

REDUCING FORMATION-KEEPING MANEUVER COSTS FOR FORMATION FLYING SATELLITES IN LOW-EARTH ORBIT

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ABSTRACT

Several techniques are used to synthesize the formation-keeping control law for a three-satellite formation in low-earth orbit. The objective is to minimize maneuver cost and position tracking error. Initial reductions are found for a one-satellite case by tuning the state-weighting matrix within the linear-quadratic-Gaussian framework. Further savings come from adjusting the maneuver interval. Scenarios examined include cases with and without process noise. These results are then applied to a three-satellite formation. For both the one-satellite and three-satellite cases, increasing the maneuver interval yields a decrease in maneuver cost and an increase in position tracking error. A maneuver interval of 8-10 minutes provides a good trade-off between maneuver cost and position tracking error. An analysis of the closed-loop poles with respect to varying maneuver intervals explains the effectiveness of the chosen maneuver interval.

INTRODUCTION

Currently, formation flying spacecraft control is being extensively researched. This paper presents a strategy to reduce the amount of control needed for formation keeping within the framework of a linear-quadratic-Gaussian (LQG) controller. By varying the maneuver interval, the trade off between maneuver cost and position tracking error is discovered.

Speyer¹ first introduced a decentralized LQG control method. Carpenter^{2,3} applied this work to formation flying satellites, and further expanded it to deal with both time-invariant and time-varying systems. Carpenter, Folta, and Quinn⁴ investigated the decentralized framework for the applicability of autonomous formation flying control for the EO-1 mission to follow Landsat-7. In addition, Sparks⁵ studied the long-term ΔV for a relative circular formation at an 800 km altitude orbit. Orbital rendezvous is a related problem to formation flying. Kluever and Tanck⁶ looked at reducing ΔV for a geosynchronous orbit rendezvous problem using constant thrust magnitudes and varying thrust lengths, as well as varying maneuver intervals.

This paper will address tuning an LQG controller by adjusting the cost of the state tracking error in an effort to minimize the maneuver cost. These results will then be applied to a scenario that drives one satellite from random initial conditions to the origin. Further reductions to maneuver cost will be achieved by varying the maneuver interval. Finally, the one-satellite results will be tested on a three-satellite formation simulation.

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SYSTEM MODEL AND CONTROLLER DESIGN

Both Kaplan⁷ and Carpenter³ describe the dynamics for formations of closely spaced satellites in low-Earth orbits. In summary, for each formation, an imaginary satellite, or hub, is in a circular low-Earth orbit. This hub defines a reference frame with radial, in-track, and cross-track components. The radial component is in the direction from the central body (Earth) to the hub, the in-track component is in the direction of the hub's motion, and the cross-track component is in the direction of the orbit normal. Hill's equations give the mathematics of the relative motion between the real satellites and the hub.

Controller Design

Carpenter built a standard LQG controller for the relative motion of the formation with respect to the hub. The cost function to be minimized for this problem is

$$J = \frac{1}{2} E \left[\left\{ \left\{ X(t_N) - X^R(t_N) \right\}^T S_N \left\{ X(t_N) - X^R(t_N) \right\} + \int_{t_0}^{t_N} \left[\left\{ X(t) - X^R(t) \right\}^T W(t) \left\{ X(t) - X^R(t) \right\} + \sum_{j=1}^K \left[U^j(t) - U^{Rj}(t) \right]^T V^j(t) \left[U^j(t) - U^{Rj}(t) \right] \right] dt \right\} \right], \quad (1)$$

subject to the dynamic constraint

$$\dot{X} = AX + BU + w, \quad (2)$$

where X is the state vector consisting of the positions and velocities for each satellite in the formation, U is the control vector, and w is the process noise that has power spectral density

$$Q = \begin{bmatrix} Q^j & & \\ & \ddots & \\ & & Q^j \end{bmatrix}$$

$$Q^j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.81e-6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9.81e-6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.81e-6 \end{bmatrix}. \quad (3-a,b)$$

$U-U^R$ is the maneuver cost, and $X-X^R$ is the state tracking error. Specifically for this application,

$$\begin{aligned} U &= \Delta V \\ U^R &= 0 \\ S_N &= 0 \end{aligned} \quad (4-a,b,c)$$

