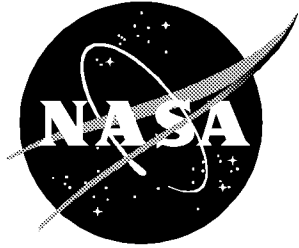


NASA/CR-2001-210661



A Study of Failure Criteria of Fibrous Composite Materials

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March 2001

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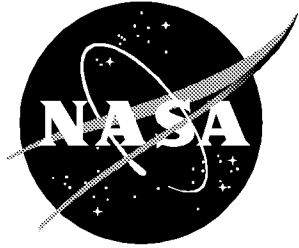
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Prepared for Langley Research Center
under Cooperative Agreement NCC1-24

March 2001

Acknowledgments

The author of this report acknowledges the efforts of Karen Jackson and Sotiris Kellas for making his stay at NASA Langley possible during the summer of 1999 and also to John Whitesides and Jessie Coates of George Washington University, through which the stay was organized. The author would like to thank all members of the Crash Dynamics Group of the Structural Dynamics Branch for their welcome and collaboration during the time spent in Hampton. Finally, thanks to all members of the Elasticity and Strength of Materials Group at the Engineering School of the University of Seville and to Carlos Davila from NASA Langley for comments on the final version of the report, in particular to J. C. Marin for his help in generating the graphs for the report.

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1.0. Introduction

This report was prepared during the two-month period the author spent at the Impact Dynamics Research Facility at NASA Langley Research Center and was completed at the University of Seville Engineering School, Seville, Spain.

The work developed at NASA Langley was originally conceived in two phases:

1. Evaluation of existing composite failure criteria in the MSC.Dytran code, a nonlinear finite element code used for crash simulation and other transient dynamic structural problems.
2. Exploration of the possibilities for modification of material and failure models to account for large deformations, and progressive failure and interaction of damage accumulation with stress-strain response of laminated composites.

The greater part of the time at NASA Langley, after an inspection of the MSC.Dytran user manual, was devoted to performing a bibliographical review of existing failure criteria of composites. The need for this revision was also motivated by the lack of a theoretical manual supporting the aforementioned program.

The papers considered most interesting for the objective of this report have been commented upon in section 2.

The failure criteria included in the code under consideration are discussed in section 3. Some decisions made in this code do not immediately indicate apparent consequences; therefore, it was necessary to perform some calculations and to represent some graphs to show the implications of such decisions. This information is presented in section 3.

A critical summary of the present procedures to perform analysis and design of composites is presented in section 4.

The study of the most important historical proposals presented to consider failure of the lamina of a fibrous composite and the more recent modifications proposed created some doubts for this author about the adequacy of the existing failure criteria. Also, some numerical analyses needed to be performed to answer pertinent questions concerning the basis for some of the proposed criteria. A summary of these ideas, which is a proposal of studies to be developed, is presented in section 5.

Finally, the most noteworthy aspects of the report, as well as some ideas for future developments, are summarized in section 6.

2.0. Review of Recent Works

A review of recent publications (refs. 1–53) connected with the research topic featured in this report has been completed. Initially, the study covered work done from 1993 to 1998. About 400 references appeared in the initial review, from which approximately 100 were selected. When studying this information, selected older papers were required and were included in the review.

Finally, 53 references have been included in this report. To facilitate the reading of this information, a certain classification of the papers has been attempted, while recognizing the limitations from which any classification can suffer. To facilitate the connection among the ideas in the different papers, the

references have been organized by date (within each group), although with some exceptions, particularly when they refer to the same author.

All the references reviewed represent excellent works in their own field of application. The majority of comments that might have negative connotations are connected with the aspects of interest in this review, which obviously might or might not be of interest to the authors of the works reviewed.

The comments that appear in this section were generated when reading each of the papers. Some of them may seem to be repetitive when the report is read in its entirety. They might have been complemented or envisaged differently in the course of drawing up this report, as can be appreciated by reading the rest of it.

2.1. Papers Proposing Failure Criteria

Summary–Reference 1

In reference 1, authors Z. Hashin and A. Rotem (1973) propose two failure mechanisms: one based on the failure of the fiber and the other based on the failure of the matrix. The first is governed by the longitudinal stress, with reference to the fiber orientation, and the second is governed by the transversal and tangential stresses to the fiber.

It is noticeable that, even though the authors do not distinguish matrix failure and interface failure, they do not mention this fact. This approach is plausible, but some explanations might have been given.

Comments

There is an assumption here that has influenced failure criterion proposals associated with nonfiber breakage mechanisms. Once it is assumed that there are two modes of failure, the one not associated with the fiber breakage (in this paper, it is called the matrix failure mode) is assumed to be capable of being approximated by a quadratic function of transversal and tangential stresses. That the material is insensitive to the direction of shear stress cancels out the coupling term between the two stresses, leading to

$$(\sigma_T/Y)^2 + (\tau/S)^2 = 1$$

This expression has been used extensively since the paper by Hashin and Rotem (1973) was published and was initially proposed in a very simplistic way. In the expression, σ_T is the stress transverse to the fiber, and Y is its allowable value, while τ is the tangential stress and S is its allowable value.

What does not seem to be a very appropriate procedure is to detect the values of some parameters involved in the criterion (particularly the strength associated with σ_T and τ), by applying the criterion to different tests and then checking whether the criterion fits the experimental results of these tests very well. Specific procedures ought to be followed to calculate the strength associated with σ_T and τ and, with these values measured, the criterion should then be applied and the predictions checked with the experimental results.

The authors propose the criterion in the context of a fatigue study. The criterion is proposed for unidirectional laminates. Nothing is said either about stress interaction between laminas in nonunidirectional laminates and its effects on failure (delaminations) or about degradation of properties when the limit values of the criterion are reached.

Summary–Reference 2

The incorporation of failure mechanisms into a failure criterion is the main aim of reference 2, the paper by Z. Hashin (1980). The author proposes two failure mechanisms: one associated with the fiber and the other associated with the matrix, distinguishing, in both cases, between tension and compression.

It has to be mentioned that once this distinction based on the failure mechanism is made, the author's strategy to deduce the criterion is to apply logical reasoning to reach an applicable criterion, rather than to continue with the mechanism of failure to establish the macrovariables associated with it and to propose a criterion based on them. Thus, for the matrix, the author proposes a quadratic criterion because, on the one hand, a linear criterion underestimates, in his experience, the strength of the material and, on the other hand, a polynomial of higher degree would be too complicated to manage. The author specifically states that "it is unfortunate that the quadratic nature of stress-energy density has at times led to physical interpretation of quadratic failure criteria."

The author recognizes some limitations and noncoherent consequences derived from the choice of the polynomial expression to represent the failure of the matrix. The most serious is that it implies that failure occurs at the maximum transverse shear plane, which is difficult to accept as a general conclusion.

Comments

The proposal of Hashin has, in this author's opinion, been of great value in the field of composite materials. The main contribution of this paper is to clearly establish the need to design failure criteria based on failure mechanisms, while it also presents a critical view of the Tsai approach-based failure criterion. This fact is recognized in the literature but it is not mentioned that this contribution is all that Hashin proposed based on mechanisms of failure. The rest of his proposal, as has been mentioned in the summary, is based on logical reasoning rather than micromechanics, an approach with which Hashin's criterion is not connected.

Hashin's critical view of his own proposal and also the explanations about alternative paths he explored are very much appreciated. Although he uses a 3D criterion, Hashin limits the scope of his proposal, as is mentioned in the title, to unidirectional laminates.

The author particularizes the 3D criteria to a plane stress case. It is interesting to note that there are some differences with respect to the proposal made by the author in the paper he coauthored with Rotem in 1973. The first difference affects the criterion that predicts tensile fiber mode in which the effect of the shear stress parallel to the fiber has been incorporated. The second difference affects the compressive matrix mode of failure, the change being due to including information that a quasi-isotropic state of stress would produce the failure at a value much higher than the nominal strength to a stress transverse to the fibers. This fact prevents the criterion from being quadratic, which is the only rule used for other modes of failures. A discussion of the implications of these two modifications, at least from a predictive point of view, would have been welcome.

Summary–Reference 3

In reference 3, S. E. Yamada and C. T. Sun (1978) propose a failure criterion of a lamina in a laminate by means of the expression

$$\left(\frac{\sigma_{11}}{X}\right)^2 + \left(\frac{\sigma_{12}}{S_{is}}\right)^2 = 1$$

where σ_{11} and σ_{12} are the longitudinal and tangential stresses, X is the strength of the lamina in the fiber direction, and S_{is} is the in situ shear strength of the lamina.

To propose this criterion, the authors make the following two assumptions:

- The laminate is assumed to fail when all the laminae have failed with cracks along the fiber direction.
- The failure strength of a lamina is taken from a cross-ply laminate.

Comments

There are several interesting points in this proposal that have led to the survival of this criterion. (The scheme to estimate failure of composites by Chang and Chang, described later, uses this criterion for fiber-matrix shearing and fiber breakage.)

Perhaps most interesting is the recognition of the different behavior in terms of the strength of a single lamina in a laminate, what the authors called in situ properties. Although this author personally found the name confusing (a concept such as this one is required because the failure is expressed in terms of different variables associated with those controlling the mechanism of failure), it is obviously important to point out that the shear strength of a laminate is about two to three times higher than the lamina shear strength. What can be found lacking in the criterion proposal are the fundamentals of such a proposal. For instance, the role of σ_{12} in the failure of the fibers which the interaction of the criterion predicts is not immediately understood. As the authors recognize, the criterion they propose is not applicable for failures dominated by the strength perpendicular to the fiber.

Summary–Reference 4

Reference 4, by F. K. Chang and K. Y. Chang (1987), describes a finite element approach to deal with the study of fiber composites. The authors present a failure criterion and a property degradation model that are of interest to the present study.

The failure criterion considers the possibility of having matrix cracking, fiber-matrix shearing, and fiber breakage, the last two being governed by the same criterion, based on the Yamada-Sun failure criterion. The possibility of having nonlinear behavior is taken into account by means of a cubic term in the strain-stress relation that involves a parameter being determined experimentally.

With reference to the property degradation model, for matrix cracking failure, all the properties are reduced to zero except E_x , whose value is maintained. For fiber and/or fiber-matrix shear failure, E_x and G_{xy} degenerate according to a Weibull distribution, whereas the other two parameters are reduced to zero.

The predictions of final tensile strength, for several stacking sequences, are compared with experimental results, and the evolution of the damage predicted by the model is also shown.

Comments

Although the authors discuss three modes of failure, the way in which they treat them in effect reduces to two types of failure: matrix and fiber failure. Fiber-matrix shear failure is treated as a fiber failure, which should have required further explanation by the authors.

The model involves several parameters, and little discussion is presented about the way in which they are determined. It is noticeable that the authors apply the value of S_C , the shear strength, as Yamada and Sun (ref. 3) suggested in their criterion for the cross-ply laminate.

The finite element modelization performed for a problem with a stress concentration is surprisingly poor, and it is noticeable that the authors do not comment on this fact. It may not affect the value of the final tensile strength, but it will obviously affect the onset of the damage, a question of primary interest in the presence of a stress concentration. In any case, if the final tensile strength value is not affected by the discretization, perhaps the authors should have discussed that fact.

Information about the evolution of the damage with increased load is also lacking. Had such information been available, it would have been easier to understand the influence of the parameters involved in the damage growth, leading to the ultimate strength of the laminate. Since the model and the determination of the parameters involved in it are so complicated, it would have been helpful to have some quantitative (or at least qualitative) ideas of the influence of parametric values in the final predictions.

The part of this paper that refers to failure criteria is very similar to what the MSC.Dytran program assigns to the Chang criterion. One difference is that Dytran does not consider a fiber-matrix shearing failure mechanism, but this omission is irrelevant due to the treatment, similar to fiber breakage, that Chang gives to this mechanism. Another difference is that Dytran distinguishes, for the matrix failure, between tension and compression and selects the Hashin criterion for a matrix in compression, taking as shear strength, the value managed by Chang in his nonlinear model. See reference 5.

Summary—References 5 and 6

The two papers by Chang, Scott, and Springer (refs. 5 and 6) were reviewed because of Chang's references in his previous paper (ref. 4) to the failure criteria that were used in references 5 and 6. The Yamada and Sun criterion (ref. 3) is used for tension or compression. To take into account nonlinear behavior, Sandhu's strain energy failure criterion is incorporated into the Yamada-Sun criterion in the shear term.

Comments

Yamada and Sun emphasized in their proposal that their criterion was intended for cross-ply laminates and specified clearly the in situ value of the shear strength. Chang et al. (refs. 5 and 6) used the Yamada and Sun criterion for any type of stacking sequence, the shear strength that appears in the expression of the criterion being referenced in this paper to the shear strength of the laminate.

Summary—References 7 and 8

References 7 and 8 (Shahid and Chang) will be discussed together because, although they were published two years apart (in 1993 and 1995), they represent, with a different level of explanation, the same work.

A complete procedure of laminate analysis under biaxial loads is presented, involving a constitutive modeling and a damage accumulation prediction. The constitutive modeling considers matrix cracking failure, fiber-matrix shearing failure, and fiber breakage. The material properties at a certain value of the applied load are assumed to be a function of a crack density function, whose saturation value is defined as the one at which the rate of change of effective strength, with respect to crack density, becomes small. A degradation model is proposed to evaluate the values of stiffness properties at a certain instant of the degradation process, but the model has several material parameters which are difficult to determine experimentally.

The failure criteria employed (the authors call them the damage growth criteria) are based on Hashin's criteria but consider three mechanisms of failure: matrix cracking, fiber-matrix shear-out, and fiber breakage. With the exception of the strength in the fiber direction, the remaining properties involved in the criteria are assumed to be dependent on the crack density function.

This model is implemented in a finite element code allowing large deformations, and the numerical predictions are compared with experimental results obtained from other authors, fitting quite well with them.

Comments

The work of Shahid and Chang represents a complete approach for dealing with the problem under consideration. The only aspects not included are these: no criteria are proposed for a compressive state of stress, and nothing is said about interlaminar stresses and their effects. The authors mention that edge effects are not included in the analysis model, which may lead to overestimating the strength of the laminate, depending upon whether the stacking sequence is affected by the edge effects.

It is interesting that although the failure criteria Shahid and Chang use are based on Hashin's criteria, Hashin only considers two modes of failure: fiber or matrix, whereas they consider three: fiber cracking, matrix failure, and fiber-matrix shear-out. Note that the Hashin fiber breakage criterion is applied here as the fiber-matrix shear-out criterion, whereas fiber breakage is proposed to be governed only by σ_{11} , as in the Hashin-Rotem criterion.

It is also interesting that Chang changes significantly what he calls the constitutive model of the material with respect to the model used in 1987. Now, three effective and different modes of failure appear. The matrix failure is considered, as in the 1987 model. The fiber breakage is governed exclusively by σ_x , and the matrix-fiber shear failure is governed by the 1987 model, which, in turn, coincides with the Hashin criterion for fiber breakage in tension! Such a change clearly calls for some explanation by the authors, the present situation leading the reader to feel less than confident with either the first or the second proposals.

In general, a procedure such as the one proposed in these papers (or in the preceding one) has two disadvantages:

1. There are seven material-dependent parameters involved in the approach: (a) the crack density function, (b) the crack density saturation of a ply, (c) the shape of the parameter dictating the rate of stiffness degradation due to fiber-matrix shear-out failure, (d) the extent of the predicted fiber failure area, (e) the fiber interaction length for a unidirectional composite, (f) the parameter which controls the rate of material degradation due to fiber failure, and (g) the coefficient of shear nonlinearity. All parameters ought to be determined experimentally and proved to be, at the

least, a property of the material. The difficulty of the experimental determination of the parameters involved in the model leads to an indirect determination, that is, to determining their values using the model, which may convert the approach to a curve fitting.

2. There are two disadvantages to implementing the approach in an existing finite element code such as MSC.Dytran: First, the difficulty of implementing such a complicated model in a program (the codes usually only allow one to vary the failure criterion), and second, the time of running an actual structure, not a simple panel, may be prohibitive.

In this author's opinion, to take advantage of a model like this, certain additional work ought to be performed. The model should be used extensively with many different composites to investigate which of the variables involved in the formulation really controls the behavior of the material and more important, to get some simple ideas about the behavior of the laminate. Next, a certain macromechanical model ought to be designed to try to represent the behavior obtained in the micromechanical analysis in a manner that design engineers can use.

Summary–Reference 9

In reference 9, Christensen (1988) develops a 3D stress-strain relation for a transversely isotropic material that can be expressed in a regular part, plus in a term, which only includes the effect of the deformation in the direction of the fiber, a term that the author relates to an effect of the reinforcement. The relation is thus written in terms of four material parameters (E_1 , E_2 , and ν_{12} , plus a term that represents the effect of the reinforcement). To reach such a conclusion, the author must assume different relations between properties of the material at the macromechanical level.

The form obtained by this apparent constitutive equation leads the author to propose a failure prediction based on two different criteria, one associated with the failure of the reinforcement and the other associated with the failure of the system matrix-interface.

Although the two expressions of the criteria can be expressed in terms of stresses, the criteria are strain based.

Comments

The reasoning and procedure that Christensen follows in developing his formulation is very elegant and requires a good knowledge of mechanics in the field of lamination theory. Nevertheless, even when disregarding the degree of acceptance one may have of the assumptions, the author needs to discuss the sequence of relationships that lead to the constitutive law. It is questionable, when dealing with a nonhomogeneous material, to reason in terms of dilatation or distortion at the macromechanical level. The need to express the criterion in terms of strains is not completely clear, and it is not apparently based on physical considerations.

Christensen disagrees with the other authors, Hahn, Ericsson, and Tsai, when he considers it relevant that σ_{11} appears to be involved in the prediction of matrix failure. The others conjectured that σ_{11} can be neglected in a failure criterion involving matrix action. This important point will be addressed in section 5. The author recognizes that the proposed criterion focuses upon failure at the level of the individual lamina and must be supplemented by an auxiliary criterion to deal with delamination conditions in a laminate.

Summary–Reference 10

A new criterion based on macro in-plane stresses is presented by Christensen (ref. 10). The criterion considers two modes of failure: one associated with the failure of the fiber and another associated with the yield of the matrix. Four material parameters (two in each mode of failure) are required. The key assumption is that the macro stress σ_{11} affects only the fiber failure and not the matrix failure. Predictions are compared with the results of other authors and discrepancies are justified.

Comments

As described previously, there is fairly wide use of the criterion that is based on strains (see Christensen (1988), ref. 9). The first part of the paper is nevertheless devoted to explaining why it is better to use a stress-based failure criterion. It is also noticeable that the key point of the deduction of this criterion (the independence of σ_{11} in the matrix failure) was, in the opposite sense, one of the relevant aspects of the first criterion.

In addition to the hypothesis previously mentioned, the deduction of the criterion assumes that the matrix can yield under hydrostatic tensile stress but not under hydrostatic compressive stress. There is also no yield in the fiber under hydrostatic compression—all this in the range in which $\sigma_{22}^C \ll \sigma_{11}^T$ (the procedure involves the properties of the material satisfying $\sigma_{12}^Y = \sigma_{22}^C/2$).

The aim of this criterion's proposal is to take into account the presence of hydrostatic pressure, although not, as the author recognizes, under extreme conditions. Nothing is said about the appropriate degradation procedure that is to be coupled with the proposed failure criterion. The paper represents an interesting academic exercise but does not seem to be fully developed enough for implementation in a computer design program.

Summary–Reference 11

Hart-Smith (ref. 11) presents his theory of the failure of fibrous composite materials, which is based initially on a maximum shear stress criterion. The criterion is first presented for isotropic materials, then particularized for orthotropic laminae, with different cutoff lines being proposed for different types of failure that can appear, depending on the specific properties of the material and the configuration of the laminate and/or loading.

Comments

In reference 11, Hart-Smith (1989) presents his approach for dealing with failure of composite materials. In papers published later, the author complains about the lack of the physically based mechanisms of failure of Tsai and related criteria. However, in some way, Hart-Smith's proposal presents the same feature: the criterion is based on the failure of isotropic material and is then extended to orthotropic behavior, superimposing several cutoff lines that take into account the experimental behavior observed.

It is noticeable that different judgments can be made when comparing predictive and experimental results. When comparing Hart-Smith's data with the experimental results of Swanson et al., who used tubes, a lack of agreement is observed for loading cases leading to points close to the 45° cutoff in the fourth quadrant. Whereas Sun et al. question the necessity of having this cutoff, Hart-Smith "...suspects that the second batch of tubes was either made with more precision or of stronger material..." The problem is that this cutoff is one basis of the Hart-Smith proposal.

Reading and understanding the Hart-Smith proposal is difficult. What began as a single, simple rule, became a complicated set of rules, partially applied to different cases whenever the author tried to cover the behavior of different composite laminates.

Summary–Reference 12

In reference 12, Feng (1991) assumes that unidirectional fiber reinforced laminates behave as a transversely isotropic material, and he considers the case of finite elasticity. The stress state is written as a function of the invariants of the stress tensor, and a general failure criterion up to quadratic terms is generated. Assuming two uncoupled modes of failure (matrix and fiber modes) and associating the terms of the criterion to each of them, the general equation can be separated into two expressions, each having three material dependent parameters.

The criterion is finally particularized for the case of infinitesimal strains, showing a similar expression to the Tsai-Wu criterion.

Comments

In this proposal of a criterion for composite materials, the author emphasizes the dependence of the properties on the direction rather than on the nonhomogeneous character of the material. The criterion then could be applied to homogeneous transversely isotropic materials. The only concession to the particular behavior of composites is the consideration of two modes of failure, the assumption of uncoupling between them allowing the author to cancel one term of the general expression of the criterion, which is actually an extension of the Tsai-Wu criterion that accounts for finite deformations. This fact, which could be considered one of the most attractive aspects of the proposal, has doubtful application to composites, which may be loaded in large displacement conditions but rarely suffer, in macroscopic terms, large strains.

Summary–Reference 13

The paper by Pang, Pandian, and Bradshaw (1992), reference 13, presents two ways of simplifying the Tsai-Wu criterion. The first simply requires a knowledge of the fiber and resin properties, and the second requires, in addition, the performance of a single longitudinal tensile test. Errors of 25.4 and 15.9 percent are obtained in the two methods, respectively, with the vinyl-ester-fiberglass material tested.

Comments

The existing knowledge about composite materials makes it difficult to accept simplistic approaches that might be adequate for other materials. Thus, the approach followed in this paper and the reasoning used, at the micromechanical level (according to the authors), seems rather questionable.

Summary–Reference 14

The Yeh-Kim criterion (ref. 14) for isotropic materials is adapted to predict composite failures. The approach is the Tsai-Wu type but allows a piece-wise definition (by quadrants in the stress space), in an attempt to reflect, in a better way than the Tsai-Wu approach, the particular features of composites.

Comments

A small degree of flexibility in the possibility of adapting the failure prediction to actual behavior in composite materials can be gained with this approach (when compared with a classical tensorial approach), but to achieve it, a more detailed testing program would have to be carried out.

Summary–Reference 15

A new criterion is proposed by Chandler, Campbell, and Stone (1995) in reference 15. Separate failures for the fiber and the matrix are proposed. Fibers are predicted to fail when a critical rupture value is reached. The matrix is expected to break when the Mohr-Coulomb criterion is satisfied. A comparison is made with maximum stress and Tsai-Hill criteria for off-axis loading between 0° and 90° .

