Investigation of Nonlinear Pressurization and Modal Restart In MSC/NASTRAN for Modeling Thin Film Inflatable Structures

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Abstract

This paper is written for the purpose of providing an introduction and set of guidelines for the use of a methodology for NASTRAN eigenvalue modeling of thin film inflatable structures. It is hoped that this paper will spare the reader from the problems and headaches the authors were confronted with during their investigation by presenting here not only an introduction and verification of the methodology, but also a discussion of the problems that this methodology can ensue. Our goal in this investigation was to verify the basic methodology through the creation and correlation of a simple model.

An overview of thin film structures, their history, and their applications is given. Previous modeling work is then briefly discussed. An introduction is then given for the method of modeling. The specific mechanics of the method are then discussed in parallel with a basic discussion of NASTRAN s implementation of these mechanics. The problems encountered with the method are then given along with suggestions for their work-a-rounds. The methodology is verified through the correlation between an analytical model and modal test results of a thin film strut. Recommendations are given for the needed advancement of our understanding of this method and ability to accurately model thin film structures. Finally, conclusions are drawn regarding the usefulness of the methodology.

Introduction and Background

Inflated cylindrical struts constructed of Kapton polyimide film have considerable practical application and potential for use as components of inflatable concentrator assemblies, antenna structures, and space power systems.

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Because of their importance, it is of great interest to characterize the dynamic behavior of inflatable cylinders both experimentally and analytically. It is very helpful to take a building-block approach to modeling and understanding inflatable assemblies by first investigating in detail the behavior of components such as struts. The polyimide film material used for construction of such cylinders (and the film-to-pressurized-air interaction) is highly nonlinear, with modulus varying as a function of frequency, temperature, and level of excitation. Thus, the primary motivation of tests and analytical modeling efforts was to determine and understand the response of inflatable cylinders for different pressures, film thicknesses, and boundary conditions.

In recent years, inflatable structures have been the subject of renewed interest for space applications such as communications antennae, solar thermal propulsion, and space solar power. A major advantage of using inflatable structures in space is their extremely light weight. This makes inflatables a good match for solar thermal propulsion because of the low thrust levels available. An obvious second advantage is on-orbit deployability and subsequent space savings in the launch configuration. A recent technology demonstrator flight for inflatable structures was the Inflatable Antenna Experiment (IAE) that was deployed on orbit from the Shuttle Orbiter. Although difficulty was encountered in the inflation/deployment phase, the flight was successful overall and provided valuable experience in the use of such structures (Ref. 1).

The Solar Orbital Transfer Vehicle (SOTV), discussed in Ref. 2, is a planned technology demonstrator flight for solar thermal propulsion. The basic concept behind solar thermal propulsion is to utilize sunlight or solar energy as a means of heating a working fluid (propellant) to provide thrust at increased specific impulse. As described in Ref. 3, thrust is produced by expanding the heated propellant through a nozzle. No combustion occurs, and the thrust level is low. For this reason, solar thermal propulsive systems are mainly applicable for orbital transfer vehicles.

Another technology demonstration program for solar thermal propulsion is the Solar Thermal Upper Stage (STUS), which is described in Refs. 4-6. The engine system envisioned for the STUS is designed to utilize hydrogen propellant to produce a thrust level of about 2 lbf. Two inflatable parabolic collectors could be used that would be rotated and gymballed for focusing sunlight into an absorber cavity (Ref. 6). The collectors would be inflated after separation of the upper stage from the launch vehicle.

In Fig. 1, a prototype inflatable solar concentrator (Refs. 7-8) is shown that consists of a torus/lens assembly supported by three struts. This concentrator is constructed of Kapton polyimide film, with epoxy as the primary adhesive for joints. In practical applications, the Fresnel lens of such a concentrator assembly would focus sunlight into a collector near the fixed ends of the struts. Solar energy stored in the collector could be utilized to heat a propellant as described previously. The inflatable struts are attached to a base plate by means of three cylindrical appendages. These hollow appendages also allow inflation of the concentrator assembly through air hoses connected at each strut. It can be seen that inflatable cylindrical struts are critical components of structural assemblies for practical applications. In view of their importance, structural dynamic and static behavior of typical inflated polyimide struts needs to be investigated.

