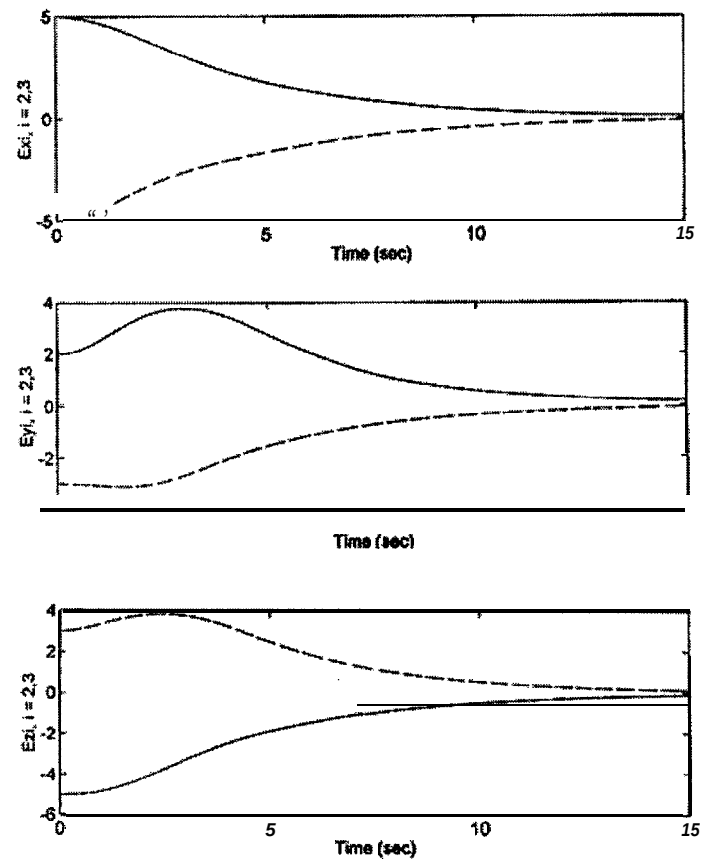


Figure 3a. Positional tracking errors (m) *vs.* time.

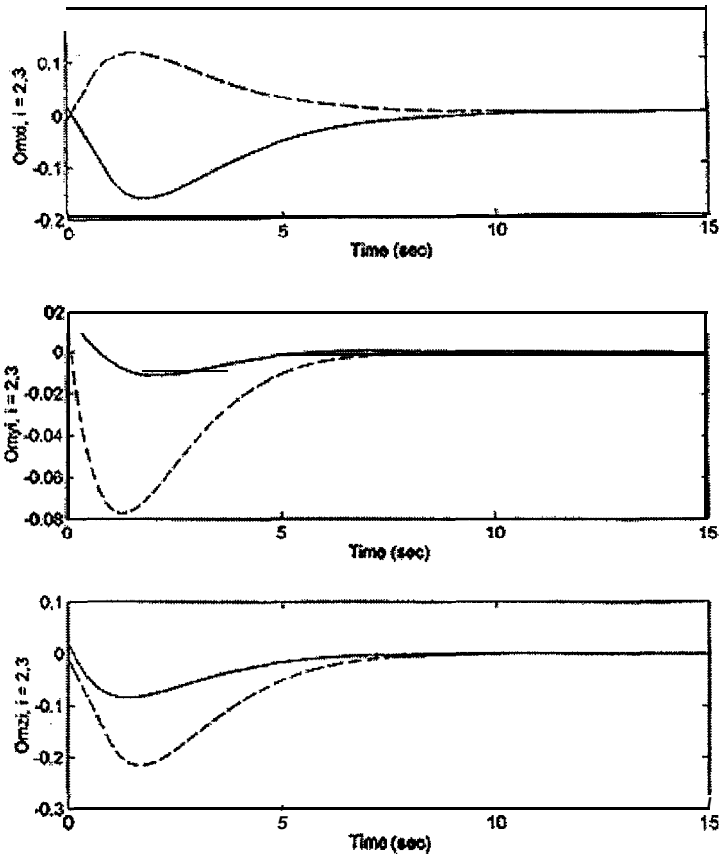


Figure 3b. Angular velocities (m/see) vs. time.

