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Bandu N. Pamadi, Patrick C. Murphy, Vladislav Klein and Jay M. Brandon

NASA Langley Research Center
Hampton, VA 23681

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PREDICTION OF UNSTEADY AERODYNAMIC COEFFICIENTS AT HIGH ANGLES OF ATTACK

Bandu N. Pamadi,* Patrick C. Murphy,† Vladislav Klein,‡ and Jay M. Brandon§
NASA Langley Research Center, Hampton, VA-23681.

Abstract

The nonlinear indicial response method is used to model the unsteady aerodynamic coefficients in the low speed longitudinal oscillatory wind tunnel test data of the 0.1 scale model of the F-16XL aircraft. Exponential functions are used to approximate the deficiency function in the indicial response. Using one set of oscillatory wind tunnel data and parameter identification method, the unknown parameters in the exponential functions are estimated. The genetic algorithm is used as a least square minimizing algorithm. The assumed model structures and parameter estimates are validated by comparing the predictions with other sets of available oscillatory wind tunnel test data.

Nomenclature

$a(\alpha), a_1^*(\alpha), a_2^*(\alpha)$	Indicial parameters in Model I and Model II
$b_1(\alpha), b_1'(\alpha), b_2'(\alpha)$	Indicial parameters (exponents) in Model I and II, 1/sec
c_1, c_2, c_3	Constants in the algebraic model for C_{Lq}
$\bar{c}$	Mean aerodynamic chord (mac), m
d_1, d_2, d_3	Constants in the algebraic model for C_{mq}
C_L	Lift coefficient
$C_{L\alpha}$	Lift-Curve-Slope, $\frac{dC_L}{d\alpha}$, per rad
C_{aq}	Rotary derivative in pitch, $\frac{\partial C_a}{\partial\left(\frac{q\bar{c}}{2V}\right)}$, $C_a = C_L$ or C_m, per rad
C_m	Pitching moment coefficient
$C_{m\alpha}$	Slope of pitching-moment-coefficient curve, $\frac{dC_m}{d\alpha}$, per rad
$C_{m\dot{\alpha}}$	Acceleration derivative in pitch, $\frac{\partial C_m}{\partial\left(\frac{\dot{\alpha}\bar{c}}{2V}\right)}$, per rad
F_2, F_3, F_α	Deficiency functions in Model I and II
J	Cost function as defined in Eq. (23)
k	Reduced frequency, $\frac{\omega\bar{c}}{2V}$
M	Mach number
q	Pitch rate, rad/sec
t	Time, sec
T_p	Period of oscillation, sec
V	Velocity, m/sec
$\bar{x}_{ref}$	Distance to moment reference point (in terms of mac) from nose, m
$\bar{x}_{cg}$	Distance to center of the planform area (in terms of mac) from nose, m
α	Angle of attack, deg

*Senior Aerospace Engineer, Vehicle Analysis Branch, Aerospace System Concepts and Analysis Competency, Associate Fellow, AIAA.
†Senior Aerospace Engineer, Dynamics and Control Branch, Airborne Systems Competency, Senior Member, AIAA.
‡Emeritus Professor, Joint Institute for Advancement of Flight Sciences, George Washington University, Associate Fellow, AIAA.
§Senior Aerospace Engineer, Vehicle Dynamics Branch, Airborne Systems Competency, Senior Member, AIAA.

ω	Frequency of oscillatory motion in pitch, rad/sec
τ	Dummy variable for time

Acronym

GA	Genetic Algorithm

Introduction

With the advent of innovative high alpha control devices such as three-axes thrust vectoring and forebody vortex controllers, it has become increasingly attractive for fighter aircraft to exploit the full potential offered by flight at high angles of attack (at or beyond stall) to achieve air superiority in close-air combat. Since aircraft development programs tend to rely heavily on simulations to reduce the risk and cost of flight test programs, accurate modeling of such motions assumes critical importance.[1] The traditional aerodynamic approach based on time invariant stability and control derivatives proves to be inadequate for predicting such aircraft motions because it does not account for unsteady aerodynamic effects that are significant at such flight conditions. The problem of modeling unsteady aerodynamic effects at high angles of attack is a formidable task because the flow field is characterized by extensive flow separations, vortices, vortex interactions and possible vortex bursts over the wing and tail surfaces. In view of this, generalized unsteady aerodynamic models that are suitable for high angles of attack flight simulation are not yet available.

NASA Langley Research Center has conducted low speed static and forced oscillation tests on the 0.1 scale model of the F-16XL aircraft at high angles of attack in the Langley 12-Foot Low Speed Wind Tunnel. Using these experimental data and the indicial response method,[2,3] an attempt has been made in this paper to develop aerodynamic models suitable for predicting unsteady aerodynamic coefficients in the longitudinal forced oscillatory motion of the F-16XL aircraft model at high angles of attack.

The indicial response method provides a fundamental approach to the problem of modeling unsteady aerodynamic effects. One significant advantage of this approach is that when the indicial response to a particular forcing mode is known, e.g., due to angle of attack, the cumulative response to an arbitrary forcing mode can be obtained using the superposition principle in the form of Duhamel integral. By definition, an indicial function is the response to a change in the surface boundary condition, say angle of attack, that is applied instantaneously and held constant thereafter. The indicial response is a mathematical concept and as such is difficult to determine by an experiment. In general, the indicial response can be assumed to consist of two distinct components, the noncirculatory loading and the circulatory loading. The noncirculatory loading arises due to the sudden change in the surface boundary condition such as a sudden change in the angle of attack. It assumes a large peak value (theoretically infinity in incompressible flow) initially but subsequently decays rapidly. At present, results based on the linear piston theory are used to define the initial value of the noncirculatory loading. For two-dimensional airfoils in incompressible flow, Lomax[4] has presented analytical expressions for the early part of the decay of the noncirculatory loading. For arbitrary configurations, such information is not available. The circulatory loading is initially zero and builds up gradually to its final steady state value that can be obtained in a relatively easier fashion either by CFD methods or static wind tunnel tests. The difficulty lies in modeling the decay of noncirculatory loading and the build up of the circulatory loading. Approximating the decay of the noncirculatory loading and the build up of circulatory loading with exponential functions and using linear indicial response method, Beddoes,[5] and, Leishman[6] have obtained satisfactory results for the oscillatory motion of two-dimensional airfoils at low angles of attack in subsonic flow. In the linear indicial response method, the indicial response is independent of angle of attack. In other words, if we know the indicial response at one angle of attack, the same is applicable for all other angles of attack as long as aerodynamic coefficients are linear with angle of attack.