This paper will go on to give an overview of previous modeling of inflatable cylinders. An introduction is then given for the present method of modeling. The specific mechanics of the method are discussed in parallel with a basic discussion of NASTRAN s implementation of these mechanics. The problems encountered with the method are then given along with suggestions for their work-a-rounds. The methodology is verified through the correlation between an analytical model and modal test results of a thin film strut. Recommendations are given for the needed advancement of our understanding of this method and ability to accurately model thin film structures. Finally, conclusions are drawn on the usefulness of the methodology.

Review of Previous Work with Inflated Beam-Like Structures

A number of investigators have considered the use of inflatable cylindrical beams. Perhaps the earliest was Frei Otto (Ref. 9), who in 1962 published ideas for inflated tubular frames for use in structures such as orbiting platforms. A more recent proposed application involves the use of inflatable beam segments to replace solid segments of the Space Shuttle remote manipulator system and thus reduce storage space and inertia of the arm (Ref. 10).

Several papers on static structural analysis of inflated cylinders describe different techniques such as linear shell theory, and nonlinear and variational methods (Refs. 11-15). Fichter (Ref. 16) developed nonlinear equations for twisting, bending, and stretching of pressurized thin-wall cylindrical beams. The Fichter equations, when linearized with internal pressure and axial force set to zero, reduced to the Timoshenko beam equations. Comer and Levy (Ref. 17), Webber (Ref. 18), and Main, et al. (Ref. 19) modeled an inflated beam using an Euler-Bernoulli approach, but with modified material longitudinal modulus and beam moment of inertia. Wrinkling behavior of the material was accounted for by the assumption of no load-carrying capability when longitudinal stress in the beam cross-section reached zero.

Additional work of significance involves rigidization of inflated beam structures. One proposed concept is the use of injected foam which fills the cylindrical beam cross-section, subsequently hardens and thus rigidizes the structure. This approach is discussed in Ref. 20.

Very little work has been done in dynamics analysis of inflatable beams. Leonard (Ref. 21) indicated that elastic beam bending modes could be utilized in approximating lower-order frequencies of inflatable beams. Main, et al. wrote a very significant paper describing results of modal tests of inflated cantilever beams and the determination of effective material properties (Ref. 22). Changes in material properties for different pressures were also discussed, and the beam model was used in a more complex structure. The paper demonstrated that conventional finite element analysis packages could be very useful in the analysis of complex inflatable structures.

Finally, Refs. 23 and 24 describe closed-form and finite element beam representations of inflatable cylinder dynamics, along with shell-element models for comparison. It was found that frequency-dependent modulus, or bending stiffness EI, was required for the beam models to adequately represent the dynamic test results. Shell models preformed reasonably well with linear material properties. It is noted however, that the film modulus was varied in the shell models of Refs.23 and 24 to approximate the effects of pressure and still allow for linear eigensolutions. A more realistic and satisfying approach is to directly include pressure loading in the modeling and modal analysis. Such an approach is investigated in this paper, and is described in the following sections.

Methodology Overview

The purpose of this investigation was to verify the usefulness of a procedure for dynamically analyzing thin film inflatable structures in MSC/NASTRAN that takes into account the increase of structural stiffness due to internal pressure. Ordinary eigenvalue analysis in NASTRAN does not allow for the effects of loads applied on a model during analysis. Since a true modal characterization of a structure by definition is free from active loads, this assumption is acceptable. In the case of inflatable structures though, this is not the case. The stiffness of the structure is directly related to the internal load. Therefore a standard modal analysis that does not account for internal pressure will give incorrect results.

In order to work around this problem, the proper stiffness matrix for the modeled structure in NASTRAN analysis can be generated in a nonlinear static analysis. Through a restart procedure this generated stiffness matrix can then be imported into an eigenvalue analysis.

NASTRAN Nonlinear Static Analysis

The key to this methodology, and ironically the most difficult step, is the nonlinear static analysis. In order to more fully understand the problems that arise during this procedure, and the reasons their work-arounds are successful, an understanding of NASTRAN s nonlinear static analysis procedure is necessary. This procedure provides NASTRAN the ability to handle geometric nonlinearities, kinematic nonlinearities, follower forces, large displacements, displacement variable loads, material nonlinearity, nonlinear stiffness relationships, buckling analysis, contact analysis and changing boundary conditions. For our purposes, only the abilities of the procedure that are applicable to our analysis will be discussed. These abilities are the stiffness updating and follower forces scheme including its iteration and convergence procedures.

