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Aeroelastic Response of Swept Aircraft Wings in a Compressible Flow Field

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ABSTRACT

The present study addresses the subcritical aeroelastic response of swept wings, in various flight speed regimes, to arbitrary time-dependent external excitations. The methodology based on the concept of indicial functions is carried out in time and frequency domains. As a result of this approach, the proper unsteady aerodynamic loads necessary to study the subcritical aeroelastic response of the open/closed loop aeroelastic systems, and of flutter instability, respectively are obtained. Validation of the aeroelastic model is provided, and applications to subcritical aeroelastic response to blast pressure signatures are illustrated. In this context, an original representation of the aeroelastic response in the phase – space is displayed, and pertinent conclusions on the implications of a number of selected parameters of the system are outlined.

NOMENCLATURE

a_n	Dimensionless elastic axis position measured from the midchord, positive aft
c_n	Chord length of wing, normal to the elastic axis, $2b_n$
$C_{L\alpha_n}$	Lift-curve slope
h, h_0	Plunging displacement and its amplitude, respectively
I_y, $\bar{r}_\alpha$	Mass moment of inertia per unit length of wing and the dimensionless radius of gyration, $\sqrt{I_y/mb_n^2}$, respectively
l	Wing semi-span measured along the mid-chord line
l_a, m_a	Dimensionless aerodynamic lift and moment, $L_a b_n/mU_n^2$ and $M_a b_n^2/I_y U_n^2$, respectively
m, μ	Wing mass per unit length and wing/air mass ratio, $m/\pi\rho b_n^2$, respectively
N	Load Factor, $1+h''/g$
$P_m, \wp_m$	Peak reflected pressure in excess and its dimensionless value $P_m b_n/mU_n^2$, respectively
r	Shock pulse length factor
$s, \mathcal{L}$	Laplace transform variable and operator, respectively
$S_\alpha, \bar{\chi}_\alpha$	Static unbalance about the elastic axis and its dimensionless counterpart, S_y/mb_n, respectively
t, τ_0, τ	Time variables and dimensionless time, $U_n t/b_n$ respectively
U_∞, U_n	Freestream speed and its component normal to the elastic axis, respectively
V_n	Dimensionless free-stream speed, $U_n/b_n\omega_\alpha$
Z	Vertical displacement in z direction
α, α_0	Twist angle about the pitch axis and its amplitude, respectively
ζ_h, ζ_α	Structural damping ratio in plunging, $c_h/2m\omega_h$ and in pitching, $c_\alpha/2I_y\omega_\alpha$, respectively
η	Dimensionless coordinate along the wing span, $\bar{y}/l$
Λ	Swept angle (positive for swept back)
ξ	Dimensionless plunge coordinate, w/b_n

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τ_p	Dimensionless positive phase duration of the pulse, measured from the time of the arrival
ϖ	Plunging-pitching frequency ratio, ω_h/ω_α
ω, k	Circular and reduced frequencies, $\omega b_n / U_n$, respectively
ω_h, ω_α	Uncoupled frequency in plunging, $\sqrt{K_h/m}$ and pitching, $\sqrt{K_\alpha/I_y}$, respectively

1. INTRODUCTION

The modern post cold-war combat aircraft is likely to be exposed during its operational life to more severe environmental conditions than in the past. This implies that, while being in flight, its structure should sustain pressure pulses due to fuel explosions, shock waves, sonic-boom, etc. It clearly results that the analysis of the aeroelastic response in the precritical range should be addressed in the various flight speed regimes i.e. from the incompressible to the hypersonic one.
In this paper, using the time domain representation in conjunction with the concept of indicial function[1-2], the relevant unsteady aerodynamic loads necessary to approach the aeroelastic response of swept aircraft wings in various flight speed regimes, such as the incompressible, compressible subsonic, supersonic and hypersonic ones are obtained. Such a representation of unsteady aerodynamic loads is required towards determination of the aeroelastic response to various pressure signatures[3-4] in the subcritical flight speed regime. Herein the idea of the *modified strip theory* initiated by Yates[5] has been further developed as to address the problem of the aeroelastic response of swept aircraft wings in various flight speed regimes. In this context, an original *phase – space* representation, provides full information about the behavior of the aeroelastic system. As a by-product of the response analysis, the conditions resulting in the occurrence of the flutter instability can be obtained. A validation of this procedure towards determination of the flutter instability boundary was done via the study of the aeroelastic eigenvalue problem, and an excellent agreement was reached. Herein the case of a swept wing (see Fig. 1), featuring the plunging and pitching degrees of freedom is considered.

2. GENERAL THEORY

2.1. Aerodynamic Loads

An unified approach based upon the use of the aerodynamic indicial functions enabling one to determine the aerodynamic lift and moment in subsonic compressible and supersonic flight speed regimes is developed. The aerodynamic loads in the various flight speed regimes are obtained via the use of the appropriate indicial functions[6-7] and their application toward determination of the aeroelastic response and flutter instability boundary is addressed in the paper.

2.1.1. Subsonic Compressible Flight Speed Regime

A significant work toward developing appropriate analytical expressions for indicial functions in a subsonic compressible flow has been carried.
In Refs. 3 and 4 it was shown that, for subsonic compressible flight speed regime, the circulation around the airfoil is determined by a new set of indicial functions, different of the Wagner's function.