Comments

This proposal can be allocated to the Hashin and Rotem approach. The main criticisms to the proposal come from the description of matrix failure (nothing is said about the interface) and from the manner in which the authors evaluate the terms involved in the Mohr-Coulomb criterion, whose expression is

$$\left(\frac{\sigma_I}{S_T} \right) + \left(\frac{\sigma_{III}}{S_C} \right) = 1$$

The strength of the matrix in tension is taken as double the strength of the composite in a transverse direction to the fibers. The strength in compression is taken (in absolute value) as 3 times the strength in tension.

With reference to the stresses, the value of σ_1^m (the stress in the matrix perpendicular to the fiber orientation) is evaluated by applying the rule of mixtures, and the stresses σ_2^m and τ_{12}^m are assumed to be equal to the nominal stresses in the lamina.

There is no basis for understanding the proposal's detail. What is noticeable is the close agreement between the predictions of this criterion and the maximum stress and Tsai-Hill criteria that the authors have previously discarded, showing that they are able to produce counterintuitive results. Comparison with biaxial test results are mandatory to discriminate between the acceptability of failure criteria.

Summary–Reference 16

The first part of the paper by Kopp and Michaeli (1996), reference 16, is devoted to presenting a case (a $\pm 45^\circ$ tube in torsion) in which the interfiber failure is the dominant mode of failure. The hypotheses of the more recent Puck criterion are presented next. The hypotheses basically state the following:

- Interfiber failure in a plane parallel to the fibers is governed by the three components of the stress vector associated with such a plane, namely the normal stress acting on that plane and the two tangential stresses, one parallel and the other perpendicular to the fiber direction.
- The two shear stresses always promote fracture, whereas normal stress promotes fracture if it is a traction and impedes it if it is a compression.

Next, the two expressions of the criteria are written, and an explicit expression of the fracture plane is given. Finally, some experiments for verification of the proposed criteria are presented.

Comments

Reference 16 has been reviewed and included in this report in conjunction with reference 17 (Kroll and Hufenbach (1997)) who refer to the Kopp and Michaeli paper for details about factors appearing in their version of the criterion for the failure of the interface in compression.

Unfortunately, the paper by Kopp and Michaeli was a bit confusing for several reasons. The authors do not explain the meaning of the symbols involved in the expressions that appear in the paper. The expressions of the criteria do not coincide with those of Kroll and Hufenbach, and it is not clear whether the authors are just explaining the Puck proposal or presenting their own proposal (revised by Puck).

Apparently there are four German groups working on a project whose main objective is the generation of a fracture criterion for composites. Most of the publications are in German and the proposals intersect with each other. The publications of these groups will have to be followed in the future.

Nevertheless, the basic ideas of the criterion that appear in reference 16 coincide with those in the paper by Kroll and Hufenbach. Once again, what is clear to this author is that these proposals are a distinct advance in the direction of Hashin’s proposal. What is less clear is that micromechanical arguments are again invoked to distinguish between two modes of failure. Once this distinction is made, the material is treated as homogeneous. A polynomial expression is then proposed and is modified to account for obvious facts stemming from reasoning and with experimental support. The line of continued reasoning at the micromechanical level, when proposing an interfiber failure criterion, seems to be underexplored.

Summary–Reference 17

Kroll and Hufenbach, reference 17 (1997), argue for the use of a physically based failure criterion: Hashin’s criterion and Puck’s criteria, a first version published in 1969 and an updated version published in the 1990’s. The authors (ref. 17) dedicate maximum attention to the case of fiber-matrix failure and, in particular, to the case of compressive stresses perpendicular to the plane of failure. They introduce a criterion that takes into consideration the positive effect of the compressive stresses, due to “internal material friction,” in the fiber-matrix mechanism of failure.

$$\left(\frac{\tau_{nl}}{S - p_{nl}\sigma_n} \right)^2 + \left(\frac{\tau_{nt}}{S_T - p_{nt}\sigma_n} \right)^2 = 1$$

They present results associated with tension/compression-torsion tests and high-pressure tests.

Comments

The reference 17 study carried out and partially described in this paper is of undoubted importance in the approach of failure criteria based on failure mechanisms. What is not understandable is that the editor of the journal has allowed crucial parts of the proposal to be referred to in German publications. Understanding the facts is of particular importance when proposing the failure criterion for fiber-matrix

in the compression region. There are aspects of the proposal associated with the action plane resistance that must be clarified for a correct understanding and application of this modified version of Hashin's criterion. These aspects address one of the questions that Hashin himself classified as able to be improved—the estimation of the plane of fracture in the fiber-matrix failure mechanism.

Although this author understands and agrees with the sense of the criterion modification Kroll and Hufenbach propose (the experimental results clearly lead to this modification), he has some reservations about the reasons invoked for such a modification (i.e., “internal material friction”). What does this modification mean in referring to nonhomogeneous materials? How is this effect implemented in the criterion? Accepting the quadratic form of the coupling between τ_{nt} and τ_{nl} , what does not seem to be coherent is altering the denominator (where parameters of the materials appear) with the stresses, which, in all the criteria, are placed in the numerators and are different for each load case.

The criterion requires the determination of a parameter to quantify the contribution of the compressive lateral stresses to failure. What is questionable is determining the value of these two parameters by using the same tests employed to show the correct agreement between experimental and predicted results. The tests are, on the other hand, correctly selected and have a sufficiently complex state of stresses to be considered representative.

The authors do not clarify the stacking sequence they are using, but the context of the paper leads the reader to think they are primarily concerned with unidirectional laminates. It would then be necessary to extend the study to nonunidirectional laminates and to study the role of delaminations in the strength predictions. The authors do not make any reference to the necessity of using a degradation procedure in the design of a general laminate.

Summary–Reference 18

In reference 18, Echabi and Trochu (1997) propose Kriging to generate a failure envelope. Kriging is a statistical technique that was proposed by Krige in 1951 to evaluate natural resources. Its purpose in mathematical terms is to estimate the value of a model function when given a set of measurements at certain locations and when the derivatives along certain directions at certain locations are also known.

Applying this technique, the authors generate the failure envelopes for a unidirectional graphite-epoxy composite and for a graphite-epoxy fabric. The failure envelopes obtained are compared with other predictions and also with already published experimental results.

Comments

Without giving the particulars of a Kriging procedure, which are beyond the scope of this review, it can be said that the failure envelope is generated by three sets of series expansions. The shape of the terms is defined by the designer to fit the experimental results better, and the coefficients are determined by using the information available about the failure of the particular composite for which the failure envelope is to be generated. These mathematical problems can be solved in different ways (see for instance París, Picon, Marin, and Cañas, *Exp. Mechanics*, Vol. 37, No. 1, March 1997, pp. 45–55) for a similar problem in photoelasticity. The important feature of Kriging seems to be that it allows one to impose locally some restrictions based on the observed behavior of the material and is what gives the authors license to allocate their procedure to a failure-based criteria group.

Some aspects of the mathematical procedure might be further investigated. In particular, the use of the same terms in the series expansion as the number of restrictions needed to be reflected in the

criterion seems to be a very useful procedure (Picon, Paris, Cañas, and Marin, *Eng. Fracture Mechanics*, 51(3), 1996, pp. 505–516), although the envelopes shown by the authors do not seem to be affected by this decision.

In any case, the approach proposed by the authors (ref. 18) is, in general, a suitable one. It has been successfully used in other problems, for instance, in the estimation of the maximum bending moment that a reinforced concrete section can suffer or the generation of the failure envelope in the seams of a welded joint. The difference is that in other cases, as in the two mentioned, a general failure criterion has been found independently of the materials used. In the case of the welding joints, for instance, the criterion changes from one country to another, but its expression is independent, for each country, of the materials used. In the case of composites, it is hard to accept that only one general envelope, a function of material-dependent parameters, can be generated. If the Kriging procedure has to be developed each time for each material, it is obvious that this procedure would limit seriously the applicability of the approach, which would then be reduced to research purposes; however, it would also be worthwhile. Design engineers who need a more straightforward approach would not find it usable.

2.2. Papers/Publications Revising Existing Approaches

Summary–Reference 19

As in many of his papers, Hart-Smith (ref. 19 (1992)) is devoted to demonstrating the inability of the Tsai criterion and its innumerable clones to take into consideration different modes of failure for composite laminates. The 10 percent rule criterion, already proposed by the author, is again reconsidered as an engineering approach to deal with composite laminates.

Comments

Considering the particular way in which the author presents his ideas, there are, in this author's opinion, many valuable points in this paper and in the ideas advocated by the author.

The most important is that it is a serious mistake to believe that anisotropic failure criteria for homogeneous material can be applied to fibrous composite materials, considering the important point that these composite materials behave macroscopically when they are anisotropic and not considering the nonhomogeneous character of the composites. However, a failure criterion must be associated to a failure mechanism, and in composite materials, they appear at the micromechanical level. When one reaches this level, it is obvious that the problem is the number of variables involved in the formulation and the difficulty of measuring them.

The ideal way to proceed, though clearly not possible, would be to identify the failure mechanisms. Each of them ought to be characterized by one parameter whose value was determined by one test that is able to activate, in isolation, the mechanism under consideration. Other measurements on other tests would only lead to repetitive information. Once the mechanisms had been identified, it would be necessary to look for the macromechanical variables most closely associated with those mechanisms.

The need to perform tests at the biaxial level is obvious, although the exclusive nature of these tests is not so clear.

Summary—References 19 and 20

In reference 20, Hart-Smith (1993) answers the question which appears in the title of his paper—in the sense that fibrous composite failure modes must be superimposed and not interacted by showing a couple of nonsensical consequences of the interacted character of a criterion. This question is partially revised in his previous, more detailed paper (ref. 19).

Comments

Hart-Smith has developed a theory to take into account failures in composite materials. The author explains in reference 20 that he is going to try, in a series of short papers, to show how the classical Tsai and similar criteria are not adequate to deal with the prediction of the behavior of composites in terms of strength. In reference 20, two examples are given. The most illustrative one shows that stronger submarine hulls could be fabricated from composite materials with degraded transverse tension strength.

Summary—Reference 21

The equations representing failure theories of isotropic and nonisotropic materials, with a brief explanation of the main features of each theory, are listed in reference 21. For the case of composite materials, 30 theories were found during the study for failures of both the lamina and laminate level, and 12 theories were described for postfailure behavior.

Comments

The involvement of the author in the positive and negative aspects of the different proposals is slight, and actually the objective of the paper is not a critical evaluation of the theories of failure but a survey of them.

Summary—Reference 22

The aim of this contribution by van Wamelen, Johnson, and Haftka (1994, ref. 22) is to compare the ability of two schemes: the Tsai scheme (to predict composite strength) and the Hart-Smith criterion to design laminates. The Tsai scheme uses an approach that involves the use of Tsai-Wu criteria and a degradation procedure to estimate last-ply failure. A procedure is used to detect the load configurations leading to the greatest differences between the Tsai and Hart-Smith predictions. For practical reasons, the load case that maximizes differences between Hart-Smith and the Tsai prediction that corresponds to an N_x (plus an N_{xy} of about one half) dominant load is selected. The results show an enormous overestimation of failure strength for Hart Smith's approach (66.36 percent error) and a reasonable underestimation (−9.66 percent error) for Tsai's approach.

Comments

Papers of this kind that try to distinguish between different proposals are clearly needed. The problem, as the authors themselves recognize, is that matrix failure may play an important role in the mechanism of failure in the combination of the laminate and the load selected. Hart-Smith's criterion, as the author clearly states, is a criterion intended for laminates, in which the breakage of the fibers dominates the failure of the laminate. The authors of this study have therefore been more concerned about selecting a configuration to force (to extremes) the differences in both approaches than about selecting a case of applicability for both criteria. In any case, because the Hart-Smith criterion is not able to predict the

behavior of a laminate like the one analyzed by the authors [0/90/45/0/90/−45/90/0]_S, it is impossible to consider this proposal as a general purpose failure criterion.

Summary–Reference 23

Bogetti, Hoppel, and Drysdale (1995), reference 23, offer a final report of a research project that implements (in a code) the simplified thick laminate theory of Chou et al. (1972). The theory is essentially based on two assumptions: (1) in-plane strains are assumed to be uniform through the thickness of the laminate, and (2) stress components associated with the out-of-plane direction are assumed to be uniform.

Eight failure criteria at the lamina level are considered: (1) Von-Mises-Hencky, (2) maximum stress, (3) maximum strain, (4) hydrostatic pressure adjusted, (5) Tsai-Wu, (6) Christensen (1988), (7) Feng, and (8) a modified Hashin's failure criterion. First and progressive ply failures at the laminate level are considered.

Comparisons of predicted 3D properties, as well as the strength of several laminates, are provided.

Comments

It is striking that the detailed presentation of the theory and the program contrasts with the very small part of the report which is devoted to a comparative study of the eight failure criteria that are implemented. Worth mentioning is a new version of Hashin's criterion for failure of the matrix in compression. In connection with the modifications of Sun et al. and Kroll et al. (already commented upon in this report), the authors (ref. 23) take a different option: they cancel the contribution of σ_{22} to the failure, but they do not modify the contributions of the other two terms. With this assumption, they achieve excellent agreement in the predictions for one material and very poor results for another material, which leads to the conclusion that something in terms of materials properties and/or stress state must be added to the terms of the criterion.

Worth discussing, in any case, would be the advantages of such an approach as the one followed here by the authors. It would have been a good occasion for them to compare, in terms of calculation time, the advantages with respect to nonsimplified thick-laminate theories.

Summary–Reference 24

Reference 24 (Sun, Quinn, Tao, and Oplinger, 1996) is the final report of a research project, the main purpose of which is to compare predicted results by using the most widely used failure design criteria, and also by using experimental results when they are available. The authors distinguish between lamina and laminate failure analysis.

At the lamina level, the authors use six criteria selected from an AIAA failure criteria survey. The percentage of people using each criterion appears in parentheses: maximum stress and strain criteria (30 percent and 22 percent, respectively), Tsai-Hill (17 percent), Tsai-Wu (11 percent), and Hashin and Hashin-Rotem (20 percent), which is shared with all other criteria. At the lamina level, the criteria are compared for cases of bidirectional stresses, off-axis loading, and pure shear. From the experimental data analyzed, it can be observed that lamina shear strength increases as the σ_{22} stress increases in the compressive range, a fact not easily discerned by the criteria studied.

The authors conclude this section by proposing a separate failure mode criterion for the fiber and matrix, which is actually a modification of the Hashin-Rotem criterion, to take into account the beneficial role that a compressive stress σ_{22} can have on matrix failure under compression.

At the laminate failure level, the authors first revise the two possibilities of stiffness reduction based on a parallel spring model or on an incremental reduction model, the first being extensively used in the report. They then discuss the use of a ply-by-ply discount method or the sudden failure method when partial failures start to appear at different plies, as well as the final truncated maximum strain envelope criterion by Hart-Smith.

Next, Sun et al. study the case of laminate failure under biaxial loading and unidirectional off-axis loading. By comparing the available experimental data, they conclude that the interactive criteria (Hill-Tsai and Tsai-Wu) and the Hashin criterion (which couples σ_{11} with σ_{12} in the failure of the fibers) underestimate ultimate failure, and what is more, are not able to predict the trend of the experimental results. On the other hand, maximum stress, strain, and the Hashin-Rotem criteria all perform quite well for the cases the authors have studied. Additionally, these criteria are relatively insensitive to inaccurate lamina strengths Y and S , have properties of disperse values within themselves, and are difficult to evaluate.

The authors also take into account the need to consider a ply-by-ply discount process in laminate failure, whether or not matrix failure occurs. They conclude that matrix failure happens, although they refer to an inspected specimen [0/90/0] that has been applied at 95 percent of the failure load.

Comments

Of all publications reviewed, this one (ref. 24) is most closely related to the objectives of the present report. The conclusions are plausible and well supported. Because the work presented has been detailed in the previous summary, only a couple of points, worthy of further discussion, will be considered in these comments. At the lamina level, the authors propose a “new” failure criterion for the matrix in compression, whose expression is

$$\left(\frac{\sigma_{11}}{X}\right) = 1 \quad \text{for fiber failure}$$

$$\left(\frac{\sigma_{22}}{Y}\right)^2 + \left(\frac{\tau_{12}}{S - \mu\sigma_{22}}\right)^2 = 1 \quad \text{for matrix failure}$$

where μ is a material parameter.

The expression for matrix failure prediction is a modification of Hashin’s (1995) criterion that has also been modified in a similar way to the one proposed here by Puck and some German groups (see ref. 17, Kroll and Hufenbach (1997)). It can, of course, be considered a modification of Hashin’s criterion. The second term in the denominator of the τ_{12} term is said to take into account the increase that a negative value of σ_{22} has on the fiber-matrix interface shear strength. Two questionable decisions are made in the expression the authors propose. If σ_{22} plays a positive part in the failure, it does not make sense to keep the first term in the failure criterion because the higher the value of σ_{22} , the higher the positive contribution in the second term, and also the higher the negative contribution of the first term. In fact, Kroll et al. cancel, in their proposal, the first term of the expression. The second inconsistency is

common to the Kroll proposal and is the decision to mix strengths and stresses at the same level (in the denominator).

The second observation is that nothing is included in the models about delaminations, which in many of the tests ought, at first sight, to play a significant (or at least some) role. Does the fact that good agreement between experimental and predicted results is reached imply that delamination does not play an important role from a quantitative point of view? Some considerations about this question ought to be addressed by the authors.

A similar thought arises from the consideration of residual stresses during curing. The authors mention this phenomenon at the laminate level (without making any particular decision about it), but nothing is mentioned about the lamina level. With reference to the particular mechanism of failure to which they dedicate special attention, this fact ought to have been taken into consideration.

Summary–Reference 25

Echabi, Trochu, and Gauvin, 1996 (ref. 25) present a review of the most recent failure criteria proposed. They divide them into two sets. The first set groups criteria not based on failure modes: polynomial, tensorial, quadratic, cubic, and parametric. The second set groups criteria based on failure modes: maximum stress or strain criteria, Hashin's criterion, Hart-Smith's proposal, and Kriging, this last one being the approach followed by the authors in their personal proposal of a criterion, presented separately in reference 18 (previously reviewed in this report).

Comments

The way in which the authors group the criteria included in the revision is rational. The omission of some former proposals and approaches is not explained unless it is understood that they consider them worthless, which is by no means apparent. The revision is distorted because the authors are directly involved in one of the approaches they review (Kriging), to which they devote a great deal of attention.

It is positive to hear their concern for the consideration of residual stresses in a failure criterion (although nothing specific is proposed), and it is noticeable that there is no mention of out-of-plane effects in the failure of a laminate.

Summary–Reference 26

Reference 26 (Bower and Koedam, 1997) presents a study of the limits of the coefficients of the tensor polynomial failure criterion in quadratic (Tsai-Wu), cubic (Wu), and quartic forms, based on the convexity of the failure surface in the stress space.

Comments

The most interesting result is that the cubic form, considered an improved form of the familiar quadratic criteria, is not able to generate a convex failure surface.

2.3. Papers Dealing With Out-of-Lamina Failures

Summary–Reference 27

Reeder (ref. 27) first presents a redesigned mixed-mode bending test apparatus. Next, he explains the way in which toughness is evaluated and applied to three different composites, followed by a review of how the most plausible mixed-mode delamination criterion is performed. A new bilinear criterion is proposed, based on the appearance of the delamination fracture surfaces, which suggest a change in the failure mechanism near a ratio of $G_I/G_{II} = 1$. Subsequently, the results for the three materials tested are compared with the predictions of five failure criteria: power law, exponential hackle, exponential K_I/K_{II} , interaction, and bilinear criteria.

Comments

Reference 27 is an excellent work, very carefully set out and developed. The experimental work is of particular relevance. The need to propose a bilinear criterion is not perhaps clear. Obviously, the advantage is that it can be easily managed as compared to other former criteria that also fit the experimental data well.

The problem of all criteria considered in the paper is that, being based on a fracture mechanics approach, they are difficult to implement in a computer code for design of composites at the same level as in-plane failure criteria (e.g., Hashin's criterion).

Summary–Reference 28

Reference 28 (Wang and Socie (1993)) is an experimental paper, in which the behavior of a certain class of composite material under compressive loads is studied. Unidirectional and cross-ply laminates are considered. Three failure mechanisms were found: (1) A material failure mode that included matrix shear failures and fiber failures by either microshear or bending, (2) delaminations, and (3) global buckling.

The main conclusions of the tests were these:

- For both unidirectional and cross-ply laminates, the failure mode and the failure strength and strain in one direction were not affected by the stress or strain in the normal direction.
- Biaxial compressive strength is adequately described by the maximum stress or strain failure theory.
- Delamination is the lower strength damage mode compared to the kink band and shear failure modes.
- Large sections and lower lateral support enhance the possibility of having delamination and then lower strength. Smaller sections and strong lateral support reduce stresses associated with delamination, thus allowing the activation of high-strength failure modes.

Comments

Papers like reference 27 are necessary for the understanding of those mechanisms of failure which may lead to tailoring better composite materials and are indispensable for proposing realistic failure

criteria at the macromechanical level. One noticeable feature is the authors' explanation for the increasing attention given to the study of failure of composites under compressive loads.

The work seems to be well developed and the paper well written. The conclusions seem to be in agreement with the results obtained; however, there is a crucial point in the evaluation of the significance of the paper that deserves a comment. On the one hand, the authors maintain that the strength of the laminate is well described by the maximum stress and strain theory which is based on in-plane variables. On the other hand, they point out that it is the delamination, governed by out-of-plane stresses, that is the mechanism of failure. This apparent contradiction ought to be explained.

Summary–Reference 29

In this abstract (ref. 29) by Reedy and Mello (1996), a method is described in which a special element is placed between two sublaminates to consider the appearance of delamination. The element is implemented in a general finite element program to perform crashworthiness studies.

Comments

This abstract describing interesting aspects of the inclusion of modeling delamination in a finite element analysis is promising. The fact that the size of the elements does not play an important role in the results obtained is relevant. The implications, from a numerical point of view, of the introduction of this intermediate element should be clarified. Also, the sequence in which these elements are introduced in the model (where and when and in accordance with what criteria) should be explained. This work is worth following in the future.

Summary–Reference 30

In reference 30 (de Moura et al. (1997), an element able to represent delaminations is developed and implemented in ABAQUS. The element (1) considers rupture and delamination growth when a failure criterion is satisfied, and (2) models the contact between the two delaminated surfaces avoiding interpenetrations. The failure criterion employed by de Moura et al., 1997 (ref. 30) is the Quadratic Delamination Criterion in terms of stresses and strengths. The authors recognize that the modeling approach proposed requires a great deal of computing time and that some refinements have to be made to avoid some problems when sliding between delaminated interfaces takes place.

Comments

It is clear that a delamination criterion in terms of stresses is appropriate from a finite element modeling point of view. As a general reflection, what is difficult to accept from a mechanical point of view, is that the same criterion may, at the same time, be used to predict the onset of the delamination as well as its growth. What is more, it may be used, in the presence of a delaminated area, to predict delamination growth in the presence of a contact zone (nonsingular stresses) and in the absence of it (singular stresses).

Summary–Reference 31

The composite behavior of laminates with delaminations (through-the-width and embedded) is studied by Kim (1997) in reference 31 using a finite element modeling approach. The author identifies three different modes of instability in his study.

Comments

The emphasis in this paper lies in a different direction from that of the present report. It is assumed, in the postbuckling analysis which was done, that a delamination exists and that its growth is ignored. We believe that “such an assumption is valid for overall compression behavior of composite laminates.”

Summary–Reference 32

Fleming and Boonsuan (ref. 32) attempt to verify the actual possibilities of including delamination in a finite element crash code. Three models are considered: the tied connections with the Force-Based Failure Model, the Cohesive Fracture Model, and the Virtual Crack Closure Technique Model. The three models are implemented in MSC.Dytran code, and the double cantilevered beam (DCB) specimen, from which experimental results are available, is taken as a test problem for comparison.

