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APPLICATION OF AEROELASTIC SOLVERS BASED ON NAVIER-STOKES EQUATIONS

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Introduction
The propulsion element of the NASA Advanced Subsonic Technology (AST) initiative is directed towards increasing the overall efficiency of current aircraft engines. This effort requires an increase in the efficiency of various components, such as fans, compressors, turbines etc. Improvement in engine efficiency can be accomplished through the use of lighter materials, larger diameter fans and/or higher-pressure ratio compressors. However, each of these has the potential to result in aeroelastic problems such as flutter or forced response. To address the aeroelastic problems, the Structural Dynamics Branch of NASA Glenn has been involved in the development of numerical capabilities for analyzing the aeroelastic stability characteristics and forced response of wide chord fans, multi-stage compressors and turbines.

In order to design an engine to safely perform a set of desired tasks, accurate information of the stresses on the blade during the entire cycle of blade motion is required. This requirement in turn demands that accurate knowledge of steady and unsteady blade loading is available. To obtain the steady and unsteady aerodynamic forces for the complex flows around the engine components, for the flow regimes encountered by the rotor, an advanced compressible Navier-Stokes solver is required. A finite volume based Navier-Stokes solver has been developed at Mississippi State University (MSU) for solving the flow field around multistage rotors. The focus of the current research effort, under NASA Cooperative Agreement NCC3-596 was on developing an aeroelastic analysis code (entitled TURBO-AE) based on the Navier-Stokes solver developed by MSU. The TURBO-AE code has been developed for flutter analysis of turbomachine components and delivered to NASA and its industry partners. The code has been verified, validated and is being applied by NASA Glenn and by aircraft engine manufacturers to analyze the aeroelastic stability characteristics of modern fans, compressors and turbines.
Summary of Accomplishments

Version 4 of the TURBO-AE code was developed, checked and delivered to NASA and its industry partners. In this version of code, all possible inter-blade phase angles are analyzed using a single blade passage by incorporating phase-lagged boundary conditions. Both direct-store and Fourier-decomposed methods have been implemented into the code for the phase-lag analysis. In the direct-store method, all the relevant information are stored for lagging (time-shifting) the passage boundary conditions by the appropriate phase. Since the flow variables for the passage fluid boundaries need to be stored for the oscillation cycle, this method can become prohibitive in regards to memory requirements. Memory requirements on CRAY computers can be reduced by using the solid-state devices (ssds) to read and write instead of storing the variables within the core memory. This option, however, is not feasible within a work-station environment. To alleviate the problem on work-stations, a Fourier-decomposition analysis method was implemented. In this method, the time variation of the flow variables at the passage boundary was decomposed into their Fourier coefficients and only relevant coefficients were stored. This significantly reduces the storage requirement, however, the computational time is increased as Fourier decomposition as well as reconstruction of flow variables from the stored Fourier coefficients are required at each time step.

Both of these methods were implemented into the TURBO-AE code. The code has been verified by applying it to a helical fan and released to NASA and its industry partners. Some of the results obtained from this version of the code are summarized in Refs. [1,2]. Further code validation results are summarized in Ref. [3].

Modification of the TURBO-AE, code in order to perform forced response calculations was started. The 3-D unsteady boundary conditions, developed by researchers at UTRC [2], currently does not allow disturbances to enter the computational domain. In other words, it only allows disturbances to propagate out of the domain. Work was also started with the ultimate aim of understanding the unsteady boundary conditions and modifying them for forced response calculations. This modification requires coding changes to allow for prescribed disturbances from outside the domain to propagate into the computational domain.

Drs. R. Srivastava and M. A. Bakhle, both of the University of Toledo, held two workshops, for the industry partners and the Air Force, at NASA Glenn. The workshops provided the potential users of TURBO-AE all the relevant information required to prepare the input data, execute the code, interpret the results and benchmark the code on their computer systems. Technical support was also provided to researchers at NASA and to the industry partners who are currently using the code.
References
Phase-Lagged Boundary Condition Methods for Aeroelastic Analysis of Turbomachines - A Comparative Study

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ABSTRACT
In the present work a comparative study of phase-lagged boundary condition methods is carried out. The relative merits and advantages of time-shifted and the Fourier decomposition methods are compared. Both methods are implemented in a time marching Euler/Navier-Stokes solver and are applied to a flat plate helical fan with harmonically oscillating blades to perform the study. Results were obtained for subsonic as well as supersonic flows. Results for subsonic inflow showed good comparisons with published results and between the two methods along with comparable computational costs. For the supersonic inflow, despite the presence of shocks at the periodic boundary results from both the methods compared well, however, Fourier decomposition method was computationally more expensive. For linear flowfield Fourier decomposition method is best suited, especially for work-station environment. The time-shifted method is better suited for CRAY category of computers where fast input-output devices are available.

INTRODUCTION
Numerical aeroelastic analysis methods have been developed using both two-dimensional and three-dimensional aerodynamics. Srivastava et al. (1998) have provided a good reference for these methods. The methods based on two-dimensional aerodynamics are fast but ignore the real physical effects of three-dimensional flows. The three-dimensional analyses capture all the required physics but are much more computationally expensive.

More so for analyses of inter-blade phase angles (IBPAs) requiring large number of blade passages to be included in the analysis. To reduce computational time, especially for smaller IBPA vibrations, several phase-lagged methods have been reported in the past. Erdos et al. (1977) were the first to develop an analysis based on direct-store method. The direct-store method stores all the relevant fluid properties over the oscillation cycle which is applied with appropriate lag for the IBPA being analyzed. Though, no loss in fidelity occurs, the method requires significant additional memory and becomes prohibitive for large three-dimensional problems. Another method, known as time-inclined computational plane approach, was proposed by Giles (1988) primarily to overcome the problem encountered in rotor-stator applications where no final periodic state exists. This method requires transforming the original governing equation to account for the tilting of the time plane. He (1989) proposed a shape-correction method for applying the phase-lagged boundary conditions that did not have the storage penalty associated with the direct-store method. In the shape-correction method, the variation of fluid properties over an oscillation cycle is decomposed into its Fourier coefficients and only the coefficients are stored. These coefficients are used later to regenerate the fluid properties as required. Later, He and Denton (1994) extended the method to three-dimensions. Peitsch, Gallus and Weber (1996) proposed a variation of the direct-store method to reduce the storage requirements, using a “foothold technique” that stores the fluid properties only at certain foothold points. The properties at other points over the oscillation cycle is obtained by interpolation from the nearest foothold point.