In this paper, the nonlinear indicial response method developed by Tobak[2,3] has been used to analyze the low speed, longitudinal oscillatory wind tunnel data of the F-16XL aircraft model at high angles of attack. The linear indicial method is not suitable for high angles of attack because, in general, the aerodynamic coefficients vary nonlinearly with angle of attack. In the nonlinear indicial response method, the indicial response depends on angle of attack. In Tobak's[2,3] formulation, the time varying part of the indicial response is represented by the "deficiency" function. Thus, the task now becomes one of modeling the time history of the deficiency function. In this paper, exponential functions are used to approximate the time history of deficiency function. First, we use a model with two term exponential functions, one to approximate the decay of noncirculatory part and the other to approximate the decay of circulatory part. This two term exponential representation is called model I and is physically consistent because it explicitly models the decay of both noncirculatory and circulatory components of the deficiency function. According to Tobak,[2,3] the initial magnitudes of the noncirculatory and circulatory components are fixed by the theory. Therefore, the task is to model the decay of noncirculatory and circulatory components of the decay function.

Secondly, we use one term exponential function to approximate the complete deficiency function as proposed by Klein[7,8,9] et al which is called Model II in this study. This one term model has a simple structure and is computationally efficient because it can be expressed in the form of an ordinary differential equation as opposed to the convolution integral form of Model I. By comparing the predictions of Model II with those of Model I, it is proposed to evaluate the validity of Model II for predicting the longitudinal oscillatory unsteady aerodynamic coefficients of the F-16XL aircraft model at high angles of attack.

In this paper, parameter identification is used to determine the unknown parameters in the exponential functions of Model I and Model II. Some adjustments have been introduced in Model I to account for uncertainties in the use of the static wind tunnel data that is needed in Model I and to account for deviations from linear piston theory due to nonlinearities at high angles of attack. Only one set of longitudinal oscillatory test data is used for parameter estimation purposes. The other sets of available oscillatory test data are used to validate the parameter estimates. The genetic algorithm is used to minimize the sum of squares of error between the estimated and measured time histories of lift or pitching moment coefficients.

Analysis

Consider the problem of estimation of unsteady lift and pitching moment coefficients. According to the nonlinear indicial function formulation of Tobak,[2,3] the time varying lift and picthing moment coefficients during oscillatory motion in pitch, for first order accuracy in frequency, can be expressed (in Tobak's[2,3] notation) as

$$C_L(t) = C_L(\infty, \alpha(t), q=0) + \left(\frac{q\bar{c}}{V}\right)C_{Lq} - \int_0^t F_2(t-\tau; \alpha(\tau), q=0)\dot{\alpha}(\tau)d\tau \tag{1}$$

$$C_m(t) = C_m(\infty, \alpha(t), q=0) + \left(\frac{q\bar{c}}{V}\right)C_{mq} - \int_0^t F_3(t-\tau; \alpha(\tau), q=0)\dot{\alpha}(\tau)d\tau \tag{2}$$

where $\dot{\alpha} = \frac{d\alpha}{dt}$. In this formulation, the terms like $\dot{q}$ and $\dot{\alpha}q$ are ignored since they will be of second order in frequency. The first term on the right hand side of Eqs (1) is the lift coefficient in steady flow conditions $(t \to \infty)$ and at an angle of attack equal to $\alpha(t)$ and zero pitch rate. Similarly, the first term on the right hand side of Eq.(2) is the pitching moment coefficient under the steady flow conditions. Both these coefficients can be obtained either from static wind tunnel tests or CFD methods. The second terms on the right hand sides of Eq. (1) and (2) contain the familiar damping in pitch derivatives. Usually, the isolated value of C_{Lq} or C_{mq} is not known because the small amplitude forced oscillation wind tunnel tests that are normally used to obtain pitch damping derivatives give the combined value of $(C_{Lq} + C_{L\dot{\alpha}})$ or $(C_{mq} + C_{m\alpha})$. In such tests with $q = \dot{\alpha}$, it is not possible isolate values of C_{Lq} or C_{mq}. In view of this, both C_{Lq} and C_{mq} are assumed unknown. The third terms on the right hand sides of Eq.(1) and (2) with convolution integrals represent the unsteady lift and pitching moment coefficient respectively and contain the unknown deficiency functions F_2 and F_3 which depend on angle of attack and the elapsed time $t - \tau$. According to Tobak,[2,3]

$$F_2(\alpha, t) = C_{L\alpha}(\alpha, t=\infty) - C_{L\alpha}(\alpha, t) \tag{3}$$

$$F_3(\alpha, t) = C_{m\alpha}(\alpha, t=\infty) - C_{m\alpha}(\alpha, t) \tag{4}$$

where $C_{L\alpha}(\alpha,t)$ and $C_{m\alpha}(\alpha,t)$ are the nonlinear, time dependent indicial responses, and $C_{L\alpha}(\alpha, t=\infty)$ and $C_{m\alpha}(\alpha, t=\infty)$ are their steady state values. Therefore, as $(t \to \infty)$, $F_2(\alpha,t) = F_3(\alpha,t) = 0$.

Let

$$y_1(t) = C_L(t) - C_L(\infty, \alpha(t), 0) - \left(\frac{q\bar{c}}{V}\right)C_{Lq} \tag{5}$$

$$y_2(t) = C_m(t) - C_m(\infty, \alpha(t), 0) - \left(\frac{q\bar{c}}{V}\right)C_{mq} \tag{6}$$

so that

$$y_1(t) = -\int_0^t F_2(t-\tau; \alpha(\tau), q=0)\dot{\alpha}(\tau)d\tau \tag{7}$$

$$y_2(t) = -\int_0^t F_3(t-\tau;\alpha(\tau), q=0)\dot{\alpha}(\tau)d\tau \tag{8}$$

We observe that $y_1(t)$ and $y_2(t)$ denote the unsteady components of time varying lift and pitching moment coefficient respectively.

Model I

We assume that deficiency functions $F_2(\alpha,t)$ and $F_3(\alpha,t)$ consist of two term exponential functions as given by

$$F_2(\alpha,t) = C_{L\alpha}(\alpha)e^{-b_1(\alpha)t} - \left(\frac{4}{M}\right)e^{-b_1'(\alpha)t} \tag{9}$$

$$F_3(\alpha,t) = C_{m\alpha}(\alpha)e^{-b_2(\alpha)t} - \left(\frac{4}{M}\right)(\bar{x}_{ref} - \bar{x}_{cg})e^{-b_2'(\alpha)t} \tag{10}$$

