Analysis of Graphite Reinforced Cementitious Composites

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ABSTRACT

This paper describes analytical methods that can be used to determine the deflections and stresses in highly compliant graphite-reinforced cementitious composites. It is demonstrated that the standard transform section method fails to provide accurate results when the elastic modulus ratio exceeds 20. So an alternate approach is formulated by using the rule of mixtures to determine a set of effective material properties for the composite. Tensile tests are conducted on composite samples to verify this approach; and, when the effective material properties are used to characterize the deflections of composite beams subjected to pure bending, an excellent agreement is obtained.

Laminated composite plate theory is also investigated as a means for analyzing even more complex composites, consisting of multiple graphite layers oriented in different directions. In this case, composite beams are analyzed by incorporating material properties established from tensile tests. Finite element modeling is used to verify the results and, considering the complexity of the samples, a very good agreement is obtained.

INTRODUCTION

Strategically embedding layers of graphite mesh in a highly compliant cementitious matrix produces a composite material with relatively high tension and compressive properties as compared to steel-reinforced structures fabricated from a standard concrete mix. [1] Although these composite systems are somewhat similar, the methods used to analyze steel-reinforced composites often fail to characterize the behavior of their more advanced graphite reinforced counterparts. Since only a few investigations have been carried out on the materials in question, there is a pressing need to develop analyses tools suitable for design purposes.

OVERVIEW OF TEST PROGRAM

A finite element model of a multi-layered composite beam was produced and run with boundary conditions for a beam in pure bending. The same beam was analyzed using the transform section method. The elastic modulus ratio was varied from 1 to 80 for each analytical method.

The results of the analyses diverged as the elastic modulus ratio increased, and the observation was made that the traditional transform section method of analysis does not work well for ratios greater than 20.

The next step was to design, produce, instrument, and test a number of highly compliant graphite reinforced cementitious composite specimens.

A relatively flexible concrete was designed with the following mix proportions: Portland cement (393 kg/m^3), latex (78.2 kg/m^3), acrylic fortifier (24.3 kg/m^3), micro-balloons (154 kg/m^3), and water (247 kg/m^3). The air content by volume in a standard compression test cylinder was 14 percent and the water to cement ratio of the mix was 1.19. Tension and compression tests revealed 28-day tensile and compressive strengths of 1.77 MPa and 4.80 MPa, respectively. The elastic modulus and Poisson's ratio were 0.8 GPa and 0.28, respectively.

The composite samples were reinforced with a layer of a non-impregnated graphite mesh having 3k fibers per tow spaced at 3.18 mm intervals. Each tow was 0.19 mm thick by 1.07 mm wide. The fibers were held in place using 0.08 mm diameter Kevlar strands. The elastic modulus and tensile strength of the graphite were 231 GPa and 3.65 GPa, respectively.

A Plexiglas mold was used with a graphite tensioning device to produce a 2.54 mm thick "cookie" sheet of composite material having a single layer of mesh positioned in the center. Porosity was controlled by working with small amounts of cementitious material and floating it in the mold. The cookie sheet was cut into 5 cm wide by 35 cm long strips using a band saw. The strips were machined using a router and template into test specimens having geometries that matched ASTM composite standards. The specimens were sanded to adjust the thickness and polish their surfaces. Then, strain gages were attached to the front and back surfaces to account for bending and/or twist.

After "path-finder" specimens were used to verify the test procedure and instrumentation, tensile tests were conducted to determine the elastic modulus (2.8 GPa) and Poisson's ratio (0.14). Iosipescu specimens were used to determine the shear modulus (517 MPa).

The rule of mixtures was applied in an attempt to predict these effective material properties. As a result, it was demonstrated that a modified transform section approach could be used to analyze simple composite beams.

A MathCad solution sheet was developed to make the required calculations. When the material properties predicted by the rule of mixtures were compared to test data, it was found that the elastic modulus in the direction of loading, E_x, compared well to the test

data. However, Poisson's ratio, the shear modulus, and the transverse modulus, Ey, did not compare well. This result was somewhat anticipated, since other investigators have reported this trend while studying composite lay-ups made from traditional materials.