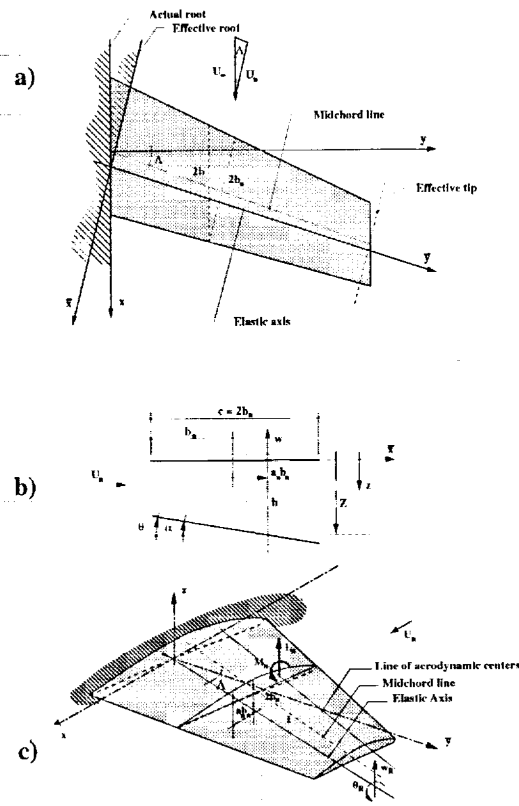


Fig. 1. a) Non-uniform swept wing; b) Airfoil section; c) 3-D view of a swept wing.

In contrast to the incompressible case, the indicial functions in subsonic compressible flow are not analytic, except for limited instants of time. Following the formulation in Ref. 8, in our approach, new indicial functions, for the plunging and pitching degrees of freedom are presented. Via the use of the four indicial functions defined for a two-dimensional lifting surface, the corresponding indicial lift and moment can be written as:

$$L_y(\tau)=\tfrac{1}{2}C_{L\alpha}\rho U_\infty^2 S\langle h_0'/b\rangle\langle\phi_c(\tau)\rangle; \quad (1a)$$

$$M_y(\tau)=C_{L\alpha}\rho U_\infty^2 Sh_0'\langle\phi_{cM}(\tau)+\tfrac{1}{2}(a+1)\phi_c(\tau)\rangle, \quad (1b)$$

$$L_q(\tau)=C_{L\alpha}\rho U_\infty Sqb\langle\phi_{cq}(\tau)-\tfrac{1}{2}(a+1)\phi_c(\tau)\rangle, \quad (1c)$$

$$M_{yq}(\tau)=C_{L\alpha}\rho U_{\infty}S(2b)q_0 b$$
$$\left\langle \phi_{cMq}(\tau)+\tfrac{1}{2}(a+1)\left(\phi_{cq}(\tau)-\phi_{cM}(\tau)\right)-\tfrac{1}{4}(a+1)^2\phi_c(\tau)\right\rangle \quad (1d)$$

The expressions presented in Eqs. (1) have been adapted and extended as to approach the swept aircraft wing aeroelastic problems. Herein, the quantities in brackets $\langle\cdot\rangle$, identify the indicial functions for the compressible flight speed regimes; h_0' is the plunging velocity and q_0 is the indicial angular velocity; (L_y, M_y) and (L_q, M_{yq}) correspond to the lift and moment due to plunging (in y direction) and pitching (around the elastic axis), respectively. The unsteady lift associated with the compressible flight speed regime can be described as:

$$L_a(\tau)=-C_{L\alpha}b\rho U_{\infty}^2\int_{-\infty}^{\tau}\phi_c(\tau-\tau_0)\left(\alpha'(\tau_0)+\frac{h''(\tau_0)}{b}\right)d\tau_0$$
$$-2C_{L\alpha}b\rho U_{\infty}^2\int_{-\infty}^{\tau}\phi_{cq}(\tau-\tau_0)\alpha''(\tau_0)d\tau_0 \quad (2)$$

As concern the aerodynamic moment, $M_a(\tau)$ this can be obtained from Eq. (2) by replacing ϕ_c and ϕ_{cq} by $-2b\phi_{cM}$ and $-2b\phi_{cMq}$, respectively.

In order to have an unique formulation in both the incompressible and compressible flight speed regimes, the indicial functions appearing in the aerodynamic lift and moment, Eq. (2), have to be expressed in a form which is similar to that used in the incompressible flight speed range[8], where the elastic axis is located at ab back from the mid-chord. To this end, their modified expressions have to be used. These are:

$$\phi_c(\tau)\Rightarrow\phi_c(\tau); \quad (3a)$$
$$\phi_{cM}(\tau)\Rightarrow\phi_{cM}(\tau)+\tfrac{1}{2}(a+1)\phi_c(\tau); \quad (3b)$$
$$\phi_{cq}(\tau)\Rightarrow\phi_{cq}(\tau)-\tfrac{1}{2}(a+1)\phi_c(\tau); \quad (3c)$$
$$\phi_{cMq}(\tau)\Rightarrow\phi_{cMq}(\tau)+\tfrac{1}{2}(a+1)\left(\phi_{cq}(\tau)-\phi_{cM}(\tau)\right) -\tfrac{1}{4}(a+1)^2\phi_c(\tau) \quad (3d)$$

The validation of the new indicial function is provided in the frequency domain, where the unsteady aerodynamic coefficients are determined, and the comparisons with those available in the classical literature[9-13] are provided in Fig. 2. The following representations for the unsteady aerodynamic lift is postulated:

$$L_a(k,\tau)=\rho U_{\infty}^2 k^2 b\left(\frac{h}{b}L_1+\alpha L_2\right), \quad (4a)$$

where

$$L_1=iH_1+H_4, \qquad L_2=iH_2+H_3. \quad (4b)$$

Herein, the dimensionless unsteady aerodynamic coefficients H_i have been introduced, and the reduced frequency k, has been included as to render the quantities in brackets dimensionless. As concern the aerodynamic moment, $M_a(\tau)$ this can be obtained from Eqs. (4) by replacing L_i and H_i by M_i and A_i, respectively.

