Human Exploration Missions Study

Space Surveillance Telescope Transfer to and Station at a Halo Orbit At the Earth-Sun Libration Point L2

MSFC/Alpha Technology, Inc.

FINAL REPORT

REF: Order Number H-32390D

January 31, 2001

Submitted to:
Larry Kos
Advanced Concepts Department (TD30)
Space Transportation Directorate
MSFC, AL 35812

By Vincent A. Dauro, Sr.
ALPHA TECHNOLOGY, INC.
3322 South Memorial Parkway, Suite 630, Huntsville, AL 35801

INTRODUCTION

This study was undertaken to determine mission profile and delta velocity requirements to place a telescope at the Earth-Sun libration point L2. The program "Integrated Mission Program" (IMP), was selected to be used in the investigation.

A description of IMP and its capabilities may be found in the Addenda. The Addenda also contains the libration halo equations, constants and other parameters. Comments regarding the chaotic nature of numerical integration near the libration points are also attached in the Addenda.

A basic two stage S/C with a simple mission profile was selected. This profile is shown immediately below.

SPACE SURVEILANCE TELESCOPE MISSION PROFILE
TBD DAY TRANSFER WITH COURSE CORRECTION AT 2-3 DAYS

DEPART FROM NODE OF PARKING ORBIT (TLIB)
160 NM : INCLINATION 23.45 DEG

DROP 1ST STAGE

MIDCOURSE

HALO ORBIT INSERTION

DROP 2ND STAGE

The original study began with a space craft (S/C) weight of 6200 lbs. This was raised scaling by 10 to 62000 lbs. The two S/C are described in Table 3 and 6 respectively. The study results will be shown for both vehicles.

An important part of the study was to identify opportunites for departure and arrival. The initial study focused on a constant date of arrival. Trip times of 20, 30, 40, 60, 80, and 100 days were investigated to arrive on 25 Sep 2011. Later a constant trip time of 100 days to arrive at selected dates in September, October, November, and December was studied.

TRANSFER (TLIB)

In a classical Newtonian two body system, the optimimum transfer from one circular orbit to another is a Hohmann transfer of 180 degrees. In an oblate system, an exact 180 degree transfer in inertial space cannot be accomplished by a single burn at the departure point, but must have a midcourse to account for the transfer plane's regression. Or a transfer can be made to the regressed apsides. Our system is a three body oblate system, with perturbations that also pose restrictions. Two techniques for surmounting these restrictions are discussed on the next page.

DEPARTURE-ARRIVAL POINTS

Another requirement was an algorithm to be used in selecting departure date, time, orbit, position etc to arrive after varying trip times. The departure and arrival points, Table 1, 2, were selected using the following as an algorithm.

Select a transfer time and an arrival date. This choses a departure date and time. The departure date/time is used as program time zero, and sets the coordinate system that will be used for the calculations.

With program zero time set, use the insertion event in IMP to insert a target vehicle at the arrival time at the L2 point. From the output, note the arrival geocentric Latitude and the inertial Longitude. Note also whether the arrival azimuth is ascending or descending. The departure point is set to the opposite side of the Earth in plane with the Earth-Sun.

Using IMP INSERTION, option 0, set the main S/C at the negative of the arrival Latitude, at the arrival inertial Longitude + 180 degrees, and at the departure altitude of 160 NM. Set the azimuth flag to compute the proper direction of the orbit velocity. Input an inclination of 23.45 degrees. Then if velocity is set to 0., IMP will set circular velocity and azimuth.

This yields an almost inplane departure with close to 180 degrees of transfer. A small coast can then set the transfer less than 179 degrees so that it can be solved. This coast can also be used for optimization. A second technique to get away from the 180 degree point is to offset the longitude by a few degrees. This accounts for the transfer plane's regression rate varying as the S/C moves from the oblate gravity field to a spherical field. The second method was not used other than a few selected test cases.

Table 1 assumes a constant arrival date 25 Sep 2011 is desired and shows departure Lat/Lon for several trip times.

TABLE 1. CONSTANT ARRIVAL DATE 25 SEP 2011 E/S L2

TRIP TIME	DEPARTURE				ARRIVAL			
DAY	DATE	LAT	LON(G)		DATE	LAT	LON(IN)	
20	SEP 05	-.731	107.68	DSC	SEP 25	.731	-72.32	ASC
30	AUG 26	"	117.54	DSC	"	"	-62.46	
40	AUG 16	"	127.40	DSC	"	"	-52.60	
60	JUL 27	"	147.11	DSC	"	"	-32.89	
80	JUL 07	"	166.82	DSC	"	"	-13.18	
100	JUN 17	"	186.53	DSC	"	"	6.53	

Table 2 assumes a constant trip time of 100 days to arrive at the dates shown.

TABLE 2. 100 DAY TRIP MIXED ARRIVAL DATES E/S L2

DEPARTURE			ARRIVAL		
DATE	LAT	LON(G)	DATE	LAT	LON(IN)
JUN 7	3.146	187.418 DSC	SEP 15	-3.146	7.418
JUN 17	-.731	186.530 DSC	SEP 25	.731	6.532
JUL 07	-8.398	185.043 DSC	OCT 15	8.398	5.043
AUG 07	-18.399	184.691 DSC	NOV 15	18.399	4.691
SEP 07	-23.298	187.400 DSC	DEC 16	23.298	7.400

TLIB TRANSFERS TO L-2

A simple method of breaking the initial orbit and moving out of the Earth's major influence was desired. Two stages are used. It is planned to use a fixed size first stage for the TLIB burn, drop it and then after 2-3 days do a midcourse correction with the second stage. There are several techniques for the TLIB burn.