Here, $C_{L\alpha}(\alpha)$, $\left(\frac{4}{M}\right)$ and $C_{m\alpha}(\alpha)$, $\left(\frac{4}{M}\right)(\bar{x}_{ref} - \bar{x}_{cg})$ are the terms fixed by the theory,[2,3] $b_1(\alpha)$, $b_1'(\alpha)$, $b_2(\alpha)$ and $b_2'(\alpha)$ are unknown indicial parameters which are assumed to be functions of angle attack, $\bar{x}_{ref}$ is the distance (in terms of mean aerodynamic chord, mac) from the nose to the moment reference point, and $\bar{x}_{cg}$ is the distance (in terms of mac) from the nose to the center of the planform area of the vehicle. The determination of all unknown parameters will be discussed later. The first term on the right hand side of Eqs. (9) or (10) approximates the decay of circulatory component. To evaluate this term, we need to know the "theoretically" correct value of static lift-curve-slope, $C_{L\alpha}(\alpha)$ or the pitching-moment-curve slope $C_{m\alpha}(\alpha)$. However, to account for uncertainties in determining a "theoretically" correct value of $C_{L\alpha}(\alpha)$ or $C_{m\alpha}(\alpha)$ using static wind tunnel tests, we introduce a multiplier $a_1^*(\alpha)$. This unknown parameter is assumed to be a function of angle of attack (with suitable upper and lower bounds). The second term in Eq. (9) approximates the decay of noncirculatory component with an initial value equal to $\left(\frac{4}{M}\right)$. This initial value is based on the linear piston theory and is supposed to be independent of planform shape of the body. The noncirculatory component is assumed to be uniformly distributed over the entire planform surface of the body so that the resultant force acts at the center of the planform area giving a moment arm ($\bar{x}_{ref} - \bar{x}_{cg}$) as used in Eq.(10). However, as noted by Tobak,[10] this result is applicable mainly for flat plate like surfaces at zero angle of attack. Therefore, for application to complex aircraft configurations such as F-16XL operating at high angles of attack, it is necessary to derive a suitable initial value for the nonciculatory component. This, however was not attempted in this study. Instead, to account for the deviations from linear piston theory, an unknown empirical multiplier $a_2^*(\alpha)$ is introduced in the second term. With these assumptions,

$$F_2(\alpha,t) = a_1^*(\alpha)C_{L\alpha}(\alpha)e^{-b_1(\alpha)t} - a_2^*(\alpha)\left(\frac{4}{M}\right)e^{-b_1'(\alpha)t} \tag{11}$$

$$F_3(\alpha,t) = a_1^*(\alpha)C_{m\alpha}(\alpha)e^{-b_2(\alpha)t} - a_2^*(\alpha)\left(\frac{4}{M}\right)(\bar{x}_{ref} - \bar{x}_{cg})e^{-b_2'(\alpha)t} \tag{12}$$

Substituting in Eqs. (7) and (8), we obtain,

$$y_1(t) = -\left[\int_0^t a_1^*(\alpha(\tau))C_{L\alpha}(\alpha(\tau))e^{-b_1(\alpha(\tau))(t-\tau)}\dot{\alpha}(\tau)d\tau - \left(\frac{4}{M}\right)\int_0^t a_2^*(\alpha(\tau))e^{-b_1'(\alpha(\tau))(t-\tau)}\dot{\alpha}(\tau)d\tau\right] \tag{13}$$

$$y_2(t) = -\left[\int_0^t a_1^*(\alpha(\tau))C_{m\alpha}(\alpha(\tau))e^{-b_2(\alpha(\tau))(t-\tau)}\dot{\alpha}(\tau)d\tau - \left(\frac{4(\bar{x}_{ref} - \bar{x}_{cg})}{M}\right)\int_0^t a_2^*(\alpha(\tau))e^{-b_2'(\alpha(\tau))(t-\tau)}\dot{\alpha}(\tau)d\tau\right] \tag{14}$$

Note that when the "theoretically correct" value of $C_L(\alpha)$ and $C_m(\alpha)$ are known, $a_1^*(\alpha) = 1$, and for flat plate like surface operating at low angles of attack, $a_2^*(\alpha) = 1$.

The rotary derivatives are assumed as follows:[9]

$$C_{Lq} = c_1, \qquad \alpha \le 20\ deg \tag{15}$$

$$C_{Lq} = c_1 + c_2\left(\frac{\alpha - 20}{57.3}\right) + c_3\left(\frac{\alpha - 20}{57.3}\right)^2; \quad 20\ deg \le \alpha \le 70\ deg \tag{16}$$

$$C_{mq} = d_1 + d_2\left(\frac{\alpha}{57.3}\right) + d_3\left(\frac{\alpha}{57.3}\right)^2, \quad 0 \le \alpha \le 70\ deg \tag{17}$$

where c_1, c_2, c_3, d_1, d_2, and d_3 are unknown constants.

Model II

We approximate the complete deficiency function by a single exponential term[7,8,9] as follows:

$$F_\alpha(\alpha,t) = a(\alpha)e^{-b_1 t} \tag{18}$$

where $F_\alpha(\alpha,t)$ denotes the deficiency function for the unsteady lift or pitching moment coefficient. With this assumption, the unsteady lift or pitching moment coefficient is given by

$$y(t) = -\int_0^t a(\alpha(\tau))e^{-b_1(t-\tau)}\dot{\alpha}(\tau)d\tau \tag{19}$$

Here, $a(\alpha)$ is an unknown parameter that is a function of angle of attack and b_1 is an unknown constant. Note that the parameters $a(\alpha)$ and b_1 will have different values for lift and pitching moment deficiency functions. The time derivative of $y(t)$ is given by

$$\dot{y}(t) = -\frac{d}{dt}\left[\int_0^t a(\alpha(\tau))e^{-b_1(t-\tau)}\dot{\alpha}(\tau)d\tau\right] \tag{20}$$

$$= -\left[\int_0^t (-b_1)a(\alpha(\tau))e^{-b_1(t-\tau)}\dot{\alpha}(\tau)d\tau + a(\alpha(t))\dot{\alpha}(t)\right]$$

or,

$$\dot{y}(t) = -b_1 y(t) - a(\alpha(t))\dot{\alpha}(t) \tag{21}$$

Thus, Eq. (21) is an equivalent ordinary differential equation for the convolution integral form (Eq.(19)) of the unsteady lift coefficient $y(t)$. Given the initial value $y(0)$, this differential equation is much more convenient to solve compared to the evaluation of the convolution integral in Eq.(19).

If we assume b_1 to be a function of angle of attack, the convolution integral for $y(t)$ which still has the ordinary differential equation of the form given in Eq.(21) is given by the following expression:[9]

$$y(t) = -\int_0^t a(\alpha(\tau))e^{\int_\tau^t -b_1(\alpha(\xi))d\xi}\dot{\alpha}(\tau)d\tau \tag{22}$$

We note that the convolution integral in Eq.(22) is more complex than the convolution integral in Eq.(14) but we need not evaluate it. Instead, we can evaluate $y(t)$ more easily by solving the ordinary differential equation as given in Eq. (21) with b_1 replaced by $b_1(\alpha)$. In this study, the functional forms for these two parameters are not prescribed a priori. Instead, such functional forms are determined during the estimation process.

The model structures for C_{Lq} and C_{mq} as given in Eqs. (15) to (17) for Model I were also used for Model II.

