

PREPRINT/14/32  
2001 027 937

p 12  
515-772

# Comment on “Heterodyne lidar returns in the turbulent atmosphere: performance evaluation of simulated systems”

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## Abstract

The explanation for the difference between simulation and the zero-order theory for heterodyne lidar returns in a turbulent atmosphere proposed by Belmonte and Rye [Appl. Opt. **39**, 2401, (2000)] is incorrect. The theoretical expansion is not developed under a square-law-structure function approximation (random wedge atmosphere). Agreement between the simulations and the zero-order term of the theoretical expansion is produced for the limit of statistically independent paths (bi-static operation with large transmitter-receiver separation) when the simulations correctly include the large-scale gradients of the turbulent atmosphere.

## 1 Introduction

The effects of refractive turbulence on heterodyne or coherent Doppler lidar have been investigated by theoretical methods [1, 2, 3, 4, 5] and numerical simulations [6, 7, 8, 9, 10]. The theoretical results are important for verification of the simulation algorithms and for calculations in parameter regimes where simulations are not feasible such as conditions of large path-integrated refractive turbulence (strong scattering). The two methods are complementary and the valid parameter space for each method is a critical issue for evaluation of Doppler lidar performance.

The most important statistical quantity is the signal-to-noise ratio ( $SNR$ ) which can be written as [4, 11]

$$SNR(R) \propto C(R) \quad (1)$$

for diffuse or aerosol targets at range  $R$  where  $C(R)$  is the coherent responsivity which is given by

$$C(R) = \lambda^2 \int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, R) j_{BPLO}(\mathbf{p}, R) \rangle d\mathbf{p} \quad (2)$$

where  $\lambda$  is the laser wavelength,  $j_T(\mathbf{p}, R)$  and  $j_{BPLO}(\mathbf{p}, R)$  are the random intensities of the transmit and back-propagated local oscillator (BPLO) beam, respectively,  $\langle \rangle$  denotes ensemble average over the random refractive turbulence and the random phases of the backscattered fields at the target, and  $\mathbf{p}$  denotes the two-dimensional transverse coordinate at the target. Here,  $j_T(\mathbf{p}, R) = |e_T(\mathbf{p}, R)|^2$  and  $j_{BPLO}(\mathbf{p}, R) = |e_{BPLO}(\mathbf{p}, R)|^2$  where

$$e_T(\mathbf{p}, R) = \int_{-\infty}^{\infty} e_L(\mathbf{u}, 0) W_T(\mathbf{u}) G(\mathbf{p}; \mathbf{u}, R) d\mathbf{u} \quad (3)$$

$$e_{BPLO}(\mathbf{p}, R) = \int_{-\infty}^{\infty} e_{LO}^*(\mathbf{u}, 0) W_R(\mathbf{u}) G(\mathbf{p}; \mathbf{u}, R) d\mathbf{u} \quad (4)$$

and  $e_L(\mathbf{u}, 0)$  is the normalized laser transmit field at the transmit telescope aperture  $W_T(\mathbf{u})$ ,  $e_{LO}(\mathbf{u}, 0)$  is the normalized LO field at the receiver telescope aperture  $W_R(\mathbf{u})$ , and  $G(\mathbf{p}; \mathbf{u}, R)$  is the Green's function for propagating the fields through a random atmosphere under the Fresnel approximation which can be written as a Feynman path integral [4, 12, 13, 14]. Combining these equations produces [4]

$$C(R) = \lambda^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e_T(\mathbf{u}_1, 0) e_T^*(\mathbf{u}_2, 0) e_{BPLO}(\mathbf{u}_3, 0) e_{BPLO}^*(\mathbf{u}_4, 0) \Gamma_4(\mathbf{p}, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, R) d\mathbf{u}_1 d\mathbf{u}_2 d\mathbf{u}_3 d\mathbf{u}_4 d\mathbf{p} \quad (5)$$

where

$$\Gamma_4(\mathbf{p}, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, R) = \langle G(\mathbf{p}; \mathbf{u}_1, R) G^*(\mathbf{p}; \mathbf{u}_2, R) G(\mathbf{p}; \mathbf{u}_3, R) G^*(\mathbf{p}; \mathbf{u}_4, R) \rangle \quad (6)$$

is the fourth moment Greens function for wave propagation through the random atmosphere. For typical atmospheric conditions, there is no exact solution for  $\Gamma_4$  and various approximations and series expansions have been produced. One of the most basic series expansions is a Taylor series expansion.

