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INERTIAL EFFECTS IN SUSPENSION DYNAMICS

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ABSTRACT

The present work analyses the dynamics of a suspension of heavy particles in shear flow. The magnitude of the particle inertia is given by the Stokes number $St = m\dot{\gamma}/6\pi\eta a$, which is the ratio of the viscous relaxation time of a particle $\tau_p = m/6\pi\eta a$ to the flow time $\dot{\gamma}^{-1}$. Here, m is the mass of the particle, a is its size, η is the viscosity of the suspending fluid and $\dot{\gamma}$ is the shear rate. The ratio of the Stokes number to the Reynolds number, $Re = \rho_f \dot{\gamma} a^2/\eta$, is the density ratio ρ_p/ρ_f. Of interest is to understand the separate roles of particle (St) and fluid (Re) inertia in the dynamics of suspensions. In this study we focus on heavy particles, $\rho_p/\rho_f \gg 1$, for which the Stokes number is finite, but the Reynolds number is sufficiently small for inertial forces in the fluid to be neglected; thus, the fluid motion is governed by the Stokes equations. On the other hand, the probability density governing the statistics of the suspended particles satisfies a Fokker-Planck equation that accounts for both configuration and momentum coordinates, the latter being essential for finite St. The solution of the Fokker-Planck equation is obtained to $O(St)$ via a Chapman-Enskog type-procedure, and the conditional velocity distribution so obtained is used to derive a configuration-space Smoluchowski equation with inertial corrections. The inertial effects are responsible for asymmetry in the relative trajectories of two spheres in shear flow, in contrast to the well known symmetric structure in the absence of inertia. Finite St open trajectories in the plane of shear suffer a downward lateral displacement resulting from the inability of a particle of finite mass to follow the curvature of the zero-Stokes-number pathlines. In addition to the induced asymmetry, the $O(St)$ inertial perturbation dramatically alters the nature of the near-field trajectories. The stable closed orbits (for $St = 0$) in the plane of shear now spiral in, approaching particle-particle contact in the limit. All trajectories starting from an initial offset of $O(St^{1/2})$ or less (which remain open for $St = 0$) also spiral in. The asymmetry of the trajectories leads to a non-Newtonian rheology and diffusive behavior. The latter because a given particle (moving along a finite St open trajectory) suffers a net displacement in the transverse direction after a single interaction. A sequence of such uncorrelated displacements leads to the particle executing a random walk. The inertial diffusivity tensor is anisotropic on account of differing strengths of interaction in the gradient and vorticity directions. Since the entire region (constituting an infinite area) of closed orbits in the plane of shear spirals onto contact for finite St, the latter represents a singular surface for the pair-distribution function. The exact form of the pair-distribution function at contact is still, however, indeterminate in the absence of non-hydrodynamic effects. It should also be noted that finite St non-rectilinear flows do not support a spatially uniform number density owing to the cross-streamline inertial migration of particles.

Inertial Effects in Suspension Dynamics

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Problem & Motivation

- **Understand the effects of inertia on the behavior of concentrated suspensions.**

- **Inertial effects are important in many natural and industrial suspensions flows, e.g., fluidized beds, drilling fluids, debris flow, etc.**

- **There is a considerable body of knowledge for small particle suspensions – low Reynolds numbers – and a quantitative understanding/predictive ability is emerging. The advances have come from an intimate interplay among experiment, theory and simulation.**

- **For suspensions where inertial forces are important much less is known and our predictive ability is severely limited.**

Need for microgravity

There are two distinct inertial effects in suspensions:

- **Particle inertial: characterized by the Stokes number**

$$St = \frac{\rho_p U a}{\eta}$$

- **Fluid inertial: characterized by the Reynolds number**

$$Re = \frac{\rho_f U a}{\eta}$$

- **Of course, $St = \rho_p / \rho_f Re$, but we would like to vary independently the effects of particle and fluid inertia. This can only be done in microgravity because of the gravitational settling that occurs when the particle and fluid densities do not match.**

Related work

- **Considerable body of work on very dilute dispersions ($\phi << 0.1$) of heavy particles moving through and modifying turbulent flows. No (or very limited) particle-particle interactions.**