The authors conclude that the onset of delamination is reasonably well captured, whereas the propagation presents some discrepancies between experimental and predicted results, propagation being much more rapid in the computation than in the experiments.

Comments

The work developed and presented in this paper is a necessary step prior to including delamination in a finite element code for design of actual composite structures against crash. Two limitations, referenced by Fleming and Boonsuan, clearly appear in this task. First, the mesh requirements for accurate delamination modeling (particularly when using the crack closure technique) are very demanding in the case of a crash analysis. Second, the existing knowledge is very limited with reference to the modeling of material properties involved in the problem. Thus, although the previous experience of the authors and their initial feeling is addressed to the use of the virtual crack closure technique, they finally propose, and it seems a very realistic proposal, that at the present moment, with all the limitations in mind, the force-based tied connection may be the most affordable way of treating the problem.

Another problem is the limitation that an existing finite element program such as MSC.Dytran presents for implementation of external models. This fact obviously limits the capacity for performing several analyses in real structures.

Two aspects finally require a further comment. The first is that the authors performed the study with the presence of a crack 45 mm long. In an actual problem, the existing inherent cracks cannot be modeled, and the behavior of the approaches analyzed (some of them are unable to work without the presence of a crack) ought to be studied. The second aspect refers to the presence, at the same time, of in-plane failures and delamination failures (because the latter can affect the former), and the strategy to be followed in the analysis has to be carefully thought out. In fact, the use of a force-based tied connection approach would require the presence of interelement surfaces at all the surfaces susceptible to delaminations, which, in computational terms, would be prohibitive if the whole structure were to undergo it. The design of a finite element with a possibility of degradation of properties reflecting the presence of delamination inside the element seems to be an affordable way to accomplish the task, but such an approach has not as yet been developed.

2.4. Papers Comparing Different Approaches or Carrying Out Studies on Particular Cases

Summary–Reference 33

Cohen et al. (1995) present an in-depth study (in ref. 33) to predict the strength of thick multifastener composites. With reference to the objective of this report, the study considers the average stress criterion proposed by Witney and Nuismer. The effects of the three stresses (radial, circumferential, and shear) are quadratically combined.

Comments

The quadratic combination of effects would require the determination of the strengths to radial, circumferential, and shear stresses in an independent form and for any angle, which does not seem to be an easy task to accomplish. The authors assume that the hoop stress is the dominant stress around the hole, which simplifies the problem. The determination of the strength of the unnotched laminate and of the characteristic distance involved in the criterion is not very well explained, and these values play a crucial role in the results and consequently in the evaluation of the criterion's adequacy.

Summary–Reference 34

In reference 34, Cheung and Scott (1996) use MSC.Dytran for the analysis of damage in carbon fiber composite panels. They use the Chang and Chang failure criterion considered in the program, setting the nonlinear parameter α of the model equal to zero.

Comments

Although the title and the summary of the paper suggest that it might be of interest for this report, the authors place more emphasis on impact modeling features than on the influence of failure criterion on the results.

Summary–Reference 35

Reference 35 (Ireman et al. (1996)) is an overview of the work developed up to the moment of publication by the GARTEUR Group, a European group formed by research institutes and industries from six different countries. Both the tests and the modeling that are being carried out are described. Different codes (ABAQUS, LS-DYNA3D, SAMCEF, STRIPE, DYN-CRACK) as well as different approaches (strain energy-based failure criteria and damage theory) are used in the different GARTEUR groups.

Comments

The final objective of the ongoing work is to make proposals for improvements in prediction rules. The work is not finished, but the creation of this research group has generated significant expectations regarding the conclusions of the work being developed. Close attention will have to be paid in the near future to the publications arising from this effort.

Summary–Reference 36

Two approaches are presented in reference 36 (Fish et al. (1997)) to deal with the complete analysis of composites. The micromechanical approach is based on the mathematical homogenization theory and divides the analysis into two stages. In the first, a nonlinear macroscopic analysis is carried out by creating a database at Gaussian points in the critical regions. Next, critical unit cells are studied at the micromechanical level.

The macromechanical approach is only roughly explained in this paper. The basic idea of the proposal is to enrich the kinematics of the shell elements to take into account 3D (edge) and delamination effects. Delamination indicators have to be developed to predict the critical regions; therefore, enriched shell elements would be used only when and where it is necessary to do so.

Comments

Although nothing is said in this paper about the failure criteria of fibrous composite materials, reference 36 has been included in this review because it addresses the whole problem of analyzing a composite structure. To reach a micromechanical level of analysis in the design does not seem to be a very practical way to proceed, both for numerical reasons (although the features they present in terms of computational time are excellent) and for the limited existing knowledge of failure at the micromechanical level.

The idea of treating edge effects and delaminations by adapting the shape functions to the damage without modifying the mesh seems to be the most promising way to proceed. This idea is only a scheme of the approach they are working on. The publications of this group ought to be followed.

Summary–Reference 37

A progressive failure methodology is implemented in Program COMET (see ref. 37, Sleight, Knight, and Wang (1997)). Three failure criteria are used: maximum strain, Christensen's criterion (1988), and Hashin's criterion. Four types of failure are considered: fiber failure, matrix failure, shear failure, or an interaction of failure modes. The progressive failure follows Pifko's work. Two examples are analyzed: a rail shear test panel and a compression-loaded composite panel, the numerical predictions being compared with experimental results.

Comments

Reference 37 is an excellent and necessary numerical study. The description of the progressive failure criterion that was followed and the influence of the parameters that were employed in the degradation of the material ought nevertheless to have received a more detailed discussion. The influence of the values of these parameters on the results is a key factor for the acceptance of a failure criterion.

The numerical model does not take into consideration interlaminar stresses and associated modes of failure (delamination). Nevertheless, the numerical predictions are in close agreement with the experimental results, which for this case implies that the mechanisms of failure associated with these stresses are not involved in the actual failure of the panels studied. The close agreement between the predictions of Hashin's and Christensen's criteria has to be pointed out. A description of the procedure followed in the selection of the parameters involved or the adjustment procedure followed would have been welcome.

As the authors recognize, a study involving interlaminar stresses and associated failure criterion ought to be developed to prove the applicability of this numerical scheme to general laminates subjected to general loads.

Summary–Reference 38

A procedure to design composite structures is presented and evaluated in reference 38 (Moas and Griffin (1997)). The procedure is based on a finite element analysis and uses Tsai-Wu and maximum strain as the failure criteria. Two constitutive damage criteria are considered in this paper. The first is a ply degradation model which considers that the damage is complete and irreversible, and that it is implemented by setting certain material constants to zero, depending on the three failure modes considered: fiber, matrix, and shear failures. The second is a statistical strength model that assumes that the stiffness loss associated with the fiber tensile failure is proportional to the probability of fiber failure; Rosen's weakest-link fiber model is being used to determine the probability of failure of a fiber bundle. The procedure is applied to the study of an actual structure: a frame representing ring stiffeners for rotorcraft fuselage.

Comments

A complete study such as the one carried out in this paper involves making many decisions about different aspects of the problem. One must also consider that the isolation of the effects of each of them is difficult, but at the same time necessary, to exploit the full capability of the study and to deduce all possible consequences from it.

This paper is not well balanced in the description of the different aspects of the study, making it difficult to assess the consequences that were obtained from it. There are several nonrelevant experimental aspects that are extraordinarily well explained, whereas some procedural aspects are explained only roughly.

To illustrate this point, the authors do not mention, for example, whether the experimental results obtained from the frames were repetitive or not. Also, they consider three failure mechanisms, but they do not explain how they associate the values obtained from the criteria used to the presence of the failure mechanisms.

It is mentioned that a particular strain gauge did not produce good results, but these results are not shown. The authors present several factors that may contribute to the disagreement between predicted and measured results. The most obvious of these is, in this author's opinion, not having modeled the interaction between the frame and the applied load by means of a contact zone, particularly considering that there is a vertical displacement in the frame that measures 5 in., even though the zone of interest is confined to vertical displacements of 1.5 in. The lack of this type of modeling makes it impossible to check the validity of the approach followed in the zone of major interest: the zone where the load is applied.

Moreover, the range of vertical displacements clearly suggests that delaminations ought to play an important role in the energy absorption of the frame. However, while not considering it, the authors achieved quite good agreement between experimental and numerical predicted results, suggesting that, against what might originally be expected, the role of this mechanism in the strength behavior of the structure is not very important, a question unfortunately not evaluated in the paper. The different results obtained with the different failure and damage criteria for two different materials also ought to have been analyzed in greater depth.

Studies of the kind developed in reference 38 are absolutely necessary to discriminate between the different theories of failures and damage progression. The impression one gets from this work is that it was an excellent opportunity to have compared different approaches to designing composites and that, considering the total amount of work, not much more time would have been involved.

Summary–Reference 39

In reference 39 (1996), Hinton, Soden, and Kaddour study the experimental behavior of filament wound GRP tubes for three different orientations of the fibers under loads that originate a biaxial state of stress. The results (failure initiation and final failure) are compared with the predictions by using first-ply lamina failure and a simple degradation procedure.

The authors point out two necessities in further research: (1) develop a more accurate prediction of the residual values of E_T , ν_{LT} , and G_{LT} in a cracked laminate, and (2) determine whether a lamina has a unique set of strengths since they are influenced by the layup configuration within which the lamina is situated.

Comments

Works like reference 39 are absolutely necessary and will represent an obligatory reference for future models in predicting failure mechanism of composites. It is obvious that finding actual fracture strengths with a factor of 10 for failure predictions clearly shows the need for research in this field and the unexplored possibilities of these materials.

One point that needs attention, nevertheless, is the enormous relative difference found between predictions and experimental values that are between $\pm 75^\circ$ (reasonable predictions) and $\pm 55^\circ$ (very conservative predictions). Such results emphasize the need for deeper knowledge of the failure mechanism of these materials.

Of the two research requirements called for by the authors, the first, in particular the stress interactions, requires a better understanding of the failure mechanism, which can only be accomplished by micromechanics analysis that is focused on the aspect under consideration.

With reference to the second aspect of future research required, it is expressed, although the authors do not use the term explicitly, in terms of in situ properties. The author of this report understands that the use of the generalized term, in situ properties, may lead to some misunderstandings. It would then be preferable to describe this necessity as the establishment of a reliable prediction of actual strength properties of a lamina (in a laminate under an actual 3D state of stress) but calculated under a nominal 2D state of stress. The objective described in this manner, the achievement of which is by no means certain, would explain what, in this case, would be under the in situ property term.

2.5. Papers From Industry

Summary–Reference 40

Reference 40 (Berczynski (1996)) reviews the actual procedure followed by the Boeing group in developing the design of the composite structure of a helicopter. The author places great emphasis on the reliability assessment of composite elements and on the need to treat both material properties and loads by means of a probabilistic analysis.

Comments

It is very interesting, in this paper, to observe the design details of an actual composite structure, details which are particularly well explained here. Some of these details are interesting enough to account for damage originated by low-energy impacts such as an open-hole-compression allowable. Also interesting is the simple methodology used, in which the allowable strain in each of the fiber directions is dependent upon a parameter called the “angle minus loaded plies,” allowable fiber direction strains being expressed as a linear function of this parameter. Although the concept of this parameter is not clearly explained in the paper, the impression is given that the design rule is even simpler than the 10 percent rule proposed by Hart-Smith. The author explicitly states that this method does not require any failure criterion. Also noticeable is the number of coupons tested during the design of the structure (9700).

Summary–Reference 41

In reference 41, Feillard (1999) presents a complete study to evaluate the possibility of modeling crash behavior of actual composite structures. Most interesting is that the modeling of the composite material used in the actual structures was based on experimental tests that showed (1) a clear elastic-plastic behavior with a constant strain-hardening, (2) maximum stress/strains much higher than those obtained in static loading, and (3) that the yield stress increases with the strain rate. A Johnson-Cook mechanical load was selected to represent a behavior with these features.

With reference to the foam, its mechanical response was assumed not to be strain dependent, considering that the energy absorption capacity of foam is much lower than that of the composite.

Comments

Many decisions and hypotheses have to be made to perform an analysis such as the one addressed in this study. The study is handled quite carefully, and most of the decisions are based on realistic and mechanically based reasons. The most risky assumption made is that the high-speed failure of the composite is driven by tensile properties, a decision resulting from the difficulty of performing high-speed compression tests.

The author addresses, in detail, the modeling approach of the connection between the composite and the foam, whereas the modeling of delamination and its role in the energy absorption mechanism, which ought to be an aspect of major concern in the study, does not seem to be very clear at all.

The author mentions the high scatter in the experimental results in composite materials. This fact, inherent at the present moment of technology in fabrication of composites, is mentioned at token level. Nothing is said about the scatter in the global response at the real structure size level, an aspect that is very expensive to evaluate but very interesting to know.

2.6. Composite Structures Design Papers

Summary–Reference 42

Antonio, Marques, and Goncalves (1996) propose (in ref. 42) an optimization procedure to design laminates. The authors use the Tsai-Wu criterion, and the degradation model is based on Tsai’s totally degraded ply model. The degradation of the mechanical properties of each ply is performed, keeping the

longitudinal ply stiffness constant. The remaining elastic stiffness constants are reduced by a degradation factor.

Comments

Although the objective of the paper focuses on design optimization of laminates, taking the weight as the objective cost function, this paper has been reviewed to check first the criterion and degradation procedure used, and second, to learn whether the authors performed any study of the influence on the results of the parameters involved in the model.

Summary–Reference 43

In reference 43, Morton and Weber (1997) address the design of composite laminates by using the methodology of heuristic redesign (one in a series of related papers). The rules of heuristic redesign are based on stiffness requirements or first-ply failure requirements. The authors refer to an unpublished paper for interlaminar failure considerations.

Comments

The procedure proposed by the authors is simple in engineering terms. This paper was included in the review because this author assumed that the design procedure had led Morton and Weber to reach some decisions on failure criteria. However, with reference to the object of study developed in this report, the code presented by the authors considers the maximum stress and strain criteria and the Tsai-Hill and Tsai-Wu criteria, although all results presented in this paper use the maximum strain criterion. No reduction in properties due to accumulated damage is considered by the authors, their objectives being concentrated on the use of a program based on a heuristic design.

Summary–Reference 44

A procedure based on a finite element analysis to predict the ultimate strength of an actual structure is proposed and applied to a composite aircraft wing structure in reference 44 (Liu and Mahadevan (1998)). According to the authors' explanation, three ply-level modes of failure are considered: matrix cracking, fiber failure, and delaminations. A degradation of the system due to progressive failure is modeled. Although it is not clearly specified in the paper, the authors consider that failure criteria involving the function of stresses associated with each different mode of failure are the most appropriate for use in progressive failure analysis. Results obtained from the application of the procedure to the design of a wing structure are shown.

Comments

The most interesting aspect of the paper is a simplified procedure in which it is assumed that the laminate can accumulate damage, although the failure of the fibers at one ply leads to the failure of the laminate. Unfortunately, the authors include in the results the estimation of the final strength with only this simplified method and not with the regular one. A discussion about the exact way in which the failures (particularly delaminations) are evaluated would have been interesting.

Two considerations would be relevant: (1) comparison with experimental results or at least with other design procedures, and (2) evaluation of the simplified procedure proposed for other stacking sequences or other structures.

2.7. Miscellaneous Papers

Summary–Reference 45

In reference 45, Hart-Smith (1987) proposed that the off-axis tension test used to evaluate the shear strength dominated by the resin be generalized and used to characterize fiber-dominated laminates. In particular, the author proposed that the in-plane shear strength of a $\pm 45^\circ$ laminate should be estimated as one half the tensile strength of a $0/90^\circ$ laminate, which is a much easier test to perform.

Comments

It is noticeable that the main source of criticism that Hart-Smith finds in other proposals (basically Tsai's and related proposals) is also applicable here. It is not clear that the measurement of in-plane shear strength that Hart-Smith is proposing to carry out alternatively by means of a $\pm 45^\circ$ test corresponds to the same state of stress as that existing in the original tests.

Summary–Reference 46

The foundations of a continuous representation of the damage in a composite envisaged as an equivalent continuous medium are presented in reference 46 (Talreja (1987)). After revising the actual damage that a composite usually suffers (matrix cracking, crack coupling interfacial debonding, delamination, fiber breaking, and fracture of the laminate), the author proposes two modes of damage with their corresponding damage tensor: intralaminar and interlaminar cracking. An experimental verification of the proposed model closes the paper.

Comments

This paper was reviewed to examine the foundations of the damage mechanics theory that supports some of the integral failure approaches in composites that are reviewed in this report. Although the trial is smart from a scientific point of view, the approach is questionable for design engineering purposes due to the number of assumptions required and because the parameters that are involved in the model are difficult to measure.

Summary–Reference 47

In reference 47, Christensen (1990) provides a critical evaluation of several theoretical micromechanics models, in particular, the Differential Methods, the Generalized Self-Consistent Method, and the Mori-Tanaka Method.

Comments

This paper is basically concerned with particulate reinforced materials and is therefore not significant for the study carried out in this report.

Summary–Reference 48

Reifsnider and Carman (ref. 48 (1992)) review, in this paper, the general features of damage in composites and the use of micromechanics, summarizing the basic damage mechanisms in these

materials and giving ideas about the practical use of the information generated from micromechanical analysis. A brief and particular reference is made on viscoelastic behavior of the matrix.

Comments

The paper (ref. 48) aims to summarize the developments in the micromechanical approach of the study of composites by the first author and his group. The paper is very well written, as usual, but some of the ideas are ambiguous and therefore difficult to judge. There are a couple of points worth analyzing. The first concerns the fact that “Ply level representations of the evolution of damage and the consequent changes in properties are not sufficient, and certainly not optimum for the discussion of the evolution of the strength (of composites).” If this statement suggests that micromechanics is the right alternative tool for such a purpose, it seems to be unrealistic from a design point of view.

On the other hand, this author absolutely agrees with the authors when they say, “The simulation (in terms of micromechanics) can also be used to design composite material systems by determining the sensitivity of performance to the details of the constituents and their arrangements.”

Summary–Reference 49

In reference 49, Ladeveze (1992) presents his adaptation of damage theory for composite materials. The author uses a mesoscale of modeling that envisions a composite formed by layers with interfaces between them. The damage accumulation has then to be referred to these elements.

Comments

The damage approach to designing composite structures is a complicated theory that is still in progress. The author recognizes that “to be complete, the sensitivity to imperfections, to large defects has to be studied.”

Summary–Reference 50

Zhu, Sankar, and Marrey (ref. 50 (1996)) take a unit cell and use finite elements to perform a typical macromicromechanical study. Assuming that properties of stiffness and strength of the components (fiber, matrix, and interface) are known, they evaluate the elastic constants and the strength properties of the lamina and also different failure envelopes, which are compared with classic failure criteria. In the micromechanical strength analysis, different combinations of failures are considered. Fiber and matrix may fail in accordance with maximum stress or Von-Mises criteria, and the interface may fail by the maximum normal or shear stress criterion.

Comments

Reference 50 has been included in this section because the purpose of this study is not clear to the author of this report, in spite of its suggestive title. There are, independently of the objectives of the paper, several questionable aspects in the analysis. The first aspect is the selection of the failure criterion of the components, particularly that corresponding to the interface. Nothing is said about how the particular figures that represent the failure of the interface in mode I or II are selected in terms of nominal stresses. The inclusion or otherwise of the interface failure leads to differences of up to 400 percent in load cases in which this mechanism of failure plays an important role in the failure of the composite.

The second questionable aspect is the criteria that determine when the composite fails at the micromechanical level. To establish the composite failure by the failure of one element does not seem to be a very realistic assumption. Some damage evolution procedure, not difficult to implement at the micromechanical level used, in an attempt to detect loss of rigidity in the cell element, appears to be a more realistic way to estimate the failure of the composite.

Micromechanic analysis is a tool that can be used for academic purposes, but in terms of helping in the design of composite structures, it ought to be used differently, in this author's opinion.

Summary–Reference 51

In reference 51, Swanson and Lee (1996) attempt to show, by means of a simplified analysis, the role of hydrostatic pressure in the compressive strength of fiber composites. The model developed assumes the presence of wavy fibers and that the failure of the composite appears when the strengths of the matrix or of the interface are exceeded.

Comments

The results confirm that the presence of hydrostatic pressure leads to an increase in the compressive strength of the composites, a fairly intuitive conclusion if one comes down to the micromechanical level. The failure in compression is originated by the presence of cracks running parallel to the fibers. If a load is applied to close these hypothetical cracks, it seems obvious that more load in the direction of the fiber will be required to make these cracks grow. A simple micromechanical model could be carried out to quantify this effect.

Summary–Reference 52

The title of reference 52, *Methodology for Residual Strength of Damaged Laminated Composites*, by Tang, Shen, Chen, and Gaedke (1997) represents its content exactly. The proposed methodology includes (1) a model to predict impact damage based on bending strain energy density, (2) a failure criterion, (3) a scheme to store information about damage characterization, and (4) an algorithm for the residual strength analysis of a laminate with delaminations. Examples comparing predicted and experimental results are included.

Comments

Because it is a complete methodology, the proposal described in the paper covers a number of details that affect aspects of composites that are not well-known. In other words, some of the assumptions may appear questionable, but it is nevertheless unquestionable that the proposal reflects a great many years of effort in this field and is basically coherent with the general requirements for an engineering procedure.

Acceptance of the bending strain energy density model requires more information about its foundations (the cancellation of the contribution of the fibers requires some explanation) and more experimental work to prove its suitability, but it has to be recognized as an ingenious approach.

The option selected to deal with the presence of delaminations, that is, using a softened inclusion, seems to be a less mechanically based criterion than the consideration of an interlaminar strain energy release rate, but it is an attractive approach, as the authors mention, in terms of applicability and implementability.

This work (ref. 52) developed by Tang et al. (1997) is serious and closely related to the problem that originated the generation of this report.

Summary–Reference 53

Reifsnider and Case (ref. 53 (1998)) apply their critical element model to damage tolerance (prediction of remaining strength) and durability (life prediction) of different composite systems. The critical element model assumes “that a representative model can be chosen such that the state of stress in that volume is typical of all other volumes in the laminate and that the details of stress distribution and damage accumulation in that volume are sufficient to describe the final failure resulting from specific failure mode.” The procedure distinguishes between critical elements (their failure controls the failure of the laminate) and subcritical elements (their failure affects the state of stress of other elements but does not lead to the failure of the laminate). The scheme, in conjunction with a damage model, is applied to predict the behavior of laminates.

Comments

Reference 53 is a recent paper (1998) that basically attempts to show the applicability of a code developed by the authors to predict residual strength and durability. Concepts are well explained and open questions are realistically addressed.

Using a failure criterion is of no particular interest in this paper (or apparently in the work of this group). They suggest using an appropriate stress criterion such as maximum stress or Tsai-Hill.

3.0. Failure Criteria in MSC.Dytran

The user manual of MSC.Dytran in version 4.0 (pages 2-30 and 2-31) considers six failure criteria: (1) Tsai-Hill, (2) Tsai-Wu, (3) modified Tsai-Wu, (4) Maximum stress, (5) Hashin, and (6) Chang.

Major emphasis will be placed in this section on the last two criteria in the previous list for two reasons: (1) The author of this report, as will be explained in the next section, has more trust in what could be referred to as failure-mode-based failure criteria, to which category the last three criteria could be assigned; and (2) there are some details concerning the application of these two criteria (in particular the last one) that are worthy of comment. The remaining four criteria can be understood and applied with the help of any basic composites book.