The nonlinear static analysis utilized in this study involved the updating of the stiffness matrix throughout the procedure over a nonlinear curve. NASTRAN is able to do this through its iteration process (Fig. 2). This process involves applying a predetermined fraction of the total load on the structure. Through this load, and the initial geometry and property values of the loaded elements, initial values for the displacement and elemental forces are obtained. In the situation that the elemental forces and applied load are not in equilibrium, the iteration is repeated with updated values for the displacement and elemental forces. These values are calculated through the change in displacement due to the difference between the applied load and elemental load. Until equilibrium is met, this cycle is repeated. The point at which equilibrium is achieved is called the convergence point. It is at this point that the stiffness matrix is updated from the last iterated displacement and applied load. After convergence of the initial applied load, the analysis advances and applies a greater fraction of the combined load. A solution is obtained when convergence occurs for the entire load. NASTRAN offers several stiffness updating schemes, iteration schemes, advancing schemes, and convergence criteria. It is the manipulation and use of these schemes that is the greatest challenge of this methodology.

It must be stated that NASTRAN is limited in its nonlinear analysis ability. For the thin film structures investigated, the convergence criteria are extremely sensitive and do not allow for great tolerances. Due to this limitation, the authors found that it was necessary to artificially constrain the models in order to limit the number of overall degrees of freedom in the model and the direction of the displacements due to the applied loads (Fig. 3). Excess in either of these categories was found to limit the possibility for the analysis to converge and a solution to be obtained. These constraints were strategically placed at nodes throughout the model on the film elements. Only rotational degrees of freedom were allowed at these nodes. This method was found to be acceptable for the analysis. It was found that the artificial constraints only marginally changed the final values of the updated stiffness matrix even though the constraints modified the displaced shape of the pressurized model (Fig. 4). The reason for this is believed to be that the stiffness of the elements is due to the magnitude of the displacements and not the direction of them. Even though the geometry of the elements was altered, the strain and stress of those elements remained mostly unchanged and the stiffness matrix was mostly unaltered. If two adjacent nodes on an element are constrained this of course will not be true. Therefore AUTOSPC must be turned off for this procedure.

The magnitudes of the elemental normal forces rely greatly on the curvature of that element. The greater the angle of curvature, the larger the component of the tensile forces that can act against the applied load. Left alone, an element will displace rather than increase its curvature, causing convergence problems in NASTRAN due to excess displacements. The constraints allowed the elements to increase their rate of curvature in relation to the load applied, rather than displace as a rigid body. This increase in curvature is still translated into strain though in a different direction, allowing stress and thus updated stiffness to remain mostly unaltered in comparison to an unconstrained pressurization.

Once the artificial constraints are in place, the nonlinear static analysis is performed. In this analysis NASTRAN allows for the manipulation of the parameters and schemes of the analysis in its NLPARM and NLPCI entries in the bulk data section of the model. Many convergence problems can be overcome through the manipulation of these parameters. The authors found that it was adequate to only alter the convergence criteria and their margins of error. Therefore the NLPCI entry was not investigated. Further investigation into the manipulation of other parameters is certainly needed. NASTRAN offers to the user the option of choosing between displacement, energy, or load as the criterion from which it will base its convergence. The authors found it to be useful to activate all three in order to allow analysis advancement from whichever criterion converges first. It was also found useful to increase the error tolerances for these criteria from their default. Dilution of the resulting accuracy is thus traded off with the greater chance at convergence on a solution. It was also found that too large an error tolerance was just as fatal as one too small for achieving convergence. It is thought that too large a tolerance leads later iterations away from the convergence point.

NASTRAN Modal Restart

Upon the completion of the nonlinear static analysis, and the convergence of a solution, an eigenvalue restart is performed (Fig. 8). The restart is activated through the RESTART command in the executive control section of the file. This command imports the MASTER file from the nonlinear analysis and claims it as its own. From this MASTER file, the restart uses the original geometry from the bulk data section and the stiffness matrix from a chosen iteration. In the bulk data section of the restart, the iteration or NMLOOP of the nonlinear static analysis stiffness update can be chosen. In this section the artificial constraints discussed previously must be removed. This is done through removing the appropriate lines in the sorted bulk data form the nonlinear analysis. The appropriate line numbers are found in the sorted bulk data echo in the nonlinear f06 file. Remaining sections of the restart are organized as an ordinary eigenvalue analysis. Within the RESTART command it is useful to KEEP the old MASTER file for later use. Ref. 25 provides an overview of NASTRAN s capability and file structure.