- **Very nice, and ever expanding, body of work on granular flows – essentially infinite *Re* and *St* – that is, there is no fluid.**

- **Limited experiments on the rheology of inertial suspensions, e.g., Bagnold's (1954) experiments are still the most complete.**

- **Emerging simulation studies of concentrated inertial suspensions (e.g., D.D. Joseph and coworkers).**

- **Theory and simulation for finite *St*, zero *Re* suspensions in sedimentation and shear by D.L. Koch and coworkers.**

This work

- Analytical theory for zero Re, small St dilute suspensions in shear flow.

- Dynamic simulation of zero Re suspensions in shear flow for arbitrary St and ϕ.

- Dynamic simulation for nonzero Re, St, and ϕ in shear flow.

Analytical theory: $Re = 0$, $O(St)$

Fokker-Planck Equation: $P_N(\boldsymbol{x}, \boldsymbol{u}, t)$

$$\frac{\partial P_N}{\partial t} + \nabla_x \cdot \dot{\boldsymbol{x}} P_N + \nabla_u \cdot \dot{\boldsymbol{u}} P_N = 0$$

$$\dot{\boldsymbol{x}} = \boldsymbol{u} \quad , \quad \dot{\boldsymbol{u}} = \boldsymbol{m}^{-1} \cdot \left[-\boldsymbol{R}(\boldsymbol{x}) \cdot \boldsymbol{u} + \boldsymbol{F}^O(\boldsymbol{x}) - \boldsymbol{D}(\boldsymbol{x}) \cdot \nabla_u \ln P_N \right]$$

hydrodynamic drag

shear/interparticle force

$$\boldsymbol{F}^O = 6\pi\eta a \dot{\boldsymbol{\Gamma}} \cdot \boldsymbol{x}$$

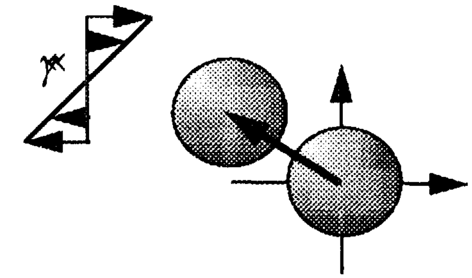
Brownian motion

$$\boldsymbol{D} = kT\boldsymbol{R} \cdot \boldsymbol{m}^{-1}$$

Integrate out the momentum variables using the method of multiple scales (Chapman-Enskog–type expansion) to derive the configuration-space Smoluchowski equation with inertia.

Method of Multiple Scales

$$\left.\begin{array}{lcl}\tau & \sim & m/6\pi\eta a \\ t & \sim & \dot{\gamma}^{-1}\end{array}\right\} \quad St = \frac{m\dot{\gamma}}{6\pi\eta a} \ll 1$$



Conditional Probability

$$P(\boldsymbol{x}, \boldsymbol{u}, t, \tau) = g(\boldsymbol{x}, t) P'(\boldsymbol{u}, \tau | \boldsymbol{x}, t)$$

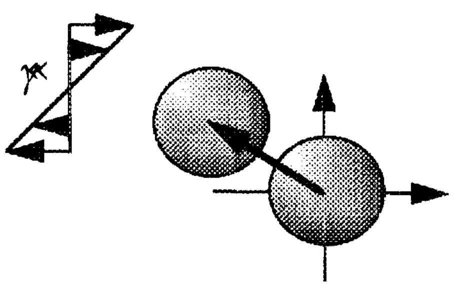
Expand: $$P'(\boldsymbol{u}, \tau | \boldsymbol{x}, t) = P'^{(0)} + St\, P'^{(1)} + \cdots$$