A modified transform section approach was developed using the Ex value obtained from the rule of mixtures, and the approach was used to predict the deflection of a composite beam in pure bending constructed using a single layer of graphite reinforcement. Then, a similar approach was used to analyze a multiple layered beam subjected to pure bending. Finally, the approach was extended to include nonlinear material properties. All of the results obtained compared well to test data.

The next step in the test program was to apply laminated composite plate theory to highly compliant graphite reinforced lay-ups, and make comparisons with test data and finite element analyses.

A MathCad solution sheet was developed to make the required calculations. When the method was applied to a composite plate with known material properties, the results were found to be more accurate that those derived using other well established solution methods. But the MathCad results agreed well with those obtained using a very reliable MSC/Nastran model. The method for analyzing beams with various ply angles and lay-ups is described below.

COMPOSITE LAMINATED PLATE THEORY

The constitutive equations for graphite reinforced cementitious laminated plates were derived from classical laminated plate theory. The resulting equations required the simultaneous solution of the following 6x6 matrix:

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\
A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\
A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} \\
B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\
B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} \\
B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33}
\end{bmatrix}
\begin{bmatrix} \varepsilon_{0x} \\ \varepsilon_{0y} \\ \gamma_{0xy} \\ K_x \\ K_y \\ K_{xy} \end{bmatrix}
$$

$N_x, N_y,$ and N_{xy} = Force in x, y, and xy direction, respectively.

$M_x, M_y,$ and M_{xy} = Moment about x, y, and xy axis, respectively.

ε_{0x} = Midplane Strain in x direction.

ε_{0y} = Midplane Strain in y direction.

γ_{0xy} = Midplane Shear Strain in xy direction.

K_x = Plate Curvature along the x direction.

K_y = Plate Curvature along the y direction.

K_{xy} = Plate Curvature along the xy direction.

A   = Extensional Stiffness Matrix.

B   = Coupling Stiffness Matrix.

D   = Bending Stiffness Matrix.

The system represents six equations with six unknowns. The A, B and D matrix are derived from the orientation and the lay-up of the composite beam. The force and moment matrix are defined as limit loads in order to solve for the remaining unknowns. To find the effective elastic modulus in the x direction, E_x, for example, it is assumed that a unit force is applied in that direction. Then, the system of equations is solved for the mid-plane strains and curvatures.

Previous authors [2,3] have reduced this system of equations by eliminating terms that are not on the diagonal of the matrix. We achieved an increase in accuracy by including all the terms in a rigorous solution of the system. Our technique is applicable to both symmetric and non-symmetric composite lay-ups.

As mentioned previously, a MathCad [4] solution sheet was to used to perform the calculations. The effective elastic moduli in the x and y directions are given from the constitutive equations:

$$ E_x = \frac{N_x}{t\,\varepsilon_{ox}} \quad \text{and} \quad E_y = \frac{N_y}{t\,\varepsilon_{oy}} \quad \text{where } t = \text{beam thickness.} $$

Similarly, the shear modulus is given by the equation:

$$ G_{xy} = \frac{2\,N_{xy}}{t\,\gamma_{0xy}} \quad. $$

The normal and shear strains in the above equations correspond to mid-plane values.