Except for the Giles’ “time-tilting” method, which requires transforming the governing equations, the above
methods are based on either the direct-store method or the shape-correction method. The direct-store method requires large in core memory for storing the flowfield properties over the oscillation cycle. On a CRAY type machine where solid state devices (SSDs) can be used for fast input-output, the memory requirements of the direct-store method can be minimized. The Fourier decomposition method does not require large data storage, however, additional computational time is required to Fourier decompose and then regenerate the fluid properties using stored coefficients. Unlike the direct-store method, it relies on superposition making the problem essentially linear. Linearity may be of concern for problems where strong vibrating shocks are present at the periodic passage boundary. Further, because of the lag associated in enforcing the "phase-lag", the rate of convergence also becomes an issue for the two methods. Clearly, there are advantages and disadvantages associated with both the methods. It is not clear if one method is superior to the other. The present study attempts to highlight the advantages of one method over the other, given the problem of interest and resources at hand. Towards this goal, the primary objective of the present study is to implement both phase-lagged methods into one solver. Both the methods are then applied to identical problems in order to investigate their relative advantages.

In the present work, the two methods are implemented within the TURBO-AE code. The TURBO-AE code is currently under development at NASA Lewis Research Center. The details of the code along with several results have been reported by the authors of this paper in Bakhle et al. (1996, 1997). This analysis can analyze flutter for all the possible IBPAs. Srivastava et al. (1998) implemented the time-shifted method based on direct-store method and validated and verified the analysis with previously published results. In the present work the Fourier decomposition method with multiple updates per oscillation cycle is implemented within the TURBO-AE and the program is applied to a flat plate helical fan. The obtained results are compared for accuracy and computational efficiency for each of the two methods. To reduce computational cost, results are obtained using inviscid calculations.

THE TURBO-AE CODE

The aeroelastic solver TURBO-AE is described in brief in this section. Interested readers may refer to Bakhle et al. (1996, 1997) for greater details. The details of the time-shifted boundary conditions are described in Srivastava et al. (1998). The Fourier decomposition method is based on the method proposed by He (1989). TURBO-AE is based on an Euler/Navier-Stokes unsteady aerodynamic solver TURBO (Janus 1989), for internal flow calculations of axial flow turbomachinery components. TURBO-AE can model multiple blade rows undergoing harmonic oscillations with arbitrary IBPAs. The aerodynamic loads are obtained by solving the unsteady Euler or Navier-Stokes equations. For the viscous calculations, Reynolds-averaged Navier-Stokes equations are solved. The Baldwin-Lomax equations are used to model the turbulence. The aerodynamic equations are solved using a finite volume scheme. Flux vector splitting is used to evaluate the flux Jacobians on the left hand side. The right hand side fluxes are discretized using the higher order Total Variation Diminishing (TVD) scheme based on Roe's flux difference splitting. Newton sub-iterations are used at each time step to maintain the higher accuracy. Symmetric Gauss-Sidel iterations are applied to the discretized equations. A newly developed three-dimensional non reflecting boundary condition (Montgomery and Verdon 1997) is applied at the upstream and downstream boundary. The blade motions are at a prescribed frequency, and are simulated using a dynamic grid deformation technique. The grid is updated at each time step by recalculating the grid using linear interpolation, assuming the far field boundaries to be fixed. The grids on the casing are, however, allowed to slide along the casing. The aeroelastic characteristics of the rotor are obtained by calculating the energy exchange between the vibrating blade and its surrounding fluid. A positive work on the blade indicates instability (Bakhle et al. 1996).

NUMERICAL RESULTS

Sample results obtained from TURBO-AE for phase-lagged boundary conditions are presented in this section. Results are obtained for a flat plate helical fan configuration used by Montgomery and Verdon (1997). The fan configuration consists of 24 flat plate blades with zero thickness enclosed within a rigid cylindrical duct with no tip-gap. At mid-span, the stagger angle is 45° and the gap to chord ratio is one. Results are presented for two different inflow conditions at mid-span: a subsonic relative inflow Mach number of 0.7 at zero incidence with axial Mach number of 0.495 and a supersonic relative inflow Mach number of 1.3 at zero incidence with axial Mach number of 0.9192.

The grid used for the analysis is an H-O type grid with 141 points in the streamwise direction, 11 points in the spanwise direction and 41 points in the blade to blade direction. The aeroelastic analysis is carried out by first obtaining a steady aerodynamic solution for the given conditions. From this steady solution, the unsteady solution is started by forcing the blades to undergo a harmonic motion at the given frequency, mode shape and IBPA. The unsteady aerodynamic behavior, as well as work-per-cycle
A comparison of results from TURBO-AE, a linearized Euler analysis (Montgomery and Verdon 1997) and a two dimensional linear theory (Smith 1972) is shown in Figs. 1 & 2 at the subsonic relative inflow condition. These results are reproduced here from Srivastava et al. (1998) for sake of completeness. A good comparison with linear theory and linearized analysis indicates the unsteady aerodynamic behavior predicted by TURBO-AE to be accurate.

The phase-lagged analysis was carried out next for the subsonic relative inflow condition. The blades were forced into a pitching oscillation about their mid-chord at a reduced frequency of one and -90° IBPA. The analysis was carried out using both the time-shifted and the Fourier decomposition methods. Two different analyses were performed for the Fourier decomposition method. In one of the analyses one Fourier coefficient was retained and the coefficient was updated only once per oscillation cycle. In the other analysis the coefficient was updated four times during each oscillation cycle. The multiple updates of the coefficients is expected to provide a faster convergence. The time-shifted analysis has been previously validated by Srivastava et al. (1998). The results obtained for the Fourier decomposition method are, therefore, compared with the results obtained from the time-shifted method to ascertain the accuracy of the Fourier decomposition method. These results are shown in Figs. 3 & 4. In Fig. 3 the variation of total work-per-cycle with oscillation cycle is shown. It can be seen that the Fourier-decomposition method with multiple updates per cycle is the fastest to converge with single update per cycle being the slowest. However, the three analyses eventually converge within 0.1% of each other, indicating that once convergence is achieved the results are same. This is further verified by comparing the unsteady pressure difference variation along chord at mid-span. The comparison for the three analyses is shown in Fig. 4. A very good comparison is obtained. This indicates that the results from the Fourier decomposition method are as accurate as the time-shifted analysis. It also indicates that the four updates per cycle converges much faster than single update, hence in all subsequent work four updates per oscillation cycle will be used for the Fourier-decomposition method.

For the subsonic inflow condition, the analysis was also carried out using four blade-passages in order to simulate the -90° IBPA condition. Using four blade-passage analysis provides the most accurate results as no approximations are involved. The phase lagged conditions introduce errors into the analysis especially since the conditions at the start of the analysis are not known. Comparisons with four-blade-passage analysis also provide a means to measure the benefits of phase-lagged boundary conditions in terms of savings of computer resources. The results obtained for the four-passage analysis are compared with the results obtained from the time-shifted and the Fourier-decomposition methods. The convergence of work-per-cycle for the three analysis methods is shown in Fig. 5.