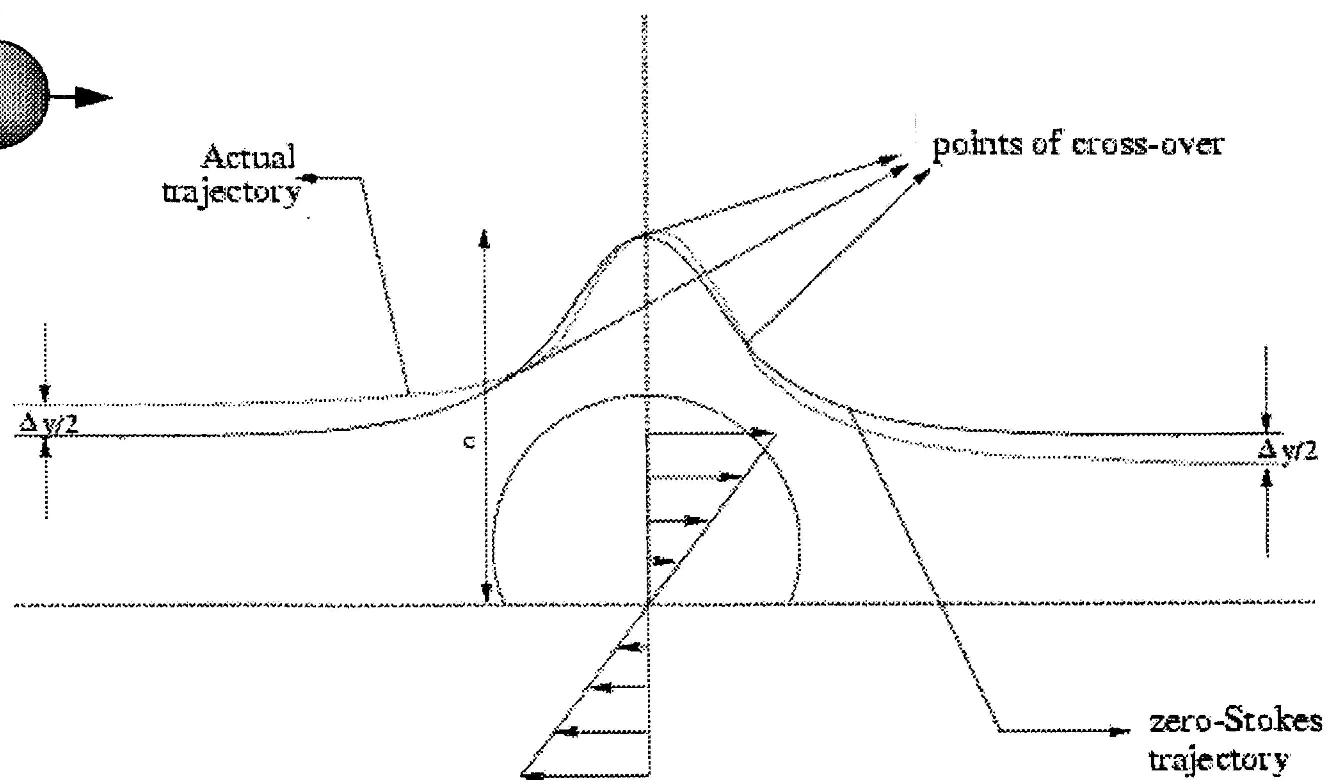
For large τ, $P'^{(0)}$ is a Maxwellian about $(\boldsymbol{u} - \boldsymbol{R}^{-1} \cdot \boldsymbol{F}^O)$, and $P'^{(1)}$ gives the first inertial correction to the Maxwellian. Integration over the velocity coordinates gives the inertially corrected Smoluchowski equation:

$$\frac{\partial g}{\partial t} + \nabla_x \cdot (\boldsymbol{R}^{-1} \cdot \boldsymbol{F}^O g) \; = \; St\, \nabla_x \cdot (\boldsymbol{V} g)$$

Simple Shear Flow: O(ϕ^2)

The pair-distribution function, *g(x,t)*, determines the suspension structure. In the absence of inertia, the microsturcture has fore-aft symmetry, the suspension is Newtonian (no normal stresses) and there is no shear-induced diffusion. Inertial breaks the symmetry as illustrated below.