Another attractive option is to numerically simulate many realizations of the random intensity on the target and determine  $C(R)$  from Eq. (2). This method has been successfully employed for many problems of wave propagation in random media [15, 16, 17, 19]. For coherent Doppler lidar applications, the results generally disagree with the first order term of the theoretical expansions [6, 7, 8] because higher-order terms of the expansion are important (these higher-order terms are numerically intensive and difficult to calculate [5]).

An explanation proposed by Belmonte and Rye [7] for the disagreement of the simulations and approximations to the zero-order theory is that the series expansion is only valid for a square-law-structure function random atmosphere. This is incorrect because such an atmosphere consists of a series of random wedges which would produce only beam tilts and therefore have the same performance as free space [20]. With this interpretation and a simulation based on an approximation for a random wedge atmosphere, Belmonte and Rye claimed agreement between the simulations and approximations to the zero-order theoretical expansion for the case of statistically independent paths (e.g., bistatic operation with a large separation between the transmitter and receiver such that the transmit and receive paths experience statistically independent turbulence). We will show that this is an incorrect interpretation and that agreement is produced for typical atmospheric conditions for the bistatic limit of the zero-order term of the series expansion, and we will clarify the procedure used to generate the series expansion.

## 2 Theory

The key quantity  $\Gamma_4$  can be written under the Markov approximation and narrow angular scattering as [14]

$$\begin{aligned} \Gamma_4(\mathbf{p}, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, R) = & G^f(\mathbf{p}, \mathbf{u}_i, R) \int D\mathbf{r}_1 \int D\mathbf{r}_2 \int D\mathbf{r}_3 \int D\mathbf{r}_4 \\ & \exp\left\{-\frac{1}{2} \int_0^R [d(\mathbf{r}_1 - \mathbf{r}_2, z) + d(\mathbf{r}_3 - \mathbf{r}_4, z) + d(\mathbf{r}_1 - \mathbf{r}_4, z) + \right. \\ & \left. d(\mathbf{r}_2 - \mathbf{r}_3, z) - d(\mathbf{r}_1 - \mathbf{r}_3, z) - d(\mathbf{r}_2 - \mathbf{r}_4, z)] dz\right\} \end{aligned} \quad (7)$$

where  $G^f(\mathbf{p}, \mathbf{u}_i, R)$  is the free-space result,

$$d(\mathbf{s}, R) = 4\pi k^2 \int_{-\infty}^{\infty} [1 - \cos(\mathbf{s} \cdot \mathbf{q})] \Phi_n(q_x, q_y, q_z = 0, R) d\mathbf{q} \quad (8)$$

$\mathbf{q} = (q_x, q_y)$ , and  $\Phi_n(q_x, q_y, q_z, R)$  is the three-dimensional spectrum of refractive index fluctuations at range  $R$ , and  $D\mathbf{r}_i$  denotes the infinite number of paths  $\mathbf{r}_i(z)$  from the transmitter-receiver plane  $z = 0$  with  $\mathbf{r}_i(0) = \mathbf{u}_i$  to the target plane  $z = R$  with  $\mathbf{r}_i(R) = \mathbf{p}$ . The Markov approximation is equivalent to replacing the random atmosphere by a series of statistically independent random refractive index screens transverse to the propagation direction and is an excellent approximation for intensity statistics such as  $\Gamma_4$  [21]

