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Fluid Physics under a stochastic acceleration field

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Contents

1 Introduction

The research summarized in this report has involved a combined theoretical and computational study of fluid flow that results from the random acceleration environment present onboard space orbiters (also known as g-jitter). We have focused on a statistical description of the observed g-jitter, on the flows that such an acceleration field can induce in a number of experimental configurations of interest, and on extending previously developed methodology to boundary layer flows.

Significant levels of residual accelerations have been detected during space missions in which microgravity experiments have been conducted [1, 2, 3]. Direct measurement of these residual accelerations has shown that they have a wide frequency spectrum, ranging approximately from $10^{-4}Hz$ to 10^2Hz. Amplitudes range from $10^{-6}g_E$ at the lowest end of the frequency spectrum, and increase roughly linearly for high frequencies, reaching values of $10^{-4}g_E - 10^{-3}g_E$ at frequencies of around $10Hz$ (g_E is the intensity of the gravitational field on the Earth's surface). A recent comprehensive survey of the effective acceleration environment onboard Space Shuttle and the Russian space station Mir is given in ref. [3]. Despite the efforts of a number of researchers over the last decade, there still remain areas of uncertainty about the potential effect of such a residual acceleration field on typical microgravity experiments, especially in quantitative terms. An improved understanding of the response of a fluid system to such disturbances would enable improved experiment design to minimize or compensate for their influence. Thus the generic goal of our research has been to gain a sufficiently broad understanding of the effect of g-jitter on fluid flow and on transport in fluid phases so as to be able to define tolerable levels of residual accelerations. Compliance with these levels would ensure that the processes under study are not appreciably distorted by the environment in which a given experiment is conducted [1, 4, 5]. In addition, it would also be useful to have error estimates of quantities measured in the presence of residual accelerations, including whenever possible some methodology for extrapolation to ideal zero gravity.

We have adopted a statistical description of the residual acceleration field onboard spacecraft, and modeled the acceleration time series as a stochastic process in time [6, 7]. The main premise of our approach is that a statistical description is necessary in those cases in which the characteristic time scales of the physical process under investigation are long compared with the correlation time of g-jitter, τ (the acceleration amplitudes at two different times are statistically independent if separated by an interval larger than τ). Progress has been achieved through the consideration of a specific stochastic model according to which each Cartesian component of the residual acceleration field $\vec{g}(t)$ is modeled as a narrow band noise. This noise is a Gaussian process characterized by three independent parameters: its intensity $< g^2 >$, a dominant angular frequency Ω, and a characteristic spectral width τ^{-1}. Each realization of narrow band noise can be viewed as a temporal sequence of periodic functions of angular frequency Ω with amplitude and phase that remain constant only for a finite amount of time (τ on average). At random intervals, new values of the amplitude and phase are drawn from prescribed distributions. This model is based on the following mechanism underlying the residual acceleration field: one particular natural frequency of vibration of the spacecraft structure (Ω) is excited by some mechanical disturbance inside the spacecraft, the excitation being of random amplitude and taking place at a sequence of

unknown (and essentially random) instants of time. A further advantage of this model is that it provides a convenient way of interpolating between monochromatic noise (akin to studies involving a deterministic and periodic gravitational field), and white noise (in which no frequency component is preferred). In the limit $\tau \to 0$ with $D = <g^2> \tau$ finite, narrow band noise reduces to white noise of intensity D; whereas, for $\tau \to \infty$ with $<g^2>$ finite, monochromatic noise is recovered. This model has been shown to describe reasonably well many of the features of g-jitter time series measured onboard Space Shuttle. In ref. [7] we analyzed actual g-jitter data collected during the SL-J mission, and studied in detail a time series of roughly six hours. A scaling analysis revealed the existence of both deterministic and stochastic components in the time series. The deterministic contribution appeared at a frequency of 17 Hz, with an amplitude $\sqrt{<g^2>} = 3.56 \times 10^{-4} g_E$. Stochastic components included two well defined spectral features with a finite correlation time; one at 22 Hz with $\sqrt{<g^2>} = 3.06 \times 10^{-4} g_E$ and $\tau = 1.09$ s, and one at 44 Hz with $\sqrt{<g^2>} = 5.20 \times 10^{-4} g_E$ and $\tau = 0.91$ s. White noise background is also present in the series with an intensity $D = 8.61 \times 10^{-4} \text{cm}^2/\text{s}^3$.