Although it has already been discussed briefly in the previous section and could also be done later on, a brief description of the fundamentals of the two criteria, as well as of their implications, will be offered next.

3.1. Hashin Criteria

Two proposals of failure criterion for fibrous composite materials that are associated with Hashin may be found in the literature. The first, reference 1 of the review, is known as the Hashin-Rotem criterion. This criterion predicts failure when one of the following equations is satisfied:

Hashin-Rotem Criterion (1973)

Fiber failure in tension

$$\sigma_{11} = X_T \quad (\sigma_{11}, X_T > 0)$$

Fiber failure in compression

$$-\sigma_{11} = X_C \quad (\sigma_{11} < 0; X_C > 0)$$

Matrix failure mode in tension

$$\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 = 1$$

Matrix failure mode in compression

$$\left(\frac{\sigma_{22}}{Y_C}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 = 1$$

where

σ_{11} is the nominal stress in the lamina in the direction of the fibers.

σ_{22} is the nominal stress in the lamina in the transverse direction to the fibers.

σ_{12} is the nominal shear stress in the plane of the lamina.

X_T is the tensile strength of the fibers.

X_C is the compressive strength of the fibers.

Y_T is the tensile strength in the transverse direction of the fibers.

Y_C is the compression strength in the transverse direction of the fibers.

S is the shear strength.

Based on observations of specimen failure in tension with different orientations of the fibers, the authors of this proposal conclude that there are only two mechanisms of failure: fiber or matrix failure. With reference to the second, they do not distinguish whether the failure is exactly at the interface or inside the matrix and thus propose that both σ_2 and σ_{12} contribute to the appearance of the failure (the proposal is in quadratic form).

The historical importance of this proposal is that it initiates a different way of approaching the generation of composites failure criteria. The authors first set out to recognize modes of failure, then to recognize the variables associated with these modes and propose an interaction between them.

The idea seems adequate for the type of materials under consideration, although it may be argued that not all failure modes that can appear in fibrous composites are covered in the proposal. It is also not clear that the variables they propose for each case are the most appropriate or in what way they combine them.

In 1980, Hashin (ref. 2) re-examined the proposal and established some modifications. There are also four expressions involved in the proposal that Hashin developed for the 3D case and then particularized for the 2D case, which is the one considered in MSC.Dytran.

Hashin Criterion 3D (1980)

Tensile fiber mode

$$\left(\frac{\sigma_{11}}{X_T}\right)^2 + \frac{1}{S^2}(\sigma_{12}^2 + \sigma_{13}^2) = 1$$

or

$$\sigma_{11} = X_T$$

Compressive fiber mode

$$|\sigma_{11}| = X_C$$

Tensile matrix mode $(\sigma_{22} + \sigma_{33}) > 0$

$$\frac{1}{Y_T^2}(\sigma_{22} + \sigma_{33})^2 + \frac{1}{S_T^2}(\sigma_{23}^2 - \sigma_{22}\sigma_{33}) + \frac{1}{S^2}(\sigma_{12}^2 + \sigma_{13}^2) = 1$$

Compressive matrix mode $(\sigma_{22} + \sigma_{33}) < 0$

$$\frac{1}{Y_C} \left[\left(\frac{Y_C}{2S_T} \right)^2 - 1 \right] (\sigma_{22} + \sigma_{33}) + \frac{1}{4S_T^2} (\sigma_{22} + \sigma_{33})^2 + \frac{1}{S_T^2} (\sigma_{23}^2 - \sigma_{22}\sigma_{33}) + \frac{1}{S^2} (\sigma_{12}^2 + \sigma_{13}^2) = 1$$

where, in addition to the previous definitions, S_T represents the transverse shear strength, the allowable value of shear stress σ_{23} (the allowable value of σ_{13} is, as for σ_{12} , S).

Hashin Criterion 2D (1980)

Tensile fiber mode $\sigma_{11} > 0$

$$\left(\frac{\sigma_{11}}{X_T}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 = 1$$

Compressive fiber mode $\sigma_{11} < 0$

$$|\sigma_{11}| = X_C$$

Tensile matrix mode ($\sigma_{22} > 0$)

$$\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 = 1$$

Compressive matrix mode ($\sigma_{22} < 0$)

$$\left(\frac{\sigma_{22}}{2S_T}\right)^2 + \left[\left(\frac{Y_C}{2S_T}\right)^2 - 1\right] \frac{\sigma_{22}}{Y_C} + \left(\frac{\sigma_{12}}{S}\right)^2 = 1$$

It must be mentioned that in this last expression, Y_C has to be taken in absolute value whereas σ_{22} keeps its sign. This condition is relevant in the case of the second term of the expression.

The last four expressions are those appearing in MSC.Dytran under the name Hashin criterion. The evolution of the Hashin criteria, although there are not too many differences between the two proposals, needs some explanation. Conceptually, both proposals divide the mechanisms of failure of a fibrous composite material into two groups.

With reference to the failure in the fiber, the only difference in the final expression concerns the contribution of σ_{12} to the failure in tension, a modification with no clear basis. Actually, although the deduction procedure of the final expression of the criterion is cumbersome, the modification comes from the quadratic interaction Hashin assumes between the components of the stress vector associated with the plane of failure. It is important to note the emphasis Hashin placed on avoiding any connection in his approach with energy concepts. Instead, his proposal is the simplest way to approximate an assumed interaction between different effects once a simple linear interaction is discarded. It is noticeable, in any case, that, in his paper, Hashin considers the possibility of discarding the contribution of σ_{12} to the tensile failure of the fiber. An explanation of the physical basis of the contribution of σ_{12} to the failure of the fibers, although a specific search has not been conducted for this topic, has not been found in the literature. It might be possible to clarify this question by means of a micromechanical analysis, followed by the appropriate tests, as will be suggested later on.

With reference to the failure in the matrix, the approach followed by Hashin in his second paper (ref. 2) is different. The reason is the impossibility of determining the plane of failure. He then assigns the failure to a quadratic interaction between stress invariants, canceling in this expression the contribution of σ_{11} , based on the fact that any possible plane of failure is parallel to the fibers and that consequently, the components of the stress vector of any of these planes do not depend on σ_{11} .

In the tensile matrix mode of failure, discarding the linear term leads to an expression that, particularized in the 2D case, is identical to the 1973 proposal. In the compressive matrix mode of failure, the linear term is not discarded, and the expression is forced to satisfy the fact that if the material fails in the presence of transversely isotropic pressure ($\sigma_{22} = \sigma_{33} = -\sigma$), this pressure can reach values much larger than the compressive uniaxial failure stress Y_C . What implication the inclusion of this idea, which in itself is acceptable in accordance with the experimental results available, has for the capacity

of predicting other failures is not clear. In any case, the most serious doubts about the criterion come from the interaction between stresses and allowables derived from the interaction between invariants. Thus, it is, at the very least, shocking to find in the 2D expression of the criterion a not-in-plane allowable S_T in an expression of a plane state of stress that apparently does not activate the mechanism of failure to which S_T is associated. A parametric study of the influence of the value of S_T in the predictions for different 2D cases should at least be carried out. Sections 3.3, 3.4, and 3.5 are devoted to this topic.

3.2. Chang Criteria

The understanding of the Chang criterion, as it appears in MSC.Dytran, is much more complicated than in the case of the Hashin criterion and requires a consideration of Chang's work, in general, starting with the Yamada and Sun criterion (ref. 3) to which he refers in his early works.

Yamada and Sun Criterion (1978)

Yamada and Sun proposed, between the two Hashin proposals, a failure criterion suitable only for cases in which the transverse strength Y of the lamina does not control the failure of the laminate

$$\left(\frac{\sigma_{11}}{X}\right)^2 + \left(\frac{\sigma_{12}}{S_{is}}\right)^2 = 1$$

where σ_{11} , σ_{12} , and X have the habitual meaning (X can be X_T or X_C), and S_{is} represents the in situ value of the shear strength, the ply shear strength measured from a symmetric cross-ply laminate.

Chang, Scott, and Springer (refs. 5 and 6) used the Yamada and Sun criterion in their papers published in 1984. In the second paper they used a nonlinear strain-stress behavior in the form

$$\gamma_{12} = \left(\frac{1}{G_{12}}\right)\sigma_{12} + \alpha\sigma_{12}^3$$

which transformed the criterion into

$$\left(\frac{\sigma_{11}}{X}\right)^2 + \frac{\frac{\sigma_{12}^2}{2G_{12}} + \frac{3}{4}\alpha\sigma_{12}^4}{\frac{S^2}{2G_{12}} + \frac{3}{4}\alpha S_{is}^4} = 1$$

In reference 5, the value of S_{is} is maintained as in the original Yamada and Sun proposal, and in reference 6, S_{is} is referenced as the actual ply shear for the laminate.

Chang and Chang (ref. 4) used the following criterion, which aims to cover the whole range of failure situations.

Chang and Chang Criterion (1987)

Matrix cracking

$$\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \frac{\frac{\sigma_{12}^2}{2G_{12}} + \frac{3}{4}\alpha\sigma_{12}^4}{\frac{S_{is}^2}{2G_{12}} + \frac{3}{4}\alpha S_{is}^4} = 1$$

Fiber matrix and/or fiber breakage

$$\left(\frac{\sigma_{11}}{X}\right)^2 + \frac{\frac{\sigma_{12}^2}{2G_{12}} + \frac{3}{4}\alpha\sigma_{12}^4}{\frac{S_{is}^2}{2G_{12}} + \frac{3}{4}\alpha S_{is}^4} = 1$$

Several modifications were made with respect to the previous situation, and new decisions were undertaken in this paper. They can be summarized in the following points:

- The failure in the matrix is proposed, taking the form mimetically from the Yamada and Sun proposal for nonmatrix failure (obviously changing σ_{11} for σ_{22}).
- The fiber breakage and fiber-matrix shearing are still assumed to be governed by the same criterion.
- The criterion is only proposed for tensile cases.

Shahid and Chang (refs. 7 and 8) proposed a new set of criteria, presented as modified Hashin criteria.

Shahid and Chang Criterion (1995)

Matrix cracking

$$\left(\frac{\sigma_{22}}{Y_T(\phi)}\right)^2 + \left(\frac{\sigma_{12}}{S(\phi)}\right)^2 = 1$$

Fiber-matrix shear-out

$$\left(\frac{\sigma_{11}}{X_T}\right)^2 + \left(\frac{\sigma_{12}}{S(\phi)}\right)^2 = 1$$

Fiber breakage

$$\left(\frac{\sigma_{11}}{X_T}\right) = 1$$

where ϕ is the crack density in the matrix.

Several drastic differences appear in this proposal, with respect to both previous Chang proposals as well as to the Hashin proposal, which this criterion presumably modified.

- With reference to fiber breakage, the contribution of σ_{12} that also appeared in the previous Chang and Hashin criteria is omitted. In fact, this proposal coincides with the Hashin-Rotem proposal.
- Nothing is said about fiber failure in compression.
- Matrix cracking in tension is maintained from the previous Chang model. Hashin-Rotem assigned this expression to fiber-matrix failure. Nothing is said in this expression about the necessity of taking S as an in situ property.
- Nothing is said about matrix failure in compression.
- The fiber-matrix shear-out failure is maintained from the previous proposal and coincides with the expression that Hashin proposes to control the failure of the fibers.

What MSC.Dytran assigns to Chang does not coincide with any of the previous Chang proposals, which does not imply that the criterion used in the code is necessarily incoherent.

Chang MSC.Dytran 4.0 Criterion

Fiber breakage

$$\left(\frac{\sigma_{11}}{X_T}\right)^2 + T = 1$$

Matrix cracking

$$\left(\frac{\sigma_{22}}{Y_T}\right)^2 + T = 1$$

Matrix compression

$$\left(\frac{\sigma_{22}}{2S}\right)^2 + \left[\left(\frac{Y_C}{2S}\right)^2 - 1\right] \frac{\sigma_{22}}{Y_C} + T = 1$$

with

$$T = \left(\frac{\sigma_{12}}{S}\right)^2 \frac{1 + \frac{3}{2}\alpha G_{12}\sigma_{12}^2}{1 + \frac{3}{2}\alpha G_{12}S^2}$$

As can be observed, a miscellanea of all previous proposals is what appears in MSC.Dytran as the Chang criterion. The main features of the criterion the code uses, in relation to the previous proposal, are the following:

- The fiber breakage criterion that is used coincides with Chang's 1987 proposal but not with his 1995 proposal. For the case $\alpha = 0$ (linear strain-stress relation), the criterion coincides with Yamada's and Sun's original proposal (without taking S in the in situ sense), and with Hashin's 1980 proposal but not with Hashin's 1973 proposal.
- The matrix cracking criterion coincides with Chang's 1987 and 1995 proposals. It coincides for the case $\alpha = 0$, with the failure assigned by Hashin in 1973 and 1980 to the fiber-matrix failure.
- The failure criterion used for a matrix in compression coincides with the expression proposed by Hashin in 1980, although it identifies S_T (the transverse shear strength) with S (the in-plane shear strength).

Several points are noteworthy with regard to the implications of using this set of expressions in the code:

- Involving σ_{12} in the fiber breakage criterion may produce underestimation of the strength of the laminate. The case $\sigma_{11} = X_T$, in the absence of σ_{12} , is obviously well captured. However, if σ_{12} reaches a certain important value (smaller in comparison, numerically speaking, with σ_{11} but significant in comparison with S), the contribution of the σ_{12} term to reaching the limit value predicted by the criterion can be significant.
- This case is of particular interest if one considers that the proponents of this criterion, Yamada and Sun, used it in connection with an in situ value of the shear strength, which is higher than the nominal shear strength of the ply. With the use derived from the proposal, this questionable contribution will be higher.
- Nothing is said about failure of the fiber in compression, an aspect of interest when studying composites subjected to impact.
- The effect of identifying S_T with S must be clarified. This identification adds even more pressure to perform the study of the influence of S_T in Hashin's criterion (mentioned previously).

3.3. Evaluation of the Implications of Taking $S_T = S$ in the Hashin Criterion

It is obvious that the error introduced with respect to Hashin's original 1980 proposal, by taking $S_T = S$, depends on the factors involved in the original expression. The two aspects of maximum concern are the stress state (relative values of σ_{12} and σ_{22}) and the relation between the actual values of S_T and S .

The first of these two aspects will be investigated by means of a parametric study considering the relation $\sigma_{12} = n\sigma_{22}$ and varying the value of n .

The second aspect to be considered would not be a problem if the value of S_T were known for typical composites. If there are problems in determining the value of S , there are more in determining the value of S_T , as Hashin mentions in the criterion proposal. The bibliography studied does not include values of this parameter. In Hashin's paper, there is one value of S_T not associated with any particular

material, and no specification of the value of S for the material considered is given. Thus, it has been considered that the most reasonable option is also to develop a parametric study in terms of the relative values of S_T with respect to S , taking S as the value corresponding to a particular material, AS/3501, whose properties are taken from S. W. Tsai, *Composites Design, Think Composites*, 1988.

Several graphs will be presented considering different values of k in the relation: $S_T = kS$ where k has been taken, for the purpose of covering the whole range of possible situations, with values greater and smaller than 1.

It is necessary, when representing the results, to make some decisions. The most informative parameter would be the relative error found when the value of S is taken instead of S_T in the expression of the criterion for the case in which the stress state leads to predicting the failure of the material; then, the procedure to build up the error graphs will be the following:

- For a certain value of n and k , the value of σ_{22} that produces the material failure [$F(S_T) = 1$] is found.
- With this value of σ_{22} (and the corresponding value of σ_{12} in accordance with the value of n considered), the value of the criterion is calculated [$F(S_T = S) = F(S)$].
- The value of the relative error is calculated in accordance with the following expression:

$$\text{error} = \frac{F(S) - F(S_T)}{F(S_T)} \quad \text{with} \quad F(S_T) = 1$$

The error is defined with respect to the theoretical value of the prediction with Hashin's proposal, $F(S_T)$, which, in accordance with the procedure outlined, is equal to 1. The error would be different if the value found by using a code in which the value of S_T has been substituted with S is taken as reference.

Figures 3.1–3.5 represent the errors for different cases of $k < 1$. For completeness, figure 3.6 represents the error for an extreme case of $k = 0.1$. In all cases, the values of $F(S)$ have been represented, the value of $F(S_T)$ being, as has previously been explained, equal to 1. The values of $F(S)$ and the error have been calculated, increasing n by 0.1; the symbols that appear are there to allow easy differentiation between both curves.

A summary of the values of the errors found for k between 0.5 and 0.9 is presented to facilitate the discussion in figure 3.7.

Inspection of the curves leads to the following conclusions:

- When $S_T < S$ ($k < 1$), $F(S) > F(S_T) = 1$, which implies that if $F(S)$ had been used as the failure criterion, the maximum stresses allowed would have had to be decreased to make $F(S)$ equal to 1. Thus, identifying S_T with S leads, in this case, to conservative predictions.
- The extreme cases, $\sigma_{12} = 0$ or $\sigma_{22} = 0$, do not have any associated error because the expression of the criterion leads to the correct prediction of the failure, $\sigma_{12} = S$ or $\sigma_{22} = Y_C$, in these cases.

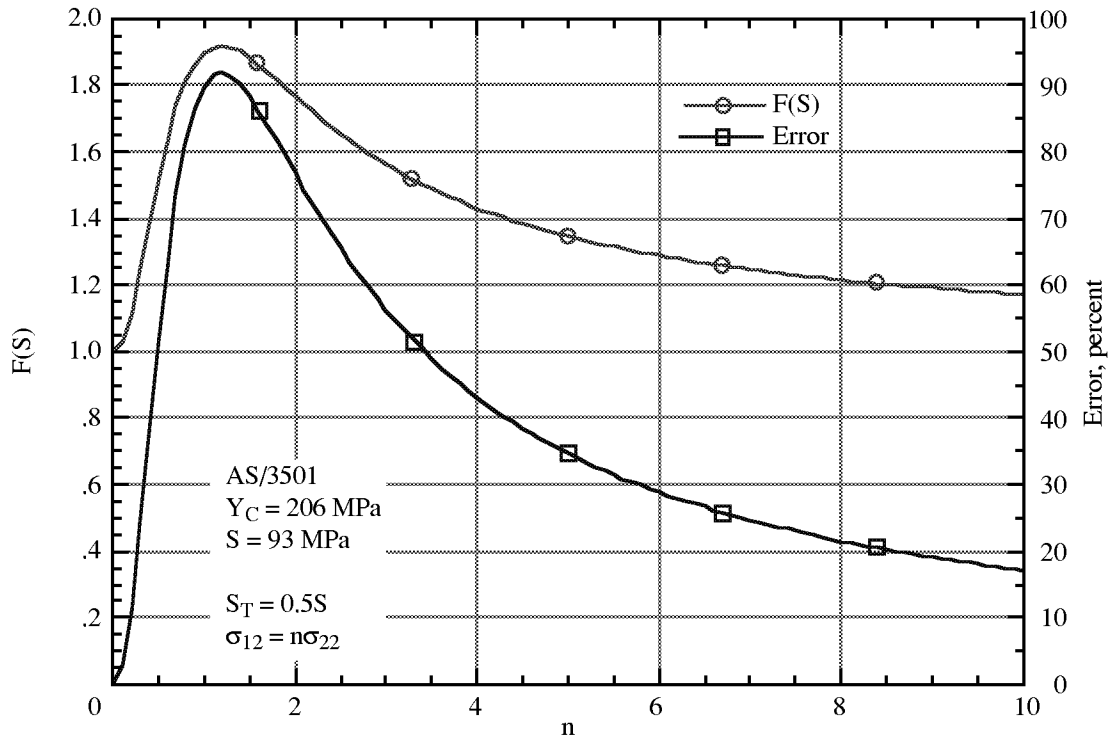


Figure 3.1. Value of $F(S)$ and relative error for case $S_T = 0.5S$.

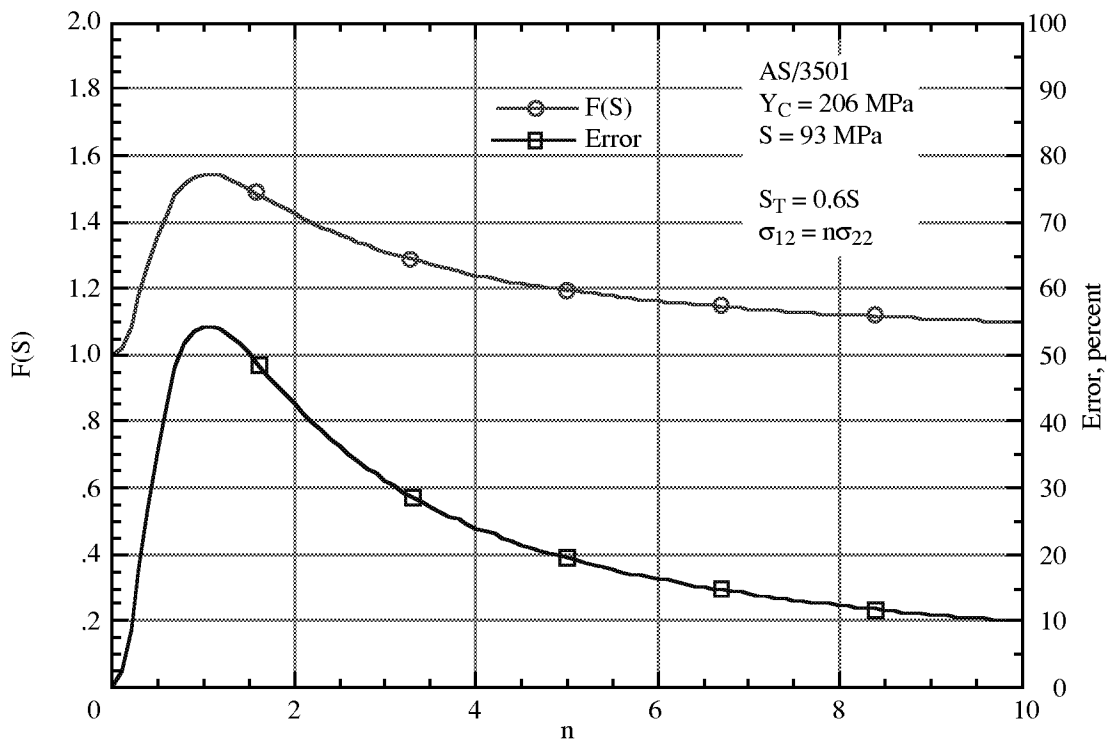


Figure 3.2. Value of $F(S)$ and relative error for case $S_T = 0.6S$.