HOHMANN

Departure point is perigee. The target apogee altitude is near the L2 radius from the Earth and can be used for optimization. Velocity at departure is calculated from closed form two body equations then modified with BELL LAB's correction for an oblate Earth. The TLIB burn is in the initial departure orbit plane.

INPLANE

An inplane burn is made such that the S/C is in an orbit intersecting the departure orbit. At the intersection, the new orbit has a velocity and flight path angle as input. Normally 10908 M/S and 0 degrees. Again the begin event time is as given above and can be used for optimization.

LAMBERT

The halo insertion point is used as a target. The departure point is the initial position. A spherical Lambert problem is solved for the velocity needed at the initial point to arrive at the target point at the time desired. This solution is basically for two bodies and cannot be a 180 degree inertial transfer. The transfer may be modified for the presence of the sun using IMP'S algorithm "NXDN" which numerically solves for a new XDN. This is referred to as an oblate Lambert. It includes the oblate Earth gravity effects, and also the Sun's. Since a Lambert solution for 180 degrees does not exist, the begin event state must insure less than 180 degrees transfer. It still may be used for optimization.

OTHER THRUST EVENTS

MIDCOURSE

An oblate Lambert is solved from the midcourse point to the halo insertion point to arrive at the time desired. The midcourse transfer angle should be between 5 to 20 degrees. And in a posigrade direction. Midcourse should be accomplished while the Earth's gravity field still predominates. If delayed, things might get squirrely in the simulation.

INJECTION

Halo velocity calculated, and burn guided such that a minimum state error is obtained. On arrival this might require a big delta V. It is very affected by the length of time the sun's gravity has to change the approach trajectory. Injection is into a retrograde Halo at a point on the LOS from L2 to the Earth. A current initial halo x is 2000 KM.

NUMERICAL RESULTS

The Integrated Mission program described on page 11 was used to integrate the equations of motion. IMP uses double precision FORTRAN and early in the study a requirement for at least Quad precision was suspected. Since this was not available, means of getting better results were investigated.

See pages 13 to 19 for a description of the basic equations used in IMP's libration simulations. In particular note the definition of a libration point. At such a point, the sum of all forces acting on a body is zero. Additionally the attraction of gravity by either the sun or the earth is very small. This means that numerical roundoff and truncation can be a chaotic factor, and indeed, they were found so.

One method for easing this problem is a modified patch conic procedure for TLIB and midcourse. The transfers are integrated with the Sun's gravity omitted from the equation of motion. On reaching the halo insertion point, it is turned back on to calculate the Halo velocity needed. In the tables below, with Sun refers to results leaving the Sun in the equation of motion. A notation of OK means that the transfer did not exceed the DV capabilty of either stage. Most of the light vehicle transfers were in the not OK status. It was expected to be difficult to design a small system to put a telescope at L2.

TABLE 3. GENERIC LIGHT VEHICLE DATA

1ST STAGE: BREAK ORBIT AND TRANSFER				3185 M/S DV	
THRUST	24000		LB		
ISP	300		SEC	80 LB/S	
FUEL		4100	LB		
DRY		825	LB		
TOTAL					4925 LB
2ND STAGE: MIDCOURSE AND FINAL INJECTION				528 M/S DV	
THRUST	1000		LB		
ISP	285		SEC	3.508 LB/S	
FUEL		220	LB		
DRY		325	LB		
TOTAL					545 LB
PAYLOAD					
TELESCOPE		700	LB		
CONTINGENCY		30	LB		
TOTAL					730 LB
TOTAL INITIAL WEIGHT					6200 LB

Basic HOHMANN transfers to radii near L2 were used to size the first stage of both vehicles.

TABLE 4. HOHMANN TRANSFERS LIGHT VEHICLE

RADIUS KM	PERIOD HRS	DV M/S	FUEL LB	TRANSFER DAYS	
1500000.	1818.4	3174.7	4092.6	37.8	
1600000.	2001.6	3176.2	4093.7	41.7	
1700000.	2190.5	3177.5	4094.6	45.6	OK
1800000.	2385.1	3178.7	4095.5	49.7	
1900000.	2585.0	3179.8	4096.2	53.8	

TABLE 5. HOHMANN TRANSFERS LIGHT VEHICLE

	HOHMANN	M/S		WITH SUN G	
TDAYS	TLIB	MID	INS	SUM	
20	3178	98	607	3883	
30	3178	74	378	3629	OK
40	3178	75	359	3612	OK
60	3176	192	384	3752	
80	3178	329	612	4120	
100	3178	398	715	4292	

TABLE 6. LIGHT VEHICLE OTHER TRANSFERS

	INPLANE 1ST FIXED		M/S	WITH SUN G	
TDAYS	TLIB	MID	INS	SUM	
20	3175	104	609	3888	
40	3175	106	329	3610	OK
60	3175	220	405	3800	
80	3175	366	574	4115	
100	3175	399	701	4275	

	INPLANE 1ST FIXED		M/S	WITHOUT SUN G	
TDAYS	TLIB	MID	INS	SUM	
20	3175	72	548	3795	
40	3175	6	254	3435	OK
60	3175	16	330	3522	OK
80	3175	40	399	3614	OK
100	3175	58	451	3685	OK