For the four-passage analysis, as expected, the total work
for all the four passages coalesce. Also, the four passage analysis takes approximately four to five oscillation cycles to converge. The time-shifted analysis, shown with dashed lines, requires approximately eight cycles, whereas the Fourier-decomposition method converged in five to six oscillation cycles. The total work from the three analysis methods converge to within 0.5% of each other. This indicates that the three methods are equally accurate. This is further confirmed by comparing the unsteady pressure difference variation, Fig. 6. Once again a very good comparison is obtained indicating accuracy is not a concern for phase-lagged boundary conditions for the conditions where flowfield can be assumed linear.

From these results it can be seen that the four-passage analysis requires the least number of oscillation cycles to converge, whereas the time-shifted analysis requires the largest number of cycles. However, the computational cost for the four-passage analysis was largest. This is because the analysis had to be carried out using four passages as opposed to a single passage for phase-lagged analyses. This reduced the problem size of the phase-lagged analysis to one fourth that of the four-passage analysis. The smaller number of cycles required for convergence do not sufficiently offset the increase in computational cost for the multiple passage analysis. Further, despite the difference in rate of convergence for the time-shifted analysis and the Fourier-decomposition method, the computational costs required for convergence are fairly comparable, see Table 1. This is because of the

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**Figure 3:** Comparison of work-per-cycle convergence for phase-lagged analysis methods for $M_{\infty}=0.7$ and -90 deg IBPA

**Figure 4:** Comparison of unsteady pressure difference variation with chord at mid-span for phase-lagged analyses for $M_{\infty}=0.7$ and -90 deg IBPA

**Figure 5:** Comparison of work-per-cycle convergence for phase-lagged analyses with multiple passage analysis for $M_{\infty}=0.7$ and -90 deg IBPA
increased computational cost required by the Fourier decomposition method to Fourier decompose and regenerate the fluid properties is offset by increased rate of convergence. It should also be noted here that the memory requirements of the time-shifted analysis can be reduced by using the SSDs available on the CRAY computers. The SSDs help reduce the memory requirements significantly for a marginal increase in the computation cost of reading and writing to the disk.

All of the above analyses were carried out on a C-90 computer. The computer resources required are tabulated in Table 1. The CPU time required per time step shows the additional cost per time-step for the Fourier decomposition method over the time-shifted method. This increase in time, however, is more than compensated for by increased rate of convergence. Also, for the current problem, using the Fourier decomposition method reduces the memory by over 50% as compared to the time-shifted analysis. Also, it can be seen that despite the faster rate of convergence for the four-passage analysis the CPU requirements are almost three times as much as that of the phase-lagged methods. This difference will increase significantly for analyzing the smaller IBPAs requiring many more blade passages for the multiple passage analysis. Especially since the rate of convergence for the time-shifted analysis is independent of the IBPA being analyzed (Srivastava et al., 1998). Also, from Table 1 it appears that the Fourier-decomposition method with multiple updates of the coefficient and the time-shifted method may be comparable in CPU requirements. Therefore, the choice of the particular method will depend on the available resources and the problem being analyzed. For problems where flow behavior can be assumed linear, superposition is not of concern, the Fourier-decomposition method is best suited irrespective of the resources available. For problems where nonlinearity may be of concern time-shifted method may have to be used, especially if SSDs are available.

The two methods were next applied to a supersonic inflow condition to help evaluate the effectiveness and problems that Fourier-decomposition method might have for flows with nonlinearities. The analysis was carried out for +90 deg and -90 deg IBPA. The flow condition is subresonant (waves propagating for supersonic relative inflow, Verdon (1989)) for the +90 deg IBPA and is superresonant (waves decay away from the blade row, Verdon (1989)) for the -90 deg IBPA with resonance at -102.1 deg. The +90 deg was analyzed using the Fourier decomposition method with one, five, and 11 Fourier coefficients, as well as with time-shifted method. A Fourier-decomposition analysis was carried out using 15 coefficients also, but the results were found to be identical to the 11 coefficient analysis.

The comparison of the total work convergence history for the four analyses is shown in Fig. 7. For sake of computational cost the analysis was carried out for only 18 oscillation cycles. Even though the flowfield is not completely converged after 18 cycles, the disturbances appear to be dying out. The 11 coefficient analysis compares very well with the time-shifted analysis.

The one and five coefficient analyses show small differences. Interestingly the total work calculated using only one coefficient compares very favorably with time-shifted analysis and also converges faster. However, significant differences are found for the unsteady pressures between the results from one coefficient analysis and the other three analyses. The first and second harmonics of the blade surface unsteady pressure differences at mid span are shown in Fig. 8. The first harmonic pressure shows good comparison between the four analyses over most of the blade chord. Over the last 10-12 % of the chord, one coefficient analysis shows differences in both real and imaginary pressures. The second harmonic of the pressure difference, on the other hand, shows significant differences between the one coefficient Fourier analysis and the other three analyses. The time-shifted, five and 11 coefficient analyses show good comparison with each other over most of the chord, with some minor differences near the trailing edge for the five coefficient analysis. The 11 coefficient analysis compares very well with the time-shifted method, hence in all future analyses 11 coefficients were used.

![Figure 6: Comparison of unsteady pressure difference variation with chord at mid-span of phase-lagged analyses with multiple passage analysis for $M_{\infty}=0.7$ and -90 deg IBPA](image-url)

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The analysis was next carried out for the -90 deg IBPA. The variation of total work is shown in Fig. 9 for both time-shifted analysis and the Fourier decomposition analysis using 11 coefficients. The analysis was again carried out for 18 vibration cycles. The time-shifted method shows convergence, whereas the Fourier decomposition method does not. Thus indicating that for this condition the Fourier decomposition method is computationally more expensive than the time-shifted method. The total work from both the methods are comparable even though the Fourier decomposition analysis has not totally converged. The first and second harmonics of the unsteady pressure difference at mid span are shown in Fig. 10. The comparison between the two methods is good except over the last 10-15% of the blade chord near the trailing edge. During parts of the oscillation cycle a shock appears in this region of the blade along with passage shocks at the fluid periodic boundaries upstream of the blades. The flowfield also showed the passage to be choked for parts of the oscillation. These flow features indicate that the flowfield behavior may be nonlinear. For the +90 deg IBPA, shocks were present but neither choked flow nor passage shocks at the fluid periodic boundaries were observed. Despite the presence of shock and choked flow conditions the method of superposition provides comparable results for this configuration. However, it should be noted here that the geometry without loading and flow turning is simplistic. Further investigations with realistic geometry needs to be carried out to understand in more detail the influence of nonlinearities on the Fourier decomposition method.
CONCLUSIONS

Two phase-lagged boundary condition methods have been successfully implemented into the TURBO-AE, an Euler/Navier-Stokes based aeroelastic solver. Both these methods along with the multiple passage analysis method have been applied to an identical geometry to investigate the accuracy and efficiency of the various methods. Comparing the results obtained from these methods it was found that all the methods provide equally accurate results, for the subsonic relative inflow condition. For the supersonic relative inflow the presence of a shock was in itself not sufficient to invalidate the Fourier decomposition method. The errors introduced by superposition were negligible if large number of Fourier coefficients were included in the analysis even for flows with shocks at the periodic boundaries and choked flow conditions.