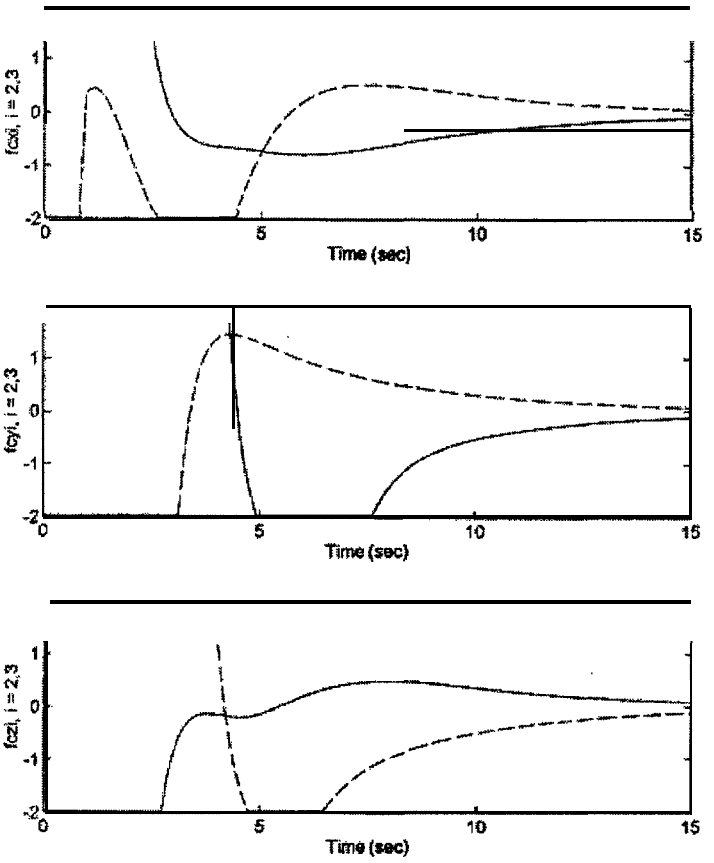
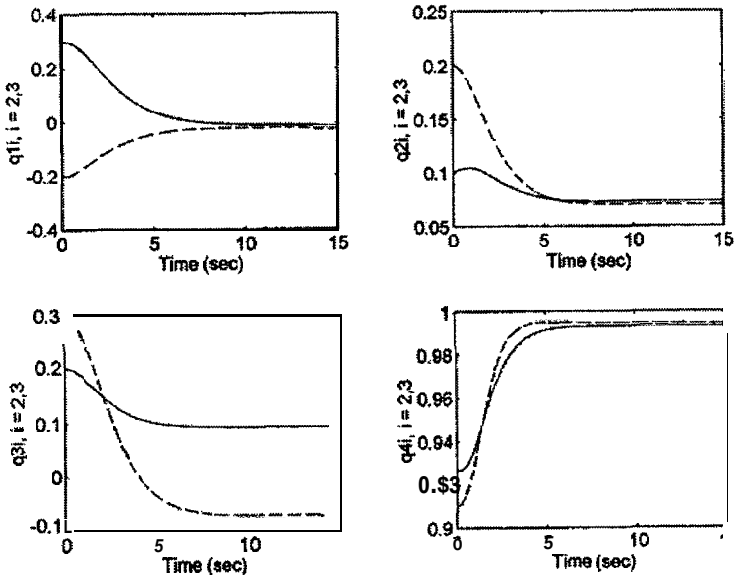


Figure 3d. Control forces (N) vs. time.



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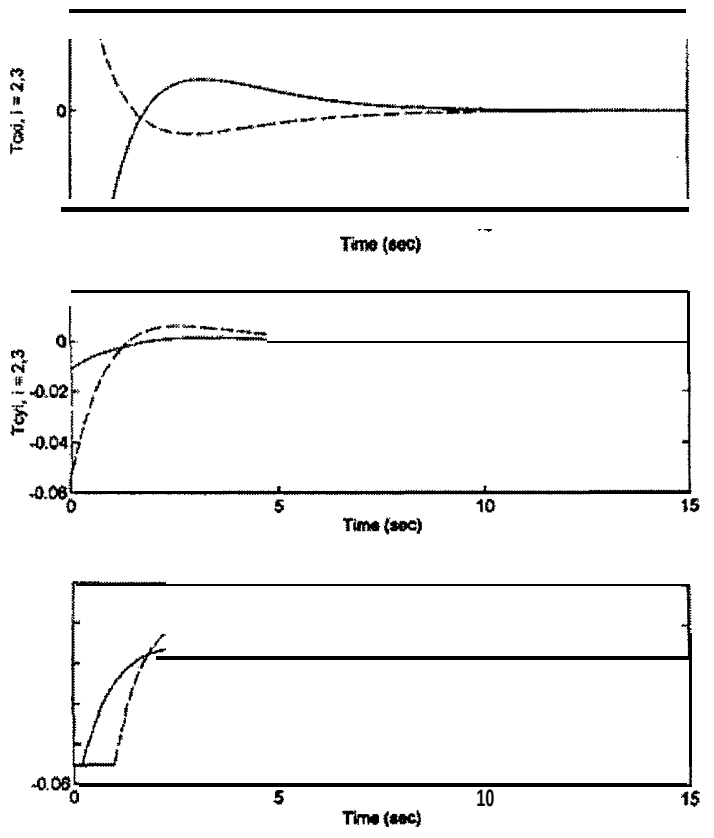

Figure 3e. Control torques (N.m) vs. time.