We next summarize our salient results during the period of the contract. Further details can be found in the publications listed in Appendix B, which are also attached.

In Section 2, we analyze the consequences of a random acceleration field on cavity flows. Here flow is assumed to be baroclinically induced in the bulk. The average vorticity in the cavity is argued to obey a random walk in time of zero average but with variance linearly increasing with time. A statistical steady state is reached, the amplitude of which is given by a stochastic Rayleigh number which we compute. Section 3 addresses the motion of buoyant particles suspended in an incompressible fluid which is being randomly vibrated. We compute the average velocity and displacement of the suspended particles. We have also examined the case of diffusion limited coarsening of a solid-liquid mixture in connection with the CSLM experiment (Coarsening of Solid-Liquid Mixtures). After the experiment was flown and the results analyzed, no effect was found in the particle distribution that could be attributed to g-jitter effects, in agreement with our predictions. Section 4 summarizes our research on the effects of stochastic modulation on oscillatory instabilities. We show that the onset of the instability is shifted due to the modulation, and that the primary bifurcation is to standing waves instead of traveling waves as is the case in the absence of forcing. This work has led to a subsequent generalization of the center manifold reduction scheme of classical bifurcation theory to a stochastic setting, which we also summarize. Finally, Section 5 summarizes more recent research on streaming flows induced by random vibration of solid boundaries. We have generalized the classical boundary layer analysis to the stochastic case, and shown that the flow produced by random vibration can penetrate anomalously large distances into the bulk. In Section 6 we use the results of section 5 concerning streaming flows near a periodically modulated boundary to study the morphological stability of a solid-melt boundary growing into a far field flow of oscillatory nature.

2 Cavity flow induced by a random acceleration field

Analytic solutions to the flow field in a laterally heated cavity have been found in the limit of large aspect ratio. We have been able to isolate a few important characteristics of cavity flow that result entirely from the stochastic nature of the acceleration field, and that would not have been obtained under a strictly periodic gravity modulation. Although the imposed acceleration field, and hence the vorticity, average to zero, the vorticity itself can be described by a random walk in time. Hence the mean squared vorticity in the center of the cavity grows linearly in time with a diffusion coefficient given by,

$$D_{eff} \propto \frac{\Delta \rho^2 \langle g^2 \rangle \tau}{\gamma^2 \left(1 + (\Omega \tau)^2\right)}.$$

where $\Delta \rho$ is the imposed density difference across the cavity, and γ is a damping coefficient arising from viscous friction (of the order of ν/L^2), ν is the kinematic viscosity of the fluid, and L the side of the cavity. This is in contrast with the case of a deterministic and time periodic acceleration in which the magnitude of the velocity remains bounded, even in the absence of viscous dissipation.

At long times, viscous friction causes the velocity field to saturate to a finite value. The flow reaches a statistical steady state of zero average velocity. The mean squared velocity is nonzero but not proportional to the Rayleigh number squared, Ra^2. Instead, the scale of the (statistical) steady state flow is characterized by a stochastic Rayleigh number R given by,

$$R = \frac{\Delta \rho L^2 \sqrt{2 <g^2> \tau}}{\kappa \sqrt{\nu}},$$

where κ is the thermal diffusivity of the fluid. The value of R can be quite different from Ra for a similar fluid, similar parameters of the cavity, and acceleration amplitude.