Three graphite reinforced cementitious beams were produced to verify the results of the solution technique. As illustrated in the schematic shown below, the beams consisted of two layers of mesh confined within the cementitious matrix.
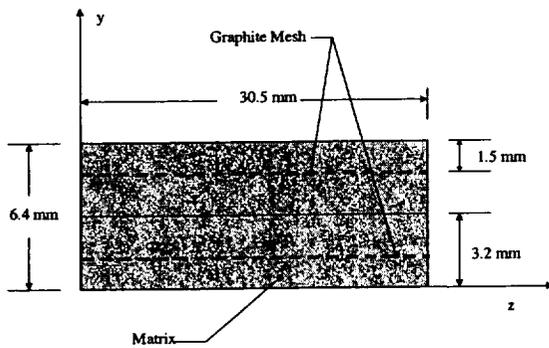
The mesh has the same elastic modulus in the x and y directions. However, because of the very large difference in stiffness between the matrix and the graphite (~300), rotating the mesh changes the effective modulus of the composite significantly. A photograph of a typical specimen is shown below.
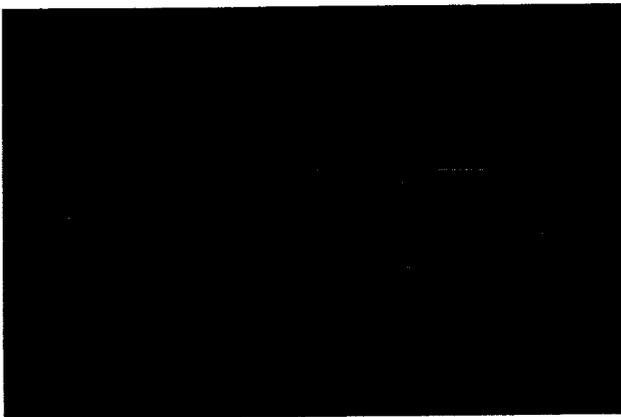
The orientation of the graphite reinforcement is [90,90] in the first beam, [45,45] in the second beam and [45,90] for the third beam. The dimensions of a beam after sanding are 22.86 cm long, 2.84 cm wide and approximately 0.635 cm thick. Since the construction process was not exact, small variations in orientation and thickness were present.

As illustrated in the following photograph, each beam was subjected to four-point bending. The ends of the beams were placed on rollers to produce the desired boundary conditions. Beam deflection data was measured using a dial gage located at the center of each beam.

Incremental loads were applied to produce a load vs. deflection curve, and the load data corrected for the variation in thickness among the test articles.

As shown in the following plot, Beam 1 [90,90] is much stiffer than Beam 2 [45,45]. As expected, Beam 3 [45,90] has a stiffness value between Beam 1 and Beam 2. The fact that the 45° layer is on the tension side in Beam 3 may explain why the behavior is close to that of Beam 2.
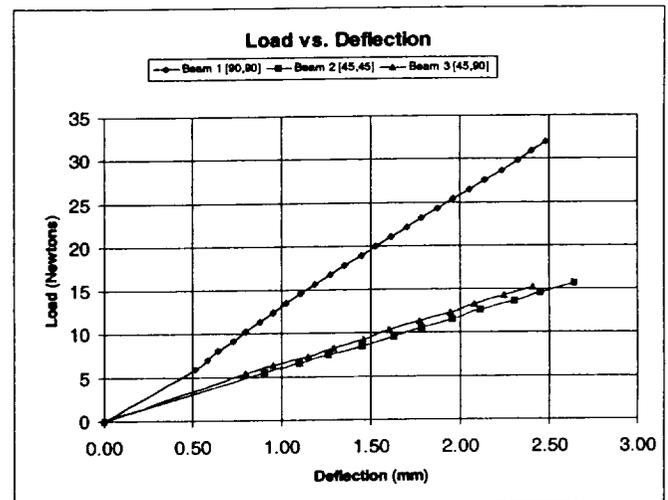
The results of the composite beam tests were compared to analytical results based on the effective elastic modulus values calculated from the constitutive equations presented previously. To achieve this, an elastic beam equation was developed to calculate the beam deflections corresponding to loading and geometry at hand. A MathCad solution sheet was used to determine the integration constants and perform the calculations required to solve the elastic beam equation. The following is an example of a typical calculation.

$$y_2(x) = \frac{1}{EI}\left(\frac{P}{2}ax^2 + C_3 x + C_4\right)$$

$$E = 3.44\ GPa,\ P = 17.2\ N$$

$$I = \frac{bh^3}{12} = \frac{(0.302)(0.006731)^3}{12} = 7.67x10^{-10}\ m^4$$

$$x = 0.111\ m$$

$$C_3 = -0.07289\frac{kg\ m^3}{s^2},\quad C_4 = 0.00063\frac{kg\ m^4}{s^2}$$

$$y_2 = -1.29\ mm$$

As shown in the following plot, the predicted deflections for Beam 1 [90,90] are in good agreement with the test results. The effective elastic modulus used in the predictions, $E_x = 2.84$ GPa, adequately predicted the behavior of this two-layer composite.