Of interest to this paper are the W and V matrices. In Equation 1 they can be time varying, but are constant matrices in this study. W is the continuous state-weighting

matrix, and V is the continuous control-weighting matrix. For this study, Equation 1 can be simplified to

$$J = \frac{1}{2} E \left[\int_{t_1}^{t_N} \left\{ [X(t) - X^R(t)]^T W [X(t) - X^R(t)] + \sum_{j=1}^K [U^j(t)^T V^j U^j(t)] \right\} dt \right]. \quad (5)$$

Carpenter³ also gives the discrete forms

$$\begin{aligned} W_d(t_{i+1}, t_i) &= \int_{t_i}^{t_{i+1}} \Phi(t, t_i)^T W(t) \Phi(t, t_i) dt \\ V_d^j(t_{i+1}, t_i) &= \int_{t_i}^{t_{i+1}} [B_d^j(t, t_i)^T W(t) B_d^j(t, t_i) + V^j(t)] dt \end{aligned} \quad (6-a,b)$$

where Φ is the state transition matrix, and B^j is the continuous control mapping whose discrete form is given by

$$B_d^j(t, t_i) = \int_{t_i}^t \Phi(t, \tau)^T B^j(\tau) d\tau. \quad (7)$$

The j in the above equations is the notation describing each node (or satellite) in the formation. The integrals in Equations 6a, 6b, and 7 are approximated as

$$\begin{aligned} W_d(t_{i+1}, t_i) &\approx [\Phi(t_{i+1}, t_i)^T W \Phi(t_{i+1}, t_i)] \cdot [t_{i+1} - t_i] \\ V_d^j(t_{i+1}, t_i) &\approx [B_d^j(t_{i+1}, t_i)^T W B_d^j(t_{i+1}, t_i) + V^j] \cdot [t_{i+1} - t_i], \\ B_d^j(t_{i+1}, t_i) &\approx [\Phi(t_{i+1}, t_i)^T B^j(t_i)] \cdot [t_{i+1} - t_i] \end{aligned} \quad (8-a,b,c)$$

except as noted below.

In this study, I simulate the control and tracking of both one-satellite and three-satellite formations with varying maneuver intervals. For all scenarios, the simulation runs for two revolutions of the hub around the earth. The maneuvers are considered ideal and impulsive.

LQR Controller Tuning

One way to reduce control effort is to tune the state weighting matrix and/or the control-weighting matrix. The relationship between these two matrices is what matters, rather than their individual values. Therefore, the control-weighting matrix can be kept constant, at identity, while the state-weighting matrix is varied. The controller is tuned without noise.

The state weighting matrix is a block diagonal matrix with each block relating to an individual satellite. All satellites are assumed to be identical, so

$$W = \begin{bmatrix} W_j & & \\ & \ddots & \\ & & W_j \end{bmatrix}. \quad (9)$$

Because the satellites are assumed identical, tuning can be done on one satellite and then applied similarly to others if necessary. For the one satellite case,

$$W = W_j. \quad (10)$$

Denoting the position weights by a and the velocity weights by b , I assume a diagonal W_j :

$$W_j = \begin{bmatrix} a & & & & & \\ & a & & & & \\ & & a & & & \\ & & & b & & \\ & & & & b & \\ & & & & & b \end{bmatrix}, \quad (11)$$

where, as a starting point,

$$\begin{aligned} a &= 0.04 \\ b &= 40000 \end{aligned} \quad (12-a,b)$$

To tune W_j , initial conditions are chosen, an a is chosen, the simulation is run, and the process is repeated until the minimum ΔV is found for that set of initial conditions. In the simulation, the satellite is driven from its initial conditions to the reference orbit. The reference orbit for one satellite is simply zero with respect to the hub. In other words, the satellite is driven from an initial offset to the origin. Total ΔV in this case is the sum of the absolute values of the control for every maneuver. I found that the magnitude of the initial displacements has no effect on determining which a is best. However, because the radial and in-track states are coupled, the relationship between the initial conditions on those two displacements does have an effect on which a is best. I performed two investigations. In the first, I studied only in-plane initial conditions, and in the second, only out-of-plane initial conditions.