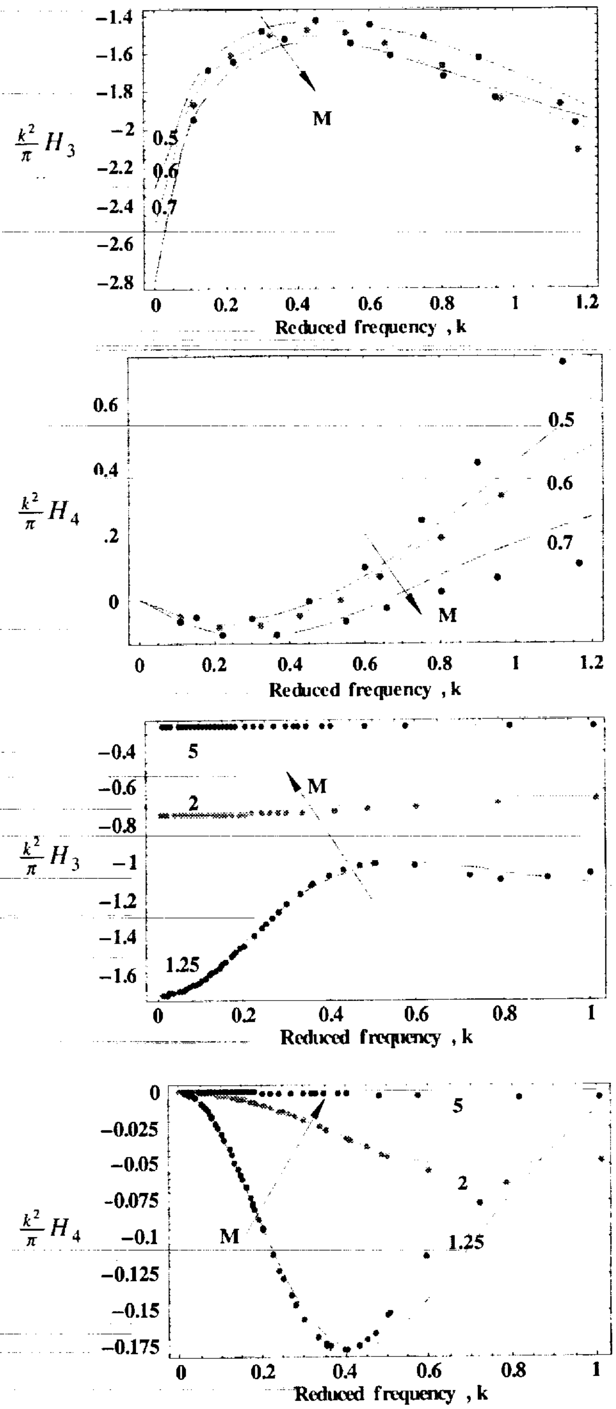


Fig. 2. Unsteady aerodynamic derivatives in: a-b) compressible subsonic and c-d) supersonic flight speed regimes. Comparisons with data from Ref. 9 through 13.

Notice that there is a considerable influence of the Mach number on the indicial functions. In contrast to the behavior of the indicial function (Wagner's function) in the incompressible flow speed that features at $\tau = 0$ an infinite value modeled as a Dirac pulse $\delta(\tau)$, in the compressible speed range a finite value for the indicial function at $\tau = 0$ is experienced. Moreover, in contrast to the compressible flight speed regime, where the aerodynamic lift and moment are obtainable through four indicial aerodynamic functions, in the incompressible regime, a single function, namely the Wagner one is sufficient for determining the lift and moment.

More recently, in Refs. 14 - 15, the concept of indicial function in subsonic compressible flow has been used, and an approximation and validation of indicial functions for any value of the Mach number in the compressible speed range was obtained. Keeping in mind the relationship between $C(k)$ and $\Phi(ik)$ for the incompressible flow-field (see Ref. 8), one can define, for the compressible flight speed regime, the analogous of Theodorsen's function in terms of the corresponding indicial functions as:

$$C_c(k) \equiv F_c(k) + iG_c(k) = ik\int_0^\infty \phi_c(\tau)e^{-ik\tau}d\tau = ik\Phi_c(ik); \quad (5a)$$

$$C_{cM}(k) \equiv M_c(k) + iN_c(k) = ik\Phi_{cM}(ik); \quad (5b)$$

$$C_{cq}(k) \equiv F_{cq}(k) + iG_{cq}(k) = ik\Phi_{cq}(ik); \quad (5c)$$

$$C_{cMq}(k) \equiv M_{cq}(k) + iN_{cq}(k) = ik\Phi_{cMq}(ik), \quad (5d)$$

where $C_c(k)$, $C_{cM}(k)$ are the compressible analogous of Theodorsen's function for plunging for lift and moment and $C_{cq}(k)$, $C_{cMq}(k)$ are the compressible analogous of Theodorsen's function for pitching for lift and moment, respectively. Also in this case, the four new complex functions, should collapse, in the incompressible flight speed regime, into a single one, namely the Theodorsen's function. Paralleling the developments carried out for the incompressible flight speed regime, for the compressible one, the lift and moment per unit span for plunging (L_y, M_y) and for pitching (L_q, M_{yq}) about the leading edge, are expressed similarly as:

$$L_y(k) = C_{L\alpha}\rho U_\infty^2 h' C_c(k); \quad (6a)$$

$$M_y(k) = 2bC_{L\alpha}\rho U_\infty^2 h' C_{cM}(k); \quad (6b)$$

$$L_q(k) = 2C_{L\alpha}\rho U_\infty b^2 q C_{cq}(k); \quad (6c)$$

$$M_{yq}(k) = 2C_{L\alpha}\rho U_\infty b^2 q C_{cMq}(k). \quad (6d)$$

It clearly appears that, whereas from the response problem, considered in the time domain, the flutter boundary is obtainable in a numerical way, for the flutter analysis in a classical sense, the aerodynamic loads have to be converted from the Laplace space to the frequency domain, replacing $s \to ik$.