For weak scattering conditions which emphasize the low-spatial frequencies (lf) of the scintillation process, the standard series expansion is a Taylor series of the quantity  $\exp(-Q/2)$  [14, 4] where  $Q = \int_0^R [d(\mathbf{r}_1 - \mathbf{r}_4, z) + d(\mathbf{r}_2 - \mathbf{r}_3, z) - d(\mathbf{r}_1 - \mathbf{r}_3, z) - d(\mathbf{r}_2 - \mathbf{r}_4, z)] dz$ . The zero-order term is given by

$$C_0^{lf}(R) = \frac{\lambda^2}{R^2} \int_{-\infty}^{\infty} O_T(\mathbf{s}, R) O_{BPLO}^*(\mathbf{s}, R) \exp[-D_S(\mathbf{s}, R)] d\mathbf{s} \quad (9)$$

where

$$D_S(\mathbf{s}, R) = \int_0^R d[\mathbf{s}(1 - z/R), z] dz \quad (10)$$

is the spherical wave structure function,

$$O_T(\mathbf{s}, R) = \int_{-\infty}^{\infty} e_T(\mathbf{r} + \mathbf{s}/2, 0) e_T^*(\mathbf{r} - \mathbf{s}/2, 0) \exp(i\mathbf{k}\mathbf{r} \cdot \mathbf{s}/R) d\mathbf{r}, \quad (11)$$

and

$$O_{BPLO}(\mathbf{s}, R) = \int_{-\infty}^{\infty} e_{BPLO}(\mathbf{r} + \mathbf{s}/2, 0) e_{BPLO}^*(\mathbf{r} - \mathbf{s}/2, 0) \exp(i\mathbf{k}\mathbf{r} \cdot \mathbf{s}/R) d\mathbf{r}. \quad (12)$$

This expression is the receiver plane version. The target plane version simplifies to [4]

$$C_0^{IJ}(R) = \lambda^2 \int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, R) \rangle \langle j_{BPLO}(\mathbf{p}, R) \rangle d\mathbf{p} \quad (13)$$

which contains the product of the average normalized intensity of the transmit and BPLO fields at the target (the statistically independent path result). This expansion was presented in Ref. [4] with the following caution: "Note that our theory was not developed under a square law structure function approximation. That pathological case corresponds to an atmosphere composed of random wedges [20], which implies there is only beam wander and no scintillation, and wave-front tilts will be self correcting for monostatic lidar". Physically, the random-wedge atmosphere produces only random beam tilt which is the same for the transmit and BPLO beams. Mathematically, substituting a square-law structure function  $d(\mathbf{r}, z) = K(z)r^2$  into Eq. (7) produces the free-space result  $G^I(\mathbf{p}, \mathbf{u}_i, R)$  because the combination of structure functions in the exponential are identically zero.

A realistic model for the atmospheric spectrum is the Hill spectrum [22, 23]

$$\Phi_n(q, z) = A C_n^2(z) q^{-11/3} f[ql_0(z)] \quad (14)$$

where  $A = 0.0330054$ ,  $q = \sqrt{q_x^2 + q_y^2 + q_z^2}$  is the magnitude of the three dimensional wave vector,  $C_n^2$  is the refractive index structure constant,  $l_0$  is the inner scale, and  $f(x) = (1.0 + 0.70937x + 2.8235x^2 - 0.28086x^3 + 0.08277x^4) \exp(-1.109x)$ . The form of the spectrum at the high wave-number region is critical for laser scintillation experiments. The Gaussian model [24, 7] for  $f(x)$  produces errors for the scintillation intensity variance [25] which is related to  $C(R)$  [4, 23]. For constant  $C_n^2$  and  $l_0$  [26]