Application of Methodology to Dynamics of Inflated Cylindrical Beams

The solution procedure described in the previous three sections was performed and ultimately verified through the modeling of previously characterized thin film inflatable struts. A typical inflated cylindrical beam or strut of the type investigated in this paper is shown in Figure 5. The beams studied are 8 ft. in length with 6 in. diameter, are constructed of Kapton polyimide film, and are sealed at the ends using styrofoam plugs. Openings in the plugs provide the means for inserting air hoses to inflate the structure to desired pressure. Epoxy was used to bond the foam plug to the polyimide film walls of each cylinder.

The beams were constructed by overlapping the edges of the polyimide sheet and placing a thin layer of epoxy adhesive along the overlapped areas to form a joint running the length of each beam. In a cross-sectional view, the bonded joint looks like a sandwich, with the epoxy layer between two layers of Kapton film. Of course, the bonded region has much higher stiffness than a nominal polyimide section of cylinder wall, and this had to be accounted for in modeling of the struts. The stiffness of the joint appears to be dominated by the epoxy layer rather than the polyimide material, partially due to the greater thickness of epoxy. Polyimide film of .002 in. and .003 in. thicknesses was utilized, and the epoxy layer was typically .01 in. thick.

Shell elements were used to model the strut s film. Non-structural masses were added to these elements to compensate for the mass of air within the structure. A single column of elements along the length of the strut was thickened to 11-_ mil to compensate for the overlap and adhesive along the seam. Solid elements were used for the end-plugs.

The results of analysis using this methodology and results of the modal test can be seen in Figure 6. Free-free cases and cantilever cases were analyzed at multiple pressures for each. As can be seen, the results fell within an acceptable margin of 6% frequency error for the free-free cases. In the cases for the cantilever setup, large errors occurred for the first modes. It is believed that these errors are due to shell wrinkling effects that were not taken into account within the finite element model. Errors in later mode shapes were believed to be the result of the nonlinear relationship between the elastic modulus and the frequencies. These comparisons are only drawn from the bending motions of the struts and do not involve the shell motions. The procedure used in the modal testing of the struts was limited in its ability to pick up shell modes, and a comparison could not be made.

It is significant to note that these accuracies were obtained in the finite models without the manipulation of the original element properties. That is, the manufacturer s modulus, density, and Poisson s ratio data were used for the polyimide film. This provides a high level of confidence in the solution approach and its ability to predict dynamic characteristics of inflatable structures. It is also significant to note that this methodology has been implemented successfully in more sophisticated NASTRAN models not discussed in this paper. Positive results from these implementations further the confidence of the authors in the usefulness and versatility of the procedure.

Advantages and Limitations of the Procedure for Modeling Thin-Film Inflatable Structures

The dynamic characterization of thin film inflatable structures falls into two major areas of research: the bending modes and the shell modes of the structure. Both types of modes have been predicted successfully through the use of beam and shell theory. Previous work done with finite element modeling of these structures has been successful in the bending modes aspect of the characterization alone. Shell modes analysis on the other hand has been limited. Proper models have had to rely on altering film modulus values to match test data and accurately model shell modes. Using this method obviously limits the analysis to only structures that have been previously characterized through modal testing. Through the use of the methodology presented in this paper, this is no longer the case. Low-order shell modes can be predicted without the previous correlation with test data.

Due to the limitations of previous modeling procedures, air mass of the structures has not been adequately characterized. The effect of this mass is readily visible through the use of the methodology presented here. Due to the extremely low mass of these structures, the mass of the air is a considerable fraction of the total mass. Therefore the mass of the air can have considerable effects on the modal frequencies.

The effects of the seam on the dynamic characteristics are also readily visible through the use of this method. Due to the extremely small values of the thickness of these structures, any variation of this thickness will have a profound effect on the eigenvalues. The seam on the struts increases the thickness of the film in that area by a factor of four. This thickness increase results in an increase in frequency for mode shapes along that seam.