3 Motion of suspended particles and coarsening of solid-liquid mixtures

We have also addressed the motion of a solid particle suspended in an incompressible fluid of different density, when the fluid is subjected to a random acceleration field. This type of motion has been termed inertial random walk because of the similarity with Brownian motion. The equation of motion of the particle is quite simple, but it illustrates some generic features of the interplay between the three time scales in this problem: the inverse characteristic frequency of the driving acceleration, a viscous decay time, and the correlation time of the noise. This relationship is best illustrated in the expression for the long time value of the mean squared velocity of the particle,

$$\left\langle v^2 \right\rangle_\infty = \frac{\Delta \rho^2 \langle g^2 \rangle \left(\gamma + \frac{1}{\tau}\right)}{\gamma \left[\left(\gamma + \frac{1}{\tau}\right)^2 + \Omega^2\right]},$$

where $\Delta \rho = (\rho_p - \rho_f)/\rho_p$, ρ_p and ρ_f are the densities of the particle and of the fluid respectively, $\gamma = 9\pi\eta/2\rho_p R^2$, with η the shear viscosity of the fluid, and R the radius of the

particle. Two limiting behaviors arise depending on whether the viscous relaxation time γ^{-1} is larger or smaller than the correlation tome of the noise. If $\tau \gg 1/\gamma$ one recovers the deterministic result according to which the velocity is inversely proportional to γ at low frequencies, and decreases as $1/\Omega$ at high frequencies. In fact, such a dependence has been widely used in the past to define tolerable levels of g-jitter as a function of frequency, and is also one of the bases for the design of the International Space Station in terms of the specifications for tolerable accelerations. On the other hand, for small τ the amplitude of the fluctuations becomes independent of frequency and proportional to $1/\gamma$ only. The significance of this result, however, is that it is more generally valid for situations in which the response of the system can be described by a single mode that responds additively to the forcing. As a reference, we mention that realistic values of the correlation time are $\tau \approx 1\ s$, whereas for example $\gamma \approx 200\ s^{-1}$ for typical colloidal particles, but can be much smaller than one for cavity flows. Finally, we note that this expression could be used to independently determine the parameters that define g-jitter. Knowledge of this sort could conceivably lead to the construction of an instrument that would complement the data set currently provided by accelerometers.

Progress has also been achieved in connection with coarsening experiments in solid-liquid mixtures. A residual acceleration field can produce a number of deleterious effects on otherwise purely diffusive controlled coarsening. We have focused on two such effects: effective inter-particle interactions induced by g-jitter, and corrections to mass transport in the liquid phase due to convective flow induced by the jitter. In the Stokes or over-damped limit, we have found that g-jitter induces an effective attraction between pairs of suspended particles at large separations, and repulsion at short separations. The same phenomenology is expected to occur between a suspended particle and a rigid wall. This effect originates in the dependence of the hydrodynamic mobility tensor on inter-particle separation, and in the fact that the same effective gravitational field is simultaneously acting on both particles. We have also found that in the same limit of Stokes flow, g-jitter induced flows act to renormalize the solute diffusivity. Both effects have been found to be negligible under the conditions appropriate for the experiment Coarsening of Solid Liquid Mixtures (CSLM) that is being carried out in microgravity.

4   Stochastic modulation of oscillatory instabilities

We have focused on the effect of g-jitter on an oscillatory instability (as it appears, for example, in double diffusive convection at negative separation ratios) [8]. In general, at a Hopf bifurcation in a periodically (and deterministically) modulated system, the trivial state loses stability to either traveling or standing waves above onset depending on the amplitude of the modulation b. For sufficiently small modulation amplitudes, traveling waves appear at a fixed value of the control parameter, a_R, independent of b. The threshold for standing waves, however, is a decreasing function of b. We have analyzed how the existence of a stochastic component in both a_R and b affects the nature of the bifurcation, as well as the stability boundaries of the trivial state. Stability boundaries have been computed by either solving the stationary Fokker-Planck equation on the center manifold of the underlying deterministic system whenever possible, or by direct numerical solution otherwise. If the modulation amplitude has a stochastic component, the primary bifurcation is always