The study also showed that the phase lagged methods significantly reduce the computational cost as compared to the multiple passage analysis. These savings will be much more significant for smaller IBPAs. The CPU cost of the time-shifted method was found to be comparable to that of the Fourier decomposition method. However, because of the large reduction in memory requirements, it is recommended that for flowfields where superposition is not of concern, Fourier-decomposition method be used. For flows with nonlinearities more study is required to further quantify the nature of the flowfield for which the Fourier decomposition method may break down.

Figure 9: Comparison of work-per-cycle history for -90 deg IBPA at $M_{\infty}=1.3$ and -90 deg IBPA

Figure 10: Comparison of unsteady pressure difference variation with chord for $M_{\infty}=1.3$ and -90 deg IBPA
ACKNOWLEDGEMENTS

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REFERENCES


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Table 1: Comparison of computer resources required by various methods for $M_\infty=0.7$ and -90 deg IBPA
Application of Time-Shifted Boundary Conditions to a 3D Euler/Navier-Stokes Aeroelastic Code

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ABSTRACT
In the present work the unsteady aerodynamic characteristics of harmonically oscillating fan blades are investigated by applying a time-shifted boundary condition at the periodic boundaries. The direct-store method is used to implement the time-shifted boundary condition in a time-marching Euler/Navier-Stokes solver. Inviscid flow calculations for a flat plate helical fan, in a single-blade passage domain, are used to verify the analysis. The results obtained show good correlation with other published results as well as with the same solver using multiple blade passages stacked together. Significant savings in computer time is realized, especially for smaller phase angles.

INTRODUCTION
The objectives of the Advanced Subsonic Technology (AST) project, funded by NASA, is to improve the efficiency of the turbomachines and reduce the NOX emissions. To satisfy these objectives, numerical aeroelastic analysis methods for turbomachinery applications are currently being developed at NASA Lewis Research Center. Numerical aeroelastic analysis methods have primarily been developed for two-dimensional cascades, representative of turbomachinery configuration (for example, Plazier 1978, Sisto 1977, Fleet 1979, and Bendiksen 1990, among others). These methods are inadequate for an accurate analysis as they ignore the strong three-dimensional flow characteristics present in a turbomachine. The large computational cost associated with three-dimensional analysis, has restricted the primary focus for the three-dimensional aeroelastic analyses, over the years, to predicting unsteady aerodynamic forces, assuming the unsteady behavior to be linear. Several methods for calculating the unsteady aerodynamic loads over vibrating blades have been reported in the literature by solving linear potential equations (Namba 1987, Chi 1993), linearized Euler equations (Hall and Lorence 1993, Montgomery and Verdon 1997) and non-linear Euler and Navier-Stokes equations (He and Denton 1994, Peitsch, Gallus and Weber 1996, and Bakhle et al. 1996 & 1997). A good review of analytical methods for turbomachinery blade vibrations is given by Chi (1993).

Some three-dimensional aeroelastic analyses of turbomachinery components have also been reported. Carta (1967), used a quasi three-dimensional approach by stacking two-dimensional strips of isolated airfoil, Williams & Cho (1991) reported an analysis based on linear panel method and solved the aeroelastic equation using a pulse and influence coefficient approach. Recently, methods based on Euler analysis have been reported by Gerolymos (1992), Srivastava and Reddy (1995) and Srivastava, Reddy and Stefko (1996). Srivastava and Reddy (1997) have also applied various aeroelastic analysis techniques for flutter analysis of a ducted rotor configuration and investigated the advantages and disadvantages of each of the methods.

With the exception of He and Denton (1994) and Bakhle et al (1997), all the other methods reported in the literature, ignore the effects of viscosity. He and Denton (1994) have used thin layer approximation to include the effects of viscosity in their calculations. To accurately model flows with separation due to stall or shock boundary layer interaction an aeroelastic analysis pro-

Presented at the International Gas Turbine & Aeroengine Congress & Exhibition
gram TURBO-AE, based on full Navier-Stokes equations, is currently being developed at NASA Lewis Research Center. The numerical analysis within the TURBO-AE program couples an aerodynamic analysis based on three-dimensional unsteady Euler/Navier-Stokes equations with a normal mode structural analysis. The aeroelastic characteristics are obtained by calculating the energy exchange between the rotor and the surrounding fluid.

The details of the TURBO-AE program along with several results have been reported by the authors of this paper in Bakhle et al. (1996, 1997). This analysis can analyze flutter for all the possible inter-blade phase angle (IBPA). The number of IBPAs possible for any given rotor is identical to the number of blades in the rotor (Lane 1956). For the non-zero IBPA calculations, the analysis stacks the required number of blade passages to simulate the motion with appropriate IBPA. This is a cumbersome and time consuming process. To improve the efficiency of the solution, a time-shifted boundary condition is being added to the TURBO-AE solver. In this method all the possible IBPA calculations can be performed using a single blade passage by applying a time-shifted boundary condition across the periodic boundaries of the passage.

Several researchers have reported methods in two-dimensions for using time-shifted periodic boundary conditions to reduce the computational domain. Erdos et al. (1977) were the first to develop a method based on direct store method. An alternative approach to the problem of the lagged periodic boundary condition was developed by Giles (1988). The time-inclined computational plane approach of Giles was primarily to overcome the problem encountered in rotor-stator applications, where no final periodic state exists. He (1989), in order to reduce the storage requirement of the direct-store method, proposed a shape-correction method, wherein, only the Fourier coefficients of the unsteady variation of the fluid properties, at the periodic boundary, were stored. The method of He (1989) was extended to three-dimensions by He and Denton (1994). Peitsch, Gallus and Weber (1996) have also reported a method for time-shifted boundary condition for their three-dimensional Euler solver. Their method is based on direct-store method, but in order to reduce the storage requirements a "foothold technique" is used. Instead of storing the fluid properties over the entire cycle, the properties are stored at a small number of foothold points. The properties at other time steps during the cycle are obtained by interpolation from the nearest foothold points.

In the present work, the time-shifted boundary condition has been incorporated within the TURBO-AE program using the direct-store method. To verify and validate the solver, it is applied to a flat plate helical fan operating in subsonic conditions. This provides for a reasonably good test case as there are some numerical results that have been published for this geometry. The flat plate helical fan, and the operating conditions are such that the mid span section of the fan has very near two-dimensional flow, at zero incidence, over it. Results obtained for the fan are compared with numerical results from linear theory (Smith 1972) and linearized Euler analysis (Montgomery and Verdon 1997), as well as with results obtained using the multiple passage option of TURBO-AE (Bakhle et al. 1997). To reduce the computational cost, the results are obtained using inviscid calculations.