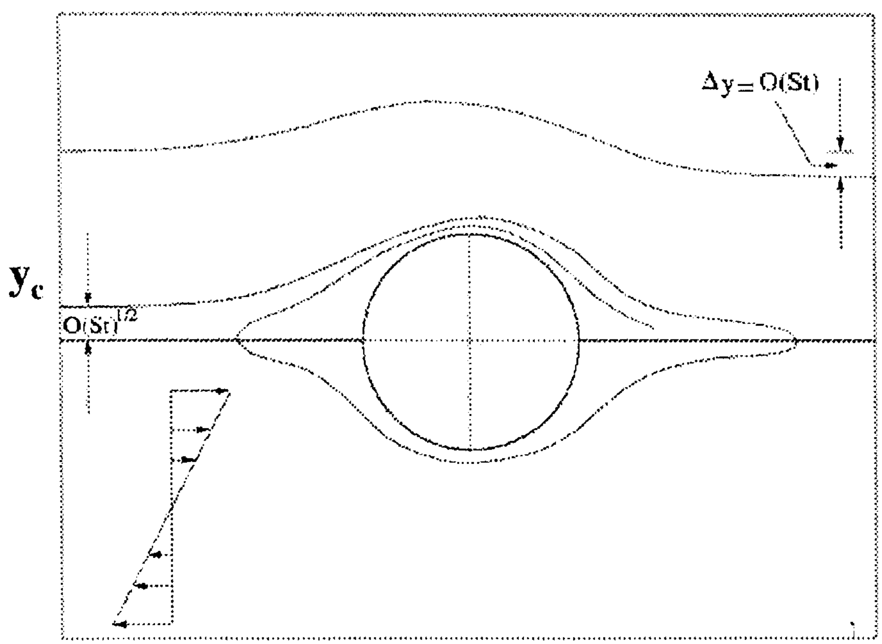




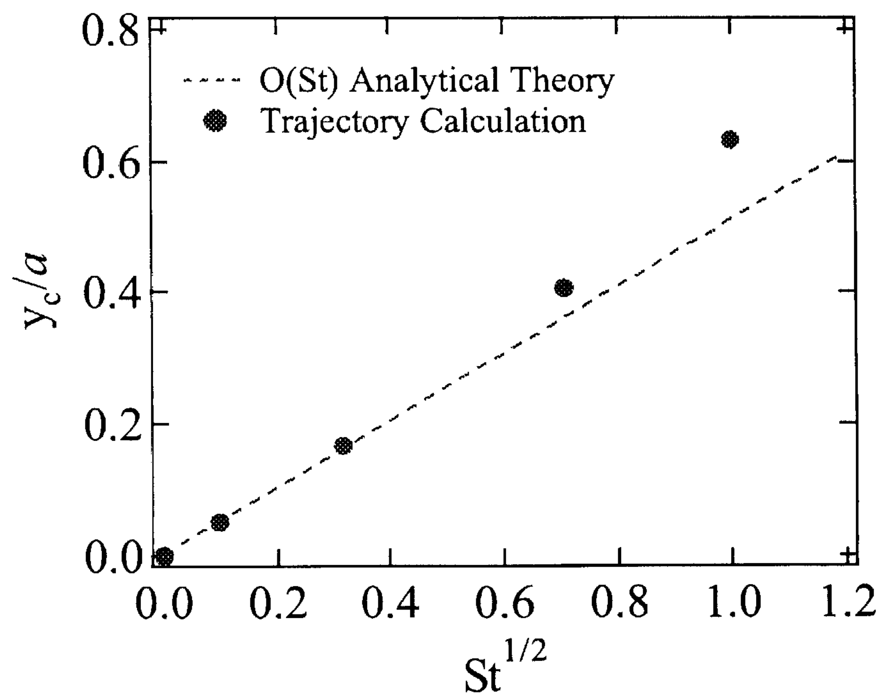
Closed Trajectories

Inertia increases the region of bound trajectories. In the plane, all particles starting off with an offset of O($St^{1/2}$) or less spiral in towards contact with the reference sphere.