Apparent limitations in this methodology were found throughout the course of the investigation. As the curvatures of the elements are important factors in the convergence of a solution, so are the angles between the elements. The initial element geometries of course do not have curvature. Thus the initial elemental forces are not due to the curvature but rather the angle between adjacent elements. Examining a group of elements as a free body diagram (Fig. 7), it becomes apparent that the reactional forces to the applied load come from the parallel component of the tensile forces applied to the center element by the perimeter elements. The smaller the angle between adjacent elements becomes, the larger the displacements become in relation to the initial load, thus retarding convergence.

Excess degrees of freedom are also a limiting factor in the analysis. This excess comes from a finer meshing of the film. Not only does a finer mesh complicate the model and lengthen the run time, but it also decreases the angles between elements along the curvature of the film. Therefore a greater number of constraints are needed. Due to this phenomenon a coarser mesh is suggested.

Another byproduct from the limiting ability of the angle between elements is the limitation in the size and curvature of the structure. A decrease of the angle between adjacent elements and an increase in the number of elements needed to properly characterize the structure is a result of a larger model. As seen above, both of these properties decrease the ability of NASTRAN to converge on a solution in its nonlinear analysis.

Triangular elements were found to be absolutely fatal to this procedure. Investigation into the reasons for this anomaly is needed. It is recommended that the use of these elements be avoided.

The thickness of the film is another limiting factor, and perhaps the controlling factor of this methodology. Elemental forces calculated during the iteration process are a function of the volume of each element. Decreasing the thickness of the film, and thus the volume of the elements and elemental forces, causing a greater number of iterations to be needed for a convergence. This causes convergence problems and decreases the likelihood of NASTRAN reaching a solution.

Recommendations And Conclusions

The investigation and verification of the nonlinear static analysis with a modal restart has been discussed in this paper. The purpose of this investigation is to verify the usefulness of the methodology through the creation of a working application. This completed, an in-depth investigation is now needed to not only further our understanding of this procedure, but to also further our understanding of modeling thin film inflatable structures. Specifically, an investigation in the use of nonlinear material properties with this solution approach is required. An understanding of the dynamic characteristics and stability of thin film structures is intrical to their advancement as a technology.

It is the hope of the authors that this paper may be used as a reference for the use of the methodology and work-arounds presented. The usefulness of such a procedure to further our finite element modeling ability of thin film inflatable structures has been discussed. This usefulness is reflected in the procedure s ability to model effects from seams, air mass, and stiffness due to pressurization, and to represent bending modes and shell modes adequately.

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Figure 1. Concept for Solar Thermal Propulsion System Utilizing Inflatable Components

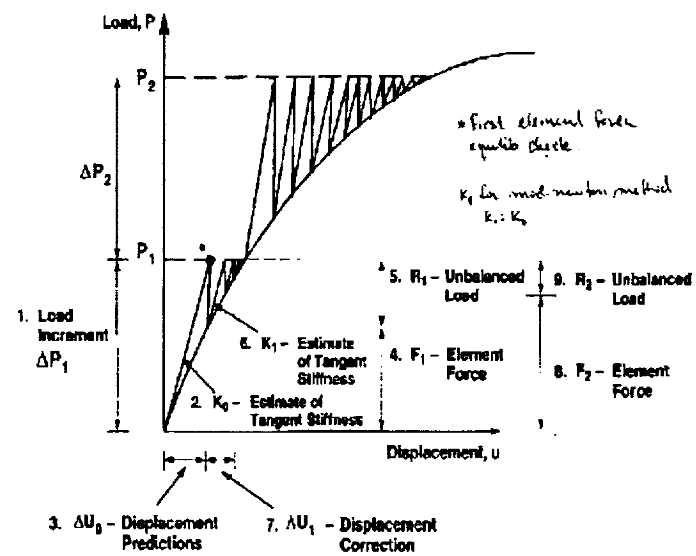

Figure 2. Nonlinear Solution Procedure Utilized in MSC/NASTRAN for Static Load-Deflection and Stiffness Matrix Update