Determination of Unknown Parameters

In this study, one set of experimental data was used to estimate the unknown parameters in Model I and Model II and other sets of available experimental data were used to assess the quality of predictions made using the estimated parameters. The genetic algorithm (GA) was used as a parameter identification algorithm because GA provides a convenient environment to estimate the unknown parameters when their functional dependence (model structure) is not known a priori. A brief description of the GA is given in the following:

The genetic algorithms are non derivative search procedures based on models of processes in natural genetics. At each iteration, the search takes place by sampling a coding parameter set from a random distribution, and applying the parameter iterates to a population of function evaluations. The performance of the population elements is used to modify the probability distribution for sampling in the next iteration. Algorithms of this class are known to be very robust and not particularly vulnerable to termination at local minima. More information on GA may be obtained in Refs 11 and 12.

The procedure used to determine the unknown parameters is as follows: First, subdivide the angle of attack range of interest into a convenient number of segments or nodes, say n, and use a linear interpolation to determine the parameter value in between the nodal points. In GA environment, the sub intervals need not be equal. The next step is to specify suitable values for upper and lower limits at each nodal point for $a_1^*(\alpha)$, $a_2^*(\alpha)$, $b_1(\alpha)$, $b_2(\alpha)$, $b_1'(\alpha)$ and $b_2'(\alpha)$, for Model I and $a(\alpha)$ and $b_1(\alpha)$ for Model II and upper and lower limits for other unknown parameters such as c_1, c_2, c_3, d_1, d_2, d_3 for both models. The GA generates a population whose elements are assigned randomly generated values for each of the unknown parameters that lie within the specified upper and lower limits. Thus, having assigned values to all unknown parameters in Model I or Model II, the time histories of $C_L(t)$ or $C_m(t)$ are generated for each population element. Let the estimated time history be denoted by $\hat{C}_L(t)$ or $\hat{C}_m(t)$. Then define,

$$J = \sum_{i=1}^{N}\left[C_a(t_i) - \hat{C}_a(t_i)\right]^2 \tag{23}$$

where, $C_a = C_L$ or C_m, and, $t_i = 1,...N$ correspond to the values of time at which the oscillatory data on $C_L(t)$ or $C_m(t)$ is available. The problem is to estimate the nodal values of the unknown parameters in Model I or Model II so that the sum of the squares of the error J is minimized. The GA was used as a least square minimization algorithm with J as the cost function.

In the longitudinal oscillatory tests of the F-16XL aircraft model, the angle of attack varied from 0 to a little over 70 deg. To cover this range of angle of attack, we use $n = 37$ ($\Delta\alpha = 2$ deg) for Model I and $n = 16$ ($\Delta\alpha = 5$ deg) for Model II. More number of nodes were used for Model I for better definition of the parameter variations. Further, we have to specify certain GA specific parameters such as population size, probability of crossover and probability of mutation. In this study, we use population size 30, probability of crossover 0.95 and probability of mutation 0.01. If the GA reduces cost as the iterations progress, then the GA was allowed to continue. Otherwise, the GA was stopped, the values of lower and upper limits were redefined and the GA was started again. Since the GA has no defined convergence criterion to end the iteration process, the GA was stopped when it became evident that further continuation would not result in any improvement in J.

The convolution integral was evaluated using the trapezoidal rule with a step size of 0.0001 s. The integration of the ordinary differential equation was done using a forth order Runge-Kutta method with a fixed step size of 0.02 s. All the computations were done in MATLAB.

Results and Discussion:

The above formulation for evaluating unsteady lift and moment coefficients based on indicial response method was applied to the analysis of longitudinal oscillatory data of the 0.1 scale model of the F-16XL aircraft obtained in the Langley 12-Foot Low Speed Wind Tunnel. A three-view sketch of the 0.1 scale model of the F-16-XL aircraft is presented in Fig. 1. Using this sketch, the center of the planform area $\bar{x}_{cg}$ was found to be approximately at 1.0090 m. The moment reference point is located at a distance of 0.9932 m from the nose. The mean aerodynamic chord (mac) of the 0.1 scale model of the F-16XL aircraft is equal to 0.753 m. Additional information on the test model and test facility may be found in Refs. 9 and 11.

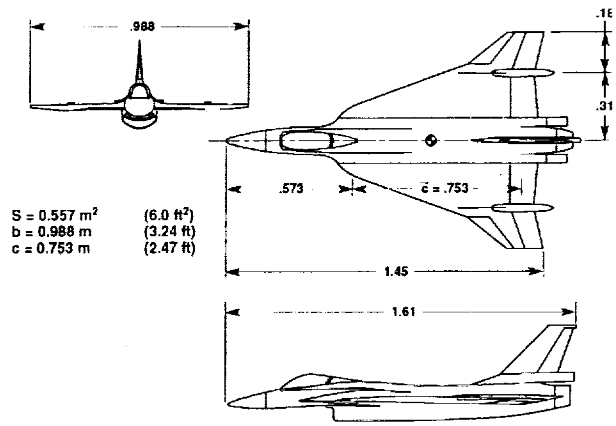

Figure 1. Three-View sketch of the F-16XL aircraft model.

In the oscillatory tests, the mean angle of attack was close to 35 deg and the oscillation amplitude was also about 35 deg so that the angle of attack varied from 0 to a little more than 70 deg during one oscillation. The test section speed was around 17.5 m/sec that corresponds to a Mach number of about 0.05. The period of oscillation T_p varied from 12 s to 1.33 s so that the corresponding reduced frequency k was in the range 0.011 to 0.101. For the purpose of parameter estimation, the oscillatory wind tunnel data corresponding to $T_p = 2.38$ s ($k = 0.056$) was used. The rest of the available oscillatory test data were used for validation of parameter estimates.

The variation of the lift and pitching moment coefficients from static wind tunnel tests on the F-16XL aircraft model are presented in Fig. 2. Using numerical curve fit to these data points, the lift-curve-slope $C_{L\alpha}$ and slope of the pitching-moment-coefficient $C_{m\alpha}$ were

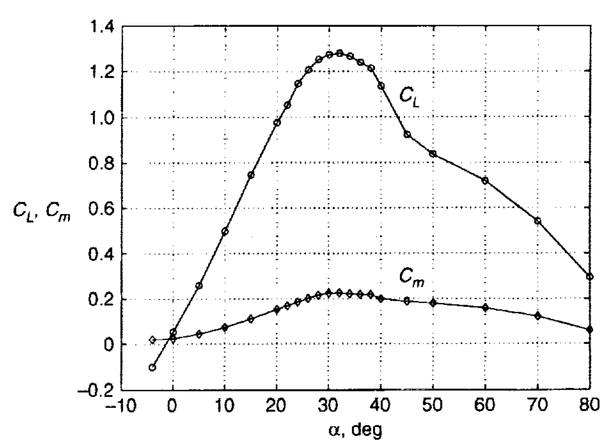

Figure 2. Static wind tunnel data of the F-16XL aircraft model.

deduced as shown in Fig. 3. However, the values of $C_{L\alpha}$ and $C_{m\alpha}$ deduced by two different methods of curve fitting such as cubic splines and cubic polynomials differed by as much ±10% for $C_{L\alpha}$ and about ±15% for $C_{m\alpha}$, especially at high angles of attack. In view of this, we assume $0.9 \le a_1^*(\alpha) \le 1.1$ in Eq. (11) and $0.85 \le a_1^*(\alpha) \le 1.15$ in Eq. (12). For better definition of these error bounds, more data points taken at much smaller intervals are needed. However, such data were not available for this study.