Typical bound trajectory

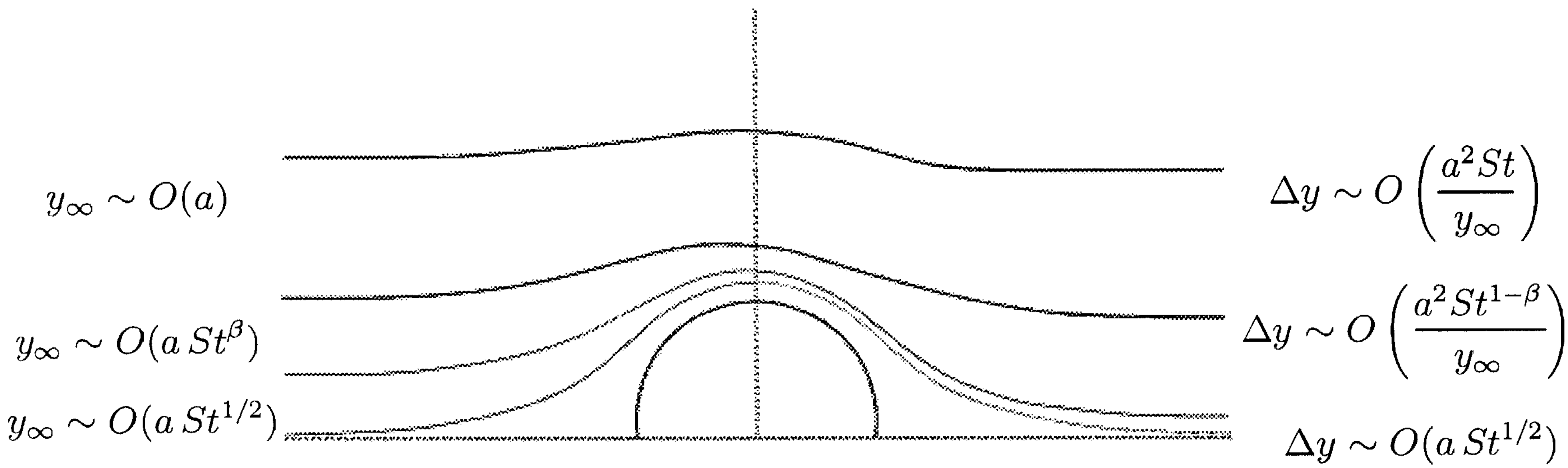


Up stream offset



Shear-induced diffusivity: D_{yy}

$$D_{yy} \sim \int \frac{(\Delta y)^2}{\tau} d(y_\infty/a) \quad , \quad \tau \sim (\dot{\gamma} y_\infty / a)^{-1}$$



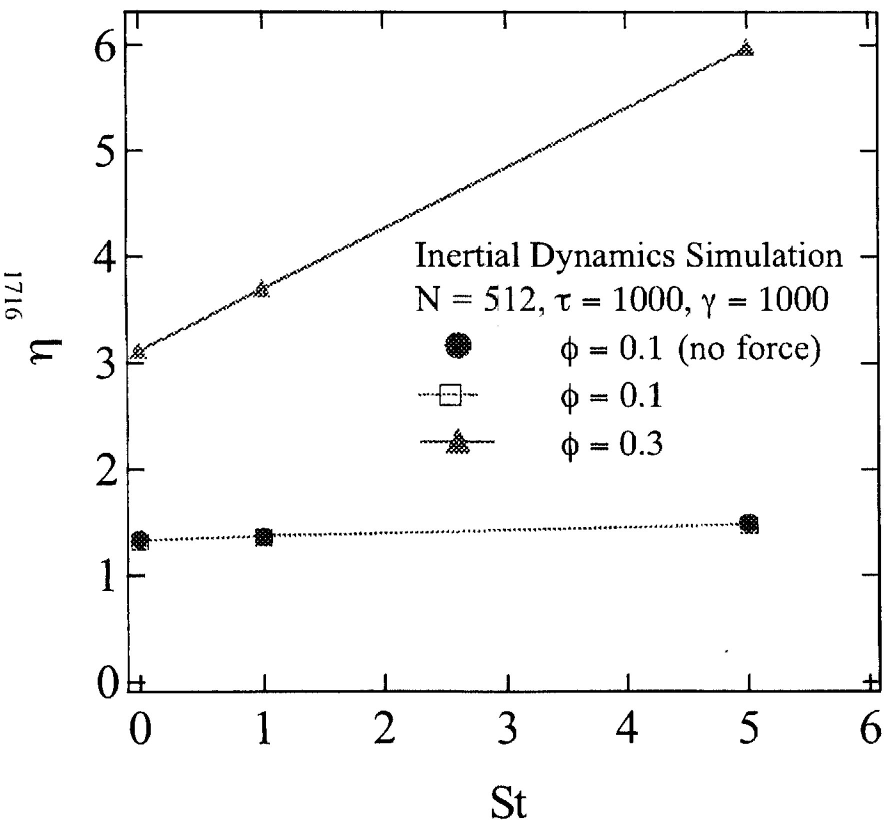
$$y_\infty \sim O(a): \quad D_{yy} \sim O(\dot{\gamma} a^2 St^2)$$