to standing waves at a value of the control parameter that depends on the intensity of the fluctuations. More precisely, and to contrast our results with the case of a deterministic periodic forcing, the onset of instability in the standing wave regime is shifted from its deterministic location, and the region of primary bifurcation to traveling waves disappears, yielding instead standing waves at negative values of the control parameter. This study has prompted a reanalysis of the procedure of adiabatic elimination of fast relaxing variables near a bifurcation point when some of the parameters of the system are stochastically modulated. Again, approximate stationary solutions of the Fokker-Planck equation have been obtained near threshold for pitchfork and transcritical bifurcations. We have found that stochastic resonance between fast variables and the random modulation may shift the effective bifurcation point by an amount proportional to the intensity of the fluctuations. We have also shown that fluctuations of the fast variables above threshold are not always Gaussian and centered around the (deterministic) center manifold as was previously believed. Numerical solutions obtained for a few illustrative examples lend support to these conclusions.

The essential aspects of the adiabatic reduction procedure in the stochastic case can be illustrated in the simple case of a second order system. Let A be the amplitude of a bifurcating mode, and B the amplitude of a second mode that is itself linearly stable near onset. A reduced control parameter α is defined such that the trivial state $A = B = 0$ is stable if $\alpha \leq 0$, and unstable otherwise. Fluctuations in α are included through a stochastic process $\xi(t)$, which we assume Gaussian, white and of small intensity κ. The evolution of the system is now stochastic and is described by the joint probability density $\mathcal{P}(A, B; t)$ at time t. The reduction procedure starts by decomposing the joint density as

$$\mathcal{P}(A, B; t) = p(B|A; t)P(A; t), \tag{1}$$

where $p(B|A; t)$ is the conditional probability density. Close to threshold, the stochastic processes A and B are small (their intensity scales with some power of κ) in such a way that characteristic values of $B/A \sim \kappa^a \ll 1$, $a > 0$. This assumption also implies that the two processes evolve over different characteristic temporal scales, fact that is reminiscent of the separation of time scales present in the deterministic limit. As a consequence, the probability densities $P(A; t)$ and $p(B|A; t)$ can be separately obtained at different orders in κ. The stationary density $P(A)$ is then used to locate the effective threshold point in the stochastic case. Below threshold, $P(A)$ is a delta function at $A = 0$, whereas above threshold there exists another normalizable solution that has some non vanishing moments. We have derived approximate expressions for the stationary probability densities $p(B|A)$ and $P(A)$ valid near threshold for the pitchfork and transcritical bifurcations. In both cases, the marginal density $P(A)$ has to satisfy a normalizability condition that is used to determine the location of onset α_c. In those cases in which $\alpha_c \neq 0$, stochastic resonance between the fast variable B and the stochastic process $\xi(t)$ is responsible for the shift away from the deterministic threshold. This result generalizes earlier analyses of the normal form equation corresponding to a pitchfork bifurcation with a fluctuating control parameter [9, 10], in which coupling to fast variables was not considered. In agreement with our results below, the absence of such coupling leads to $\alpha_c = 0$ for any intensity of the fluctuating control parameter.