	LAMBERT SPH		M/S	WITH SUN G
TDAYS .	TLIB	MID	INS	SUM
20	3202	20	611	3834
30	3237	40	369	3646
40	3291	81	320	3692
60	3345	246	426	4017
80	3375	427	592	4197
100	3402	142	493	4037

	LAMBERT SPH		M/S	WITHOUT SUN G
TDAYS	TLIB	MID	INS	SUM
20	3202	27	552	3781
30	3237	28	295	3559
40	3290	27	254	3571
60	3334	27	330	3702
80	3378	27	402	3807
100	3401	26	453	3880

TABLE 7. GENERIC HEAVY VEHICLE DATA

1ST STAGE: BREAK ORBIT AND TRANSFER				3185 M/S DV	
THRUST	60000		LB	.968 T/W	
ISP	300		SEC	200 LB/S	
FUEL		41000	LB		
DRY		8250	LB		
TOTAL					49250 LB
2ND STAGE: MIDCOURSE AND FINAL INJECTION				1130 M/S DV	
THRUST	10000		LB	.784 T/W	
ISP	285		SEC	35 LB/S	
FUEL		4250	LB		
DRY		1250	LB		
TOTAL					5500 LB
PAYLOAD					
TELESCOPE		7000	LB	PF .117	
CONTIGENCY		250	LB		
TOTAL					7250 LB
TOTAL INITIAL WEIGHT					62000 LB

See Tables 1 and 2 for the departure and arrival positions that were used for the following transfers.

TABLE 8. HEAVY VEHICLE TRANSFERS FIXED ARRIVAL DATE

	INPLANE TLIB		ARRIVAL	SEP 25		
TDAYS	TLIB	MID	INS	SUM	M/S	
20	3184	45	614	3843	WITH SUN G	OK
40	3184	105	319	3608	WITH SUN G	OK
60	3184	231	433	3848	WITH SUN G	OK
80	3184	388	600	4172	WITH SUN G	OK
100	3184	401	715	4301	WITH SUN G	OK
20	3184	74	552	3810	NO SUN G	OK
40	3184	6	252	3492	NO SUN G	OK
60	3184	29	336	3549	NO SUN G	OK
80	3184	55	408	3648	NO SUN G	OK
100	3184	75	459	3719	NO SUN G	OK