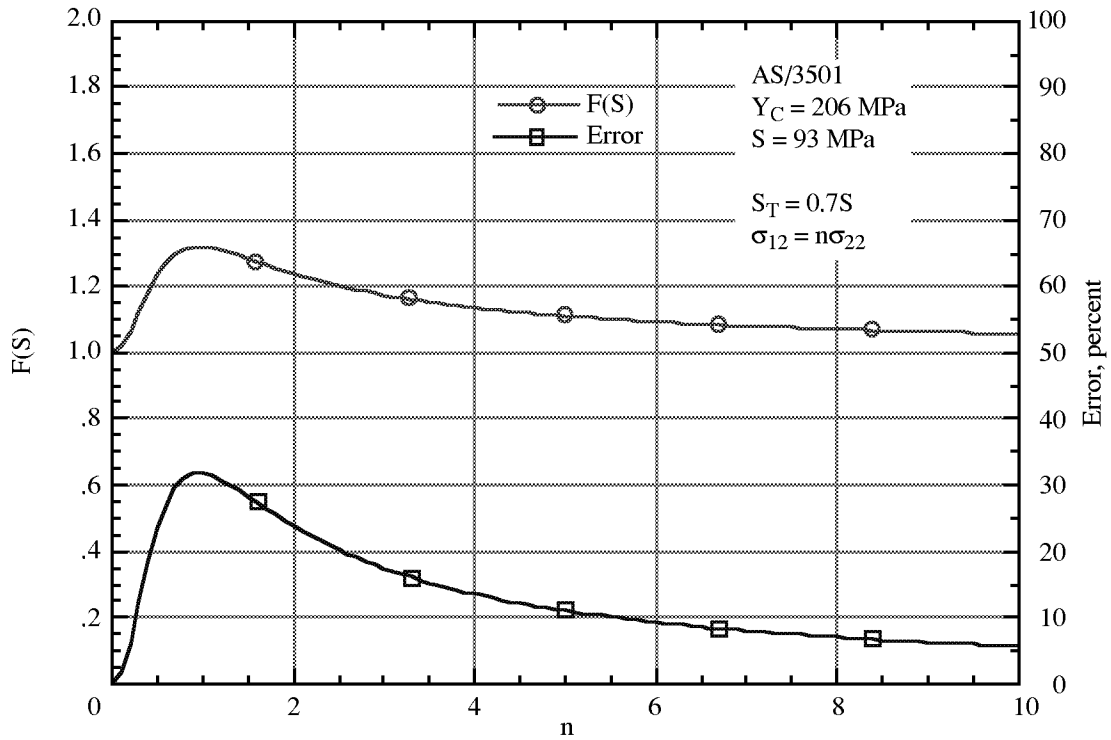


Figure 3.3. Value of $F(S)$ and relative error for case $S_T = 0.7S$.

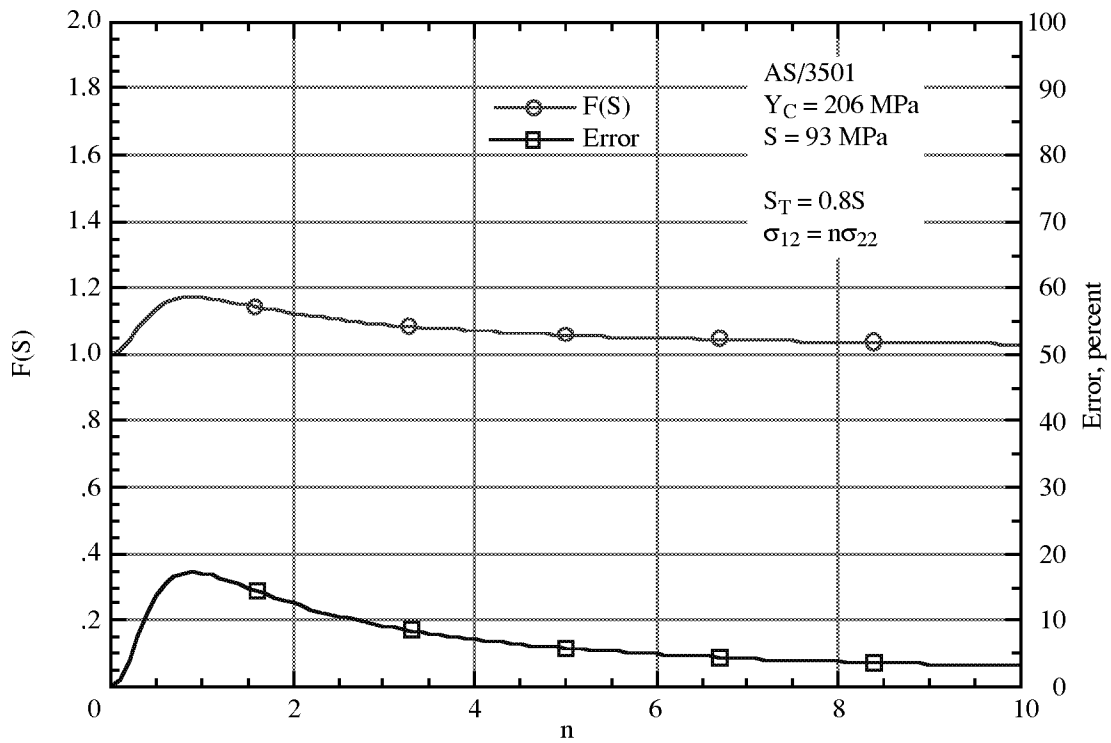


Figure 3.4. Value of $F(S)$ and relative error for case $S_T = 0.8S$.

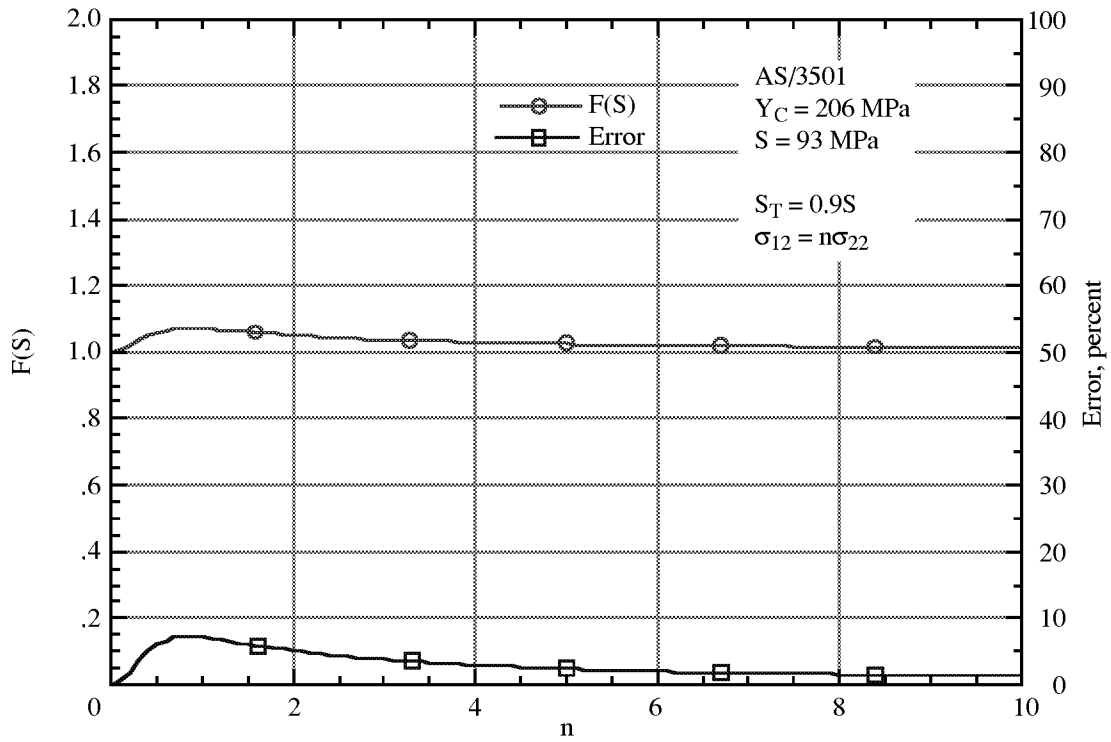


Figure 3.5. Value of $F(S)$ and relative error for case $S_T = 0.9S$.

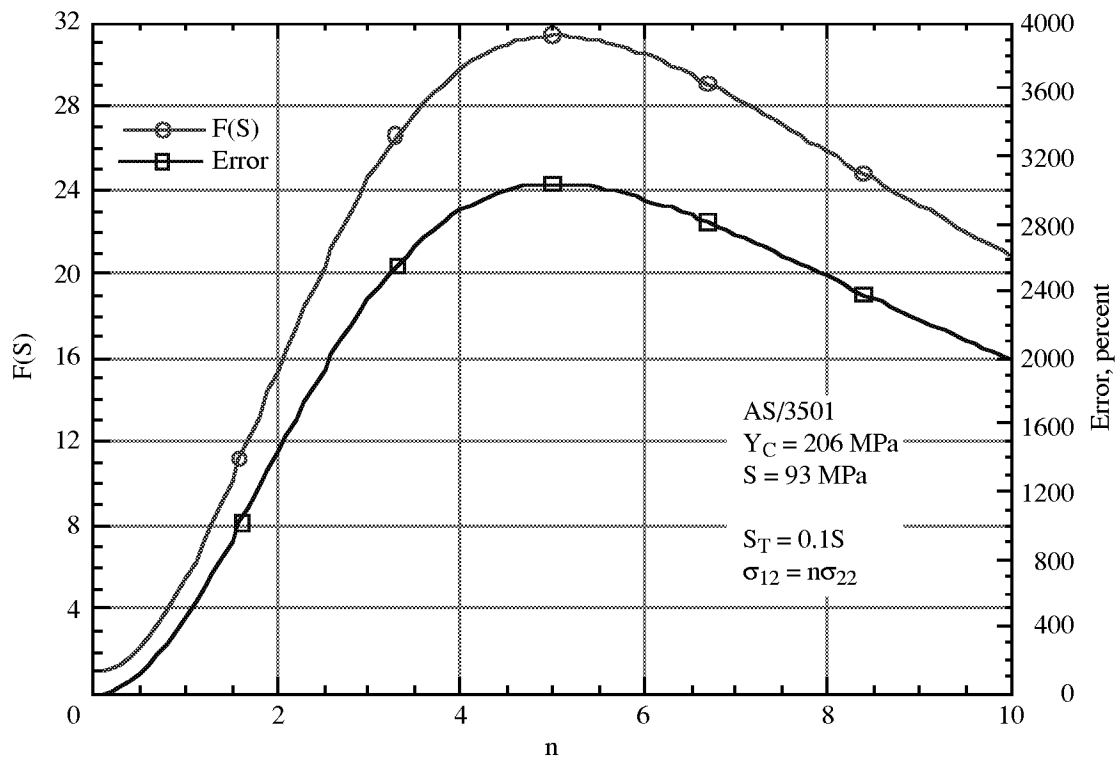


Figure 3.6. Value of $F(S)$ and relative error for case $S_T = 0.1S$.

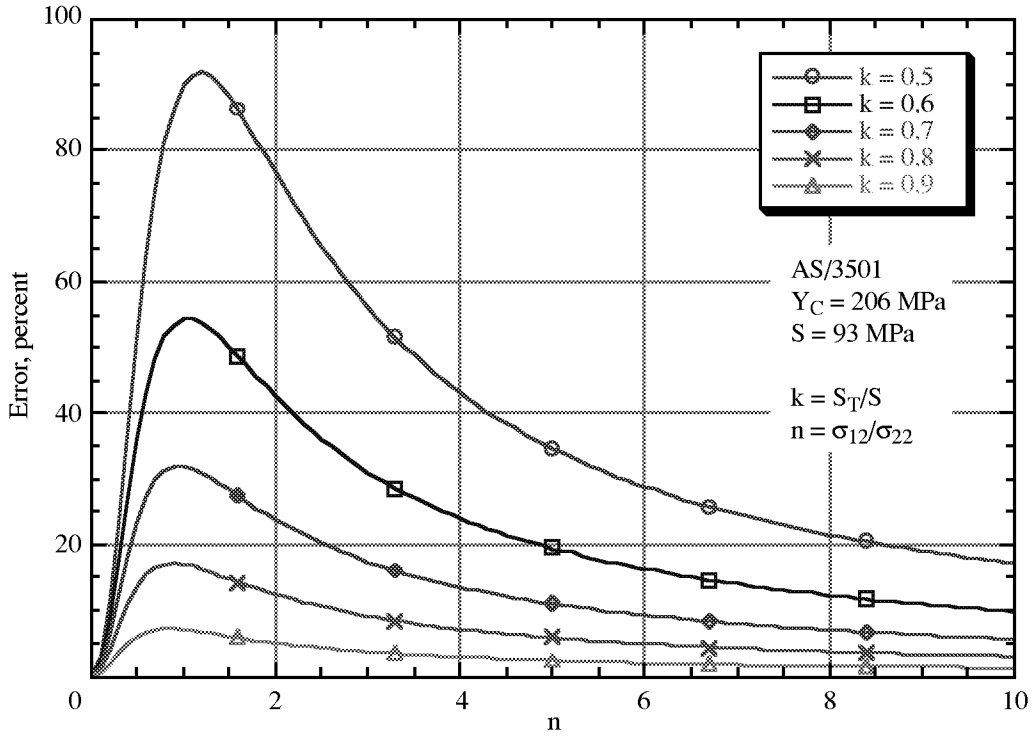


Figure 3.7. Value of relative error for different $S_T = kS$ ($k < 0$).

- For a given value of k , the error is a function of the stress state. The error is greater when σ_{12} is not dominant versus σ_{22} and starts to decrease rapidly when σ_{12} starts to be dominant. As an example, for the case $k = 0.5$, the maximum error (92 percent) appears when n is 1.2.
- If S_T is significantly smaller than S , the errors can reach very high values, as can be appreciated in figure 3.6, for the case $S_T = 0.1S$.
- When S_T is close to S , the error tends to be smaller, as can be appreciated in figure 3.7.

It is surprising that identifying S_T with S (S_T being smaller than S) leads to conservative predictions. In effect, one would expect that increasing an allowable value in a criterion would lead to higher possible values of the stresses, thus leading to unsafe predictions. A justification of the results, from a mathematical point of view, can be given by calculating the variation of $F(S_T)$ with respect to S_T . The expression of this variation is

$$\frac{\partial F(S_T)}{\partial S_T} = -\frac{\sigma_{22}(\sigma_{22} + Y_C)}{2S_T^3} > 0$$

having assumed that $\sigma_{22} < 0$ and that the absolute value of σ_{22} is smaller than the absolute value of Y_C .

The variation of $F(S_T)$ that is found with respect to S_T explains mathematically why increasing an allowable value of the material leads to conservative predictions. This fact, nevertheless, adds, from a physical point of view, still more doubts about the appearance of an out-of-plane allowable (S_T) in predicting the failure of the plane.

Figures 3.8–3.12 represent the values of $F(S)$ and the errors for different cases of $k > 1$. Figure 3.13 represents a summary of the errors in the predictions for the five values of k considered in the previous figures.

An inspection of the curves presented leads to the following conclusions:

- When $S_T > S$ ($k > 1$), $F(S) < F(S_T) = 1$, which implies that if $F(S)$ had been used as the failure criterion, the maximum stresses allowed could have been increased until $F(S) = 1$. Thus, identifying S_T with S leads, in this case, to unsafe predictions.
- The extreme cases, $\sigma_{12} = 0$ or $\sigma_{22} = 0$, as in the $k < 1$ case previously considered, do not have any associated error.
- For a given value of k , the influence of the stress state in the error is similar to the one described previously for the $k < 1$ case.
- The dependence of the error on the value of k is also similar to the former case, when k starts to be different from 1, as can be appreciated in figure 3.13, which summarizes all the cases studied. It can be observed in figure 3.13 how closely the errors are associated to high values of k , indicating that there is almost no variation of $F(S)$ with k when k tends to high values. This fact can be checked in the expression of the variation of $F(S_T)$ with respect to S_T replacing S_T with kS .

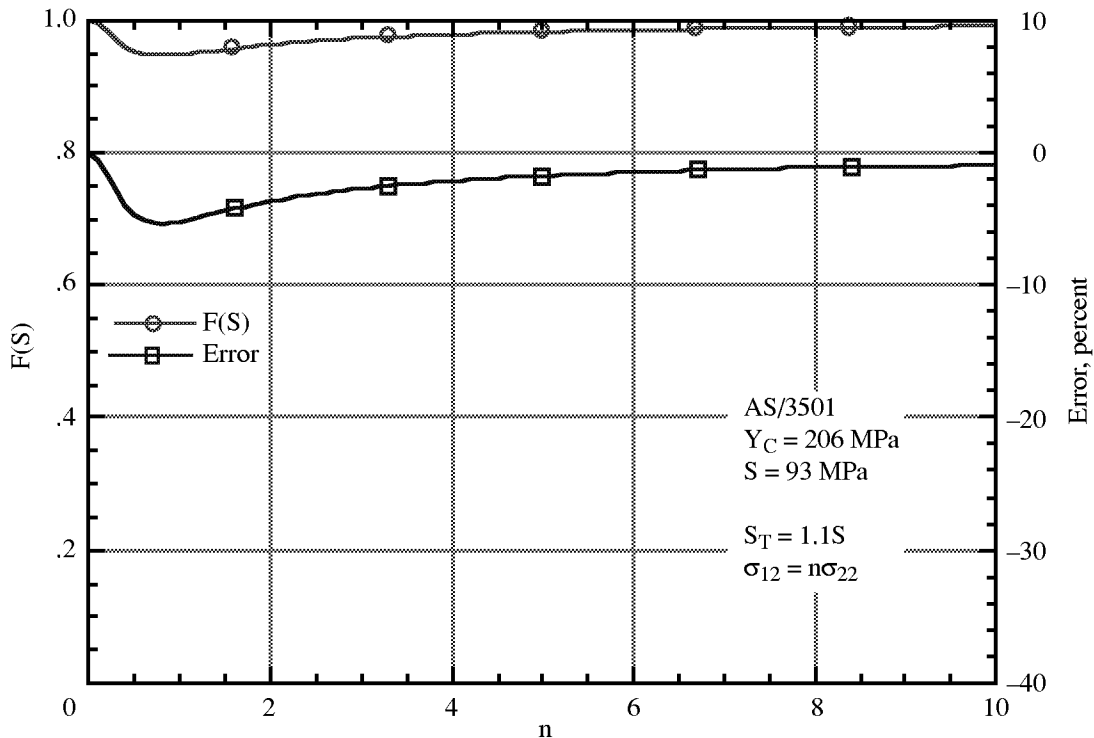


Figure 3.8. Value of $F(S)$ and relative error for case $S_T = 1.1S$.

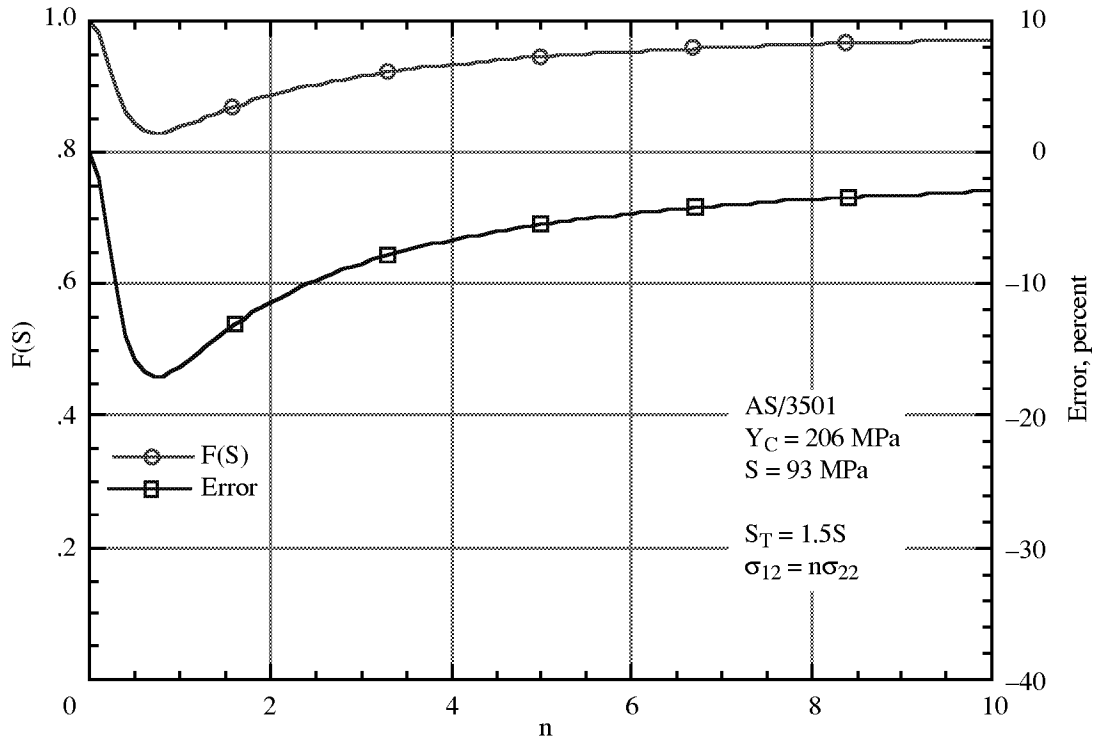


Figure 3.9. Value of $F(S)$ and relative error for case $S_T = 1.5S$.

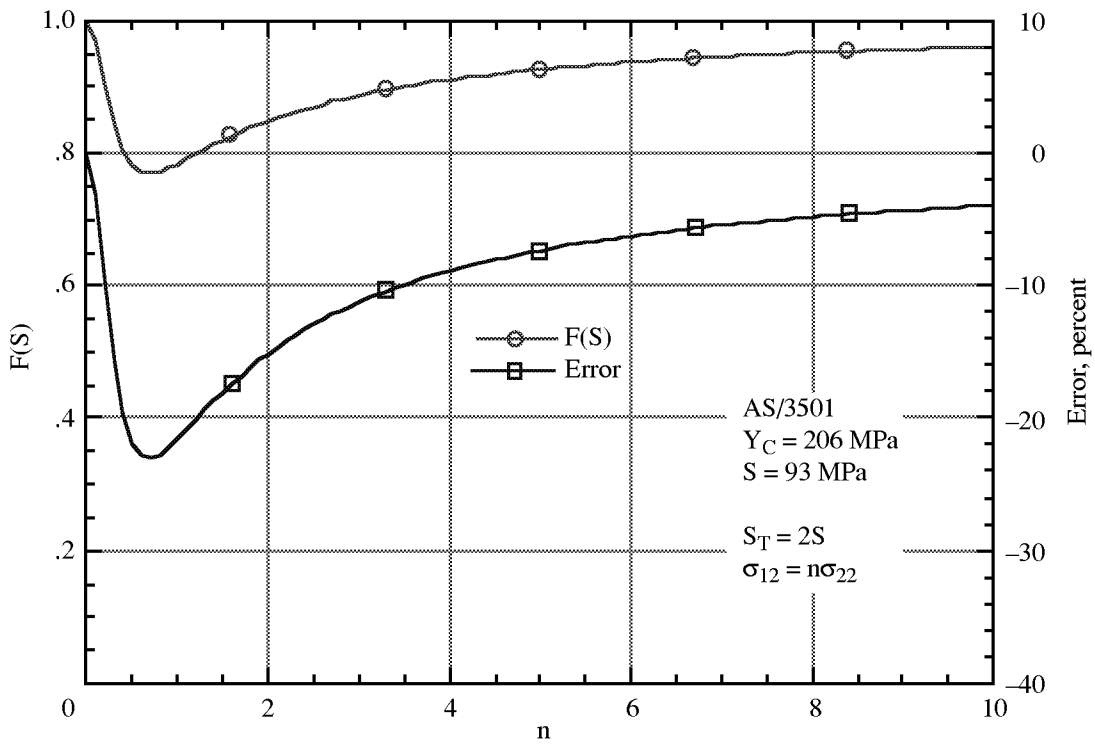


Figure 3.10. Value of $F(S)$ and relative error for case $S_T = 2S$.