5 Boundary layer flows induced by random vibration

We have examined the formation of viscous layers in a fluid which is in contact with a solid boundary that is vibrated randomly. Consider a solid boundary being displaced with a velocity $u_0(t)$ that is assumed prescribed, and modeled as a narrow band stochastic process. First, we have considered an infinite planar boundary that is being vibrated along its own plane to generalize the classical problem studied by Stokes [11]. In the monochromatic limit, the variance of the velocity field decays exponentially away from the wall, with a characteristic decay length given by the Stokes layer thickness $\delta_s = (2\nu/\Omega)^{1/2}$, where ν is the kinematic viscosity of the fluid, and Ω is the angular frequency of vibration of the boundary. Since the equations governing the flow are linear, we were able to obtain an analytic solution describing transient layer formation in the stochastic case, but only in the neighborhood of the white and monochromatic noise limits. We have shown that for any finite correlation time the stationary variance of the tangential velocity asymptotically decays as the inverse squared distance from the wall, in contrast with the exponential decay in the deterministic case. This asymptotic behavior originates from the low frequency range of the power spectrum of the boundary velocity. The crossover from power law to exponential decay is explicitly computed by introducing a low frequency cut-off in the power spectrum. When the solid boundary is planar, the flow field averages to zero (the average velocity of the boundary has been taken to be zero in all cases investigated), but its variance decays algebraically with distance away from the wall. This dependence follows from a non vanishing power spectrum of the boundary velocity at zero frequency. Introducing a low frequency cut-off in the power spectrum ω_c leads back to the classical exponential decay, with a rate that is determined by the cut-off frequency, Eq. (4). The amplitude of the decaying variance depends explicitly on the dimensionless correlation time of the boundary velocity, $\beta = \Omega\tau$. where Ω is the dominant angular frequency of the power spectrum The stationary value of the variance of the velocity is given by,

$$\left\langle u^2(z,\beta,\omega_c)\right\rangle = \frac{2\beta e^{-z(2\omega_c)^{1/2}}}{\pi(1+\beta^2)}\left(\frac{1}{z^2} + \frac{(2\omega_c)^{1/2}}{z} + \dots\right), \qquad (2)$$

where terms that are of higher order than terms retained under the assumption that both $1/z$ and $(2\omega_c)^{1/2}$ are small but independent have been neglected. For $z \gg 1$, but $z(2\omega_c)^{1/2} \ll 1$ the dominant term in (2) is

$$\left\langle u^2(z,\beta,\omega_c)\right\rangle \sim \frac{2\beta}{\pi(1+\beta^2)}\frac{1}{z^2}, \quad z(2\omega_c)^{1/2} \ll 1. \qquad (3)$$

On the other hand, if $z(2\omega_c)^{1/2} \geq 1$, the leading order term is now a function of $\zeta = z(2\omega_c)^{1/2}$

$$\left\langle u^2(z,\beta,\omega_c)\right\rangle \sim \frac{4\beta\omega_c\, e^{-\zeta}}{\pi(1+\beta^2)}\left(\frac{1}{\zeta} + \frac{1}{\zeta^2}\right), \quad \zeta \geq 1. \qquad (4)$$

Equations (3) and (4) show that at distances that are large compared with the thickness of the Stokes layer based on the dominant frequency Ω, $\langle u^2(z,\beta)\rangle$ decays algebraically with z. There exists, however, a length scale $z \sim \mathcal{O}((2\omega_c)^{-1/2})$ beyond which the decay is exponential. This new characteristic length scale is the thickness of the Stokes layer based on the cut-off frequency. This conclusion appears natural given the principle of superposition for the linear differential equation governing fluid flow in this case

