Effect of Under-Resolved Grids on High Order Methods

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There has been much discussion on verification and validation processes for establishing the credibility of CFD simulations [1-5]. Since the early 1990s, many of the aeronautical and mechanical engineering related reference journals mandated that any accepted articles in numerical simulations (without known solutions to compared with) need to perform a minimum of one level of grid refinement and time step reduction. Due to the difficulty in analysis, the effect of under-resolved grids and the nonlinear behavior of available spatial discretizations are scarcely discussed in the literature. Here, an under-resolved numerical simulation is one where the grid spacing being used is too coarse to resolve the smallest physically relevant scales of the chosen continuum governing equations that are of interest to the numerical modeler.

With the advent of new developments in fourth-order or higher spatial schemes, it has become common to regard high order schemes as more accurate, reliable and require less grid points. The danger comes when one tries to perform computations with the coarsest grid possible while still hoping to maintain numerical results sufficiently accurate for complex flows, and especially, data-limited problems. On one hand, high order methods when applies to highly coupled multidimensional complex nonlinear problems might have different stability, convergence and reliability behavior than their well studied low order counterparts, especially for nonlinear schemes such as TVD, MUSCL with limiters, ENO, WENO and discrete Galerkin. See for example [6-9]. On the other hand, high order methods involve more operation counts and systematic grid convergence study can be time consuming and prohibitively expansive. At the same time it is difficult to fully understand or categorize the different nonlinear behavior of finite discretizations, especially at the limits of under-resolution when different types of bifurcation phenomena might occur, depending on the combination of grid spacings, time steps, initial conditions and numerical treatments of boundary conditions.

By numerical experiments and using tools from dynamical system, some representative nonlinear behavior of high order schemes for under-resolved grids will be discussed [6-14]. Grid convergence study of our recently developed high order methods [15-17] (Yee et al. 1999,2000, Sjögreen & Yee 2000) will be included. Our goal is to illustrate the importance of understanding the nonlinear behavior of the scheme being used.

References

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