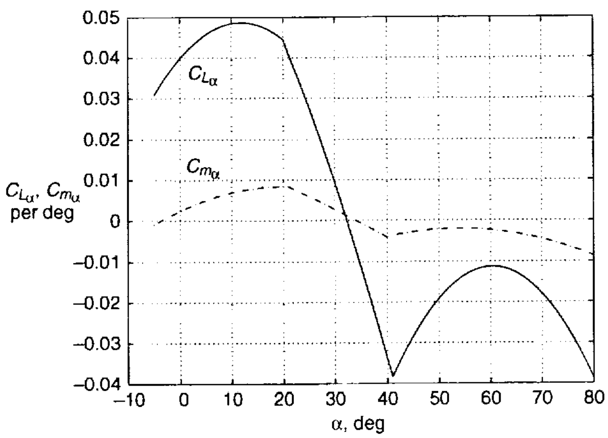

Figure 3. Lift-Curve-Slope and slope of pitching-moment-coefficient of the F-16XL aircraft model.

The unknown coefficients in Eq. (15) – (17) for C_{Lq} and C_{mq} were estimated as follows: Model I: $c_1 = -0.3844$, $c_2 = 2.3776$, $c_3 = -4.5026$, $d_1 = -0.3678$, $d_2 = -0.1616$, and $d_3 = 2.0361$; For Model II: $c_1 = -0.4240$, $c_2 = 3.3127$, $c_3 = -3.3840$, $d_1 = -1.2450$, $d_2 = -0.3806$, and $d_3 = 1.5557$.

The time history of the angle of attack for $T_p = 2.38$ s ($k = 0.056$) is shown in Fig. 4. The estimated and measured time histories of lift coefficient are shown in Fig. 5. A cross plot of $C_L(t)$ with $\alpha(t)$ is shown in Fig. 6. It is observed the estimated time histories match the measured history fairly well, with Model II giving better fit to the oscillatory test data. A cross plot of the corresponding time varying lift coefficient with time varying angle of attack is shown in Fig. 6. It was observed that the contribution of rotary derivative C_{Lq} is very small and negligible. On the other hand, the contribution of unsteady term $y(t)$ happens to be significant. It is quite possible that if more accurate values of $C_{L\alpha}$ were available, Model I may have given better result. The estimated values of the indicial parameters Model I and Model II are shown in Fig. 7(a) and 7(b). For comparison, the lift-curve-slope is included in Fig. 7(b). It is observed that the estimated values of $a_2^*(\alpha)$

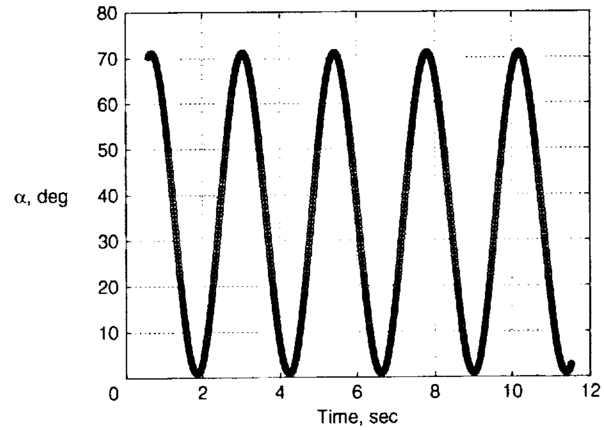

Figure 4. Time history of angle of attack, T_p = 2.38 s.

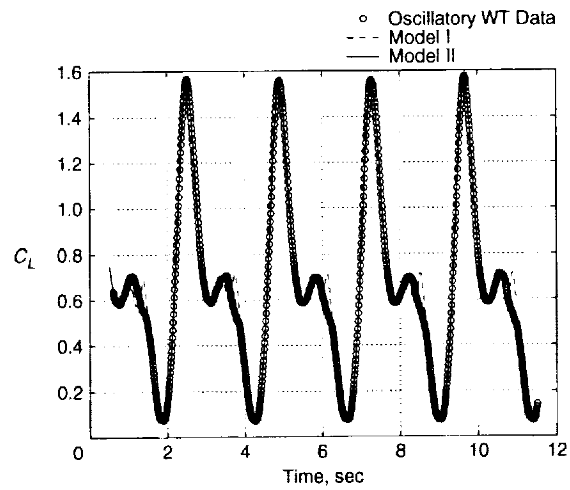

Figure 5. Time history of measured and estimated lift coefficient, T_p = 2.38 s.

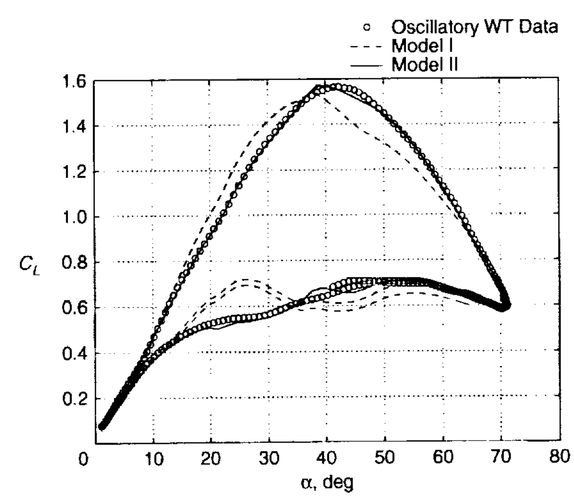

Figure 6. Comparison of measured and estimated lift coefficient, T_p = 2.38 s.

are in the range 0.38 to 0.46 suggesting that the initial peak value of the noncirculatory loading is only 0.38 to 0.46 of that based on linear piston theory.

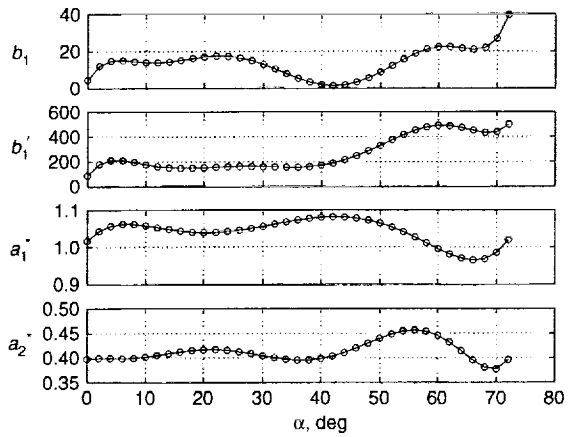

Figure 7(a) Estimated indicial function parameters in lift coefficient, Model I.