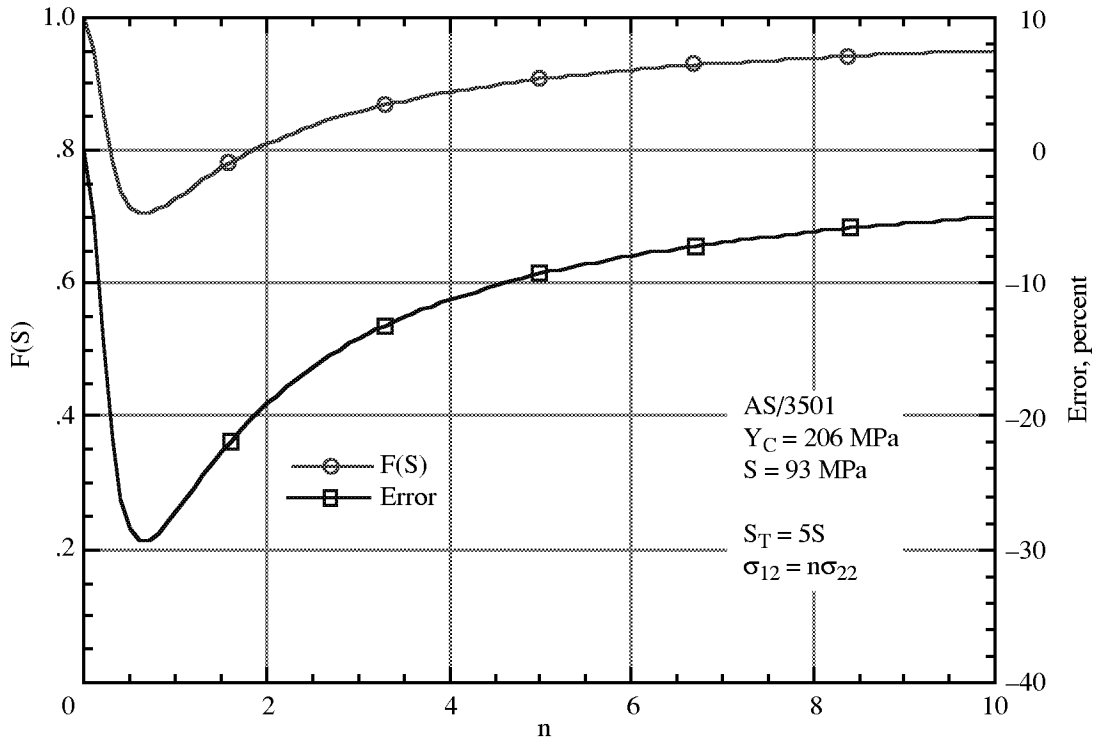


Figure 3.11. Value of $F(S)$ and relative error for case $S_T = 5S$.

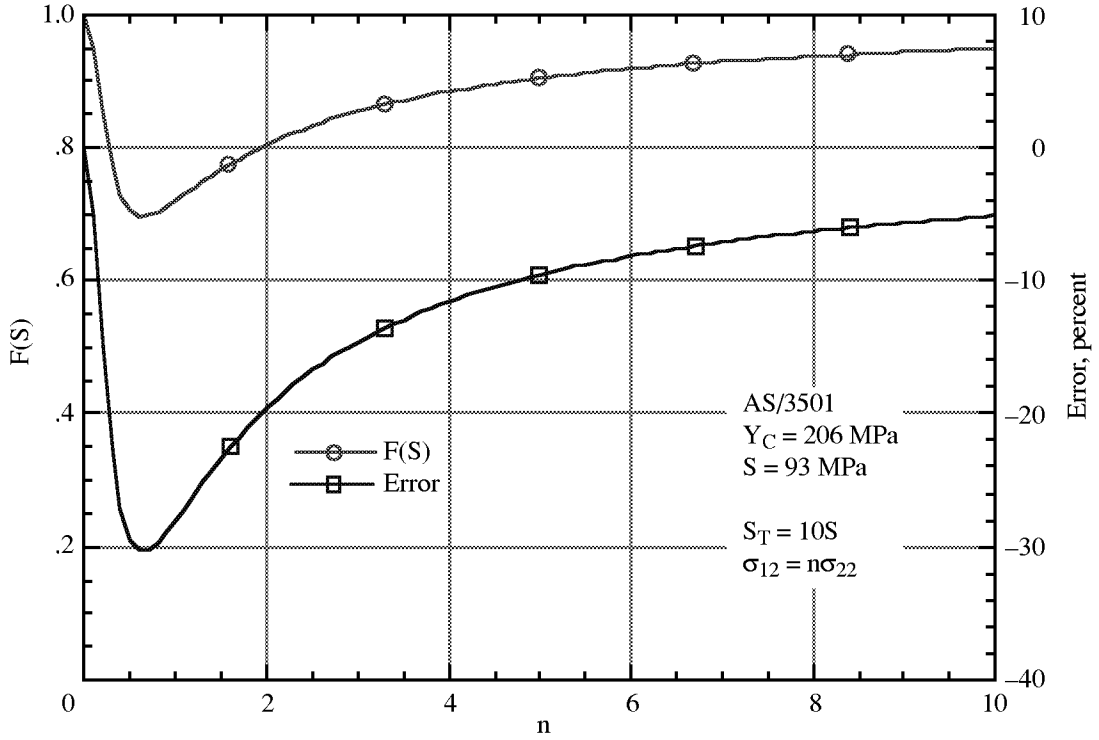


Figure 3.12. Value of $F(S)$ and relative error for case $S_T = 10S$.

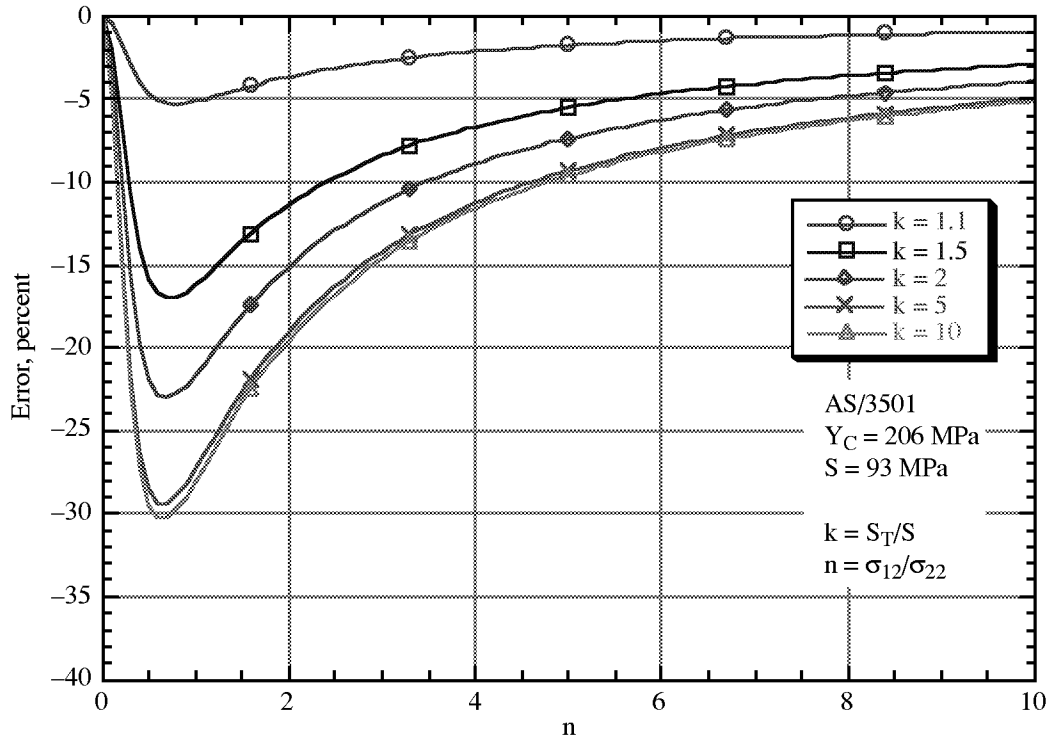


Figure 3.13. Value of relative error for different $S_T = kS$ ($k > 0$).

3.4. Sensitivity Study of Hashin Criterion to Errors in Values of S_T

The significant influence that the identification of S_T with S may have on the predictions of matrix failure in compression prompts a study of the influence that errors in the identification of the values of S_T , by direct or indirect measurement in the laboratory, may have on the predictions of Hashin's criterion. In this context it has to be mentioned that, in view of the difficulties in measuring S_T directly, it is common to take the value of S_T as that corresponding to the apparent interlaminar shear strength (ILSS).

The same procedure will be followed as was used previously. The study will again refer to the AS/3501 material. The error is now calculated in accordance with the expression:

$$\text{error} = \frac{F(S_e) - F(\text{ILSS})}{F(\text{ILSS})} \quad \text{with} \quad F(\text{ILSS}) = 1$$

where $F(\text{ILSS})$ is forced to be equal to 1, thus permitting the calculation of the stress level that satisfies the criterion. With these stress values, the value of the criterion $F(S_e)$ is calculated, S_e being the value of the interlaminar shear strength measured with a certain error e .

An equivalent situation that would lead to similar results is simply to consider that the intralaminar shear strength does not represent correctly the value of the transverse shear strength, the relative difference being denoted by e . A combination of both effects is obviously possible and is covered in the following study.

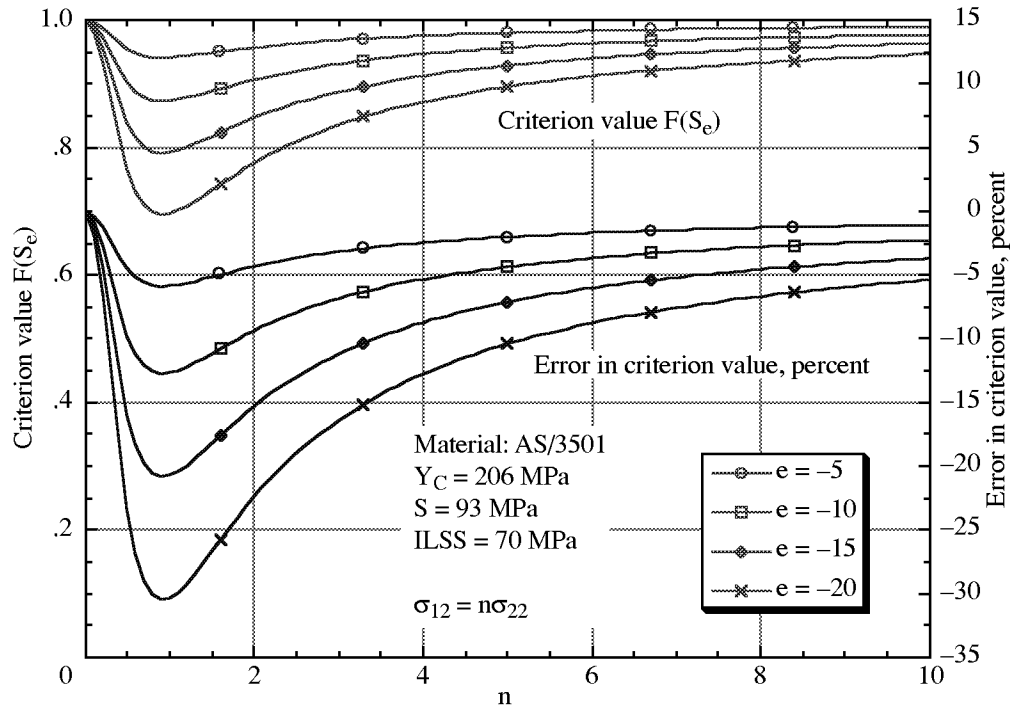


Figure 3.14. Errors in predictions of matrix failure in compression originated by underestimation of S_T .

Figure 3.14 represents the case in which there is an underestimation in the value of S_T ($S_e < ILSS$). Errors of 5, 10, 15, and 20 percent are considered. These errors (negative in the figure) and the estimation of the criterion, with the values of the stresses that led, with the correct value of S_T , to the unit value is now smaller than one (as can be observed in the figure), thus leading in all cases to unsafe predictions.

As in the previously developed study, the major errors in the predictions are associated with the case in which σ_{12} is not dominant versus σ_{22} in the stress state. As quantitative indications, the greatest errors are -5.84 percent ($e = -5$ percent), -12.69 percent ($e = -10$ percent), -20.78 percent ($e = -15$ percent), and -30.43 percent ($e = -20$ percent). All of these values correspond to values of n smaller than one ($\sigma_{12} < \sigma_{22}$).

Figure 3.15 represents the case in which there is an overestimation in the value of S_T . Errors of 5, 10, 15, and 20 percent are again considered. With these errors (positive in the figure), the estimation of the criterion, with the values of the stresses that led (with the correct value of S_T) to the unit value, is now greater than one (as can be observed in the figure), thus leading in all cases to conservative predictions.

As for the case of negative errors in S_T , the major errors in the predictions are associated with the case in which σ_{12} is not dominant versus σ_{22} in the stress state. As quantitative indications, the greatest errors are 5.03 percent ($e = 5$ percent), 9.39 percent ($e = 10$ percent), 13.19 percent ($e = 15$ percent), and 16.53 percent ($e = 20$ percent). All these values correspond to values of n close to one (σ_{12} is of the same order as σ_{22}).

As in the previous section, it is necessary to point out this inconsistency: decreasing an allowable value of the material leads to unsafe predictions.

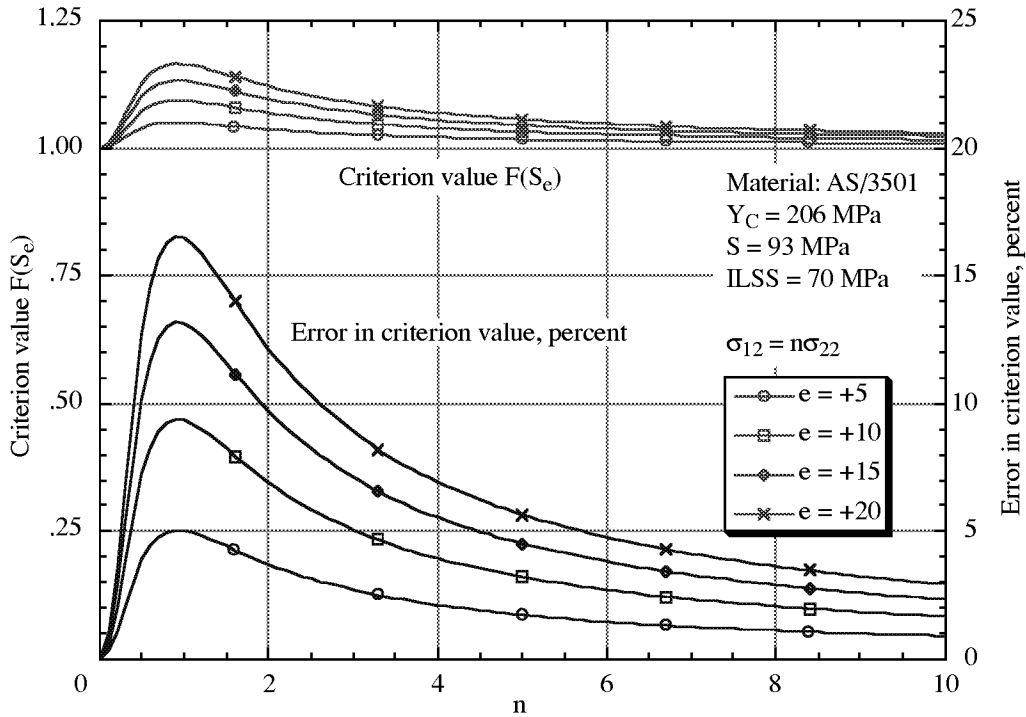


Figure 3.15. Errors in predictions of matrix failure in compression originated by overestimation of S_T .

3.5. Discussion of Hashin's Proposals on Failure of the Matrix in Compression

The results obtained in sections 3.3 and 3.4 require some additional explanations to assimilate correctly the consequences of applying any of Hashin's proposals to predict the failure of the matrix in compression.

To achieve this evaluation, the details of Hashin's proposals have to be reviewed. Although not mentioned explicitly in the first Hashin paper, the approach being followed (clearly stated in the second paper), is that there are two modes of failure (fiber and matrix), and each of them is governed by the components of the stress vector associated with the plane of failure. A further assumption is that the interaction of these components with the failure of the material throughout this plane is quadratic.

When, in the second paper, Hashin tries to apply this idea to a 3D case, particularly in the case of failure of the matrix, he finds difficulties in detecting the plane of failure and then uses a different scheme based on assigning the failure to a quadratic interaction between stress invariants. Following this scheme, Hashin finds general 3D matrix failure criteria, one for failure in tension and another for failure in compression, which obviously include the values of S and S_T . It should also be mentioned that the final two expressions for tension and compression failures are different because different hypotheses are used. In the tension failure, the linear term resulting from the interaction between the invariants is discarded, whereas it is maintained in the compression failure. Additionally, in the compression failure, the experimentally observed increment in strength is forced when there is a state of transversely isotropic pressure.

In the second paper, when proposing the 2D matrix failure criterion, the original approach (dependency of the failure on the stress vector components) could be applied in a straightforward way because the failure plane is known (parallel to the fibers). In fact, this approach would have led to the criterion

proposed in the first paper and actually does so in the case of the matrix failure in tension. Instead, in the case of the matrix failure in compression, Hashin particularizes the 3D criterion to the 2D case and then achieves an expression that includes S_T , which is not acceptable from a physical point of view, considering that it is an out-of-plane allowable. In other words, we have a failure criterion which does not involve one stress component but does involve its allowable value.

Having reached this point, it is almost mandatory to establish a connection between the two Hashin proposals for the matrix failure in compression, performing some numerical calculations.

Figure 3.16 summarizes the predictions of Hashin's criteria with the two approaches, 1973 and 1980. The value predicted by the 1973 proposal, where S_T has no influence, has been taken as a reference. What has been done is to find the value of σ_{22} and σ_{12} (for different values of n , $\sigma_{12} = n\sigma_{22}$) that lead to satisfying the criterion; then, with these stress values, the 1980 proposal has been applied for different values of k ($S_T = kS$). Only values of $k < 1$, which are more realistic, have now been included in the representation.

The values represented in figure 3.16 can be expressed in terms of the relative differences between the two proposals. These differences, taking the 1973 proposal as a reference, are represented as error,

$$\text{error} = \frac{F(80) - F(73)}{F(73)} \quad \text{with} \quad F(73) = 1$$

in figure 3.17.

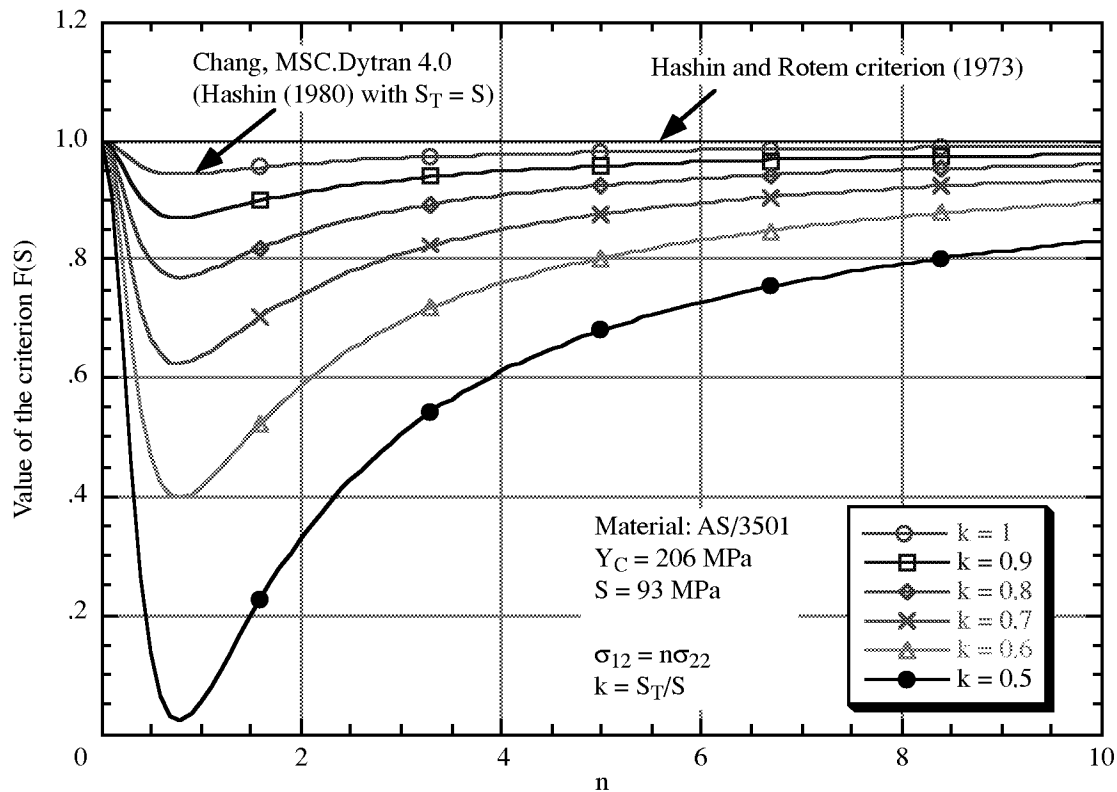


Figure 3.16. Different prediction values of Hashin's proposal for compressive failure of the matrix.

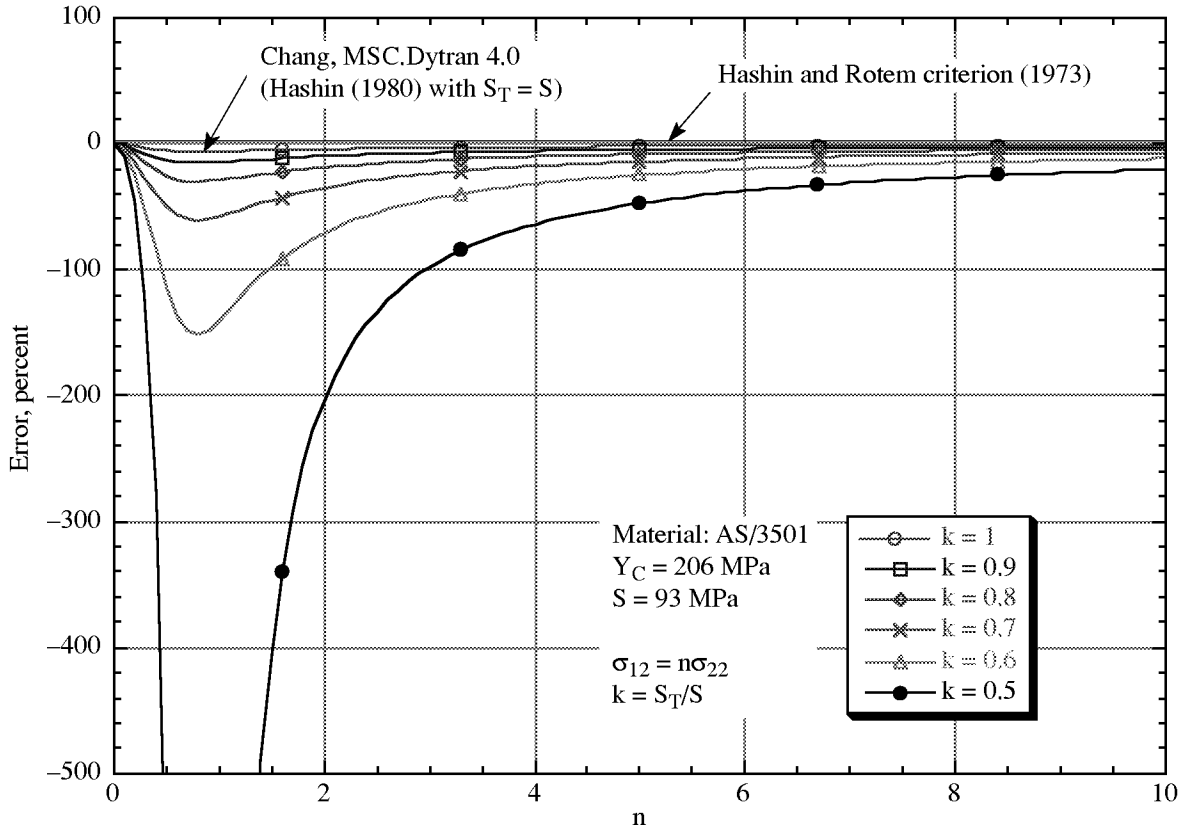


Figure 3.17. Relative errors of the prediction of matrix failure in tension with Hashin (1980), taking Hashin (1973) as a reference.

