ABSTRACT

The purpose of this investigation is to determine the feasibility of attaining and maintaining unique non-Keplerian orbit vantage locations in the Earth/Moon environment in order to obtain continuous scientific measurements. The principal difficulty associated with obtaining continuous measurements is the temporal nature of astrodynamics, i.e., classical orbits. This investigation demonstrates advanced trajectory designs to meet demanding science requirements which cannot be met following traditional orbital mechanic logic. Examples of continuous observer missions addressed include Earth pole-sitters and unique vertical libration orbits that address Sun-Earth Connection and Earth Science Vision roadmaps.

INTRODUCTION

The principal difficulty associated with obtaining continuous scientific measurements is the temporal nature of astrodynamics. As Earth and Space Science requirements become more variable, a new and innovative approach must be taken to enable unique vantagepoint missions. The Sun-Earth Connection's roadmap using a Magnetospheric Pole-Sitter and Earth Science Vision's roadmap using unique libration points orbits are examples of multiple continuous observers at unique vantage points. These requirements cannot be met using fundamental orbit design based on classical Keplerian mechanics.

To determine the feasibility of attaining and maintaining non-Keplerian orbits, an alternate mission design method based on advances in dynamical systems theory and modeling capabilities is investigated. The results from this analysis provide an innovative set of previously undiscovered 'orbits' which meet demanding scientific objectives. An investigation of the feasibility of attaining and maintaining unique non-Keplerian orbit vantage locations in the Earth/Moon environment in order to obtain
continuous scientific measurements was proposed and awarded as a Director’s Discretionary Fund activity. An association of the Goddard Space Flight Center’s (GSFC) Flight Dynamics Analysis Branch with the NASA Academy and Purdue University accomplished the work performed. This analysis related to near-Earth and Libration orbit locations includes no-thrust and constant thrust using either a standard propulsion systems or a solar sail. A dynamical systems approach is utilized to initialize vertical libration orbits. Since the objective is to provide a continuous measurement of the Polar Region; we considered the visibility duration of the Earth’s Polar Regions as the main condition of success.

Application Context

There are many NASA customers seeking unique non-Keplerian orbits. In the Space Science arena: Origins, Solar Exploration of the Universe (SEU), and the Sun-Earth Connection (SEC) have unique orbit requirements that call for stationary polar locations. Earth Science Enterprise (ESE) customers interested in enabling continuous Earth measurements also require these types of observations. Note that space science teams involving the Earth Vision’s Sensor Web which specifically calls for unique vantage locations may be unaware that the trajectory design capabilities in these unique regimes are relatively immature.

Objectives and Research Development Plan

The objective of this work was to provide NASA projects with an alternative to standard mission design with new and unique trajectories and orbits. We seek to establish the feasibility of non-Keplerian orbit applications to meet unique science requirements. Maintenance in these non-Keplerian trajectories is also addressed. Two distinct families of unique non-Keplerian locations and mission types are investigated: unique vertical lissajous orbits and non-Keplerian ‘orbits’ within the Earth’s gravitational sphere of influence.

The vertical lissajous orbits which spend time over the Polar Regions include:

- Small vertical libration orbits with z-axis amplitudes of approximately 1 million km.
- Large vertical libration orbits (see Figure 1) with z-axis amplitudes of greater than 2 million km.

Non-Keplerian orbits within the Earth gravitational sphere of influence that maintained a constant location over the Earth’s polar regions include:

Figure 1. Large Vertical Libration Orbit
• Stationary locations at a distance of 25,000 km.

• Stationary locations at a distance of 250,000 km.

A stationary 25,000km polar distance is shown in Figure 2.

The analysis is performed in two phases. Phase 1 investigated different types of non-Keplerian orbits at the Sun-Earth L₁ point. We then derived comparable Keplerian orbits for contrast, such as a lunar distance circular orbit which used a lunar gravity assist and a more traditional highly elliptical orbit that permitted polar visibility. This contrast determined the advantage if any of the vertical libration orbit in terms of polar coverage. Phase 2 addressed innovative propulsion scenarios using solar sails as a comparison to finite maneuvers to deal with the gravitational acceleration field at the Earth’s Polar Region.