We have also investigated two additional geometries in which the equations governing fluid flow are not linear, and have shown that several of the generic features obtained for the case of a planar boundary still hold. In the first case, we have generalized the analysis of [12] concerning secondary steady streaming. He found that the oscillatory motion of the boundary induces a steady secondary flow outside of the viscous boundary layer even when the velocity of the boundary averages to zero. If the thickness of the Stokes layer, δ_s, and the amplitude of oscillation, a, are small compared with a characteristic length scale of the boundary L ($\delta_s \ll L$, $a \ll L$), then the generation of secondary steady streaming may be described as follows. Vibration of the rigid boundary gives rise to an oscillatory and nonuniform motion of the fluid. The flow is potential in the bulk, and rotational in the boundary layer because of no-slip conditions on the boundary. The bulk flow applies pressure at the outer edge of boundary layer, which does not vary across the layer. The nonuniformity of the flow leads to vorticity convection in the boundary layer through nonlinear terms. Both convection and the applied pressure drive vorticity diffusion, and thus induce secondary steady motion which does not vanish outside of the boundary layer. The stationary part of the ensemble average of the secondary velocity is found to be nonzero, even though the boundary velocity averages to zero. In this case, we found that the leading contribution to the average stationary velocity diverges logarithmically with distance away from the boundary. In analogy to the planar case, the introduction of a low frequency cut-off in the power spectrum of the boundary velocity changes the asymptotic behavior qualitatively. The average stationary velocity asymptotes now to a constant, given by Eq. (5). The asymptotic velocity explicitly depends on β and logarithmically on the cut-off frequency. This asymptotic behavior is not reached until a length scale of the order of the Stokes layer thickness that is based on the cut-off frequency. It is also of interest to find the asymptotic value of the velocity away from the boundary. We find that the tangential velocity for finite but small ω_c is,

$$\partial_z \psi_1^{(s)}(\infty, \beta, \omega_c) = -\frac{3}{4} U \frac{dU}{dx} \frac{\beta}{\pi(1+\beta^2)} \left(2\beta \arctan(\beta) - \ln\left(\frac{\beta^2 \omega_c^2}{1+\beta^2} \right) \right) + \mathcal{O}(\omega_c^2), \quad (5)$$

where $U(x)$ is the far field amplitude of the flow velocity.

We have finally analyzed the case of a periodically modulated solid boundary in the limit in which the scale of the wall modulation is small compared to the thickness of the Stokes layer, and also when the spatial amplitude of the boundary oscillation is small compared with the wavelength of the wall profile. Cancellation of vorticity production over the wall boundary leads to exponential decay of the fluid velocity away from the boundary, with a decay length which is proportional to the wall wavelength, even if the zero frequency value of the power spectrum of the boundary velocity is nonzero. If the boundary wavelength is much larger than the Stokes layer thickness, we find steady streaming in the secondary flow with two or four recirculating cells per wall period depending on β. On the other hand, if the wavelength is much smaller than the Stokes layer thickness, only two recirculating cells are formed regardless of the value of β. Somewhat unexpectedly, the intensity of the recirculation can both increase or decrease with β.

6 Morphological stability analysis of directional solidification into an oscillatory fluid layer

We have used the results of the previous section to study the stability of a planar solid-melt boundary during directional solidification when the solid is being periodically vibrated in the direction parallel to the boundary (or equivalently, under a far field uniform but oscillatory flow parallel to the planar boundary). Our study is based on the natural separation of time scales between the characteristic scale for the development of the instability near threshold, and the relaxation time of the oscillatory flow for a given instantaneous interfacial configuration. The base flow field for a planar interface has a Stokes layer structure, and does not alter the base solute distribution in the melt. A small perturbation of the interface induces a secondary nonplanar flow, which we calculate to first order in the amplitude of the interface perturbation. If the displacement of a fluid element far from the interface is small compared with the critical wavelength for morphological instability, the secondary flow has harmonic, sub-harmonic and steady components. At first order in the perturbation, only steady and harmonic components couple back to the interface perturbation. For the entire flow, including the steady part (also referred to as steady streaming), we adopt a quasi-static approximation according to which the flow relaxes instantaneously for any given configuration of the solid-melt interface. The steady streaming induces convection of the base solute distribution. In addition, the oscillatory part of the secondary flow also induces an oscillatory component of the concentration field, and its nonlinear interaction with the base oscillatory flow leads to additional mean solute transport. Both contributions modify the mean solute distribution and therefore the instability threshold. The slowly varying equation governing solute transport has been derived and used to obtain the neutral stability surface for a moving solid-melt interface. We find both regions of stationary and oscillatory instability. For small ratios of the viscous to solutal layer thicknesses, s, the flow generally destabilizes the planar interface. For $s \simeq 1$, the flow stabilizes the stationary branch, but it can also excite an oscillatory instability. For large s, the effect of the flow is small.