THE TURBO-AE CODE

The aeroelastic solver TURBO-AE is described in brief in this section. Interested readers may refer to Bakhle et al. (1996, 1997) for greater details. TURBO-AE is based on an Euler/Navier-Stokes unsteady aeroelastic solver TURBO (Janus 1989), for internal flow calculations of axial flow turbomachinery components. TURBO-AE can model multiple blade rows undergoing harmonic oscillations with arbitrary IBPAs. The aeroelastic loads are obtained by solving the unsteady Euler or Navier-Stokes equations. For the viscous calculations, Reynolds-averaged Navier-Stokes equations are solved. The Baldwin-Lomax equations are used to model the turbulence. The aeroelastic equations are solved using a finite volume scheme. Flux vector splitting is used to evaluate the flux Jacobians on the left hand side. The right hand side fluxes are discretized using the higher order Total Variation Diminishing (TVD) scheme based on Roe's flux difference splitting. Newton sub-iterations are used at each time step to maintain the higher accuracy. Symmetric Gauss-Seidel iterations are applied to the discretized equations. A newly developed three-dimensional non reflecting boundary condition (Montgomery and Verdon 1997) is applied at the upstream and downstream boundary. The blade motions are at a prescribed frequency, and are simulated using dynamic grid deformation technique. The grid is updated at each time step by recalculating the grid using linear interpolation, assuming the far field boundaries to be fixed. The grids on the casing are, however, allowed to slide along the casing. The aeroelastic characteristics of the rotor are obtained by calculating the energy exchange between the vibrating blade and its surrounding fluid. A positive work on the blade indicates instability.

TIME-SHIFTED BOUNDARY CONDITIONS

In the work reported by Bakhle et al. (1997), the non-zero IBPA analysis was carried out by stacking the required number of blade passages. For smaller IBPA anal-
ysis, depending upon the number of blades in the rotor, the required number of passages could be very large and computationally prohibitive. To alleviate this problem, a time-shifted boundary condition based on direct-store method, has been implemented into the analysis. Since the time-shifted boundary condition analysis is carried out using a single passage for any IBPA, the computational cost is significantly reduced, especially for smaller IBPA analysis.

The Fourier decomposition of the fluid properties, as in the shape-correction method of He and Denton (1994), assumes linearity of the fluid properties. This could lead to inaccuracies for strongly non-linear flows at the periodic boundary such as presence of a shock. Also, since the coefficients are calculated at the end of each cycle, the application of the boundary conditions are lagged by one full cycle.

In the present analysis, the periodic boundary conditions are lagged only by the phase on one surface and by the difference of 360° and phase on the other. The reduced lag in boundary condition application helps achieve convergence faster. At the beginning of the analysis, where the lagged fluid properties are not available, the instantaneous values are used. This introduces some initial errors that are driven out of the computational domain within a few cycles of oscillation. These initial errors result in increased number of oscillations for convergence as compared to the original analysis with multiple blade passages. The large storage requirements of the direct-store method can be minimized by writing the fluid properties on to the solid state devices (ssds), rather than storing them in core memory. The ssds on CRAY C-90 allow for large input-outputs without any appreciable degradation in code performance.

NUMERICAL RESULTS

Sample results obtained from TURBO-AE time-shifted analysis for the flat plate helical fan configuration, used by Montgomery and Verdon (1997), are presented in this section. The helical fan consists of flat plate blades with zero thickness enclosed within a rigid cylindrical duct with no gap between the blade tip and the duct. The blades are twisted to maintain a zero incidence at all sections for the inflow. The inflow relative Mach number at midspan is 0.7 with the free stream axial Mach number being 0.495. The stagger angle at mid-span of the cascade is 45°, and the gap-to-chord ratio is unity for a rotor with 24 blades and a diameter of 8.448. The hub to tip ratio is 0.8.

The grid used for the analysis is an H-O type grid with 141 points in the streamwise direction, 11 grid points in the spanwise direction and 41 points in the blade to blade direction. The aeroelastic analysis is carried out by first obtaining a steady aerodynamic solution for the given conditions. From this steady solution, the unsteady solution is started by forcing the blades to undergo the harmonic motion at the given frequency, mode shape and IBPA. The unsteady aerodynamic behavior, as well as work-per-cycle is calculated for the oscillating blades.

In the first few calculations, the code is verified by obtaining the unsteady behavior of the blades undergoing either pure pitching motion or pure plunging motion. The plunging motion is normal to the blade chord at mid-span and is of constant amplitude for the entire span. The pitching motion is about the mid chord. The non-dimensional frequency of oscillation, based on blade chord for these analyses has been taken as unity. An oscillation amplitude of 0.2° for pitching and 0.1% of chord for plunging was used. It was found that 200 steps per oscillation cycle were required to eliminate dependency on time steps. Also, for the multiple passage analysis, it was found that a minimum of four to five oscillation cycles were required to obtain a converged solution. The work-per-cycle convergence is shown in Fig. 1 for three different time steps. Reducing the time step from 100 steps per oscillation cycle to 200 steps, a large difference in work is seen. However, reducing the time step further does not impact the solution significantly. Therefore, in all subsequent calculations, 200 time steps per oscillation cycle are used.

Figures 2 - 5 show the variation of the unsteady pressure along the chord at mid-span. The variation of the first harmonic of real and imaginary parts of the difference of the unsteady pressure between the pressure and suction surfaces is plotted against the chord. These results are obtained without using the time-shifted boundary conditions and are presented here for validation purposes of the unsteady calculations of the code. For the non-zero IBPA, the required number of passages were stacked for these analyses. As can be seen, good comparison is obtained with the linearized Euler analysis (Montgomery and Verdon 1997) and the analytical results (Smith 1972). These figures indicate that the unsteady behavior of the solver compares well with other published results.

To obtain the time-shifted results, the analysis was carried out for various IBPAs for both the pitching and plunging cases. Some of these results are presented here for verifying and validating the method. The total work at the end of each cycle was monitored to obtain the convergence. The variation of total work-per-cycle with vibration cycle for -90° IBPA is shown in Fig. 6. Also, shown in this figure is the variation obtained from the four passage analysis. It can be seen that the time-shifted analysis requires several more vibration cycles to reach convergence. Typically, the multiple passage analysis showed conver-
ence in 4-5 cycles, whereas, the time-shifted analysis required 7-10 cycles to reach convergence. This is to be expected. For the multiple passage analysis the boundary conditions from the start are applied appropriately at the fluid interfaces. On the other hand, for the time-shifted analysis, the lagged fluid properties at the beginning of the solution are not available, hence, inaccurate boundary condition is applied at the start of the analysis. The errors introduced because of this inherent drawback of the method, require longer to move out of the calculation domain, thus requiring more cycles of oscillations for convergence. From this figure one can also see that the work obtained from all the passages of the multiple passage analysis are same once the solution has reached convergence. It should be noted here that, even though, the multiple passage analysis requires approximately half the oscillation cycles for convergence, it has to use four blade passages. This in turn results in the multiple passage analysis requiring approximately twice the CPU time for this case.