$$D_S(s) = 8\pi^2 A C_n^2 k^2 l_0^{5/3} R H(s/l_0) \quad (15)$$

where

$$H(z) = \int_0^1 g(xz) dx \quad (16)$$

$$g(x) = \int_0^\infty q^{-8/3} f(q) [1 - J_0(qx)] dq \quad (17)$$

The field coherence length  $\rho_0$  is defined by

$$D_S(\rho_0) = 1 \quad (18)$$

and a useful approximation that we will call the square-law structure function approximation for the series expansion (not to be confused with the square-law structure function approximation for the random atmosphere or equivalently, the "random wedge atmosphere") is

$$D_S(s) = s^2/\rho_0^2 \quad (19)$$

which has been shown to have small error for calculations of the average beam intensity [1]. If  $\rho_0 \ll l_0$ , this is a very good approximation. The main motivation for this approximation is analytic expressions for important quantities such as  $C(R)$  with a Gaussian lidar model [1, 2, 4, 5].

For very strong scattering, the scintillation process consists of two components: a low-spatial frequency (large scale) component and a high-spatial frequency (HF) component. Therefore, another series expansion is required based on the high spatial-frequency (hf) component of the scintillation [4, 14]. This expansion is a Taylor series in the quantity  $\exp(-Q/2)$  where  $Q = \int_0^R [d(\mathbf{r}_1 - \mathbf{r}_2, z) + d(\mathbf{r}_3 - \mathbf{r}_4, z) - d(\mathbf{r}_1 - \mathbf{r}_3, z) - d(\mathbf{r}_2 - \mathbf{r}_4, z)] dz$ . For monostatic lidar with matched transmit and BPLO beams, the first term  $C_0^{hf} = C_0^{lf}$  [4] and the total strong scattering expression is

$$C_{SS}(R) = C_0^{lf}(R) + C_0^{hf}(R) = 2C_0^{lf}(R) \quad (20)$$

which is equivalent to the Gaussian fields assumption in strong scattering [14] and provides the lower-bound to Doppler lidar performance.

### 3 Numerical Simulation

The effects of refractive turbulence will be determined for typical ground-based atmospheric conditions using numerical simulations of the target plane intensity patterns  $j_T(\mathbf{p}, R)$  and  $j_{BPLO}(\mathbf{p}, R)$  [see Eq. (2)]. The standard algorithm [7, 15, 16, 17] propagates the field through a series of statistically independent random phase screens (the Markov approximation) which are produced by the Fast Fourier Transform (FFT) algorithm. Because the phase screens produced by the FFT algorithm are periodic, the spatial correlation is periodic and the simulation of the fields have errors for those statistics that are sensitive to large scale turbulent fluctuations (see Fig. 1 of Ref. [6]). Improved algorithms for generating phase screens with the correct large-scale statistics have been produced [6] using random sub-harmonics [18]. This algorithm (FFT-SH) has been applied to the simulation of wave propagation in random media and heterodyne lidar performance [6, 8]. The large-scale phase perturbations have been shown to produce a small effect for optimally-designed monostatic coherent Doppler lidar [6]. The statistically independent path (bistatic operation) limit can be produced by generating difference uncorrelated phase screens for the transmit and BPLO fields.

### 4 Results

The results of simulations and theory for the statistically independent path case (bistatic) are shown in Fig. 1 for the same lidar parameters of Fig. 16 in Ref. [7] (circular telescope

with diameter  $D = 0.14$  m, matched transmit and BPLO Gaussian beams with  $1/e$  intensity radius  $\sigma_L = \sigma_{LO} = 0.2836D = 0.39704$  m,  $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ ,  $l_0 = 0.01$  m,  $\lambda = 2.0 \mu\text{m}$ . The results from the most robust simulation algorithm (FFT-SH) agree very well with the exact calculation of the zero-order term Eq. (9). The results from the traditional simulation algorithm (FFT) do not agree for large ranges because the large-scale tilts are not correctly represented. This is shown in Fig. 7 of [10] where the average beam width from the traditional simulation is smaller than the theoretical calculation for ranges greater than 1000 m. The error is larger for the focused beam case [6]. The square-law structure function approximation for the exponential expression of the zero-order term also has little error [see Eqs. (9), (10), (19)]. This is to be expected because the coherence length  $\rho_0$  is less than the inner scale  $l_0$  for all ranges greater than  $R = 200$  m. The most likely explanations for the disagreement in Figs. 16-21 of [7] are the assumption of a Gaussian transmittance profile for the telescope aperture and the approximation of the coherence length  $\rho_0$  Eq. (18) by the zero inner-scale limit of the Kolmogorov spectrum [see Eq. (165) of [4]].

