Entropy Splitting for High Order Numerical Simulation of Vortex Sound at Low Mach Numbers

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A method of minimizing numerical errors, and improving nonlinear stability and accuracy associated with low Mach number computational aeroacoustics (CAA) is proposed. The method consists of two levels.

From the governing equation level, we condition the Euler equations in two steps. The first step is to split the inviscid flux derivatives into a conservative and a non-conservative portion that satisfies a so called generalized energy estimate [1]. This involves the symmetrization of the Euler equations via a transformation of variables that are functions of the physical entropy (Harten [2]). This splitting of the flux derivatives, hereafter, is referred to as the entropy splitting. The split form of the the Euler equations was found to require less numerical dissipation than its un-split counterpart in association with non-dissipative spatial central schemes [3]. Owing to the large disparity of acoustic and stagnation quantities in low Mach number aeroacoustics, the second step is to re-formulate the split Euler equations in perturbation form with the new unknowns as the small changes of the conservative variables with respect to their large stagnation values [4]. Nonlinearities and the conservative portion of the split flux derivatives are retained. This perturbation form was shown to minimize numerical cancellation errors compared to the original conservation laws [4].

From the numerical scheme level, a stable sixth-order central interior scheme with a third-order boundary schemes that satisfies the discrete analogue of the integration-by-parts procedure used in the continuous energy estimate (summation-by-parts property) is employed [5]. To suppress the spurious high frequency oscillations associated with central schemes, the characteristic filter method of Yee et al. [6] is used. The discrete scalar product is based on a diagonal matrix. The metric terms in the general coordinate transformation are discretized by the same difference operator as the flow variables leading to freestream preservation (uniform flow conservation) for the conservative portion of the split equations. The time derivative is approximated by a 4-stage low-storage second-order explicit Runge-Kutta method.

Numerical Experiments

The method has been applied to simulate vortex sound at low Mach numbers. We consider the Kirchhoff vortex, which is an elliptical patch of constant vorticity rotating with constant angular frequency in irrotational flow. The acoustic pressure generated by the Kirchhoff vortex is governed by the 2D Helmholtz equation, which can be solved analytically using the separation of variables [7].

A polar grid of 129×24 in the radial and circumferential directions, respectively, is used. The inner circular boundary with radius R almost coincides with the nearly circular Kirchhoff vortex, which is started instantaneously. Its semi-axes are $R(1 \pm \epsilon)$, and its

angular frequency is Ω. Thus, the wave number is $k = 2\Omega/c_0$. We choose $\epsilon = 0.00125$ and the Helmholtz number $\mathcal{H} = kR = 1$. The exact analytical solution at the inner circular boundary is prescribed as initial and boundary conditions.

After 100 time steps, the Kirchhoff vortex has rotated 7.5 *radians*. The computed acoustic pressure along the positive x-axis in Fig. 1 is in good agreement with the analytical solution between the inner boundary and the location ($x \approx 16$), up to which the pressure wave has travelled from the instantaneously started Kirchhoff vortex. The quadrupole structure of the acoustic pressure is correctly recovered in Fig. 2.

Discussion and Final Paper

The analytical solution is not valid for the instantaneously started Kirchhoff vortex. It is only valid for a Kirchhoff vortex, which has been rotating forever. However, since no wave is travelling from the farfield towards the Kirchhoff vortex, as long as we have stagnation conditions in the farfield, we can assume that the analytical solution for the Kirchhoff vortex rotating for infinitely long time is valid up to the wavefront of the instantaneously started Kirchhoff vortex. If the wavefront has left the domain without reflection at the farfield, the analytical solutions for the instantaneously started Kirchhoff vortex and for the infinitely long rotating Kirchhoff vortex should agree.

At $x \approx 16$, we see that in general the wavefront cannot match the infinitely long rotating Kirchhoff vortex solution, because the instantaneously started Kirchhoff vortex has zero acoustic pressure downstream of the wavefront. Therefore, we cannot expect agreement of the two solutions near the wavefront. Thus, the discrepancies between the numerical solution for the instantaneously started Kirchhoff vortex and the analytical solution for the infinitely long rotating Kirchhoff vortex have physical reasons. Of course, the numerical difficulties in treating the wavefront and the use of a second-order time discretization (with non-optimal phase error) might contribute to the discrepancies.

Two additional major issues are nonreflecting farfield boundary conditions to allow the wavefront to leave the domain without reflection, and the boundary conditions at the Kirchhoff vortex need to be implemented such that the summation-by-parts (SBP) property is not destroyed. At present, nonreflecting farfield boundary conditions have not been implemented. The so called injection method of implementing the numerical boundary conditions on the conservative and entropy variables but not the characteristic variables is used. The SBP is not guaranteed. These will be addressed in the final paper.

References

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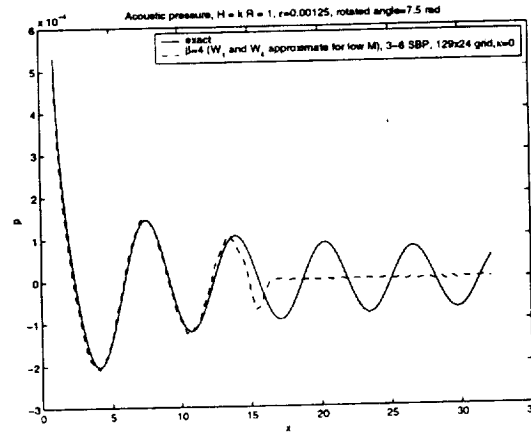
Figure 1: Comparison of exact and computed acoustic pressure for Kirchhoff vortex sound.
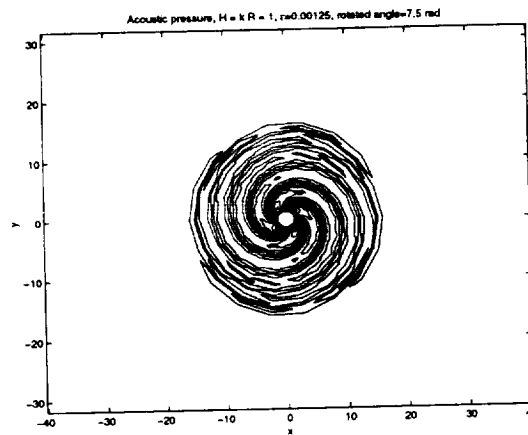

Figure 2: Computed acoustic pressure contours for Kirchhoff vortex sound.