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A   Summary of accomplishments

- Narrow band noise has been shown to describe many of the features of acceleration data collected during space missions, and the parameters defining the noise model have been obtained.

- The scale of baroclinically induced flows when the driving acceleration is random is not given by the Rayleigh number, but by a modified dimensionless group involving the intensity of the fluctuations.

- Spatially uniform g-jitter induces additional hydrodynamic forces among suspended particles in incompressible fluids. These effective forces are attractive at long distances and repulsive otherwise.

- Stochastic modulation of the control parameter shifts the location of the onset of an oscillatory instability. In addition, the bifurcation is to standing waves instead of to traveling waves as in the unmodulated case.

- Random vibration of solid boundaries leads to separation of boundary layers. In fact the boundary layer completely disappears without a low frequency cut-off in the power spectrum of the g-jitter. Steady streaming arises at long distances, similarly to the case of periodic vibration. The structure of the boundary layer flows strongly depends on the parameters of the noise.

- Steady streaming ahead of a modulated solid-melt interface enhances solute transport, and modifies the stability boundaries of a planar front. For small ratios of the viscous to solutal layer s, the flow generally destabilizes the planar interface. For $s \simeq 1$, the flow stabilizes the stationary branch, but it can also excite an oscillatory instability. For large s, the effect of the flow is small.

B Publications

Following is a list of publications that have been produced during the current funding period, and that acknowledge support from this contract.

1. J.R. Thomson and J. Viñals, "Statistical saturation of buoyant flow induced by a fluctuating acceleration". *AIAA J.* **34**, 975 (1996).

2. M. Gurtin, D. Polignone and J. Viñals. "Two-phase binary fluids and immiscible fluids described by an order parameter". *Mathematical Models and Methods in Applied Sciences*, **6**, 815 (1996).

3. W. Zhang and J. Viñals. "Square patterns and quasi-patterns in Faraday waves in low viscosity fluids". *Phys. Rev. E* **53**, R4283 (1996).

4. J.R. Thomson, J. Casademunt, F. Drolet and J. Viñals. "Coarsening of solid-liquid mixtures in a random acceleration field". *Phys. Fluids* **9**, 1336 (1997).

5. W. Zhang and J. Viñals. "Pattern formation in weakly damped Faraday waves". *J. Fluid Mech.*, **336**, 301 (1997).

6. F. Drolet and J. Viñals. "Onset of oscillatory instabilities under stochastic modulation". Phys. Rev. E **56**, 2649 (1997).

7. W. Zhang and J. Viñals. "Numerical study of pattern formation in weakly damped parametric surface waves". *Physica D* **116**, 225 (1998).

8. F. Drolet and J. Viñals. "Adiabatic reduction near a bifurcation in stochastically modulated systems". *Phys. Rev. E.*, **57**, 5036 (1998).

9. F. Drolet and J. Viñals. "Fluid flow induced by a random acceleration field". *Microgravity Sci. and Tech.* **11**, 64 (1998).

10. J. Casademunt and J. Viñals. "Stochastic modeling of the residual acceleration field in a microgravity environment". *Astrophysics and Space Science*, in press.

11. D. Volfson and J. Viñals. "Flow induced by a randomly vibrating boundary". *J. Fluid Mech.* **432**, 387-408 (2001).

12. D. Volfson and J. Viñals. "Morphological stability analysis of directional solidification into an oscillatory fluid layer". Submitted to *Phys. Fluids.* Preprint FSU-CSIT-01-19.

Not included in the list are a number of conference presentations, and papers prepared for the corresponding proceedings. We only note the presentation entitled "Fluid instabilities and materials processing in reduced gravity" at the Gordon Research Conference on "Gravitational effects in Fluids and Materials Science", held in New Hampshire in June 1997.