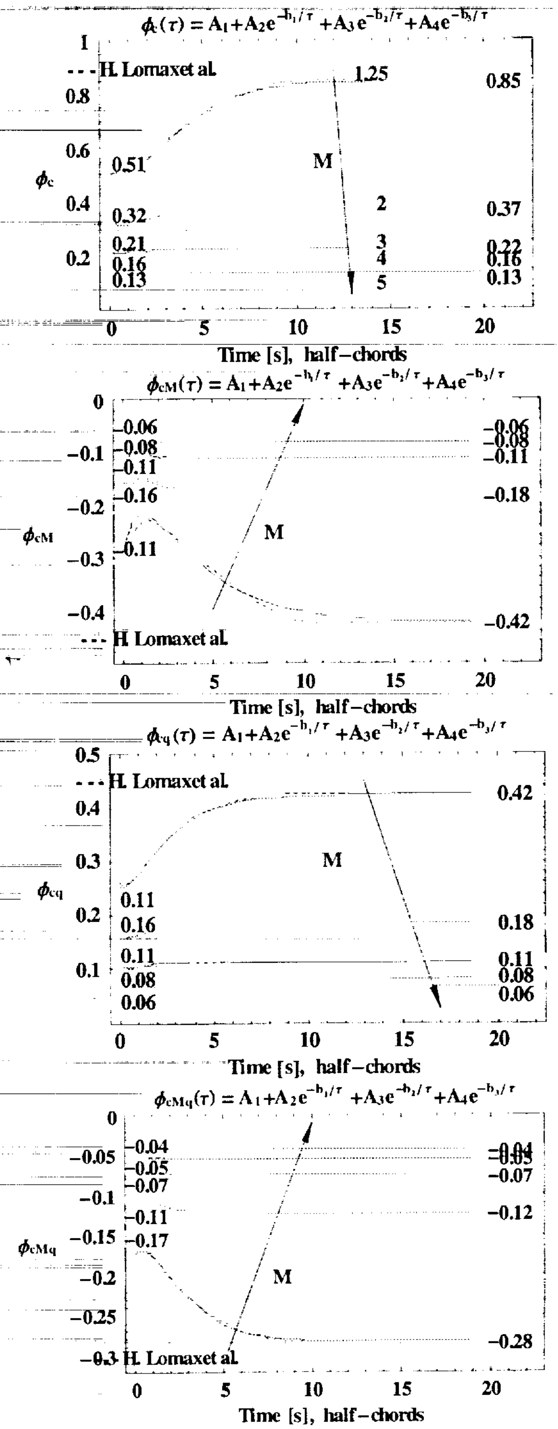


Fig. 3 Indicial lift and moment functions from plunging and pitching of a 2-DOF lifting surface. Comparisons with Refs. 2 and 8.

displayed here. The same notations, as for the subsonic compressible have been used. The new supersonic indicial functions for selected Mach numbers are displayed and compared with those provided by Ref. 8 in Fig. 3. Likewise Theodorsen's function, also in this case, the newly defined indicial functions can be expressed in terms of their real and imaginary parts. Some of these components are presented and validated in Fig. 4. In this context, the initial and final values of the Wagner's function, expressed in the time domain, can be obtained by taking the limit of $s\Phi(s)$ in the Laplace transformed space for $\tau \to 0$ and $\tau \to \infty$, respectively. From the property of the initial value, in the incompressible flight speed regime, this implies:

$$\lim_{\tau \to 0} \phi(\tau) = \lim_{s \to \infty} s\Phi(s) = \phi(0^+) = 0.5, \qquad (7a)$$

while, from the property of the final value:

$$\lim_{\tau \to \infty} \phi(\tau) = \lim_{s \to 0} s\Phi(s) = \phi(\infty) = 1. \qquad (7b)$$

The first of the above expressions, namely the property of the initial value, is valid if $\phi(\tau)$ does not contain an impulse $\delta(\tau)$. In such a case the limit of the converted function in the Laplace transformed space does not exist. The second expression, occurring from the property of the final value, is applicable only in the case where $\phi(\tau)$ and $\phi'(\tau)$ are both Laplace transformable and if the function $s\Phi(s)$ has all its singularities in the left half of the s-plane. These properties are fulfilled by the Wagner's function and by the new indicial function in the compressible flight speed regimes (see dotted lines in Figs. 4.c and 4.d).