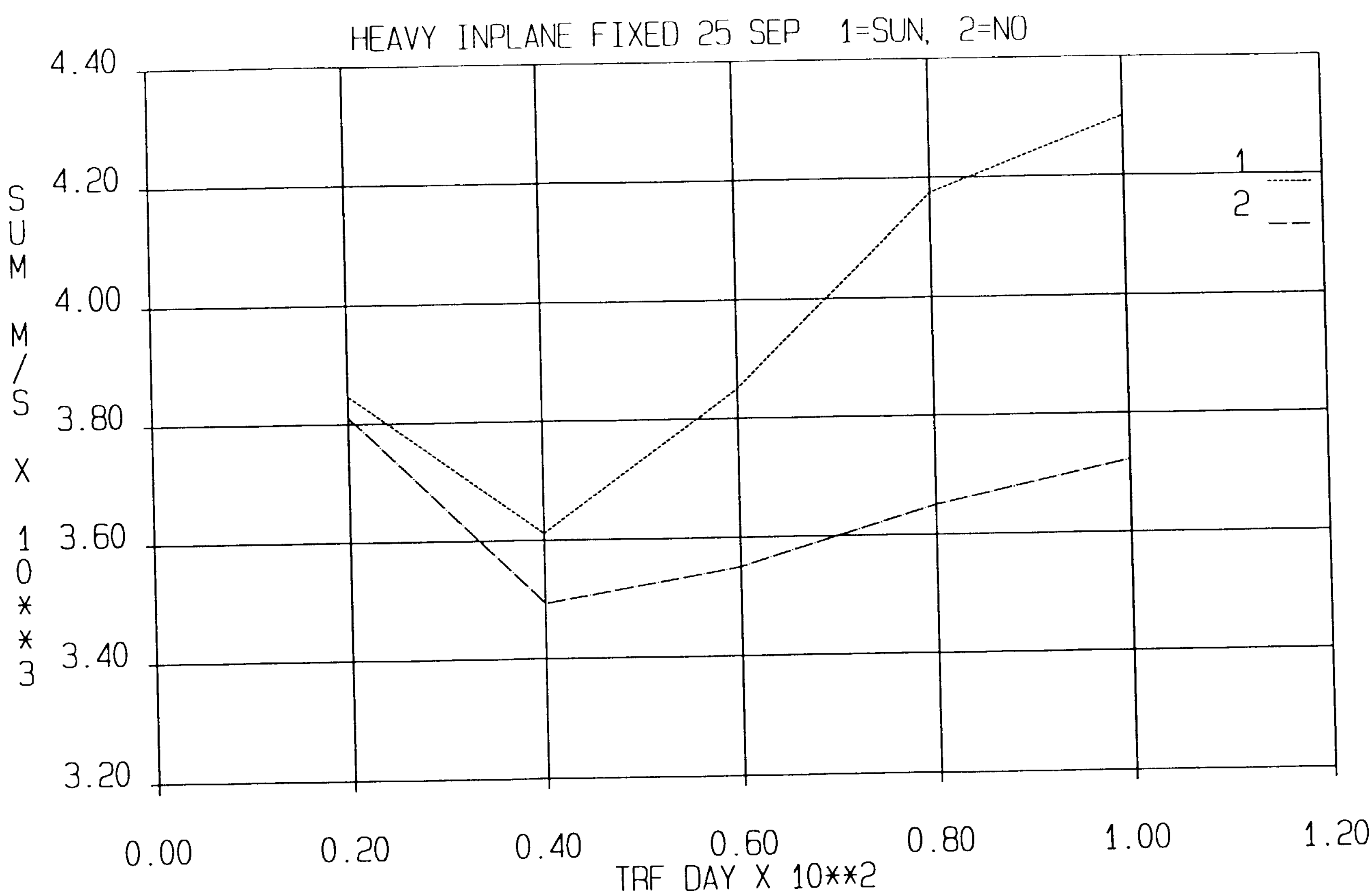
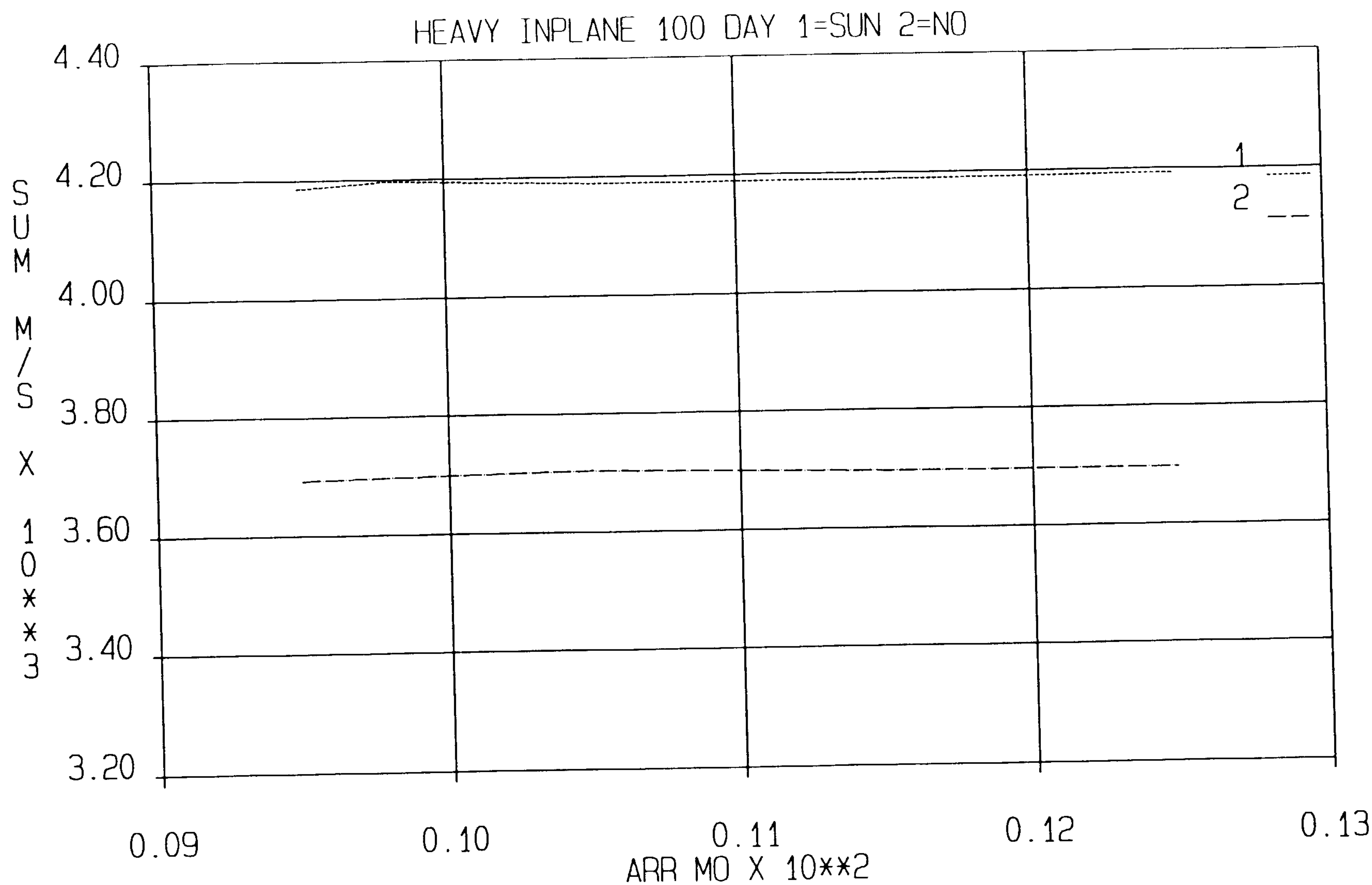

HEAVY INPLANE FIXED 25 SEP 1=SUN, 2=NO
SUM M/S X 10**3
4.40
4.20
4.00
3.80
3.60
3.40
3.20
0.00
0.20
0.40
0.60
0.80
1.00
1.20
TRF DAY X 10**2
1
2

TABLE 9. HEAVY VEHICLE 100 DAY TRANSFERS

ARRIVE	INPLANE TLIB TLIB	TLIB MID M/S	INS	SUM	
					WITH SUN G
SEP 15	3184	273	727	4183	OK
SEP 25	3184	284	727	4195	OK
OCT 15	3184	268	734	4186	OK
NOV 15	3185	261	742	4188	OK
DEC 16	3185	257	749	4191	OK
					NO SUN G
SEP 15	3184	58	449	3691	OK
SEP 25	3184	58	450	3693	OK
OCT 15	3184	56	463	3703	OK
NOV 15	3185	50	460	3695	OK
DEC 16	3185	45	465	3695	OK