The results for monostatic operation are shown in Fig. 2. The results from both simulation algorithms agree (see also [6, 8]) and are considerably larger than the zero-order strong-scattering theory. This is typical of intensity scintillation [15, 16, 19], especially with an inner scale of turbulence. It is difficult to approach the theoretical strong-scattering limit and therefore higher terms of the theoretical expansions are required. However, for the focused beam geometry and strong scattering, the simulation results converge more quickly to the theoretical predictions [see Fig. 10 of [8]]. This is a parameter regime where improved theoretical expansion could be valuable. With the development of stable coherent Doppler lidar, the measurement of  $SNR$  for hard targets has become very accurate since the relative accuracy from the speckle process is  $1/\sqrt{N}$  where  $N$  is the number of lidar shots processed. It is important to have accurate predictions from theory and simulations to understand the effects of refractive turbulence.

## 5 Summary and Discussion

The theoretical series expansion for  $SNR[C(R)]$  is produced as a Taylor series expansion and does not assume a square-law structure function approximation for the random atmosphere (random wedge atmosphere). It is difficult to evaluate the simulation results in [7] for the random-wedge atmosphere approximated by a spatial spectrum with a power-law of -4 because the spatial statistics are not a square-law structure function and are not traditional turbulence. In addition, the approximate analytic expressions for a Gaussian lidar [4] are not valid for this spectrum.

The exact calculation of the zero-order term Eq. (9) of the theoretical expansion for heterodyne  $SNR$  for the case of statistically independent paths for the transmit and BPLO beams (bistatic lidar with large separation between the transmitter and receiver) agrees with the robust simulation method (FFT-SH) (see Fig. 1) and also agrees well with the square-law structure function approximation to the exponential term of the expansion [see Eqs. (9), (10), (19)]. The traditional simulation algorithm has errors because the large-scale phase tilts are not calculated correctly and therefore this algorithm does not accurately predict performance for bi-static operation and the improved FFT-SH algorithm is required.

For monostatic lidar (see Fig. 2), the results of the simulations are larger than the predictions of the zero-order theory for weak-scattering Eq. (9) and also strong-scattering (large range  $R$ ) Eq. (20). This parameter regime requires more terms of the theoretical expansion. Numerical simulation of lidar performance is an attractive method for those parameter regimes where the simulations are valid. For other parameter regimes, better theoretical expansions may be required for accurate predictions.

## 6 Acknowledgments

This work was supported by the National Aeronautics and Space Administration, Marshall Space Flight Center, Michael J. Kavaya, Technical Officer.

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Figure 1: Normalized coherent responsivity  $C(R)/C(0)$  versus range  $R$  for a coherent Doppler lidar with statistically independent paths for the transmit and BPLO beams (bistatic limit). The exact theoretical prediction (solid line) Eq. (9) and the square-law structure function approximation (dashed line) [see Eqs. (9), (10), (19)] are compared with the results from the robust simulation algorithm (FFT-SH) and the traditional simulation algorithm (FFT).

Figure 2: Normalized coherent responsivity  $C(R)/C(0)$  versus range  $R$  for a monostatic coherent Doppler lidar. The zero-order term of the theoretical expansion for weak-scattering (solid line) Eq. (9) and strong scattering (dashed line) Eq. (20) are compared with the results from the robust simulation algorithm (FFT-SH) and the traditional simulation algorithm (FFT).