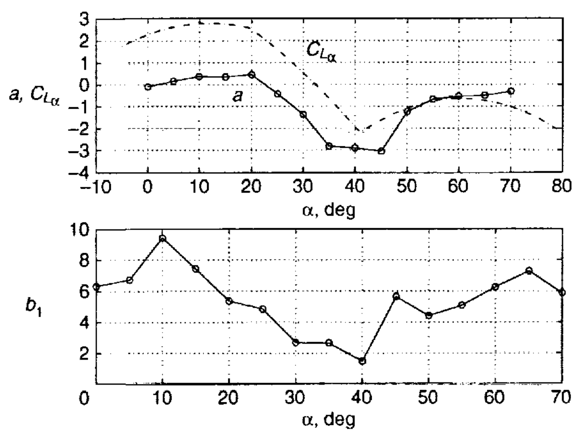

Figure 7(b). Estimated indicial function parameters in lift coefficient, Model II.

Using these parameter estimates as shown in Fig. 7, variations of the noncircular and circular components as well total loading for Model I (Eq. (11)), and total loading for Model II (Eq.(18)) were calculated for $0 \leq \alpha \leq 70$ deg. Some sample results of these variations are shown in Figs. 8 to 10. As expected, the noncirculatory component decays rapidly and the circulatory component decays gradually. The area under each of these curves is a measure of their contribution to the unsteady lift coefficient. The calculated areas are shown in Fig. 11. To first order in frequency, the area under the deficiency function is related to the parameter $C_{L\dot{\alpha}}$ or $C_{m\alpha}$ as the case may be. Thus, the process of estimating C_{Lq} and area under the deficiency function is equivalent to estimating the combined parameter

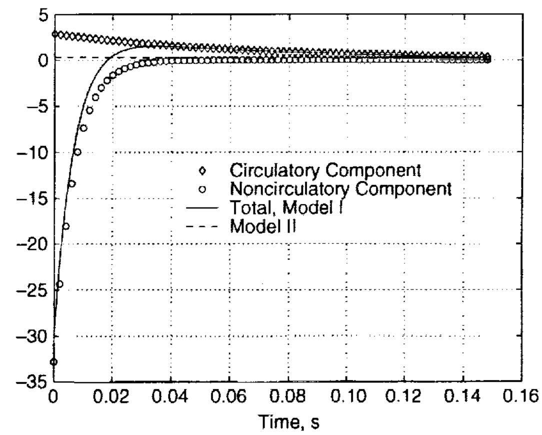

Figure 8. Estimated deficiency function components in lift coefficient, T_p = 2.38 s, α = 15 deg

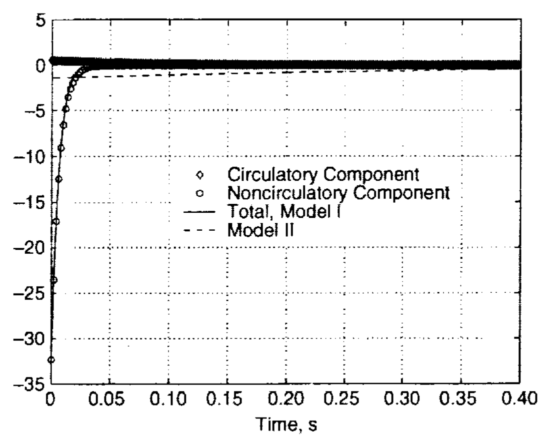

Figure 9. Estimated deficiency function components in lift coefficient, T_p = 2.38 s, α = 30 deg.

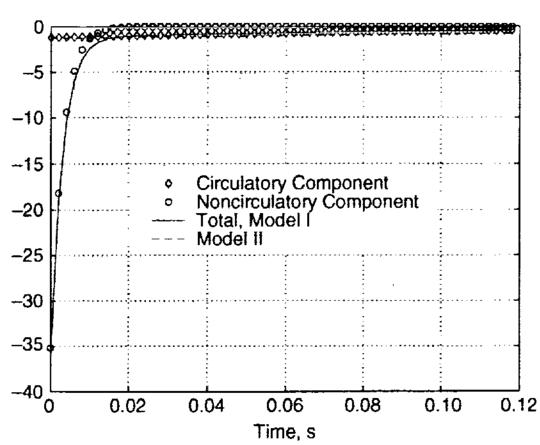

Figure 10. Estimated deficiency function components in lift coefficient, T_p = 2.38 s, α = 50 deg.

$C_{Lq} + C_{L\dot{\alpha}}$ or $C_{mq} + C_{m\dot{\alpha}}$ in small amplitude oscillatory wind tunnel tests. From Fig. 11 we observe that the contribution of noncircular component is always negative and gradually drops with increase in angle of attack. The sign of the circulatory component follows the sign of $C_L(\alpha)$. It is interesting to note that Model II follows closely the variation of the total loading of Model I which is the sum of the noncirculatory and circulatory components. Thus, the Model II with one term exponential function satisfactorily approximates the combined variation of the noncirculatory and circulatory components of Model I. Further, Model II has simple structure and is computationally efficient because the convolution integral in the deficiency function can be represented in the form of an ordinary differential equation that can be solved more easily compared to the evaluation of convolution integrals in Model I. Further, the availability of ordinary differential equation greatly facilitates the inclusion of unsteady aerodynamic terms in flight simulation computer codes.

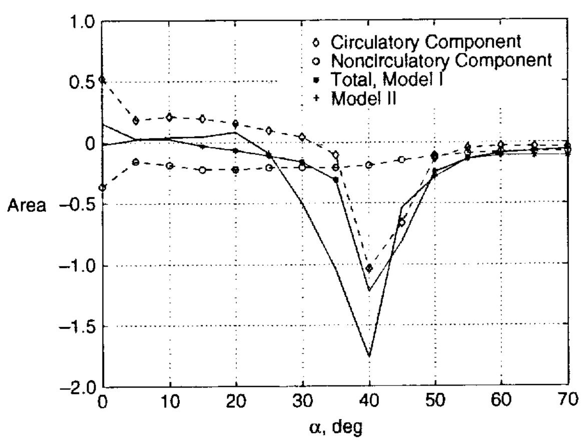

Figure 11. Variation of area under estimated deficiency function components, T_p = 2.38 s.

The estimated time histories of the pitching moment coefficient T_p = 2.38s (k = 0.0562) are shown in Fig 12. The corresponding cross plots of the time varying pitching moment coefficient with time varying angle of attack are shown in Figure 13. The estimated values of the indicial parameters for Model I and Model II are shown in Figures 14(a) and 14(b). We observe that the quality of the fit between the oscillatory pitching moment coefficient data and the estimates is not as good as that obtained for the lift coefficient. In this case also, Model II gives a better fit to the oscillatory data.