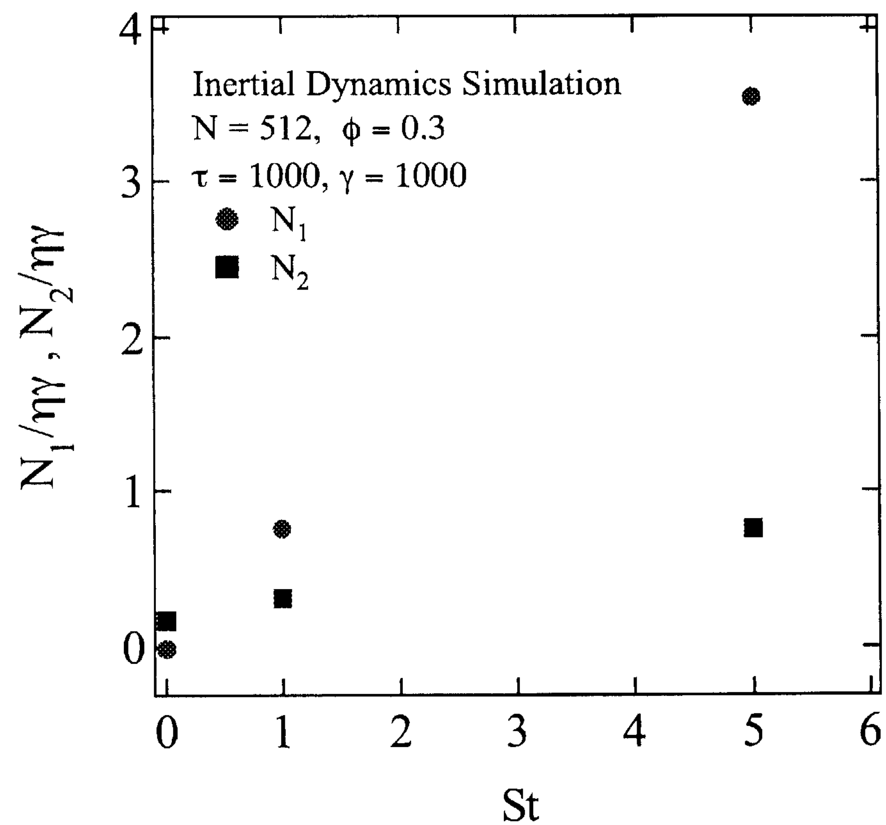
$$y_\infty \sim O(a\,St^\beta; \beta < 1/2): \quad D_{yy} \sim O(\dot{\gamma} a^2 St^2 \ln St)$$

$$y_\infty \sim O(a\,St^{1/2}): \quad D_{yy} \sim O(\dot{\gamma} a^2 St^2)$$

Inertial Dynamics Simulation

Early results on the shear viscosity and first and second normal stress differences as a function of *St*.





Conclusions

- **Derived the O(St) inertially-corrected Smoluchowski equation.**
- **In simple shear flow the region of closed trajectories is enlarged by particle inertia, and the trapped particles spiral in.**
- **The shear-induced self diffusivity in the gradient direction is predicted to scale as $St^2 \ln St$ for small St.**
- **Inertial Dynamics Simulations with an O($N \ln N$) method have been initiated.**