HEAVY INPLANE 100 DAY 1=SUN 2=NO
4.40
4.20
4.00
3.80
3.60
3.40
3.20
SUM M/S X 10**3
1
2
0.09
0.10
0.11
0.12
0.13
ARR MO X 10**2

ADDENDA

* * * INTEGRATED MISSION PROGRAM * * *

* * * IMP * * *

DEVELOPED AND PROGRAMMED

BY

V. A. DAURO, SR.

* * * ABSTRACT * * *

"IMP" IS A SIMULATION LANGUAGE THAT IS USED TO MODEL MOST PRESENT OR FUTURE MISSIONS ABOUT THE EARTH, MARS, MOON OR OTHER BODY. MISSIONS ARE USER CONTROLLED THROUGH SELECTION FROM A LARGE EVENT/MANUEVER MENU. MISSION PROFILES, TIMELINES, PROPELLANT REQUIREMENTS, FEASIBILITY AND PERTURBATION ANALYSIS MAY BE QUICKLY, ACCURATELY CALCULATED. ONE, TWO OR THREE SPACECRAFT MAY BE USED: A MAIN, A TARGET AND AN OBSERVER.

* A FEHLBERG 7/13 RUNGE-KUTTA INTEGRATOR WITH ERROR AND STEP SIZE CONTROL IS USED TO NUMERICALLY INTEGRATE THE EQUATIONS OF MOTION.

* OBLATE OR SPHERICAL GRAVITY CAN BE USED FOR THE CENTRAL BODY. ADDITIONAL EFFECTS OF SUN GRAVITY, SOLAR PRESSURE, OR MOON GRAVITY ARE AVAILABLE. WHEN ADDED, THE SUN OR MOON GRAVITY IS SPHERICAL.

* AERODYNAMIC LIFT AND DRAG WHILE IN THE EARTH OR MARTIAN ATMOSPHERE INCLUDED WHEN REQUESTED.

* INPUT/OUTPUT HAS BEEN SIMPLIFIED AND IS IN METRIC UNITS, WITH THE EXCEPTION OF THRUST AND WEIGHT WHICH ARE IN ENGLISH UNITS. INPUT IS READ FROM THE VDT KEYBOARD AND THE USER'S INPUT FILE. REAL TIME KEYBOARD INPUT HAS BEEN MINIMIZED.

* THE CODE EXECUTES EVENTS IN THE ORDER THEY ARE INPUT. AN EVENT IS ENTERED BY INVOKING ITS NAME, READING ITS OPTIONS, DATA, AND THEN EXECUTING. ON COMPLETION, THE NEXT EVENT IS STARTED.

* EVENTS/MANUEVERS MAY INVOLVE NONE, ONE, OR MULTI VELOCITY CHANGES. THRUST LEVEL, AND PROPELLANT USAGE ARE AS THE USER SELECTS. THE VELOCITY CHANGES MAY BE IMPULSIVE, OR OF FINITE DURATION WITH GUIDANCE CALCULATED INTERNALLY OR PRESET BY THE USER. ALGORITHMS FOR TWO POINT BOUNDARY VALUES PROBLEMS INVOLVING VELOCITY CHANGES AND GUIDANCE ARE AUTOMATICALLY INVOKED AS NEEDED.

* MAIN OUTPUT IS TO A USER NAMED PRINT FILE, TO A PLOT FILE, AND TO A DEBUG FILE. MAJOR MANUEVERS ALSO USE THEIR OWN PRENAMED FILES TO OUTPUT ADDITIONAL DATA IN TABULAR AS WELL AS PLOT FORM.

* THE CODE IS PROGRAMMED IN DOUBLE PRECISION FORTRAN AND WILL COMPILE WITHOUT ERRORS TO THE STANDARDS OF LAHEY (TM) FORTRAN 90.

THE PROGRAM WAS INITIALLY CODED FOR MARSHALL SPACE FLIGHT CENTER, (MSFC), SE-AERO-G, HUNTSVILLE, AL. THE AUTHOR WAS EMPLOYED BY NORTHROP SERVICES, INC., HUNTSVILLE, AL. LATER IT WAS EXTENSIVELY MODIFIED WHEN HE WAS A MISSION ANALYST AT PRELIMINARY DESIGN, PD33, MSFC. SINCE RETIREMENT HE HAS CONTINUED TO UPDATE AND IMPROVE IMP.

The following is data used and or computed in IMP to model the Lagrangian libration point L2. The spherical gravity mode of IMP and an ideal 3 body simulation, ideal constants as below, were used to validate the equations of motion on page 15.

* * * * EARTH / SUN * * * *

YEAR 2011 DATA

IDEAL LIBRATION SYSTEM S/E

GM1	M**3/S**2	0.398601200000E+15
GM2	M**3/S**2	0.132718490000E+21
MEAN R	M	149600000000.00
OMG SYS	DEG/DAY	0.98561057
	DEG/S	0.114075297544E-04
PERIOD	DAYS	365.25582281
OMG M1	DEG/SEC	0.417807419642E-02

LIBRATION DATA SYS S/E POINT 2

YEAR 2011 DATA

	YEAR MAX	MIN	DIF	P/C
RSYS KM	152098953.00	147101055.15	4997897.85	3.29
OMG D/S	0.00001170	0.00001113	0.00000057	4.89
RLIB KM	1526613.13	1476449.37	50163.77	3.29
VLIB M/S	30338.524	29835.906	502.618	1.66
BC KM	456.81	441.80	15.01	3.29