Using these parameter estimates, predictions were made for T_p = 1.33s, 1.72s, 4.0s and 12.0s for which oscillatory data was obtained. These results are shown

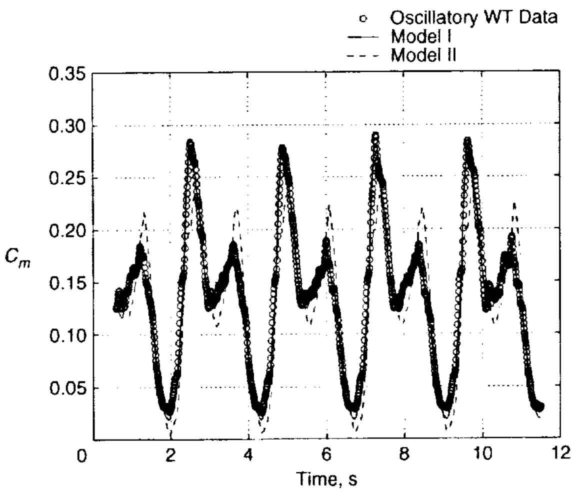

Figure 12. Time history of measured and estimated pitching moment coefficient, T_p = 2.38 s.

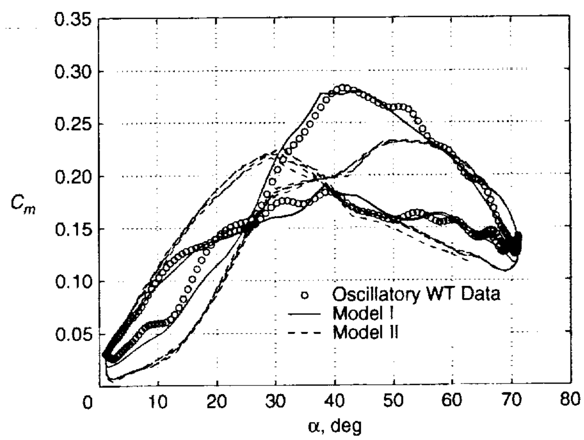

Figure 13. Comparison of measured and estimated pitching moment coefficient, T_p = 2.38 s.

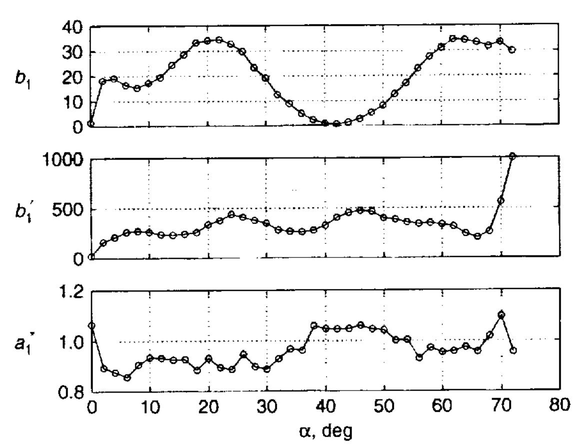

Figure 14(a). Estimated indicial function parameters in pitching moment coefficient, Model I.

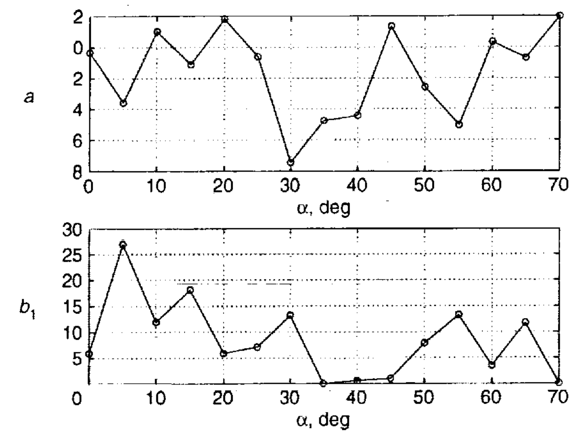

Figure 14(b). Estimated indicial function parameters in pitching moment coefficient, Model II.

in Figs. 15 to 22. It is observed that the predicted time histories of lift and moment coefficients agree reasonably well with measured oscillatory wind tunnel data, with better agreement indicated for the lift coefficient. In all these cases, Model II gives better predictions compared to Model I. However, differences do exist, particularly in the pitching moment coefficient. The quality of predictions of Model I might have been better if more static wind tunnel data at smaller intervals were available for better definition of $C_{L\alpha}$, $C_{m\alpha}$ and the upper and lower bounds for the uncertainty parameter $a_1^*(\alpha)$.

The indicial response method of Tobak[2,3] is supposed to be applicable when the lift-curve-slope is positive and the steady state of the indicial response is time invariant. However, in the present study, the lift-curve-slope goes negative beyond 30 deg angle of attack. Further, it is quite possible that the flow field over the F-16XL aircraft model at high angles of attack is characterized by extensive flow separations, vortex interactions and possible vortex bursts over wing and tail surfaces. Under such flow conditions, the assumption of time invariant steady states may not be strictly valid and the analysis should include the possible existence of time-dependent equilibrium states as suggested by Tobak et al,[14] which was not attempted in this study.

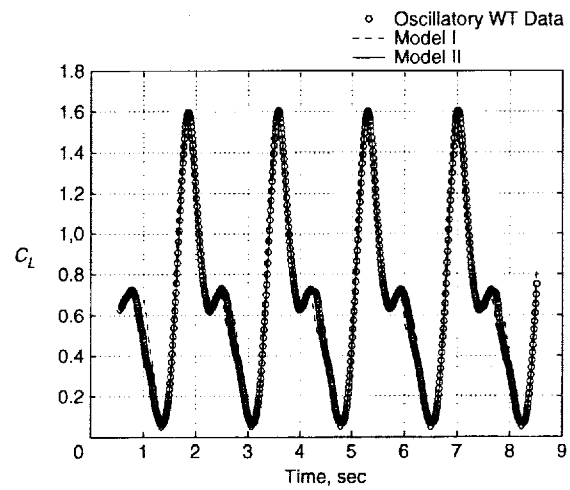

Figure 16. Time history of measured and predicted lift coefficient, T_p = 1.72 s.

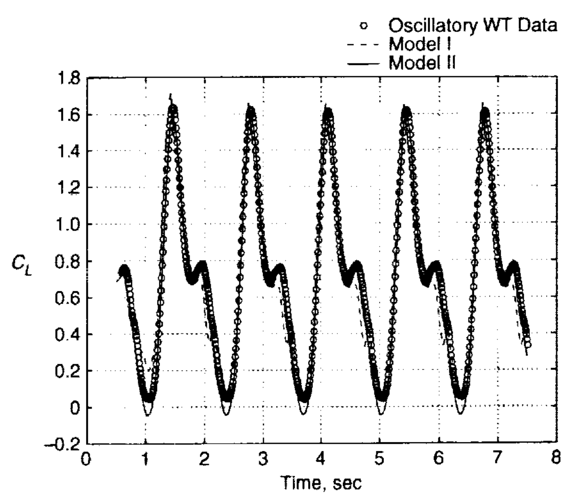

Figure 15. Time history of measured and predicted lift coefficient, T_p = 1.33 s.