After initial orbital locations were identified, comparisons of continuous polar region accesses are made and the ΔV maintenance cost is addressed. Accelerations in the Earth potential well and the related solar sail mass to area ratios to counter these accelerations were studied. Maintenance requirements for spacecraft stationed at these locations may be reduced via judicious use of location selection, perturbation modeling, and enabling technologies using solar sails, electric propulsion, and autonomous control.

**DDF and NASA Academy**

During the analysis, as part of the Director’s Discretionary Fund, NASA academy students were assigned and a grant was given to Purdue University. To examine the possibilities, they analyzed required accelerations for control of non-Keplerian trajectories at various vantage locations about the Earth. These vantage locations include geomagnetic regions (bowshock and tail), polar libration orbit distances, and polar lunar distances. To develop useful and accurate models, it is necessary to take into account all perturbations that affect the system. These include all planetary gravitational interactions, solar radiation pressure, inhomogeneities in the Earth’s gravity field, and atmospheric drag. Analysis using the GSFC program Swingby with high fidelity simulations including all perturbations is then performed. The team analyzed solar sail concepts and incorporated a control method in Swingby via finite maneuvers. Geopotential accelerations and the related solar sail mass to area ratios required to counter these accelerations were studied. Purdue University concentrated on developing the initial conditions for the vertical libration orbits using a dynamical systems approach.
UNIQUE LIBRATION ORBITS

The Restricted 3-Body Problem is a good first approximation of the solar system and for our analysis. Five stable regions, known as Lagrange points, exist in the rotating frame of reference shown in Figure 3. The \( L_1 \) and \( L_2 \) co-linear libration points are used herein.

**Initial Conditions**

A set of initial conditions was obtained for the vertical orbits from Purdue University. These conditions are modeled in Swingby and the orbital parameters optimized. This created orbital data sets for each orbit to be used in comparative analysis. Table 1 presents the initial conditions for a small and large vertical orbit about \( L_1 \). Using these conditions, simulations are run and maneuvers performed which keep the spacecraft within the proper location. The duration of the Polar Region coverage measured from the subsatellite location for these orbits is then computed over a North and South latitude range of 60 to 90 degrees. With the period of the libration orbit approximately 6 months, the insertion into the orbit should be phased so that the required polar region can be observed more easily for a given season. For example, a transfer and insertion can place the spacecraft at the most northern ecliptic location when the North pole of the Earth is tilted towards the Sun, thereby maximizing the exposure duration. Figure 4, 5, and 6 show the \( x, y, z \) views in a solar orbit.

**Table 1. Initial Conditions for Vertical Libration Orbits**

<table>
<thead>
<tr>
<th>Small Vertical Orbit Initial Cond.</th>
<th>Large Vertical Orbit Initial Cond.</th>
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</thead>
<tbody>
<tr>
<td>( x = 7.004902317657201 \times 10^4 ) km</td>
<td>( x = 1.317772822366962 \times 10^6 ) km</td>
</tr>
<tr>
<td>( y = 0 ) km</td>
<td>( y = 0 ) km</td>
</tr>
<tr>
<td>( z = 5.376155536465699 \times 10^5 ) km</td>
<td>( z = 2.688077768232865 \times 10^6 ) km</td>
</tr>
<tr>
<td>( v_x = 0 ) km/s</td>
<td>( v_x = 0 ) km/s</td>
</tr>
<tr>
<td>( v_y = -0.010798682045 ) km/s</td>
<td>( v_y = -0.206125010972 ) km/s</td>
</tr>
<tr>
<td>( v_z = 0 ) km/s</td>
<td>( v_z = 0 ) km/s</td>
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rotating coordinate frame of the small vertical orbit while Figure 7, 8, and 9 present the larger vertical orbit views.