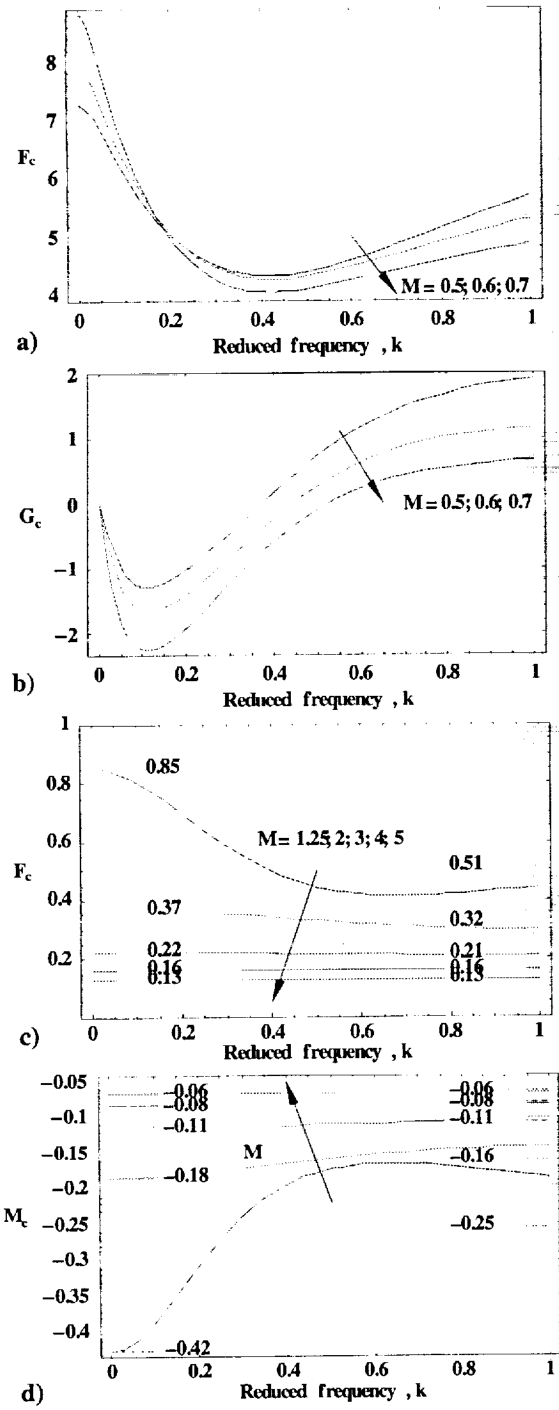


Fig. 4 Some of the real and imaginary parts of the analogous of Theodorsen's function for the subsonic (a-b) and supersonic (c-d) flight speed regimes.

2.1.2. Supersonic Flight Speed Regime

The expressions of the indicial functions for two-dimensional lifting surfaces in supersonic flow are not

2.1.3. Hypersonic Flight Speed Regime

The supersonic flight speed regime is completed with that of the hypersonic flight speed regime, and in this context, piston theory aerodynamics is used (Fig. 5).

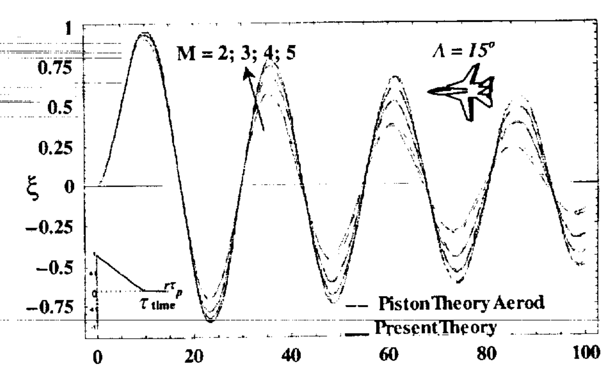


Fig. 5 Aeroelastic response time-history in the hypersonic flight speed regime to blast loads, as presented in inset. Comparison of prediction of responses based on the present and piston theory aerodynamics (Ref. 16).

2.2. Sonic-Boom and Blast Pressure Signatures

The blast and sonic-boom overpressure signature (referred to as the N-wave shock pulse) see Fig. 6, can be represented mathematically in an unified way as (Fig. 6):

$$L_b(t)=H(t)P_m\left(1-\frac{t}{t_p}\right)-H(t-rt_p)P_m\left(1-\frac{t}{t_p}\right). \quad (8)$$

Herein, $H(t)$ is the Heaviside step function; P_m denotes the peak reflected pressure in excess to the ambient one (Ref. 3-4); t_p denotes the positive phase duration of the pulse measured from the time of impact of the structure; r denotes the shock pulse length factor.

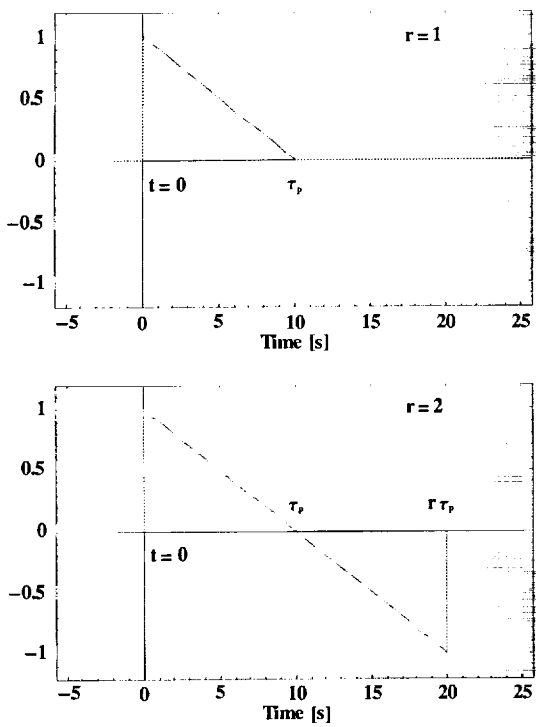


Fig. 6 Blast and sonic-boom pressure signatures.

For $r=1$ the N-shaped pulse degenerates into a triangular pulse which corresponds to an explosive pulse, and for $r=2$ a symmetric sonic-boom pulse is obtained.