Predicted vs. Test Results
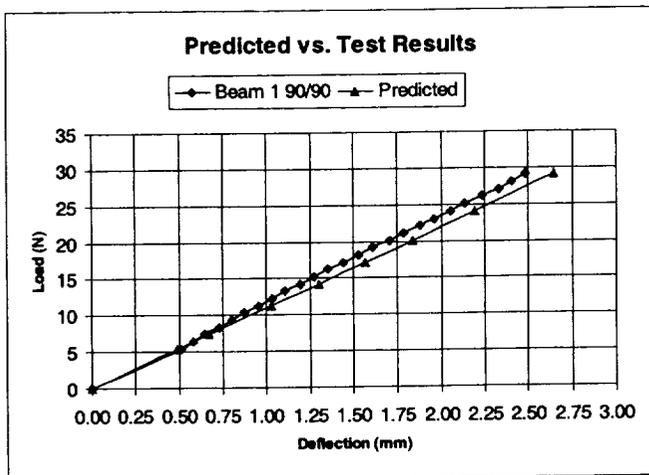
The test data for Beam 2 [45,45] also compared very well with the test results.

Predicted vs. Test Results
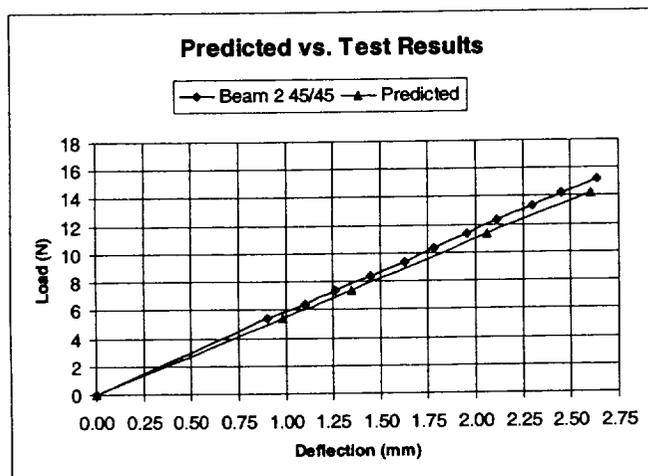
The effective elastic modulus calculated for this beam, $E_x = 1.72$ GPa, indicates that the solution methods for the constitutive equations remain accurate regardless of the orientation of the composite layers.

The test data for Beam 3 [45,90] also compared very well with the test results. This is significant, since the lay-up is non-symmetrical. The effective elastic modulus calculated for this beam, $E_x = 2.29$ GPa, was computed using the same method applied to obtain those corresponding to the symmetric cases.

Predicted vs. Test Results
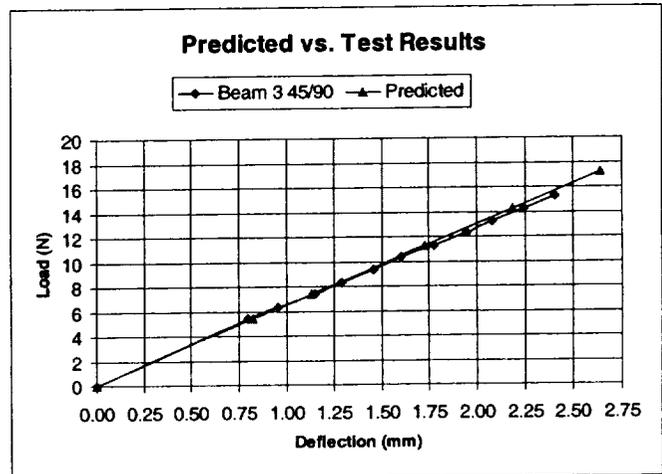
CONCLUSION

Highly compliant graphite reinforced composites can be produced by placing a relatively flexible concrete over a stiff mesh. The stress can be driven from the cementitious matrix to the reinforcement by adjusting the geometry and the system designed to absorb or store large amounts of strain energy. Although the structural behavior of these composites is somewhat different from that of traditional composites, it can be studied by using a modified transform section method or composite laminated plate theory.

REFERENCES

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