Highly Inclined Lunar and Elliptical Orbits

The next set of analysis considers orbits at a lunar orbit radius distance and an elliptical orbit for contrast to the lissajous orbit. To achieve the inclined lunar orbit, a lunar gravity assist is utilized. This gravity assist along with an insertion maneuver provided an ecliptic inclination of almost 90 degrees. This inclination was chosen to meet the
requirement of viewing the Earth Polar Regions. This orbit provides good coverage, but is temporal, albeit slower, as with traditional orbits. The viewing coverage includes both hemispheres and allows a higher science return data rate than the libration orbit. Figures 10 and 11 present the lunar orbit in three views while Figures 12 and 13 present the elliptical orbit. All views are in a solar rotating frame. As with all elliptical orbits, the time at periapsis is limited therefore the orbit apses orientation is selected to place the apoapsis in the northern polar region. For best communication coverage a Molnyia orbit can be used.

Phase 1 Results

For the cases studied, parameters of: distance from Earth over a 240 day period, the \( \Delta V \) required for orbit maintenance, and the duration and percent of the time with views between 60 and 90 degrees North or South equatorial latitudes are computed and are shown in Figures 14 through 17. Using this information it can be seen that the large libration orbit provides the best viewing geometry in terms of duration, but requires the largest distance which impacts the communication and science resolution. The orbit at
a lunar distance is the most expensive in fuel cost, requiring timing for a gravity assist as well as a large, approximately 1 km/s, $\Delta V$ for the ‘insertion’ into the mission orbit. The highly elliptical orbit is the worst in terms of temporal polar coverage. The $\Delta V$ requirements are all approximately 120 to 170 m/s per year with the exception of the additional lunar orbit insertion maneuver.

**POLE SITTERS**

The next section deals with the use of polar sitters to obtain continuous viewing. This analysis looked at placements at two different polar distances and the associated $\Delta V$ or the accelerations required to maintain this unique non-Keplerian location. The distances selected are 25,000 km and 250,000 km directly over the Earth’s North Pole. The initial conditions for these cases are presented in Table 2. The ideal condition would be one in which the orbit is stable and used minimum fuel. The stability duration was chosen to be 28 days (a lunar cycle). A view of these two cases is shown in Figures 18 and 19. The maintenance requirements were based on offsetting the accelerations in the

<table>
<thead>
<tr>
<th>Close Sitter Orbit Initial Cond.</th>
<th>Far Sitter Orbit Initial Cond.</th>
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<tbody>
<tr>
<td>$x = 0$ km</td>
<td>$x = 0$ km</td>
</tr>
<tr>
<td>$y = 0$ km</td>
<td>$y = 0$ km</td>
</tr>
<tr>
<td>$z = 2.5e+004$ km</td>
<td>$z = 2.5e+005$ km</td>
</tr>
<tr>
<td>$v_x = 0.5$ km/s</td>
<td>$v_x = 0.005$ km/s</td>
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<tr>
<td>$v_y = 0.5$ km/s</td>
<td>$v_y = 0.005$ km/s</td>
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<td>$v_z = 0$ km/s</td>
<td>$v_z = 0$ km/s</td>
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vertical or Z-axis only. This method allowed us to use a maneuver that controlled the perturbations in the z-axis direction which would result in a Earth centered traditional orbit, while only monitoring the motion in the x and y axis directions.

**Figure 18. Lower Pole Sitter at 25,000km**

**Figure 19. Higher Pole Sitter at 25,000km**

**Accelerations**

The acceleration due to gravity in the z-axis direction can be found using the following two equations.

\[
a_{\text{grav},z} = \frac{GM_\oplus z}{(x^2 + y^2 + z^2)^{3/2}}
\]

\[
\overline{a}_z = \frac{GM_\oplus}{2\alpha^2 R_\oplus^2 A^4} \left[2\lambda^2 - \frac{3}{2}\right]
\]

The acceleration in the vertical direction as a function of distance is shown in Figure 20. The acceleration under 50,000 km provides the most significant perturbation. In order to maintain this location, a maintenance strategy is employed using either a propulsion system or solar sail technology. To use solar sails, the acceleration due to the solar radiation pressure needs to cancel the gravitational accelerations. A simple model was constructed for an estimate of the required mass to area ratio needed for the appropriate acceleration level.