2.3. Subcritical Aeroelastic Response of an Aircraft Wing Featuring Plunging-Pitching Coupled Motion

Determination of subcritical aeroelastic response of swept lifting surfaces flying in a compressible flow field and exposed to time-dependent external pulses is useful in the design of wing structures and of the associated feedback control systems. The aeroelastic governing system of a swept metallic wing featuring plunging-twisting degrees of freedom and subjected to a blast pulse can be expressed as[8]:

$$EI\frac{\partial^4 w}{\partial \bar{y}^4}+m\frac{\partial^2 w}{\partial t^2}-S_y\frac{\partial^2 \theta}{\partial t^2}-L_a(\bar{y},t)=L_b(\bar{y},t); \quad (9a)$$

$$GJ\frac{\partial^2 \theta}{\partial \bar{y}^2}+S_y\frac{\partial^2 w}{\partial t^2}-I_y\frac{\partial^2 \theta}{\partial t^2}-M_a(\bar{y},t)=0. \quad (9b)$$

For the cantilevered wing, the related boundary conditions are:

$$w(\bar{y},t)=\frac{\partial w(\bar{y},t)}{\partial \bar{y}}=\theta(\bar{y},t)\Big|_{\bar{y}=0}=0, \quad \text{and} \quad (10a)$$

$$\frac{\partial^2 w(\bar{y},t)}{\partial \bar{y}^2}=\frac{\partial^3 w(\bar{y},t)}{\partial \bar{y}^3}=\frac{\partial \theta(\bar{y},t)}{\partial \bar{y}}\bigg|_{\bar{y}=l}=0. \quad (10b)$$

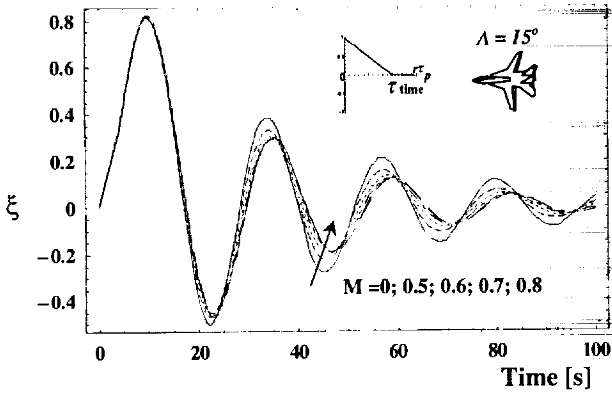
Use of shapes functions, yields the aeroelastic governing equations in dimensionless form. In the right hand side member of these equations, $L_b(\bar{y},\tau)$ denotes the external time-dependent load acting on the rigid wing counterpart that can correspond to a blast, sonic-boom, gust or shock-wave pulse. Considering this load as uniformly distributed in the chordwise direction, no moment contribution $M_b(\bar{y},\tau)$ is generated in Eq. (9). The aeroelastic governing equation can be converted in the Laplace transformed space and solved for the unknowns, $\bar{\xi}\,(\equiv\mathcal{L}(\xi))$ and $\bar{\alpha}\,(\equiv\mathcal{L}(\alpha))$; inverted back in time domain one obtain the plunging and pitching time-histories and the load factor time-history due to the sonic-boom pressure pulse. This inversion has been performed numerically via Mathematica® routine[17], and the results are depicted in the Figs. 7 – 10.

3. RESULTS AND DISCUSSION

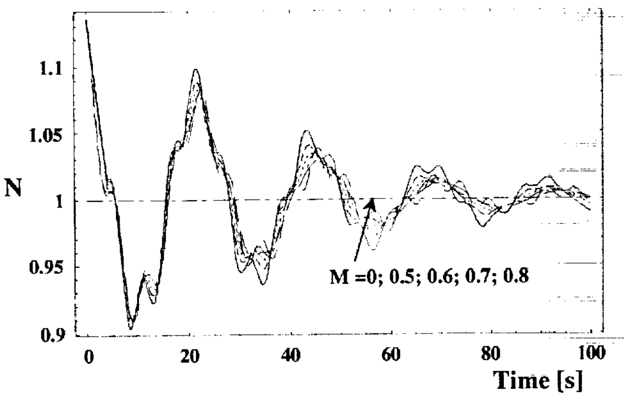
The graphs in Fig. 7 – 8 depict the dimensionless time-histories for the plunging displacement ξ, the load factor N and the pitching displacement α of a swept wing to a blast load in compressible subsonic and supersonic flow fields. In Fig. 7 the solid lines correspond to the coupled plunging-pitching motion of a 2 DOF model in the incompressible flight speed regime $(M=0)$.

The plunging-pitching coupling contributes to the reduction of the amplitude of the aeroelastic response. The load factor N has its maximum for τ = 0, when the first impulse due the blast load occurs.

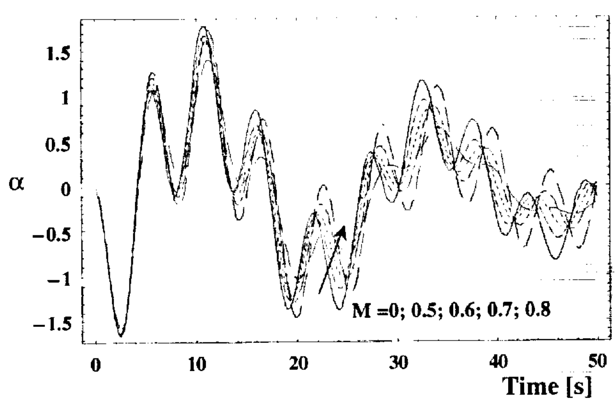
In Fig. 9 a three-dimensional plot depicting the dimensionless plunging and pitching deflection time-histories ξ of a swept aircraft wing $(\Lambda=15^\circ)$ to blast pressure signatures vs. the normalized spanwise coordinate y and the dimensionless time τ is supplied. The evolution of the aeroelastic system can be graphically illustrated by examining its motion in the *phase – space*, rather than in the real space, and recognizing that the trajectory depicted in this space represents the complete time history of the system.