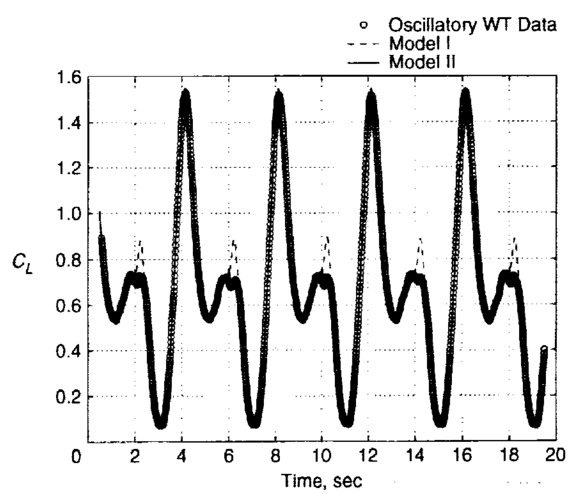

Figure 17. Time history of measured and predicted lift coefficient, T_p = 4.0 s.

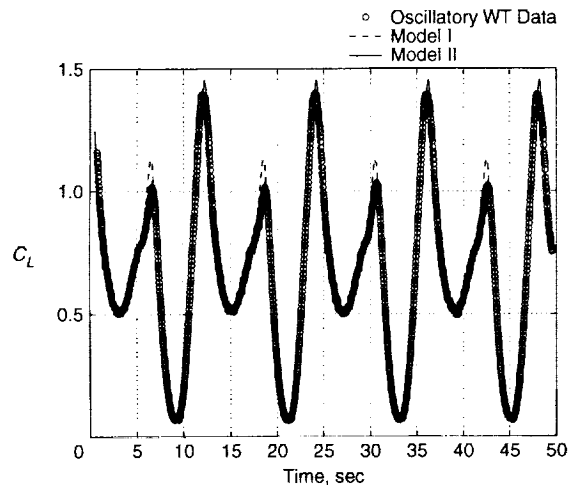

Figure 18. Time history of measured and predicted lift coefficient, T_p = 12.0 s.

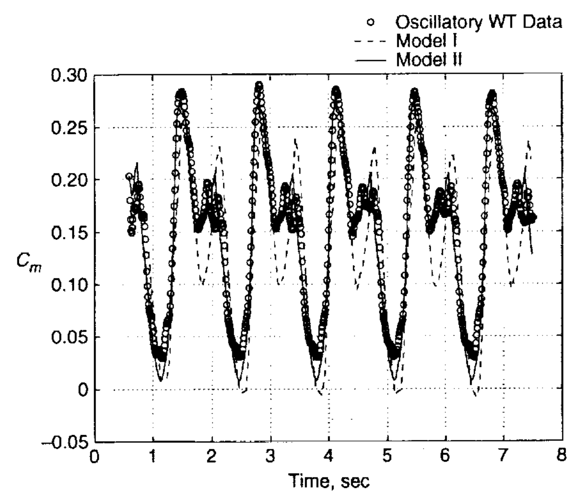

Figure 19. Time history of measured and predicted pitching moment coefficient, T_p = 1.33 s.

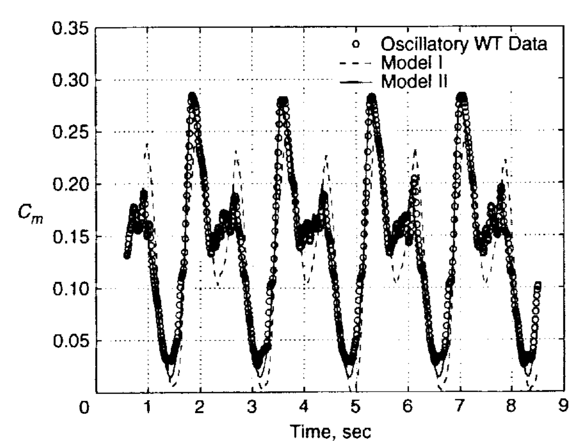

Figure 20. Time history of measured and predicted pitching moment coefficient, T_p = 1.72 s.

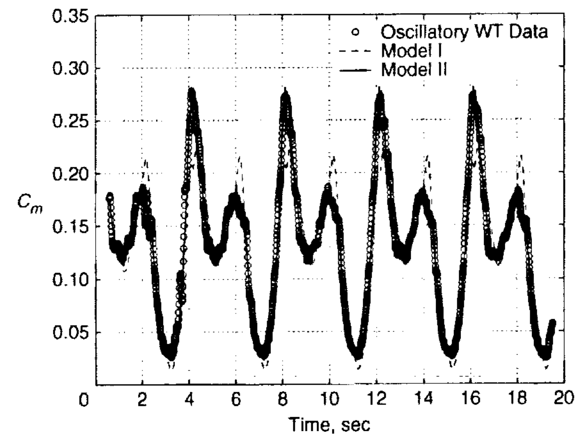

Figure 21. Time history of measured and predicted pitching moment coefficient, T_p = 4.0 s.

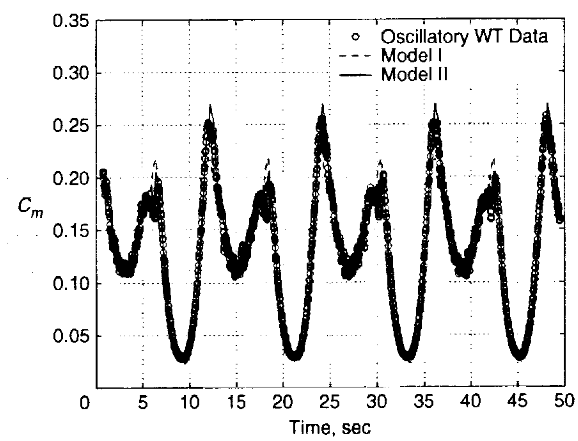

Figure 22. Time history of measured and predicted pitching moment coefficient, T_p = 12.0 s.

Concluding Remarks

The nonlinear indicial response method provides a fundamental approach to the problem of modeling unsteady aerodynamic effects at high angles of attack. In this study, two models have been used to approximate the nonlinear indicial response. Model I is based on two term exponential functions, with one term to represent the noncirculatory component and the other term to represent the decay of circulatory component. Some adjustments have been introduced to Model I to account for uncertainties in the static wind tunnel data that is needed in Model I and for deviations from linear piston theory due to nonlinearities at high angles of attack. Model II is based on one exponential function to approximate the combined variation of noncirculatory and circulatory components. The unknown indicial parameters in each of the two models were estimated using a genetic algorithm. The genetic algorithm was used as a least-square minimizing technique.

It is shown that Model II is capable of satisfactorily approximating the combined variation of the noncirculatory and circulatory components of Model I. Further, Model II is simple in structure and computationally efficient because it can be expressed in the form of an ordinary differential equation as opposed to the convolution integrals in Model I. This fact greatly facilitates the inclusion of unsteady aerodynamic terms in the flight simulation computer codes.

Acknowledgment

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