It can be observed immediately that the predictions obtained with Hashin’s 1973 proposal are conservative with respect to predictions obtained by applying Hashin’s 1980 proposal, even for the case of a material having $S_T = S$. The smaller the value of k (the greater the difference between S_T and S), the less conservative the predictions using Hashin’s 1980 proposal.

3.6. Final Comments

The results presented in this section can be summarized, with reference to the realistic case $S_T < S$, in the following way:

- First of all, a clear inconsistency of Hashin’s 1980 proposal for predicting failure of the matrix in compression has to be mentioned again. The expression of the failure of a plane parallel to the fiber includes an out-of-plane allowable, whose stress does not appear in the criterion expression. A second level of inconsistency is that to increase artificially the value of this out-of-plane allowable leads to conservative predictions.
- Taking as a reference the proposal performed in 1980 to identify S_T with S (which is the criterion that appears in MSC.Dytran assigned to Chang) would lead to conservative predictions, with significant values when σ_{12} is not dominant versus σ_{22} and particularly when S_T is significantly smaller than S .

- If the proposal performed in 1973 is taken as a reference (it has to be mentioned that it is based on the original Hashin idea that the stress vector associated with the plane is responsible for the failure of the plane), then the predictions derived from the 1980 proposal are unsafe for the whole range of possible actual values of S_T , as can be appreciated in figures 3.16 and 3.17, and consequently for the case of a material having $S_T = S$.
- It is now possible to evaluate the consequences of taking $S_T = S$ in Hashin's 1980 proposal, that is, to take $k=1$ independently of its actual value. Thus, it can be seen from figures 3.16 and 3.17 that to take $k = 1$, that is, to apply what appears in MSC.Dytran as a Chang criterion, is slightly unsafe with respect to the predictions obtained with the original 1973 proposal. Recall that experimental results have shown that this original proposal is, in turn, conservative. Several modifications recently proposed to correct the proposal are discussed in section 4. Thus, independently of the coherence of each decision taken in the process of using Hashin's 1980 proposal with $S_T = S$, there is, in some ways, a certain compensation of errors which, although it does not justify the procedure in itself, at least mitigates the uncertainties concerning its use.

4.0. Considerations on Failure Criteria of Fibrous Composite Materials

Although many comments have already been made about the different failure criteria of fibrous composites, it seems reasonable to reconsider the main ideas of the whole set of criteria proposed. When so doing, it is advisable to have them classified in a certain way, always bearing in mind that a classification helps to envisage the set of elements to be grouped together according to a certain criterion, but it also hides some other important features of the elements of the set. An appropriate start would probably be to distinguish between failure associated with a lamina and a laminate.

4.1. Failure Criteria Associated With a Lamina

The classification that seems most reasonable at present coincides with that proposed by Echabi, Trochu, and Gauvin (ref. 25). Without claiming to be exhaustive (a detailed review of the criteria up to 1994 can be found in Nahas, ref. 21), this classification would be

1. Failure criteria not directly associated with failure modes

- Tsai-Hill
- Tsai-Wu
- Modified Tsai-Wu
- All polynomial, tensorial, or parametric criteria

2. Failure criteria associated with failure modes

- Maximum strain or stress
- Hashin and Rotem
- Yamada and Sun
- Hashin

- Hart-Smith
- Puck
- Kriging

Comments on both groups will be presented herein.

4.1.1. Failure Criteria Not Directly Associated With Failure Modes

Although the use of polynomial approaches may lead to some acceptable results, they will be discarded herein as the most suitable approach to estimate the strength of fibrous composites. In fact, it does not seem that much has been learned about the evolution of failure criteria for metallic materials. The erroneous proposals made over several centuries arose from not knowing the real failure mechanism. The two most suitable criteria, Tresca and Von Mises, were based on experimental observations (Lueders lines) in performing a tension test (Tresca) and in a simplification of Tresca's criterion (Von Mises) to facilitate its use in Plasticity Theory. It was only when the mechanism of transition from elastic to plastic behavior was known that the appropriateness of these criteria was fully understood and accepted.

With composites, it might be because the task of pursuing knowledge of the failure mechanism seemed very complicated or because the requirements for designing actual structures made it necessary to have something with which to compare them. The truth is that the path followed has not quite been the right one because some of the errors generated for metallic materials have been repeated. The absence of experimental or numerical tools cannot be argued as a reason for this situation, as it could with metallic materials.

It was surprising, for someone approaching composite materials for the first time, to see proposals associated with homogeneous materials directly applied to heterogeneous materials, while presenting many of the existing book criteria of this type as a clear improvement on existing maximum strain or stress criteria because they provided smooth envelopes. There are two clear differences between metallic and composite materials: the nonhomogeneous character and the lack of isotropy of composite materials. The second change can immediately be taken into consideration in terms of macromechanical variables (the stresses) via the constitutive equations, while the first (the nonhomogeneous character) does not have any direct implication in the model generated for composites at the macromechanical level. It seems apparent that when approaching the problem of characterizing the failure of composite materials, researchers focused attention on the first aspect that contrasted with metallic materials (the lack of isotropy), while ignoring the second (the lack of homogeneity), when it is obvious, as we know from metallic materials, that it is the internal structure of the material that governs its behavior and consequently the failure.

Many counter examples have been given in the literature about the inconsistencies to which tensorial criteria may lead. One of the clearest, and probably the cruelest, is the reasoning afforded by Hart-Smith in 1993 (ref. 20) when he explains how decreasing the transverse strength of a lamina of the material leads these types of criteria to overpredict the resistance of the composite in submarine hulls subjected to bicompressive stress states.

It might seem to be a radical decision to discard, from the outset, a group of criteria that, according to the presentation of most texts, are apparently the most widely used. Sun et al. (ref. 24) include an

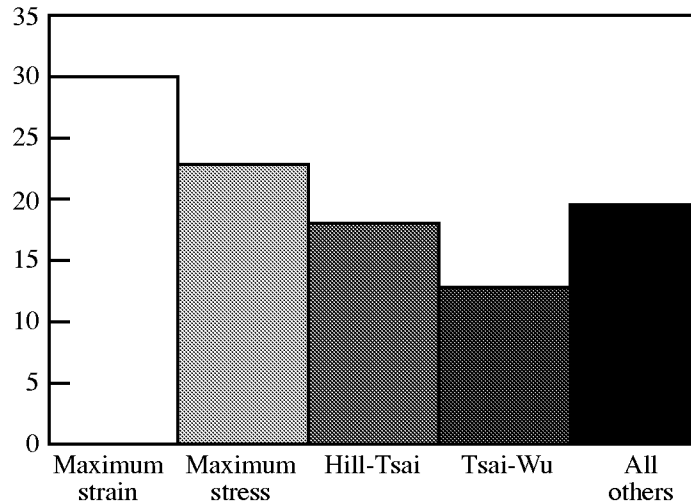


Figure 4.1. Rate of use for different failure criteria.

estimation of the use that people involved in the field of composites make of the different criteria. The information is repeated here in figure 4.1.

According to this information, around 70 percent of composite designers are not currently using Tsai-derived criteria. Also interesting is the clear preponderance of the maximum strain criterion.

The lesser use of Hashin and derived criteria, which are included in “others” in the previous representation, might seem surprising. The reason may lie in the lesser attention composite textbooks pay to this type of criterion, whereas if composite research papers are reviewed, the majority of researchers do use Hashin’s criteria.

4.1.2. Failure Criteria Associated With Failure Modes

Maximum stress and maximum strain criteria are simple, direct ways to predict failure of composites. Their major limitation is that there is no interaction between the stresses/strains acting on the lamina, underpredicting the strength in the presence of combined actions of in-plane stresses.

Hart-Smith’s proposal (ref. 11) can be allocated directly to the group of predictions of laminate strength, although it can obviously be considered here. The proposal of Hart-Smith, who argued against failure criteria not based on failure mechanisms, is not really, in this author’s opinion, a failure-mechanism-based proposal. It might be allocated to a group of experimental evidence. Sun et al. (ref. 24) have shown that the 45° cuts that appear in Hart-Smith’s proposal are not in agreement with experiments, thus underpredicting the resistance of the laminate.

Kriging is a term used to denote a procedure to generate failure envelopes that lies somewhere in between the two groups of classification. It has been placed herein to respect the decision of the authors of both this general classification and of the Kriging approach. The authors argue that Kriging is a failure-based mechanism criterion because it is allowed to orient the local shape of the failure envelope, forcing local conditions derived from the observation of the behavior of the composite under a certain combination of loads. The author of this report sees this procedure more as a phenomenological global-based failure criterion. The mechanisms of failures are not identified, but their effect can be considered when generating the failure envelope. The mathematics involved in this procedure is complicated, and

although the procedure can be used at the level of generating knowledge about composites, this author does not feel that it represents the most appropriate way to approach a general design procedure.

Nevertheless, Kriging cannot be completely excluded. There are further reflections herein on what the author of this report considers initially the most appropriate way to approach the failure criteria of composite materials. However, studies performed in the future to complete these ideas might lead to the conclusion that the mechanisms of composite failure that occur at the micromechanical level are so complicated that it is not possible to reflect them at the macromechanical level. In such a case, Kriging would appear a feasible way to implement existing knowledge of the particular behavior of each composite.

The rest of the criteria listed in this section have been discussed in the previous section, except for Puck's criterion. When dealing with this criterion, we are referring to what has recently appeared in the German composite area in connection with Puck's ideas (see refs. 16 and 17) rather than to what appears in the literature (see for instance, Nahas, ref. 21) as Puck criterion (A. Puck, *Calculating the strength of Glass fibre/Plastics laminates under combined load*, 59, 18–19, 1969). Both papers refer to the work completed recently by Puck (*Festigkeitsanalyse von Faser-Matrix laminaten-Modelle für die Praxis*, Carl Hanser Verlag, 1995), which, like the paper containing his former criterion, has not been included in the reference list because both are in German.

There is a major emphasis (by Puck and this German group) with the matrix failure criterion in compression in a 3D case. They use the same idea that Hashin explored in 1980, considering that the failure of the matrix throughout a plane is controlled by the components of the stress vector associated with the plane. Thus, they are first concerned with the determination of the plane at which the maximum value of the criterion is reached, a problem that can be solved numerically.

The second concern refers to the expression of the criterion itself. In view of some experimental results, they propose to modify the criterion, canceling the contribution of the normal stress and modifying the allowable shear strengths involved in the criterion, expressing them as a function of the compression applied transversally to the fibers.

Thus, taking as a reference an assumed failure plane parallel to the fibers, such as the one shown in figure 4.2, where the stress vector has three components, σ_n , τ_{nl} , τ_{nt} , the original Hashin criterion would be

$$\left(\frac{\sigma_n}{Y_C}\right)^2 + \left(\frac{\tau_{nl}}{S}\right)^2 + \left(\frac{\tau_{nt}}{S_T}\right)^2 = 1$$

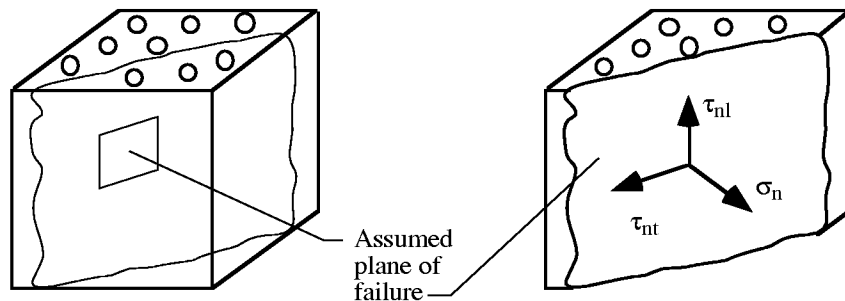


Figure 4.2. Failure of matrix in compression.

The proposal of Kroll and Hufenbach (ref. 17) (see also Kopp and Michaeli, ref. 16) is to alter the previous expression in the following way:

$$\left(\frac{\tau_{nl}}{S - p_{nl}\sigma_n} \right)^2 + \left(\frac{\tau_{nt}}{S_T - p_{nt}\sigma_n} \right)^2 = 1$$

Two modifications have been performed. First, the contribution of σ_n has been cancelled by arguing that a transverse compressive strength to the failure plane does not contribute to the initiation of the fracture. Second, the shear allowables are modified by arguing that σ_n impedes the shear fracture caused by the shear stress τ_{nl} and τ_{nt} , giving rise to apparent extra values of the shear strengths. These extra apparent allowable values would be characterized by the coefficients p_{nl} and p_{nt} , which had to be characterized experimentally and would represent internal material friction, according to the authors.

It is striking that, in a separate study, Sun et al. (ref. 24) propose a modification of the criterion very similar in form to that appearing in the Kroll and Hufenbach paper. The authors, based on experimental results and with reference to the failure of the matrix in the 2D case, propose

$$\left(\frac{\sigma_{22}}{Y} \right)^2 + \left(\frac{\tau_{12}}{S - \mu\sigma_{22}} \right)^2 = 1$$

with

$$\mu = \begin{cases} \mu_o & \sigma_{22} < 0 \\ 0 & \sigma_{22} > 0 \end{cases}$$

where μ is said to play a role similar to the friction coefficient.

The main difference between the two proposals is that in Kroll's expression the contribution of the compressive stress has been cancelled, whereas in the expression of Sun et al. it is maintained.

The proposal of Kroll et al. seems to be, on the one hand, more coherent. If a negative value of σ_n (σ_{22} in Sun's expression) is beneficial, it seems that its contribution to the prediction of the failure ought to be cancelled. On the other hand, the total cancellation of σ_n in the prediction of the failure would imply that, in the presence of only σ_n in compression, with no presence of shear stresses, the failure would never appear.

Because both proposals are based first on experimental observations and on general reasoning (this author does not see any support for the proposal at the level of the actual micromechanical failure) and because secondly they involve an experimental parameter that ought to be determined experimentally, both could lead to accurate predictions. Obviously, the two similar parameters (p 's and μ) would have different values, and μ ought to be greater than p 's to predict the failure at an equivalent level.

This fact offers the opportunity to comment on one of the big problems occurring in the field of failure approaches for composite materials. Even starting from incorrect assumptions (which does not seem to be the case in the previous situation analyzed), the final conclusion can be correct (predicting things quite accurately) because adjusting parameters are involved. One does not necessarily have to discard

approaches of this type to deal with complicated problems. Approaches of this type have been successfully used in different fields, such as the prediction of the strength of welded joints or the prediction of the maximum bending moment that a concrete section can support. The problem with composites is that there are so many ways to generate a material system that nonphysically based decisions are of very limited application. What is reasoned in section 5 is an attempt to focus failure criteria in a completely physically based manner.

4.2. Failure Criteria Associated With a Laminate

It is questionable whether it is reasonable to consider the design of a laminate without having clarified the conditions under which a lamina fails. On the other hand, it is realistic to accept that something has to be said about the behavior and prediction of laminate failure.

To organize the discussion, let us start by questioning whether the exact knowledge of what happens at the lamina level would be enough to predict the behavior of a laminate. The following reflections on this question seem to be quite clear.

- (a) A correct knowledge of the stress state in the lamina placed in the laminate would be required.
- (b) The maintenance of just the same intralaminar mechanisms of failure or the appearance of new ones, when the lamina is now subjected to the new state of stress, has to be clarified.

If the questions addressed in (a) and (b) are not clarified, the typical topic of in situ properties is called for, a scheme that, in fact, reflects a recognition that the actual situation is noncontrolled; that is, the actual state of stress and the mechanism of failure are unknown.

- (c) The inclusion of delaminations in the mechanism of failure obviously has to be considered. The question of how to do it has not been decided. To do it in terms of stresses would obviously be the simplest solution in computational terms. However, there is much more physical evidence that the phenomenon, in itself, is much more realistically controlled by the parameters associated with fracture mechanics theory, which makes the numerical simulation of the estimation and growth of the damage quite complicated. There is a definite need to find a way to combine both approaches.
- (d) Delaminations seem to be mechanisms that are clearly activated after an impact. There is not too much evidence of existing quantitative, or even qualitative knowledge about the collaboration of this mechanism in the energy absorption of the impact. In fact, it has been mentioned on several occasions during the reference review that several studies on impact achieve excellent results in terms of the similarity of damage predictions and experimental values, without considering, in the numerical simulation, the delamination mechanism of failure.

The next question, assuming that all the aspects covered in the previous points are clarified, would be whether an approach based on a progressive damage mechanism, with reduction in the values of the properties of the material, is adequate. When a progressive damage scheme is coupled with an initial failure criterion not based on a real failure mechanism, one has the impression that it is like trying to sort out an ineffectively solved problem. However, the very nature of the material and the kind of material damage progression one can “hear” when testing composite specimens may justify proceeding with such a scheme.

A subsequent point to elucidate would be whether the degradation procedure ought to be partial or total. This question, the discussion of which proves the uncertainties about the adequacy of the procedure, could probably be solved by studying numerically the consequences derived from adopting each of the two schemes, consequences which could be established by the appropriate selection of a set of tests.

As regards a degradation procedure of the properties, it would be necessary to design one associated with delamination, not an immediate possibility.

Coming back to the first question, there would always be a possibility, particularly if the questions formerly pointed to as requiring clarification could not clearly be answered, of considering a laminate failure criterion directly, which would be generated by means of the knowledge of each lamina failure mechanism. The idea, discussed in the next section, would be to extrapolate the scheme proposed about the role of micromechanics to estimate lamina failure to the next step: estimating laminate allowable values from lamina allowable values. Thus, micromechanics ought to be used to generate knowledge of lamina failure, but it is not reasonable that variables at the micromechanical level can be directly involved in the procedure of predicting lamina failure. In a similar way, a lamina failure mechanism ought to be used to generate an adequate knowledge of the behavior of laminates, but it would be desirable for variables at this level (lamina stresses) not to be directly managed in the laminate failure criterion. It would be like finding the failure envelope of efforts that originate the plastic collapse of a metallic material section without going to the stress level at each point of the section under consideration in the analysis.

5.0. Towards a Failure Criterion Based Fully on Damage Mechanisms

This section (a reflection arising from questions previously posed) attempts to elucidate whether it is realistic to develop studies aimed at generating a set of rules representing a failure criterion based fully on failure mechanisms.

This reflection will be presented following the path of an ascending series of questions about different aspects and statements, a procedure generally accepted when establishing a failure criterion for composites.

The reflection starts with the modification proposed by Puck and other German authors and by C. T. Sun and coworkers, as was previously discussed in section 4 about the failure criterion for the matrix in compression.

The first thing that attracts attention is the way in which both proposals try to take into account the accepted fact of the beneficial role of σ_{22} in the strength of the composite. In both schemes, the allowable value of the tangential stress is modified by the product of a factor times the value of the stress σ_{22} , as is indicated, for instance, for the case of σ_{23} in expression 5.1,

$$\frac{\sigma_{23}}{\sigma_{23}^a - \mu\sigma_{22}} \quad (5.1)$$

where σ_{23}^a would be the allowable value of σ_{23} .

With no other evidence available, it seems to be more coherent and equally acceptable to modify the original expression in the form defined in 5.2,

$$\frac{\sigma_{23} + \mu\sigma_{22}}{\sigma_{23}^a} \quad (5.2)$$

thus keeping the general scheme of all proposals while maintaining the numerator for the stresses and the denominator for the allowable values.

What is more, it is easy to consider the possibility of altering the former expression in the form suggested by expression 5.3 because it seems to be more reasonable to perform the modification in terms of relative values of the normal (σ_{22}) and the tangential stress (σ_{23} in this case). At least some unclear cases for extreme relative values of both stresses that would appear in the original scheme (5.1) and the modified one (5.2) would be avoided (consider particularly a case in which σ_{23} is very low and σ_{22} is very high or, in general, a fixed collaboration of σ_{22} independently of the absolute value of σ_{23}).

$$\frac{\sigma_{23}\mu(\sigma_{22}/\sigma_{23})}{\sigma_{23}^a} \quad (5.3)$$

The structure of the coefficient μ immediately leads us to consider the following question.

(1) Could the beneficial contribution of σ_{22} in the matrix failure in compression be predicted by means of micromechanics?

The problem under consideration is represented in figure 5.1 at the lamina level. For simplicity, only the stresses involved in the problem under consideration have been included in the representation.

A first approach to deal with the problem is the study of the single fiber problem represented in figure 5.2, in which a fiber surrounded by a matrix in the middle of a transverse section of a composite lamina is considered.

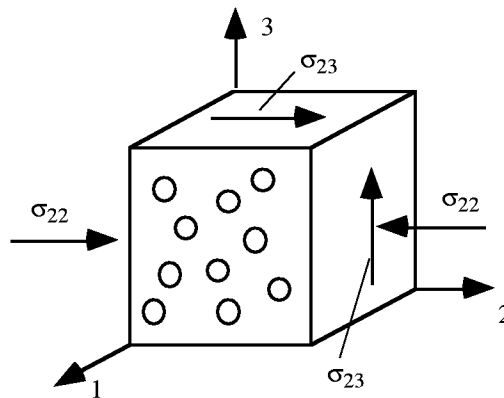


Figure 5.1. Unidirectional lamina under σ_{22} and σ_{23} stresses.

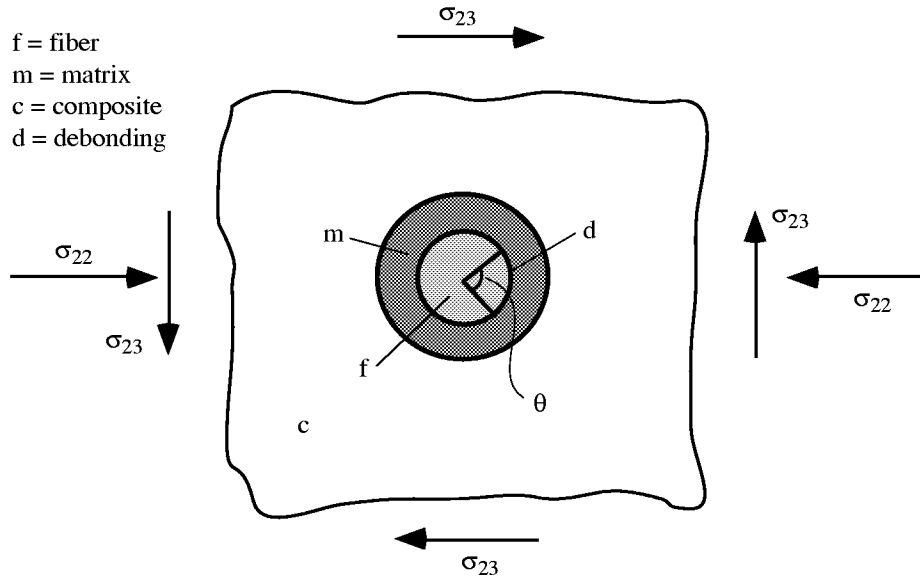


Figure 5.2. Study of transversal debonding between fiber and matrix.

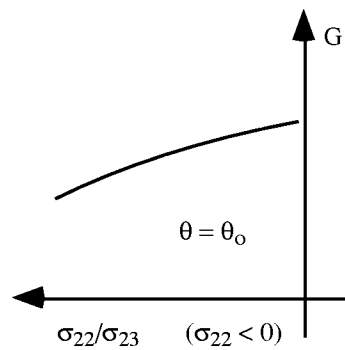


Figure 5.3. Evolution of energy release rate as a function of σ_{22}/σ_{23} for a given value of the debonding angle.