PRESENT PARAMETERS AT LIB POINT

GRAVITY	PARTIAL WRT R	FORCE	
EARTH	-0.24767952E-12	0.18284634E-03	(M/S**2)
SUN	-0.80924389E-13	0.60118772E-02	(M/S**2)
CENTRIFUGAL		0.61947235E-02	(M/S**2)

DISTANCES

SYSTEM	147103631.82	(KM)
LP TO EARTH	1476475.23	(KM)
BC TO SUN	441.80	(KM)

EXTRACTED FROM IMP USER04.MAN
USER04.MAN PAGES 14-22

* * * LIBRATION SIMULATIONS * * *
LAGRANGIAN POINTS

LIBRATION SIMULATIONS CAN BE CHAOTIC. CALCULATIONS, EVEN DOUBLE PRECISION, ARE NOT STABLE. ROUNDOFF AND TRUNCATION DEGRADE ACCURACY. FOR S/E SYSTEM, OR E/M SYSTEM, IMP IS STABLE FOR A SIMULATION OF ABOUT ONE HALO ORBIT, S/E 180 DAYS, M/E 11 DAYS. USE LONGER TIMES WITH CAUTION.

LIBRATION SIMULATIONS MAY BE

1) AN IDEAL RESTRICTED THREE-BODY SOLUTION USING CIRCULAR ORBITS, SPHERICAL GRAVITY, IDEALIZED CONSTANTS, (MU, RADII, ETC) AND EITHER

   A) CLOSED FORM EQUATIONS OR,

   B) NUMERICAL INTEGRATION OF EQUATIONS OF MOTION

2) A REAL WORLD SIMULATION USING ELLIPTICAL ORBITS, OBLATE GRAVITY, EPHEMERIDES, CURRENT CONSTANTS, (MU, RADII, ETC) AND EITHER

   A) APPROXIMATIONS USING PERTURBATION ANALYSIS OF THE IDEAL AND ASSUMED CLOSED FORM EQUATIONS

   B) NUMERICAL INTEGRATION OF EQUATIONS OF MOTION

IT IS EVIDENT THAT THE ABOVE ARE ALL USEFUL TOOLS IN STUDYING LIBRATION. 1A IS USED FOR FAST PRELIMINARY RESULTS AND AT MOST IS ONLY ADVISORY. 1B IS USED TO TEST THE IDEAL EQUATIONS AND THE RESULTS OF ROUND OFF AND TRUNCATION. 2A AGAIN GIVES FAST RESULTS AND IN GENERAL IS USED FOR PRELIMINARY PLANNING. 2B REQUIRES A LOT OF COMPUTER TIME AND IS NORMALLY USED LAST TO REFINE SOLUTIONS OBTAINED BY THE OTHER METHODS.

IMP USES NUMERICAL INTEGRATION OF THE EQUATIONS OF MOTION, AND CAN BE USED IN AN IDEAL (1B) OR PERTURBED (2B) MODES.

KEATON'S EQUATIONS AS SHOWN IN REFERENCE 4, WERE ADAPTED AND USED IN IMP FOR LIBRATION STUDIES. REFERENCES 6, 7, AND 8 WERE ALSO USED IN PREPARING THE SIMULATION. THESE ARE ALL EXAMPLES OF THE CLOSED FORM IDEAL EQUATIONS.

BY DEFINITION, THE SUM OF FORCES AT A LIB POINT IS ZERO. EACH SYSTEM HAS 5 PLACES WERE THAT CAN OCCUR. THE FIRST THREE EARTH SUN ARE ON THE EARTH SUN LINE. L1 ON THE SUNNY SIDE, L2 ON THE DARK SIDE, AND L3 ON THE SIDE OF THE SUN AWAY FROM THE EARTH, L4 IS AHEAD OF THE EARTH IN ITS ORBIT, AND L5 TRAILS THE EARTH IN ITS ORBIT. L4(L5) WITH THE EARTH AND SUN FORMS AN EQUILATERAL TRIANGLE. IMP HAS SIMULATIONS FOR L1, L2, L4 AND L5. L3 IS NOT NORMALLY STUDIED.

IT IS KNOWN THAT THE LIB POINTS 1, AND 2 ARE UNSTABLE. HALO ORBITS ABOUT L1 OR L2 CAN BE MADE STABLE IN THE RESTRICTED 3 BODY CASE.

L4 AND L5 ARE STABLE AT WHAT ARE TERMED THE TROJAN POINTS. HALO ORBITS DO NOT EXIST ABOUT L4 AND L5.

HALO ORBITS ABOUT L1 AND L2 ARE USUALLY WHAT INVESTIGATORS STUDY. WE INTEND STATIONING FOR A PERIOD OF TIME, IF PERTURBATIONS ARE PRESENT, INSTABILITY MAY RESULT. IN FACT, TRUNCATION OR ROUND OFF EVENTUALLY DISTURB THE SIMULATION OF THE SYSTEM. THEREFORE IN TESTING, WE NEED TO EVALUATE THE IDEAL AS WELL AS THE REAL WORLD.