The real and imaginary parts of the unsteady pressure difference for the two methods are compared with each other in Figs. 7 & 8 for 180° IBPA for pitching and plunging motion, and in Figs. 9 & 10 for 90° and -90° IBPAs for pitching oscillation. Also shown in Figs. 9 & 10 are results from linearized Euler (Montgomery and Verdon 1977) and linear theory (Smith 1972) for comparison purposes. In all four cases good comparison is obtained, in fact for most of the solutions the results are indistinguishable. For the 90° IBPA, problem was encountered in convergence. This case is a superresonant condition and hence had some reflections from the upstream boundary. Even though non-reflecting boundary conditions are used, the radial modes are not accounted for in the analysis. This results in some reflections from the boundary. Because of this the computations required longer for convergence. This was observed for both the multiple passage analysis as well as the time-shifted boundary condition case.

The analysis was carried out for 45° and -45° IBPA as well. These results also compared well with the linear results of Smith (1972). One interesting fact seems to emerge by comparing the convergence characteristics of the time-shifted boundary calculations - irrespective of the IBPA of motion, the convergence is achieved within seven to 10 oscillation cycles. This is shown in Fig. 11. This implies that potential savings are quite large for smaller IBPAs since they require larger number of passages for the multiple passage analysis. The break even point, in terms of computational costs, seems to be 180° IBPA. The number of oscillations for this case, required for time-shifted analysis is approximately twice that of the two passage analysis, resulting in roughly equal CPU requirements.

All of the above computations were carried out on a CRAY C-90 computer at the NAS facility of NASA. For the time-shifted analysis approximately 15 minutes were required per oscillation cycle per passage. A total of 75 Mw of memory was required for in-core storage of boundary condition data, whereas only 35 Mw were required if the ssds were used.

CONCLUSIONS

A time-shifted boundary condition has been successfully implemented into the TURBO-AE, an Euler/Navier-Stokes based aeroelastic solver. Although, the direct store method has been implemented, the large storage requirements are not needed because the data is written and read from the solid-state input-output devices on Cray computer. The solver has been verified by applying it to a flat plate helical fan geometry. The results indicate that the time-shifted boundary conditions have been implemented satisfactorily and provide solutions that are in good agreement with other published results and the multiple passage analysis.

The analysis showed that significant computational time can be saved using this procedure over the method of stacking the blade passages for analyzing the non-zero IBPA. Even though the number of oscillation cycles required for convergence are higher for the time-shifted boundary conditions, the overall computational cost is reduced significantly, since only one passage is used in the analysis. It was also found that for the present geometry, the number of oscillations required for convergence were not strongly coupled to IBPA of analysis and for the cases analyzed, the number of oscillations required were of the order of seven to ten. However, one must be cautioned that this conclusion may not hold strictly for other geometries.

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REFERENCES


Figure 1: Effect of time steps on convergence of work-per-cycle

Figure 2: Unsteady pressure difference variation with chord at mid-span for 0° IBPA pitching oscillations

Figure 3: Unsteady pressure difference variation with chord at mid-span for 180° IBPA pitching oscillations
Figure 4: Unsteady pressure difference variation with chord at mid-span for $0^\circ$ IBPA plunging oscillations

Figure 6: Comparison of work-per-cycle convergence for time-shifted analysis with multiple passage analysis

Figure 5: Unsteady pressure difference variation with chord at mid-span for $180^\circ$ IBPA plunging oscillations

Figure 7: Comparison of unsteady pressure difference variation at mid-span for $180^\circ$ IBPA pitching oscillations
Figure 8: Comparison of unsteady pressure difference variation at mid-span for 180° IBPA plunging oscillations

Figure 10: Comparison of unsteady pressure difference variation at mid-span for 90° IBPA pitching oscillations

Figure 9: Comparison of unsteady pressure difference variation at mid-span for -90° IBPA pitching oscillations

Figure 11: Comparison of work-per-cycle convergence history for various IBPA pitching oscillations
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Aeroelastic Calculations Based on Three-Dimensional Euler Analysis

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AEREOELASTIC CALCULATIONS 
BASED ON THREE-DIMENSIONAL EULER ANALYSIS 

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Abstract

This paper presents representative results from an aeroelastic code (TURBO-AE) based on an 
Euler/Navier-Stokes unsteady aerodynamic code (TURBO). Unsteady pressure, lift, and moment 
distributions are presented for a helical fan test configuration which is used to verify the code by 
comparison to two-dimensional linear potential (flat plate) theory. The results are for pitching and 
plunging motions over a range of phase angles. Good agreement with linear theory is seen for all phase 
angles except those near acoustic resonances. The agreement is better for pitching motions than for 
plunging motions. The reason for this difference is not understood at present. Numerical checks 
have been performed to ensure that solutions are independent of time step, converged to periodicity, 
and linearly dependent on amplitude of blade motion. The paper concludes with an evaluation of the 
current state of development of the TURBO-AE code and presents some plans for further development and 
validation of the TURBO-AE code.

Introduction

There is an ongoing effort to develop technologies to increase the fuel efficiency of commercial aircraft 
engines, improve the safety of engine operation, reduce the emissions, and reduce engine noise. With 
the development of new designs of ducted fans, compressors, and turbines to achieve these goals, a 
basic aeroelastic requirement is that there should be no flutter or high resonant blade stresses in the 
operating regime. In order to verify the aeroelastic soundness of the design, an accurate prediction of the 
unsteady aerodynamics and structural dynamics of the propulsion component is required. The complex 
geometry, the presence of shock waves and flow separation makes the modeling of the unsteady 
aerodynamics a difficult task. The advanced blade geometry, new blade materials and new blade 
attachment concepts make the modeling of the structural dynamics a difficult problem.

Computational aeroelastic modeling of fans, compressors, and turbines requires many simplifying 
assumptions. For instance, flutter calculations are typically carried out assuming that the blade row is 
isolated. This simplifies the structural dynamics formulation and the unsteady aerodynamic 
calculations considerably.

For an isolated blade row flutter calculation, the modeling of the unsteady aerodynamics is the biggest 
challenge. Many simplifying assumptions are made...
in the modeling of the unsteady aerodynamics. In the past, panel methods based on linear compressible small-disturbance potential theory have been used to model the unsteady aerodynamics and aeroelasticity of fans in subsonic flow; see for example [1,2]. The major limitations of this type of analysis are the neglect of transonic, vortical, and viscous flow effects in the model. These inherent limitations in the model preclude its use in a majority of practical applications. A full potential unsteady aerodynamic analysis has been used with a modal structural dynamics method to model the aeroelastic behavior of fan blades [3,4]. Although the full potential aerodynamic formulation is able to model transonic effects (limited to weak shocks), the vortical and viscous effects are still neglected. For example, the blade tip vortex, or a leading-edge vortex is not modeled. Recently, researchers [5-10] have also developed inviscid and viscous unsteady aerodynamic analyses for vibrating blades.