Figure 1 Frehlich and Kavaya

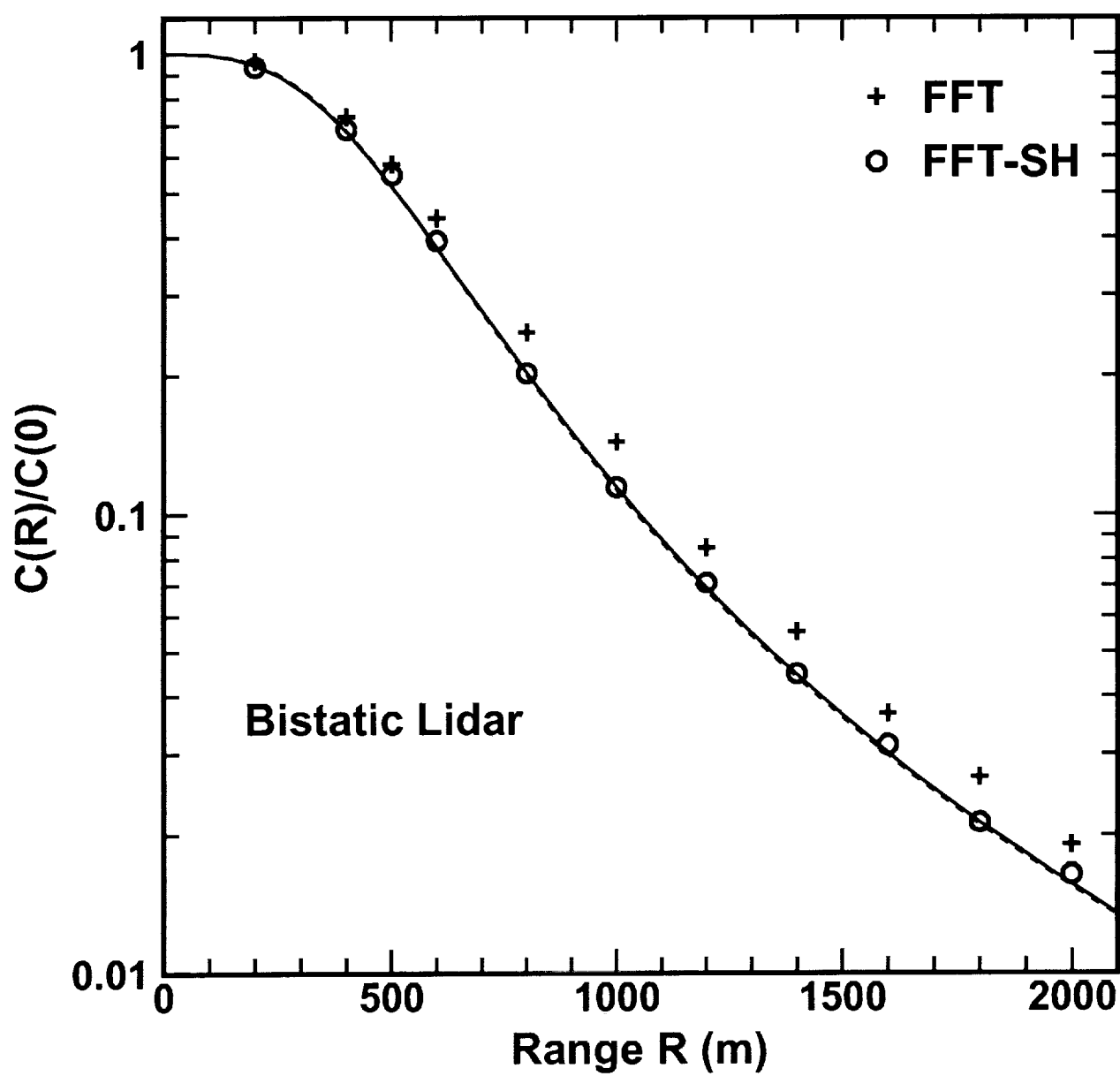


Figure 2 Frehlich and Kavaya