Figure 3. Constraint Points for Pressurization of Inflated Strut

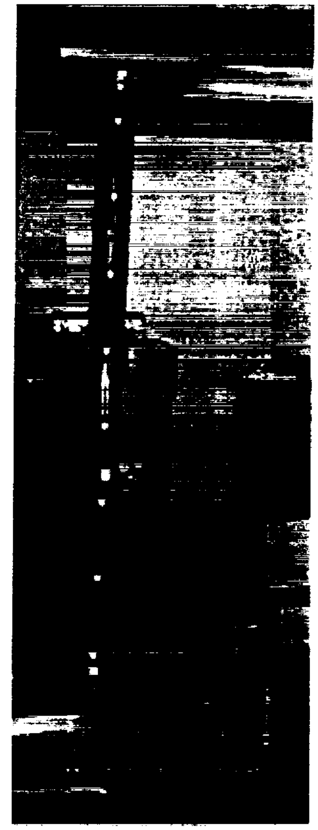

Figure 5. Dynamic Test Configuration for Inflated Cylindrical Strut

Figure 4. Pressurized Deformed Shape of Inflated Strut

Results Comparison Between Experimental and Anylitical Bending Eigenvalues				
	Experimental (hz)	*Analyitcal (hz)*	*Difference (hz)*	*%Error*
Free-Free				
Pressure = 1.4 psi				
mode 1	24.6	25.3	-0.7	-2.85
mode 2	71.4	70.1	1.3	1.82
mode3	-	-	-	
Pressure = 7.6 psi				
mode 1	24.3	22.85	1.45	5.97
mode 2	71.8	67.83	3.97	5.53
mode3	-	-	-	
Cantliever				
Pressure = 1.0 psi				
mode 1	2.78	3.61	-0.83	-29.86
mode 2	25.33	26.69	-1.36	-5.37
mode3	78.73	75	3.73	4.74
Pressure = 0.5 psi				
mode 1	2.81	4.44	-1.63	-58.01
mode 2	25.1	28.22	-3.12	-12.43
mode3	79.04	76.58	2.46	3.11

Figure 6. Comparison of Modal Results for Model and Experiment

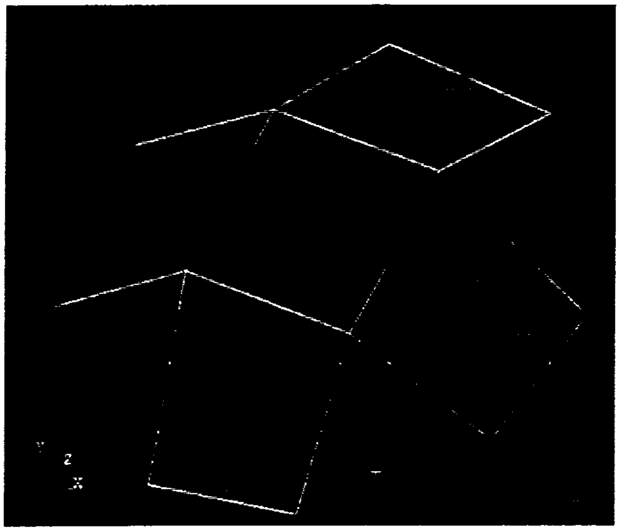

Figure 7. Quadrilateral Plate Element Deformation Under Pressure Loading

```
$ Restart Program for modes analysis using the
$ stiffness matrix of a static analysis MASTER
ASSIGN OUTPUT2 = 'case23.op2', UNIT = 12
$ Direct Text Input for File Management Section
$ Normal Modes Analysis, Database
$
RESTART VERSION=1,KEEP
ASSIGN MASTER='case22.MASTER'
$
SOL 103
TIME 600
$ Direct Text Input for Executive Control
CEND
SEALL = ALL
SUPER = ALL
TITLE = MSC/NASTRAN job MODES RESTART
ECHO = NONE
MAXLINES = 999999999
$ Direct Text Input for Global Case Control Data
SUBCASE 1
$ Subcase name : rigid
   SUBTITLE=rigid
   METHOD = 10
   SPC = 2
   VECTOR(SORT1,REAL)=ALL
   SPCFORCES(SORT1,REAL)=ALL
BEGIN BULK
PARAM,NMLOOP,12
EIGRL,10,0.,500.,20
/,2350
/,2351
/,2352
/,2353
/,2354
/,2355
$
ENDDATA
```

Figure 8. NASTRAN Runstream for Modal Solution Utilizing Stiffness Matrix from Nonlinear Pressure Solution