TESTS SHOW THAT IMP'S EQUATIONS, ARE ACCEPTABLE FOR AT LEAST ONE IDEAL EARTH/SUN, MOON/EARTH OR MARS/SUN HALO ORBIT. THAT IS, IN THE NON PERTURBED RESTRICTED 3 BODY MODE, THE PROGRAM WILL SIMULATE HALO ORBITS ABOUT E/S, E/M OR M/S LIBRATION POINTS L1 AND L2. AS A MEASURE OF THE SIMULATIONS RELIABILITY, A HALO ORBIT ABOUT THE EARTH/SUN L1 POINT, RESTRICTED 3 BODY SOLUTION, NEEDS LESS THAN 1 M/S DELTA V TO ACCOUNT FOR ROUND OFF AND TRUNCATION.

IDEAL STATIONING AT L4 AND L5 IS ALSO MODELED IN IMP, AND 90 DAYS AT THE EARTH-SUN L5 POINT REQUIRES NO CORRECTIVE DELTA-V.

AS PART OF THE INVESTIGATION, PARAMETERS FOR THE IDEAL AND PERTURBED SYSTEMS WERE CALCULATED. FOR THE EARTH/SUN SYSTEM MODELED IN IMP, THE DATA OBTAINED WAS PREVIOUSLY SHOWN.

* * * PERTURBED RESULTS * * *

STUDIES OF WHAT HAPPENS WHEN THE GRAVITY IS NON KEPLERIAN, OR ECCENTRIC ORBITS ARE USED, OR A FOURTH BODY IS PRESENT YIELD SOME INTERESTING RESULTS.

HALO ORBITS ABOUT THE E/S LIBRATION POINTS L1 AND L2 ARE FAIRLY STABLE. THEY ARE AFFECTED BY THE EARTH'S ORBIT, AND THE PRESENCE OF THE MOON, BUT THEY REQUIRE VERY LITTLE ENERGY TO REMAIN ON STATION.

HALO ORBITS ABOUT THE M/E LIBRATION POINTS, ARE AFFECTED BY THE MOON'S UNUSUAL ORBIT. CONSIDER MOON EVECTION. WHENEVER THE MOON'S VELOCITY IS DIRECTED TOWARD THE SUN, IT SPEEDS UP. IT SLOWS DOWN WHEN MOVING AWAY FROM THE SUN. THIS EFFECT IS ALSO FELT BY THE 3RD BODY IN ITS HALO ORBIT. THE SUN'S PRESENCE HAS AN EFFECT ON M/E HALO ORBITS ABOUT A LIBRATION POINT. STATIONING IS HOWEVER NOT AS DIFFICULT AS I ONCE THOUGHT. THE SUN'S EVECTION EFFECT IS EFFECTIVELY CANCELLED TO A GREAT DEGREE AND I NOW CONCLUDE THAT:

* HALO ORBITS ABOUT THE M/E LIBRATION POINTS (L1 OR L2) CAN BE USED AS TRANSPORTATION NODES. DELTA-V FOR STATIONING IS APPROXIMATELY 7 TIMES THE EARTH SUN SYSTEM.

THE FOLLOWING EQUATIONS ARE MODELED IN IMP

IDEAL LIBRATION NODE/HALO SYSTEMS
(SEE REFERENCES 4,5,6,7, AND 8)

```
B1    SMALLER BODY  WITH GRAVITY GM1
B2    LARGER   "               "    GM2
BC    BARYCENTER OF SYSTEM

L1    LIBRATION POINT    B2 >>>> L1 > B1   INSIDE
L2    LIBRATION POINT    B2 >>>> B1 > L2   OUTSIDE

                DISTANCES
RS    B1 TO B2              SYSTEM
R1    L1 TO B1
R2    L1 TO B2
R3    B2 TO BC
R4    BC TO L1

        OMEGA SYSTEM ROTATION RATE
OMEGA**2 = (GM1+GM2)/RS**3


              CALCULATIONS
 ALPHA = GM2/(GM1+GM2)
 R3 =   (ONE-ALPHA)*RS

              ITERATE R1 UNTIL SUM OF F = VERY SMALL

 R2 = F(LIB POINT, R1, RS)
 R4 = R2-RS

 F1= GM1/R1**2          GRAVITY BODY1
 F2= GM2/R2**2          GRAVITY BODY2
 F3= R4*OMEGA**2        CENTRIFUGAL FORCE AROUND BC

              SIGN F1, F2, AND F3 ACCORDING
              TO LIB POINT DESIRED AND SUM

 F1+F2+F3 = ZERO


             THEN SET KEATON'S EQUATIONS REFERENCE 4
 FSQ=(ONE-ALPHA)*(RS/R1)**3+ALPHA*(RS/R2)**3
 F=SQRT(FSQ)
 BSQ=ONE-FSQ/TWO+F*SQRT(2.25D0*FSQ-TWO)
 B=SQRT(BSQ)
 GAMH=(ONE+BSQ+TWO*FSQ)/(TWO*B)
 BOMGS=B*OMEGA
 FOMGS=F*OMEGA
 SPER= 2*PI/OMEGA                SYSTEM PERIOD
 HPER=SPER/B                     HALO PERIOD
```