![Altitude vs Vertical Acceleration](image)

**Figure 20 - Vertical Acceleration**

Tilting the sail by an angle \(\beta\) causes a component of the solar pressure to cancel the downward pull of gravity. The optimal \(\beta\) is 35.26°. Using this estimate, a formula for the mass to area ratio, \(\sigma_{a/c}\), for a solar sail is constructed from
\[ \ddot{a}_{sc} = 2\eta \frac{S_0}{c} \left( \frac{1}{R_{AU}} \right)^2 \frac{A}{m_{sc}} \cos^2 \beta \cdot \hat{e}_n \]

\[ \sigma_{sc} = \frac{\lambda^4}{2\lambda^2 - \frac{3}{2}} \left[ \frac{\eta \alpha^2}{R^2} \right] \left[ \frac{4R_{\oplus}^2 S_0}{GM_{\odot} c} \cos^2 \beta_0 \sin \beta_0 \right] \]

Where
\[ \sigma_{sc} = \frac{m_{sc}}{A} \quad \lambda = \frac{\pi}{a_{R_{\oplus}}} \quad m_{sc} = m_{sail} + m_{sat} \]

and \( R_0 \) is the radius of the earth, \( S_0 \) the solar flux, \( GM \) the gravitational constant, \( \beta_0 \) the angle between the solar radiation and sail, \( c \) the speed of light, \( \eta \) the sail efficiency, \( R \) the distance, \( \sigma \) the mass to area ratio, \( A \) the area, \( m \) are the relative masses, \( \lambda \) the normalized distance. Calculating the solar sail acceleration required for the cancellation of the gravitational acceleration leads to the data in Figure 21 which shows the ratio for all distances between 25,000km and 250,000km.

![Figure 21. Distance vs. Mass/Area Ratio](image)

**Propulsion Maintenance**

A \( \Delta V \) estimate is computed using the Swingby program with a continuous \( \Delta V \) in the \( z \)-axis direction to cancel the vertical accelerations similar to the solar sail case. The propulsion system requirements are as follows; for the 250,000km distance a \( \Delta V \) of 0.0064 m/s per second duration leading to approximately 0.5 km/s per day. At the 25,000km distance, a \( \Delta V \) of 0.596 m/s per second duration is required leading to over 50km/s per day. This \( \Delta V \) needs to be computed in terms of a fuel mass fraction as an impact statement into the selection of the payload mass and launch capability. The subsatellite pattern shown in Figures 22 and 23 for the 25,000km and 250,000km case, respectively, is presented in a rotating Earth frame.
SUMMARY

The feasibility of attaining and maintaining unique non-Keplerian orbit vantage locations in the Earth/Moon environment in order to obtain continuous scientific measurements has been demonstrated. This investigation demonstrated vertical libration orbits, polar orbits at a lunar distance, and stationary polar sitters which require a maintenance strategy. A dynamical systems approach was utilized to initialize the vertical libration orbits. The maintenance of the polar sitters is demonstrated using both a long duration ΔV and solar sail concept. It was shown that maintenance fuel for spacecraft stationed at these locations can be reduced thus enabling technology for innovative missions such as solar sails, electric propulsion, and autonomous control. The results from this analysis provide an innovative set of ‘orbits’ which meet demanding scientific objectives.

CONCLUSION

An example of continuous observer missions which include Earth pole-sitters and unique vertical libration orbits that meet Sun-Earth Connection and Earth Science Vision roadmaps has been demonstrated. Continuous polar coverage requirements can be met using a non-Keplerian approach to maximize the viewing requirement. These results provide NASA projects with an alternative to standard mission design and the related impacts. Results provide NASA projects with new and unique non-Keplerian ‘orbits’ and vantage locations as alternative to standard mission design.

REFERENCES