For aeroelastic problems in which viscous effects play an important role (such as flutter with flow separation, or stall flutter, and flutter in the presence of shock and boundary-layer interaction), a more advanced aeroelastic computational capability is required. The authors of this paper have earlier presented [11] some results from the TURBO-AE aeroelastic code. Initial calculations were restricted to in-phase (zero phase angle) blade motions and inviscid flow. In a later paper [12], results were presented for zero and non-zero phase angle motions and viscous flow. In these calculations, multiple blade passages were modeled for non-zero phase angle motions. Most recently [13], results have been presented using a single blade passage with phase-lag periodic boundary conditions to model arbitrary phase angle motions.

This paper presents unsteady pressure, lift, and moment distributions due to blade vibration over a range of phase angles for verification of the TURBO-AE aeroelastic code. For non-zero phase angle motions, phase-lag periodic boundary conditions are used. The configuration selected is a helical fan. The geometry and flow conditions are chosen to minimize non-linear and three-dimensional effects since the intent is to verify the code by comparison with two-dimensional linear potential (flat plate) theory.

Aeroelastic Code – TURBO-AE

This section briefly describes the aeroelastic code (TURBO-AE); previous publications [11-13] provide additional details. The TURBO-AE code is based on an unsteady aerodynamic Euler/Navier-Stokes code (TURBO), developed separately [14,15]. The TURBO code provides all the unsteady aerodynamics to the TURBO-AE code.

The TURBO code was originally developed [14] as an inviscid flow solver for modeling the flow through turbomachinery blade rows. Additional developments were made [15] to incorporate viscous effects into the model. This Reynolds-averaged Navier Stokes unsteady aerodynamic code is based on a finite volume scheme. Flux vector splitting is used to evaluate the flux Jacobians on the left hand side of the governing equations [14] and Roe's flux difference splitting is used to form a higher-order TVD (Total Variation Diminishing) scheme to evaluate the fluxes on the right hand side. Newton sub-iterations are used at each time step to maintain higher accuracy. Symmetric Gauss-Seidel iterations are applied to the discretized equations. A Baldwin-Lomax algebraic turbulence model is used in the code.

The TURBO-AE code assumes a normal mode representation of the structural dynamics of the blade. A work-per-cycle method is used to determine aeroelastic stability (flutter). Using this method, the motion of the blade is prescribed to be a harmonic vibration in a specified in-vacuum normal mode with a specified frequency (typically the natural frequency). The work done on the vibrating blade by aerodynamic forces during a cycle of vibration is calculated. If work is being done on the blade by the aerodynamic forces at the end of a vibration cycle, the blade is dynamically unstable, since it will result in extraction of energy from the flow, leading to an increase in amplitude of oscillation of the blade.

The inlet/exit boundary conditions used in this code are described in [16-18]. For cases in which the blade motions are not in-phase, phase-lag periodic boundary conditions based on the direct store method are used.

Results

In this section, results are presented which serve to verify the TURBO-AE code. The test configuration selected is a helical fan [16]. This configuration consists of a rotor with twisted flat plate blades enclosed in a cylindrical duct with no tip gap. This configuration was developed by researchers [16] to provide a relatively simple test case for comparison.
with two-dimensional analyses. The geometry is such that three-dimensionality of the flow is minimized.

The parameters of this three-dimensional configuration are such that the mid-span location corresponds to a flat plate cascade with a stagger angle of 45 deg. and unit gap-to-chord ratio operating in a uniform mean flow at a Mach number of 0.7 parallel to the blades. The rotor has 24 blades with a hub/tip ratio of 0.8. The inlet flow (axial) Mach number used in this calculation is 0.495, which results in a relative Mach number of approximately 0.7 at the mid-span section. The results presented are for inviscid runs of the TURBO-AE code.

The grid used for the calculations is $141 \times 11 \times 41$ in one blade passage. On each blade surface, 81 points are located in the chordwise direction and 11 points in the spanwise direction. The inlet and exit boundaries are located at an axial distance of approximately 0.7 chord lengths from the blade leading and trailing edges. To begin, a steady solution is obtained for this configuration. The steady flowfield consists of uniform flow at each radial location.

Aeroelastic calculations are performed starting from the steady solution. Calculations have been performed for harmonic blade vibration in plunging and pitching modes, separately. The pitching is about the mid-chord. The prescribed mode shapes are such that the amplitude of vibration does not vary along the span. This choice of mode shapes is meant to reduces the three-dimensionality of the unsteady flowfield for ease of comparison with two-dimensional analyses.

The vibration frequency is selected so that the non-dimensional reduced frequency based on blade chord is 1.0 at the mid-span. A study was performed to determine the sensitivity of numerical results to the number of time steps used in each cycle of blade vibration. Calculations were done with 100, 200, and 300 time steps per cycle of vibration for 0 deg. phase angle plunging motion. The time step was varied so as to keep the vibration time period (or frequency) fixed. Figure 1 shows the work-per-cycle from this study. As the flowfield reaches periodicity, it can be seen that the results are nearly identical for 200 and 300 time steps per cycle. These results differ slightly from the results for 100 time steps per cycle. Figure 2 shows the unsteady pressure difference for the same three numbers of time steps per cycle. The results for 200 and 300 time steps per cycle are indistinguishable. Based on such calculations, it was determined that 200 time steps per cycle provided adequate temporal resolution for the selected vibration frequency. All results presented here have been obtained using 200 time steps per cycle.

The non-dimensional time step used in the calculations (with 200 time steps per cycle) is 0.045, which results in a maximum CFL number of 60.5. The amplitude of blade vibrations in the calculation is a pitching amplitude of 0.2 deg. or a plunging amplitude of 0.1% chord. In all cases, calculations were continued for a number of cycles of blade vibration to allow the flowfield to become periodic. Initial calculations with phase angles of 0, 45, 90, 135, 180, 225, 270, and 315 deg. were continued for 15 cycles of blade vibration to ensure periodicity. Later calculations with intermediate phase angles (22.5, 67.5, ..., and 337.5 deg.) were continued only for 10 cycles of blade vibration due to insufficient computational resources. In an earlier study [13], it was shown that, for the various phase angles studied, the flowfield became periodic after about 7-10 cycles of blade vibration. Hence, the 10 or 15 cycles used in the present work were considered adequate to reach periodicity.

Figure 3 shows the unsteady moment about mid-chord (in complex form) for pitching blade motion about the mid-chord. These results are from the mid-span location and were calculated using the first harmonic of the unsteady blade surface pressure difference. Semi-analytical results from two-dimensional linear potential (flat plate) theory [19] are included for comparison.