COORDINATE SYSTEMS

THE INTEGRATION OF THE DEQS OF MOTION IS DONE WITH THE BODY B1 AS THE CENTER OF THE COORDINATE SYSTEM. LIBRATION NODES ARE SET IN THIS SYSTEM AND USED AS THE CENTER OF THE HALO COORDINATE SYSTEM (HX, HY, HZ).

```
HX POINTS FROM B1 TO B2
HY IS TO LEFT VIEWED FROM TOP IN ROTATION PLANE
HZ COMPLETES A RIGHT HAND SYSTEM

T0   INSERTION TIME             (ONLY AT BANG = ZERO, OR PI)
X0   INSERTION POSITION         ALWAYS POSITIVE
Y0   ALWAYS AT ZERO
Z0   OUT OF PLANE COMPONENT AT INSERTION ALWAYS POSITIVE
PHASE0  INSERTION PHASE   ZERO IF INSERT AT BANG = ZERO
                          PI   IF INSERT AT BANG = PI

T = POSITION TIME

THALO = T - T0
BANG=BOMGS*THALO+PHASE0       ANOMALY ANGLE IN HX, HY PLANE
FANG=FOMGS*THALO+PHASE0       "Z"  ANGLE WRT TO HX, HY PLANE
                              HALO POSITION
HX= X0*COS(BANG)
HY=-X0*GAMH*SIN(BANG)
HZ= Z0*COS(FANG)
                              HALO VELOCITY
HXD=-X0*BOMGS*SIN(BANG)
HYD=-X0*BOMGS*GAMH*COS(BANG)
HZD=-Z0*FOMGS*SIN(FANG)
```

* * * LIBRATION NOTES * * *

IT IS IMPORTANT THAT THE USER KNOW THE DIFFERENCE BETWEEN THE RESTRICTED THREE BODY SYSTEM (IDEAL) AND THE REAL WORLD. IN THE IDEAL SYSTEM, THERE ARE ONLY THREE BODIES PRESENT. THESE ARE IN IMP'S SIMULATION, DESIGNATED B1, B2, AND B3. THE MASSES ARE SUCH THAT

B3<<B1<B2.

B1 AND B2 ROTATE ABOUT A COMMON BARYCENTER WITH A CONSTANT RATE AND AT A CONSTANT DISTANCE. THIS IMPLIES SPHERICAL GRAVITY FIELDS FOR BOTH B1 AND B2, AND THAT B3, AT THE LIBRATION POINT, HAS A MINIMAL INFLUENCE ON THE SYSTEM.

IN THE REAL WORLD, THERE ARE PERTURBATIONS.

IN THE SUN-EARTH SYSTEM, THE EARTH IS IN AN ELLIPTICAL ORBIT ABOUT THE BARYCENTER. ALTHOUGH THE DISTANCE TO THE SUN IS SUCH THAT THE SUN APPEARS TO HAVE A SPHERICAL GRAVITY FIELD, THE THIRD BODY IS CLOSE ENOUGH TO THE EARTH SO THAT IT IS SEES AN OBLATE EARTH GRAVITY FIELD. FURTHER, THERE IS A FOURTH BODY, THE MOON, CLOSE ENOUGH TO EFFECT THE SYSTEM. HOWEVER, FOR B3 AT THE EARTH-SUN LIBRATION POINTS, THE MOON IS FAR ENOUGH AWAY SO THAT ITS EFFECTS ARE MINIMAL.

FOR THE EARTH-MOON SYSTEM, THE SUN IS A FOURTH BODY THAT DOES AFFECT THE SYSTEM, AND MUST BE ACCOUNTED FOR IN A SIMULATION. ADDITIONALLY, THE MOONS ORBIT IS 3 TIME MORE ECCENTRIC THAN THE EARTH, AND THE MOON ORBITS THE EARTH ABOUT 13 TIMES A YEAR.

* * * REFERENCES * * *

1) FEHLBERG, E., CLASSICAL FIFTH-SIXTH-SEVENTH-AND EIGHTH ORDER RUNGE-KUTTA FORMULAS WITH STEP SIZE CONTROL, NASA TR R-287, MARSHALL SPACE FLIGHT CENTER, HUNTSVILLE,AL., OCTOBER 1968.

2) ----, NATURAL ENVIRONMENT AND PHYSICAL STANDARDS FOR THE APOLLO PROGRAM AND THE APOLLO APPLICATIONS PROGRAM, MD-E 8020.008C, SE 015-001-1B, OFFICE OF MANNED SPACE FLIGHT, WASHINGTON, D. C.,

3) ----,SPACE AND PLANETARY ENVIRONMENT CRITERIA GUIDELINES FOR USE IN SPACE VEHICLE DEVELOPMENT. 1982 REVISION (VOLUME I). NASA TM-82478, JANUARY 1983.

4) KEATON, PAUL, A MOON BASE / MARS BASE TRANSPORTATION DEPOT, LOS ALAMOS NATIONAL LABORATORY, MSD434, LOS ALAMOS, NM, 1985.

5) MENDELL, W. W., ET AL, LUNAR BASES AND SPACE ACTIVITIES OF THE 21ST CENTURY, LUNAR AND PLANETARY INSTITUTE, HOUSTON, TX, 1985.

6) DESLOGE, EDWARD A., CLASSICAL MECHANICS, JOHN WILEY AND SONS, NEW YORK AND CHICHESTER, 1982.

7) SZEBEHELY, VICTOR, THEORY OF ORBITS, THE RESTRICTED PROBLEM OF THREE BODIES, ACADEMIC PRESS, NEW YORK AND LONDON, 1967.

8) FARQUHAR, ROBERT AND DUNHAM, DAVID, LIBRATION POINT STAGING CONCEPTS FOR EARTH-MARS TRANSPORTATION, NASA M002, JUNE 1986.

9) HOFFMAN, DAVID A., STATION-KEEPING AT THE COLLINEAR EQUILIBRIUM POINTS OF THE EARTH-MOON SYSTEM, JSC26189, LYNDON B. JOHNSON SPACE CENTER, HOUSTON, TX, SEPTEMBER 1993.