a) Aeroelastic time-history in plunging as affected by the pitching motion

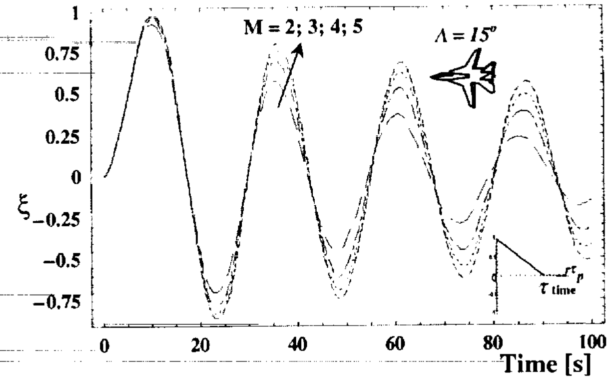


b) Acceleration time-history expressed in terms of the load factor $N(\tau) \equiv 1 + h''/g$

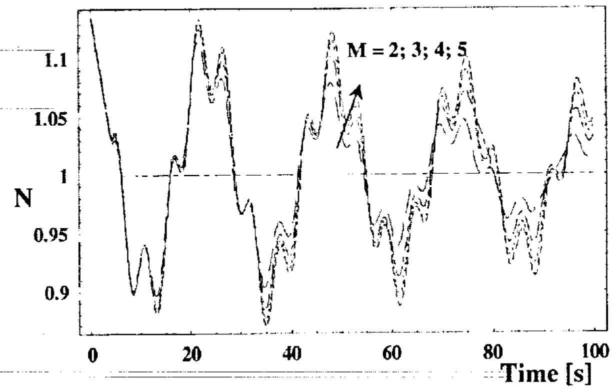


c) Twist angle time-history

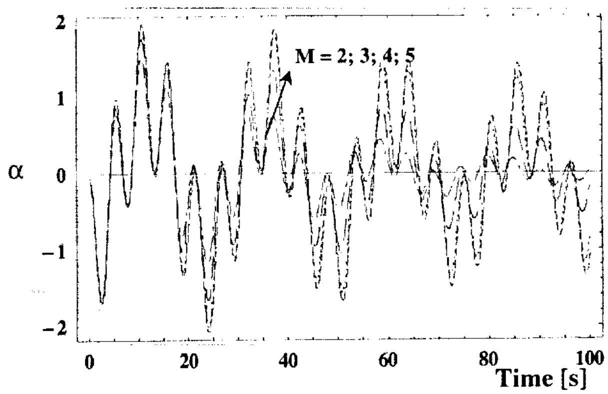
Fig. 7 Aeroelastic response time-history of swept wing $(\Lambda = 15^\circ)$ in subsonic compressible flow field to blast loads, as presented in inset



a) Aeroelastic time-history in plunging as affected by the pitching motion



b) Acceleration time-history expressed in terms of the load factor $N(\tau) \equiv 1 + h''/g$



c) Twist angle time-history

Fig. 8 Aeroelastic response time-history of swept wing $(\Lambda = 15^\circ)$ in supersonic flow field to blast loads, as presented in inset.

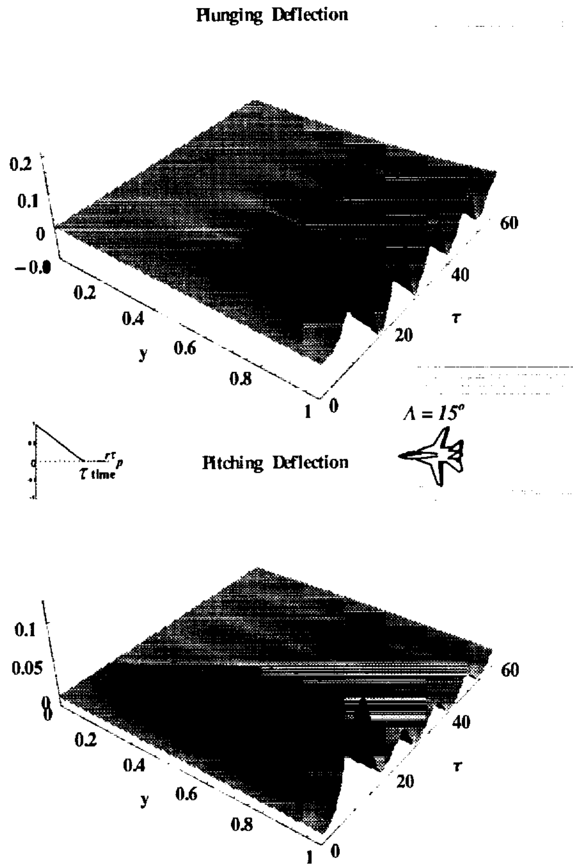


Fig. 9 Three-dimensional plots depicting the dimensionless plunging ξ and pitching α deflections time-histories of a swept aircraft wing $(\Lambda = 15°)$ to blast pressure signature, vs. the normalized spanwise coordinate y and the dimensionless time τ (M = 3).