The overall level of agreement between TURBO-AE results and linear theory is very good, with exceptions to be discussed in the following paragraph. For subsonic flows and small amplitude of blade motions, it is expected that there will be no significant difference between the Euler and linear potential results. Hence, the observed agreement is not surprising and provides a basic verification of the TURBO-AE code. It may be noted that the parameters of the present configuration were selected [16] to allow exactly this type of a verification by comparison to two-dimensional analyses.

In Figure 3, some deviation from linear theory is seen in the results for phase angles of 112.5 and 135 deg., and to a lesser extent for phase angles of 157.5 and 315 deg. All these phase angles fall near conditions of
acoustic resonance (or cut-off conditions) in the corresponding two-dimensional flat plate cascade. The acoustic resonances occur at phase angles of 107.3 and 330.6 deg.; these values are marked on the phase angle axis of Figure 3 for reference. The phase angles between these resonances are associated with sub-resonant [20] (cut-off) conditions in which all disturbances attenuate away from the cascade. No disturbances propagate in the upstream or downstream directions under sub-resonant conditions. The phase angles between 0 and 107.3 deg. and between 330.6 and 360 deg. are associated with super-resonant (cut-on) conditions in which at least one disturbance propagates in either the far upstream or downstream direction.

The significance of the sub-resonant and super-resonant conditions to computational aeroelasticity can be explained as follows. Since the typical computational domain does not extend very far from the blade row or cascade, the inlet/exit boundary conditions must minimize (or eliminate) the reflection of disturbances generated by the vibration of the blades. For sub-resonant conditions, it may be possible to reduce the reflected disturbances by moving the boundary farther away from the blade row. This is not possible for super-resonant conditions. From Figure 3, it can be seen that the results from TURBO-AE agree well with linear theory for both sub-resonant and super-resonant conditions. It may be also recalled that the computational inlet/exit boundaries are located quite near (0.7 axial chord lengths from leading/trailing edges) the blade row in the present calculations.

Figure 4 shows the unsteady lift (in complex form) for plunging blade motion. As noted for the pitching results, these results are also from the mid-span location and were also calculated using the first harmonic of the unsteady blade surface pressure difference. Results from linear potential theory are included in Figure 4 for comparison. The overall level of agreement with linear theory is good, but not as good as that for pitching motion (Figure 3). The source of such a difference between the plunging and pitching results is not understood. However, such differences in agreement have been noted by other researchers [16,17] for a different configuration. In addition, deviations are observed close to the acoustic resonances, as for pitching.

Figure 5 shows the unsteady blade surface pressure difference (first harmonic) at the mid-span location for pitching blade motion about the mid-chord. Results are presented for phase angles values between 0 and 360 deg. in steps of 22.5 deg. In each case, the linear theory results are included for comparison. In most cases, the agreement with linear theory is very good. The exceptions occur at phase angles near acoustic resonance conditions, as noted earlier in the description of the unsteady moment (Figure 3). It is worth noting that, in this case, the integrated results in Figure 3 accurately represent the level of agreement with linear theory, without obscuring any differences in the details of the pressure distributions.

Figure 6 shows the unsteady blade surface pressure difference (in complex form) for plunging blade motion. The level of agreement with linear theory is not as good as for pitching, as reflected in the unsteady lift (Figure 4). The most serious deviations from linear theory are restricted to the phase angles near conditions of acoustic resonance.

Some of the results for plunging motion (Figure 6) show an irregular (unsmooth) variation in the unsteady pressure distribution which is not seen in any of the results for pitching motion (Figure 5). This uneven variation can be seen in the plunging results in Figures 6b, 6d, 6f, 6h, 6j, 6l, 6n, and 6p for phase angles of 22.5, 67.5, 112.5, ... , and 337.5 deg. One common characteristic of these results is that these were all generated on a workstation and may therefore suffer from some precision-related numerical problem. However, it is surprising to note that the corresponding results for pitching motion (also computed on a workstation) are quite smooth and do not show such unevenness. A re-calculation of selected plunging results on a super-computer does indeed eliminate the unevenness in pressure variation, but the pressure distributions remain substantially unchanged from those presented in Figure 6.

Note that all the TURBO-AE results presented are the first harmonic components of the unsteady variations. The higher harmonics are extremely small for these calculations, indicating the linearity of the unsteady flow. Previous results [12] had shown a nonlinear dependence on amplitude for certain cases for pitching amplitudes of blade vibration of 2 deg., but not at the 0.2 deg. amplitude used in the present calculations.

To investigate the effect of some numerical parameters on the results for phase angle of 112.5 deg. (where the maximum deviation from linear...
theory is observed), the following calculations were done. The number of time steps per cycle was doubled from 200 to 400, with a corresponding halving of the time step. The unsteady pressure results showed no changes within plotting accuracy, indicating adequate temporal resolution. Similarly, the number of cycles of oscillation was doubled from 10 to 20 to examine possible lack of periodicity. No change in the unsteady pressure results was observed within plotting accuracy. The deviations in the regions of acoustic resonances may possibly be reduced by the use of finer grids. But, such a grid refinement study has not yet been performed.

Concluding Remarks

An aeroelastic analysis code named TURBO-AE has been developed and is being verified and validated. The starting point for the development was an Euler/Navier-Stokes unsteady aerodynamic code named TURBO. Some verification has been done by running the code for a helical fan test configuration. Results have been presented for pitching and plunging blade motions over a range of phase angles. The results compare well with results from a linear potential analysis. This agreement is expected for subsonic flows for which the calculations were made and for the relatively small amplitudes of blade motion.

The agreement is not as good for plunging motion as for pitching motion. The reason for this difference is not understood at present. Also, deviations are observed for values of phase angles near acoustic resonance conditions. The solutions are shown to be independent of the time step, converged to periodicity, and linearly dependent on amplitude of blade motion. This test case provides a basic verification of the TURBO-AE code. It also shows the need to perform a grid refinement study as a possible way to resolve the deviations from linear theory near acoustic resonance conditions and for plunging motion. For plunging motion, some results are affected by precision-related numerical problems, as seen from uneven pressure distributions. But, the elimination of these precision problems does not change the pressure distributions substantially, apart from making the variations smooth.

It is necessary to further verify the TURBO-AE using different standard test configurations to compare with experimental data and other code predictions. This is being done in collaboration with other researchers. Also, it is necessary that the TURBO-AE code be exercised to evaluate its ability to analyze and predict flutter for conditions in which viscous effects are significant. This work is also currently in progress.

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Figure 5: Unsteady pressure difference (first harmonic) for pitching motion.
Figure 5 (continued): Unsteady pressure difference (first harmonic) for pitching motion.
Figure 6: Unsteady pressure difference (first harmonic) for plunging motion.
Figure 6 (continued): Unsteady pressure difference (first harmonic) for plunging motion.