The most realistic way to begin studying this problem is to consider the presence of a debonding, characterized by θ in the figure, between the fiber and the matrix. This problem can be easily and properly modeled by using the Boundary Element Method (see Paris, del Caño, and Varna, *Int. J. of Fracture*, Vol. 82, No. 1, pp. 11–29, 1996). The idea would be to calculate, for each case, the fracture mechanic parameters, the energy release rate G (G_I and/or G_{II}) or the stress intensity factor (K_I and/or K_{II}) that controls the growth of the crack, and consequently, the failure of the material. Thus, curves such as the one presented in figure 5.3 for different values of the debonding angle would be generated.

The integration of the information generated from this study would permit the possibility of defining a parameter ($\mu\sigma_{22}/\sigma_{23}$). A parametric study as a function of the properties of the materials involved in the problem ought to be developed. It is expected that some aspect of the problem will be governed by geometrical features, as was observed in a similar problem studied in the previous reference.

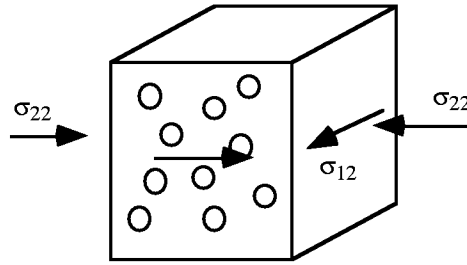


Figure 5.4. Unidirectional lamina under σ_{22} and σ_{12} stresses.

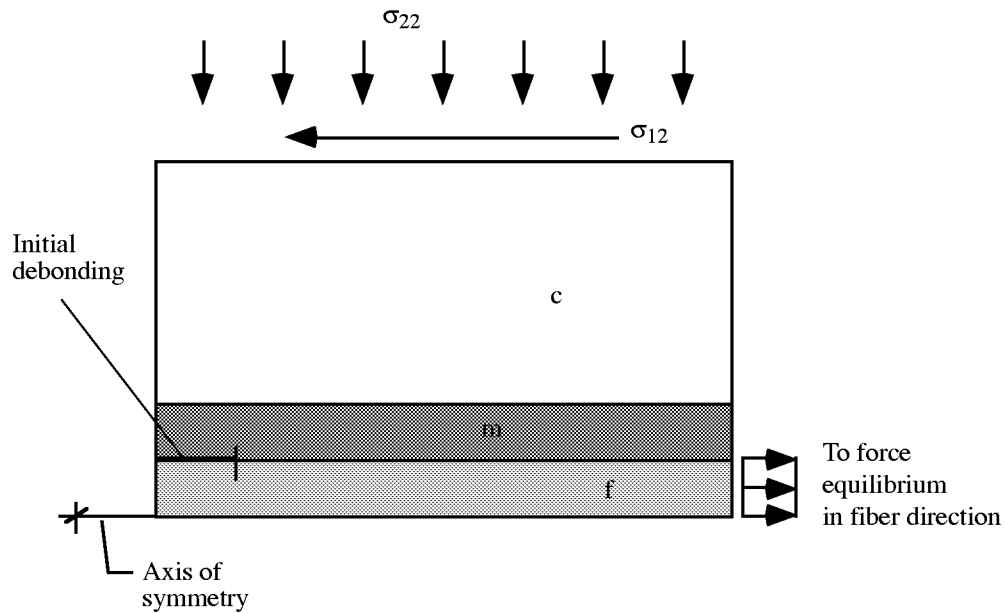


Figure 5.5. Study of longitudinal debonding between fiber and matrix.

The value of the equivalent coefficient for the interaction between σ_{22} and σ_{12} could also be estimated by means of micromechanical analysis. In this case, the problem under consideration is that represented in figure 5.4.

In this case, the most plausible 2D representation of the problem under consideration would be that represented in figure 5.5, in which an initial debonding between the fiber and the matrix, as in the previous case, would be assumed.

The calculations to elucidate whether a coefficient ($\mu\sigma_{12}/\sigma_{22}$) can be defined in terms of the geometrical considerations of the problem would be the same as those previously proposed for the case of $\mu(\sigma_{23}/\sigma_{22})$.

It has to be noted at this point that all situations are 3D. An attempt has been made to maintain the former study proposed (and the following proposals) in the 2D field (plane strain, plane stress or axisymmetric) to simplify the problem initially. The results obtained would recommend whether there is a the need to perform a full 3D study.

The approach that is followed to answer the questions formulated previously opens up new ways to study other questions that have arisen when comparing different proposals on failure criteria.

(2) Could the role of σ_{12} in fiber breakage be predicted by means of micromechanics?

In his 1980 proposal, Hashin included σ_{12} in the fiber breakage criterion of a lamina under the load represented in figure 5.6 by means of expression 5.4.

$$\left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 = 1 \quad (5.4)$$

It is not as straightforward as in the previous question to decide which 2D representation is the most appropriate to elucidate the role of σ_{12} in the fiber breakage. A 2D axisymmetric model and a plane stress model can be developed. Both are represented in figure 5.7. The application of the tangential stresses in the axisymmetrical model (only schematically represented in the figure) has to be done in such a way as not to alter the axisymmetrical conditions of the geometry.

It seems more reasonable to focus the study on a region with the stresses regularized, which would lead to a study of the geometries presented in figure 5.8.

Assuming that the failure appears by breakage of the fibers, the immediate task would be the calculation of σ_f (nominal stress at the fibers) as a function of the relation σ_{11}/σ_{12} . A further study could be done assuming the presence of a crack between the fiber and the matrix (and/or between the matrix and the composite). In any case, the situations represented are not exactly the same as those corresponding to the actual case, but the effect of the variation of σ_{12} is expected to be quite similar in all cases.

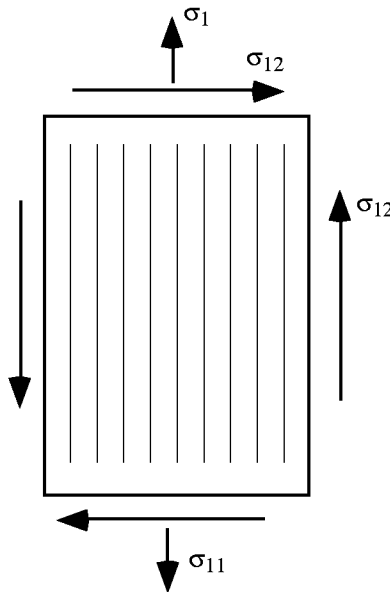


Figure 5.6. Lamina under σ_{11} and σ_{12} stresses.

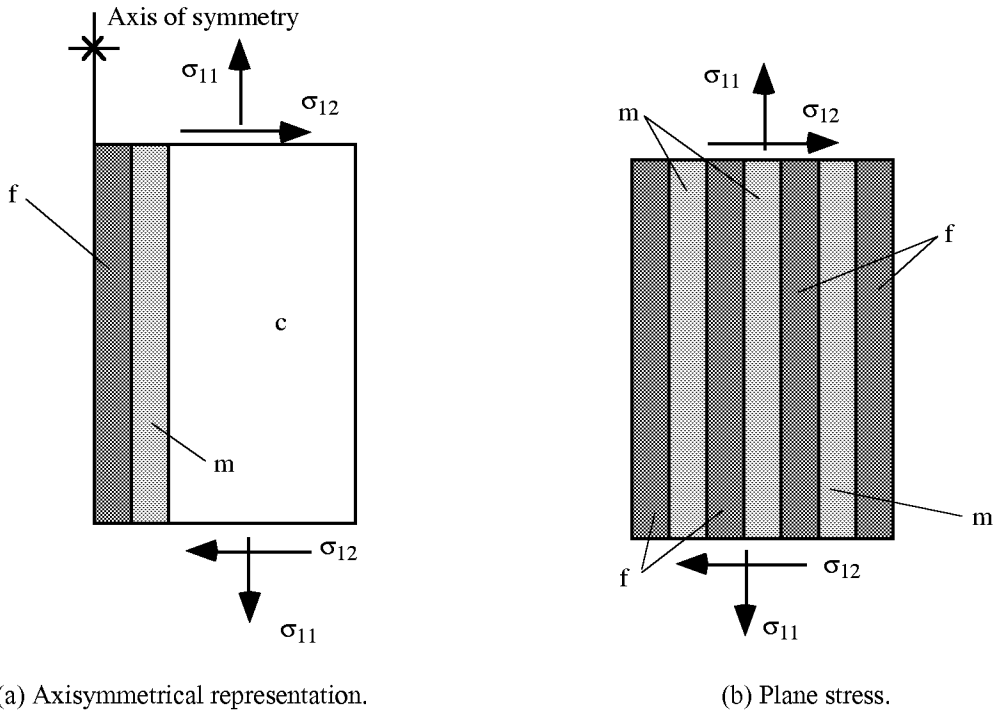


Figure 5.7. Two-dimensional representation of fiber surrounded by matrix and composite.

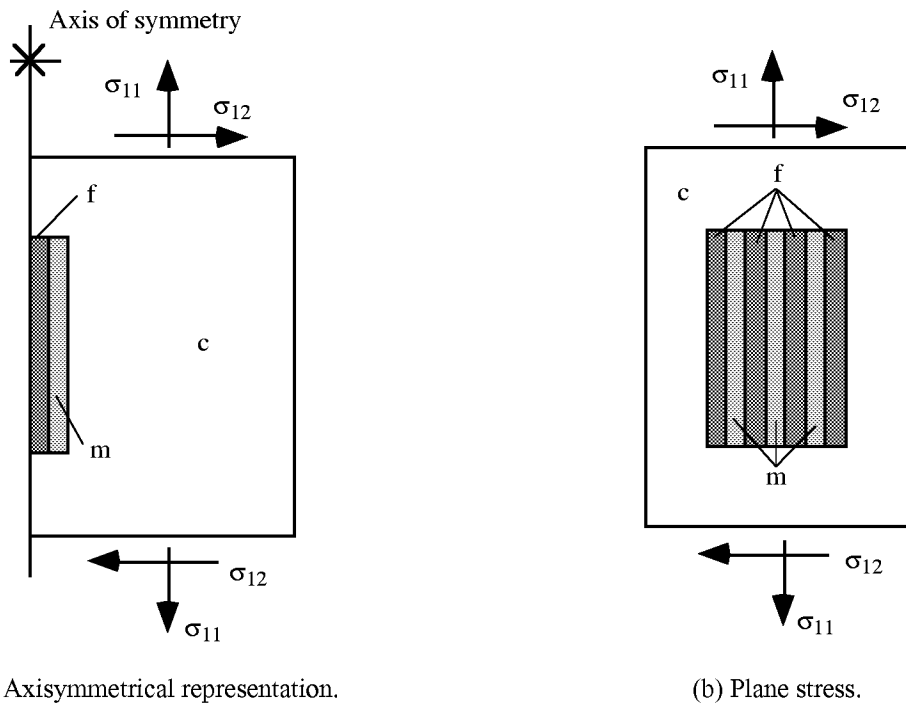


Figure 5.8. Two-dimensional regularized representation of fiber surrounded by matrix and composite.

The scheme followed to apply micromechanics to generate information about dependence of the failure on macrovariables can also be used to reexamine old questions generally accepted when addressing failure mechanisms and failure criteria.

(3) Is it reasonable to accept the quadratic combination of the components of the stress vector associated with a plane to predict the failure of the composite material at that plane?

The quadratic interaction between the stresses associated with a mechanism of failure has been generally accepted, as has been mentioned in previous sections since the Hashin proposals. For instance, with reference to the matrix failure mode in tension, the failure criterion proposed by Hashin was

$$\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 = 1$$

Let us consider, as has been previously mentioned as the most realistic way to take the failure into consideration, the presence of a crack between the fiber and the matrix. One such crack could grow longitudinally or circumferentially. To study the first case, the configuration presented in figure 5.9 is proposed and to study the second case, the configuration proposed in figure 5.10 can be considered.

With reference to the first case, longitudinal growth of the crack, it seems that the stress σ_{22} governs the growth. With reference to the second case, the same fact is even more obvious, the role of σ_{12} being irrelevant. Although a detailed calculation using previous boundary element method (BEM) models could be used to clarify the question presented, it seems that the quadratic combination of the effect of the stresses does not seem to represent what happens at the micromechanical level.

What has been considered in this case is the start of the failure. More complicated situations modeling more fibers and with cracks interacting between the fibers through the matrix ought to be considered, in a second step, for this and equivalent studies.

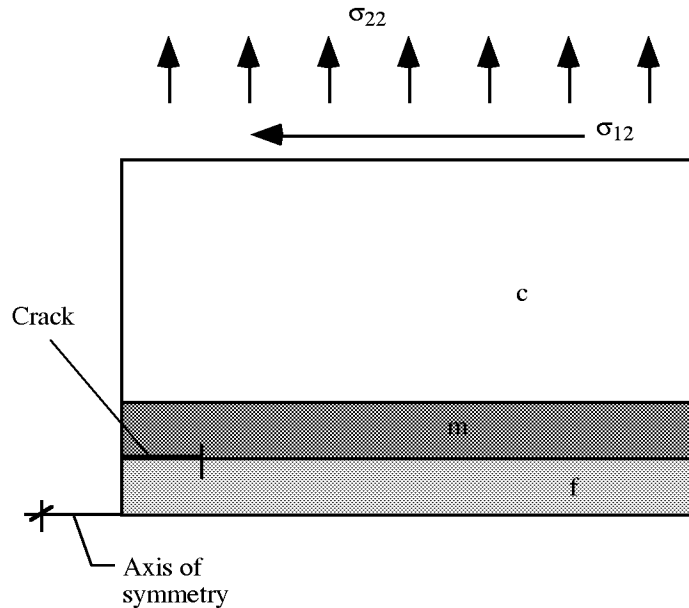


Figure 5.9. Longitudinal growth of existing crack between fiber and matrix.

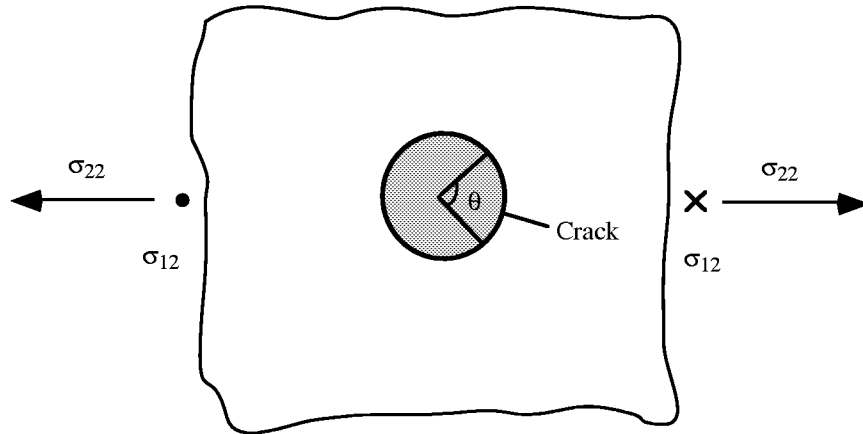


Figure 5.10. Circumferential growth of existing crack between fiber and matrix.

The previous discussion opens the possibility of considering an even more drastic question.

(4) Is the failure of a fibrous composite material at one plane governed only by the components of the stress vector associated to this plane?

This key assumption in Hashin's proposals has been generally accepted in subsequent criteria. Can micromechanic analysis shed some light on this question?

The problem could be focused on the configuration shown in figure 5.11(a). If we consider the plane of failure indicated in the figure, the regular hypothesis would lead the failure to be associated to the stresses σ_{22} and σ_{23} , whereas σ_{33} would not play any role in the failure.

A micromechanic view of the problem would lead us to consider the configuration presented in figure 5.11(b), which would be a familiar geometry already studied in this report, although the interest and the emphasis is now in a different direction.

The original question could now be formulated in the following terms: does σ_{33} have no influence in the growth of the existing assumed crack between the fiber and the matrix? The answer to the question, although pending a detailed study for different configurations, seems to be apparent: σ_{33} may increase K_I and/or K_{II} , consequently accelerating and/or delaying the growth of the crack and consequently questioning the main hypothesis on which one proposed failure criteria based on a mechanism of failure lies.

A similar comment, in the sense that out-of-plane stresses could influence the failure of a plane, can be found in the revision. Although in a different context, this influence can be verified in reference 10.

In addition to the questions addressed in this section, there is an added general problem, not usually referred to in the references, which could be studied with the micromechanical approach. That problem is the role that residual-production-curing stresses may play in the failure of a composite material. Models like those previously considered have already been used in a different context (see París, del Caño, and Varna, *Boundary Elements XX*, pp. 145–154, CMP, 1998) to check the role of the residual stresses and could be used easily in the present problem to understand the role of such stresses in the failure of fibrous composite laminates.

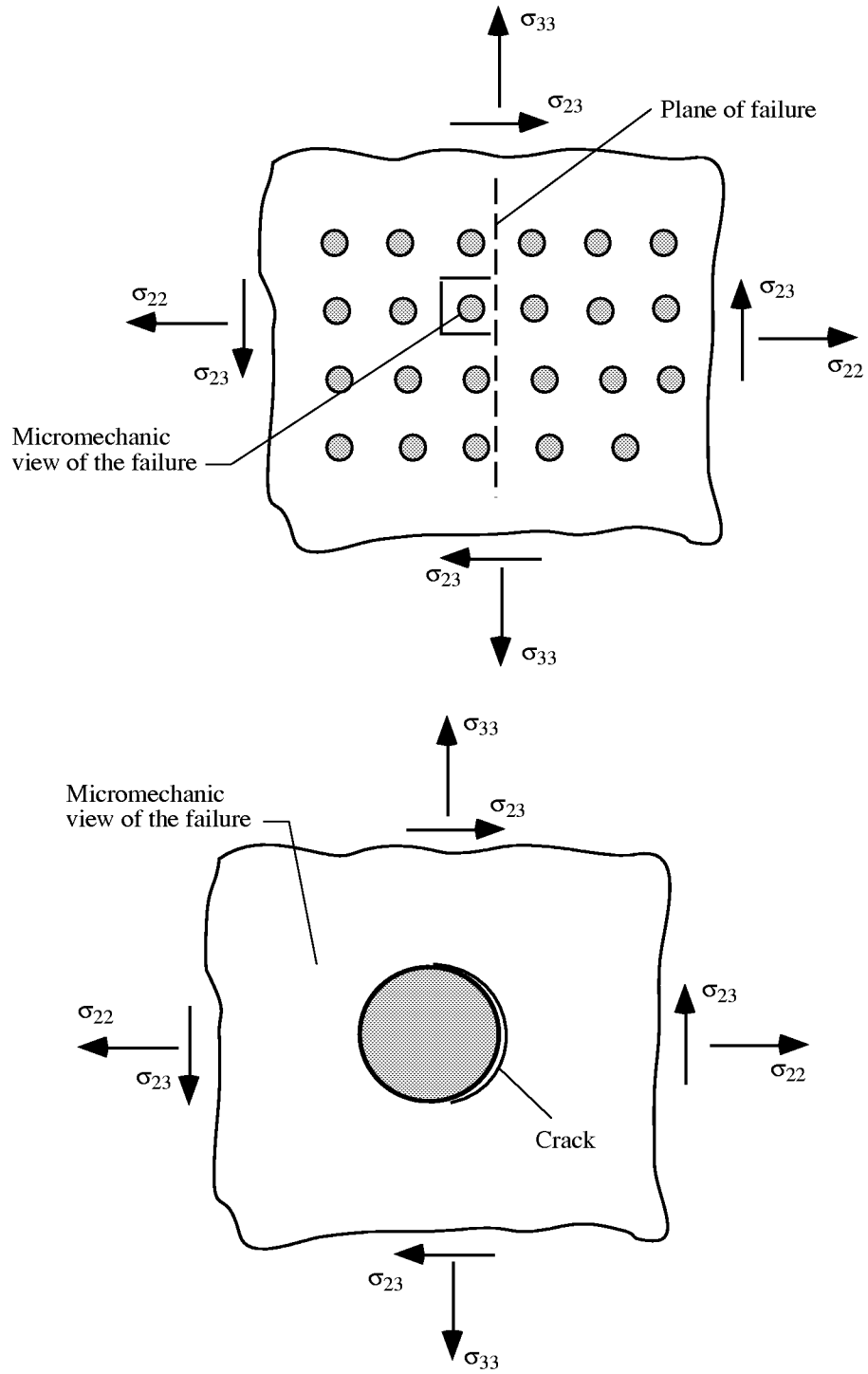


Figure 5.11. Micromechanical implications of associating failure at a plane to stresses associated with such a plane.

6.0. Concluding Remarks

At present, the failure criteria of fibrous composite materials cannot be considered a closed area of knowledge.

At the lamina level, many papers have been reviewed, demonstrating the ability or the inability of a certain criterion to predict the behavior of a particular composite, although it is impossible to recommend a criterion as the most suitable in general terms. The author of this report places most trust in what has been called in the literature “failure mechanism based criteria.” Before accepting a criterion not based on failure mechanism, the author considers that it has to be proved that this approach is impossible to follow. Such a hypothesis cannot yet be considered because of the insufficient amount of work accomplished to date.

At the lamina level, two steps ought to be considered in the development of a set of rules to generate a new fully based failure criterion for fibrous reinforced composites.

First, there should be a micromechanical study. Present knowledge suggests starting by considering at least five different mechanisms of failure: fiber breakage in tension (FBT), fiber breakage in compression (FBC), fiber matrix shearing (FMS), matrix in tension (MT), and matrix in compression (MC).

The micromechanical studies to be developed ought to clarify, for the case of a two-dimensional (2D) criterion, the following questions.

1. Role of σ_{12} in FBT and FBC mechanisms of failure.
2. Role of σ_{22} in FMS and type of interaction between σ_{11} , σ_{12} , and σ_{22} (if required).
3. Role of σ_{11} in MT and type of interaction between σ_{12} , σ_{22} , and σ_{11} (if required).
4. The MC mechanism of failure seems initially to require two cases which additionally ought to be quantitatively delimited.
 - 4.1. Case of σ_{12} dominant. Generation of a factor $f(\sigma_{12}/\sigma_{22})$ to quantify the beneficial role of σ_{22} .
 - 4.2. Case of σ_{22} dominant. Type of interaction between σ_{12} and σ_{22} .

The case of a full three-dimensional (3D) failure criterion would require similar developments for the interacting variables out of the 12 plane.

Second, there should be an experimental study. A set of tests has to be designed, dictated by the micromechanical results, to verify these results and elucidate the extrapolation of the conclusions of the necessary simple micromechanical studies to the actual lamina level. These tests must necessarily include 2D tests; in fact, they are probably the only kind of tests required.

At the laminate level, the problem is obviously more complicated because the problem involves the failure criterion of a lamina. At present, a scheme involving a degradation procedure seems to be the most realistic approach. The question of whether the properties can be degraded instantaneously or progressively still requires some further studies, which will probably not be definitive until the lamina failure criterion is completely established.

The incorporation of the delamination failure mechanism into numerical predictions seems to be necessary in cases such as impact problems, in which this mechanism plays an active role in the absorption of energy. No single present modeling procedure seems to be definitive, some because they are not affordable numerically speaking and others because they are not physically based.

The consideration of large deformations and interaction of failure mechanisms are still topics under development, and although there are several groups involved in these developments, it would seem that studying this type of problem might receive a substantial boost with clarification of the questions previously proposed for study in this report.

Seville, October 1999

7.0. References

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