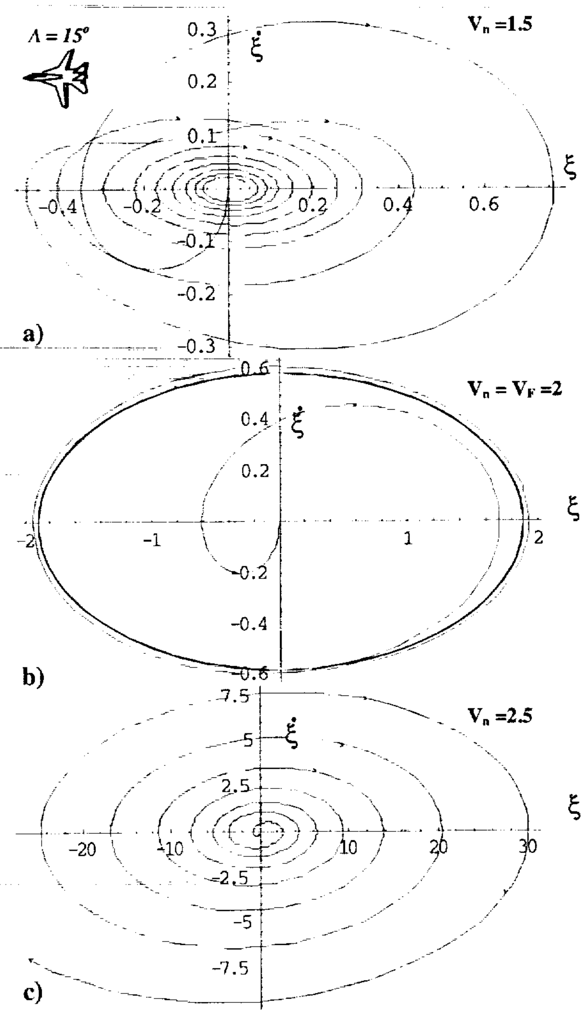


Fig. 10 Phase-plane $(\xi;\dot{\xi})$ portraits depicting the dimensionless plunging deflection time-history of a swept aircraft wing to a blast pulse, for selected values of the flight speed.

Corresponding to the condition resulting in the flutter instability (Fig. 10b and 11b), that coincides with that obtained from the eigenvalue analysis, the trajectory of motion describes an orbit with constant amplitude (*center*). For $V_n < V_F$ (Fig. 10a and 11a) as time unfolds, a decay of the amplitude is experienced, which reflects the fact that in this case a subcritical response is involved (*stable focal point*), while for $V_n > V_F$ (Fig. 10c and 11c) the response becomes unbounded, implying that the flutter instability has occurred (*unstable focal point*).

To avoid its occurrence there are two possibilities, namely, including a passive/active control methodology or acting on the sweep angle Λ. In the latter case the idea is to use the capabilities of the aircraft featuring variable sweep angle (e.g. *F-14 Tomcat*) as to reduce the severity of oscillations, and at the same time expand the flight envelope.

The critical value of the flutter speed obtained from the eigenvalue analysis of the homogeneous system of equations, and of that extracted from the aeroelastic response coincide, being $V_F = 2$. On the basis of a number of simulations, it can be concluded that the flutter predictions based on both methods are in an excellent agreement.

The way to determine the flutter speed from response becomes clear from the phase plane portrait, in which, for a certain restricted range of the flight speed (in the vicinity of the flutter speed), a periodic response with constant amplitude is experienced.

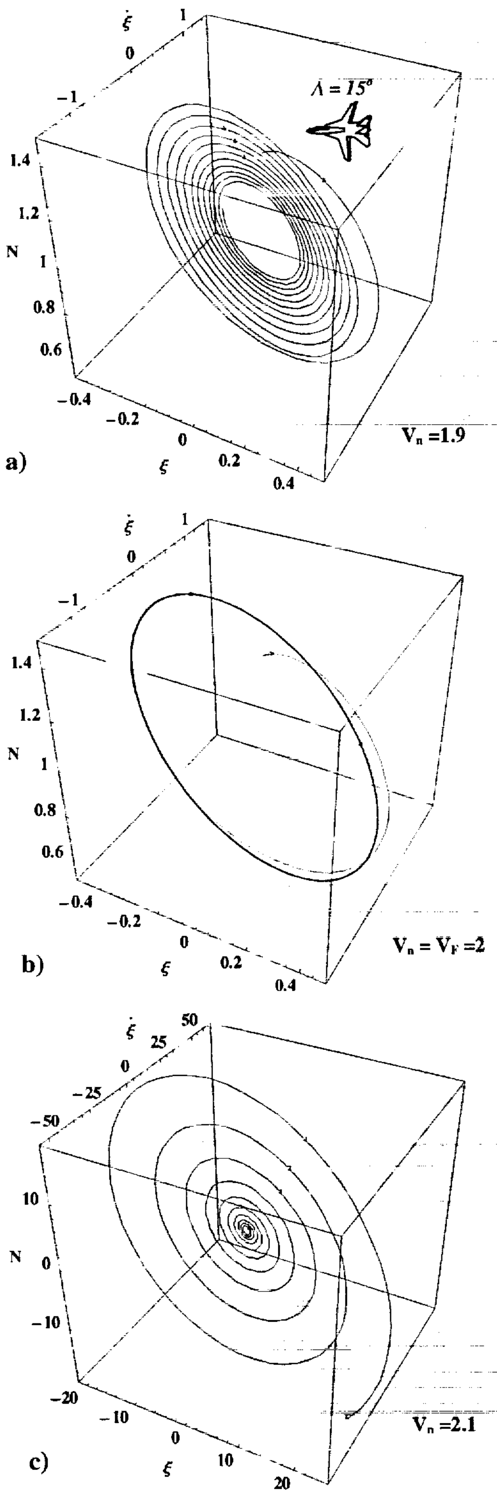


Fig. 11 Phase-space portraits $(\xi;\dot{\xi};N)$ depicting the dimensionless plunging deflection time-history of a swept aircraft wing to blast pressure signature, for selected values of the flight speed.

4. CONCLUSIONS

The versatility of the methodology presented here consisting of the approach of both the subcritical aeroelastic responses and of the flutter instability for 3-D swept aircraft wing in various flight speed regimes has been illustrated. The aeroelastic response has been represented in both the classical way, by displaying the time-histories of plunging – pitching motion and load factor, and in an original phase – space context, that provides full information about the behavior of the aeroelastic system.

Applications assessing the versatility of this approach enabling one to treat both subcritical aeroelastic response and flutter instability were presented, and, as a continuation of this study, the treatment of actively controlled 3-D wing aeroelastic problems is contemplated to be achieved in the forthcoming developments.

ACKNOWLEDGMENT

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