For the in-plane study,

$$\begin{aligned} X_0 &= [r_0 \quad i_0 \quad 0 \quad 0 \quad 0 \quad 0] \\ \alpha &= \left| \frac{r_0}{i_0} \right| \end{aligned} \quad (13-a,b)$$

where r_0 is the radial initial condition and i_0 is the in-track initial condition. Table 1 shows how the “best” a varies with α :

Table 1

α	∞	100	10	5	2	1	0.9	0.8	0.7	0.6	0.5
Best a	0.16	0.16	0.14	0.13	0.09	0.05	0.05	0.04	0.03	0.03	0.02

By choosing the best a , the total ΔV for 2 revolutions can be reduced by up to 1 m/s for some cases when the initial displacements are on the order of 500 meters. As α approaches zero (the in-track initial displacement is much greater than the radial initial displacement), the minimum ΔV occurs at a value of a less than 0.02. However, the satellite does not converge on the reference orbit within one revolution. Choosing a to be 0.02 for values of α less than 0.5 ensures convergence within one revolution, even though the ΔV is not a minimum.

Rather than implement a table lookup in the software to determine the a for a given initial state vector, two scenarios will be investigated. The first case sets α equal to one. This is a reasonable choice applicable for running random or semi-random initial state vectors. Choosing the weighting $a = 0.05$ ($\alpha = 1$), a similar process to finding the best a can be done to determine the best b . The total ΔV savings for differing b are very small, on the order of 0.01 m/s for two revolutions. Nevertheless, the best b is 39000 for

$a = 0.05$. The second case is a equal to one half. This is chosen based on the three-satellite formation reference orbit proposed by Alfriend, Schaub, and Gim⁸ for the TechSat21 program. Choosing the weighting $a = 0.02$ ($\alpha = 0.5$), the best b is once again 39000.

The process of finding the best a is repeated for the uncoupled cross-track initial displacements as well. For this case,

$$X_0 = [0 \ 0 \ c \ 0 \ 0 \ 0]. \quad (14)$$

I found the best a to be 0.03, and the corresponding best b to be 36000, regardless of the initial cross-track displacement. For the cross-track offset of 500 meters, altering a and b only resulted in total ΔV savings of 0.05 m/s for two revolutions.

PERFORMANCE EVALUATION

One-Satellite Simulation

Next, I used simulations with random initial conditions with and without process noise to determine the performance of the controller. For these cases, the state weighting matrix is chosen to be

$$W = W_j = \begin{bmatrix} 0.05 & & & & & \\ & 0.05 & & & & \\ & & 0.05 & & & \\ & & & 40000 & & \\ & & & & 40000 & \\ & & & & & 40000 \end{bmatrix}. \quad (15)$$

This choice is not the best for the cross-track displacements, but the effect is negligible once noise is introduced. First, 15 “semi-random” initial displacements are chosen in the radial and in-track directions. The offsets range from -500 meters to $+500$ meters in both directions. The set of runs over these 15 points yields statistical results that are used to determine trends. The 15 points are shown below in Figure 1 and remain the same for all subsequent cases.

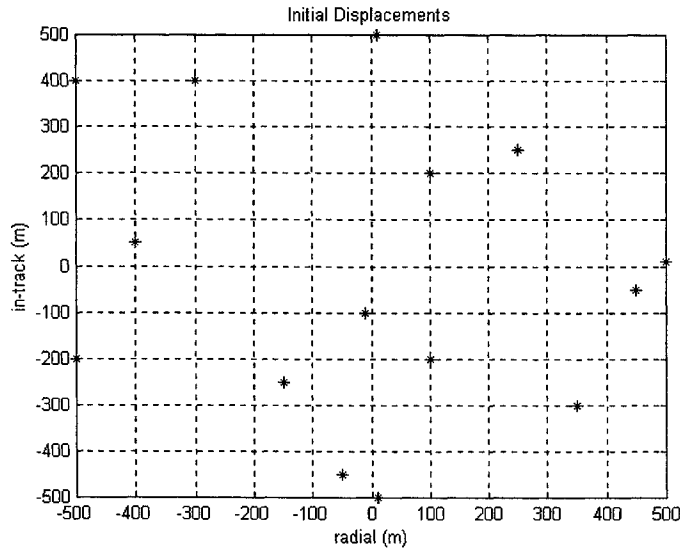


Figure 1: Assorted initial conditions for one satellite simulation

The total ΔV is defined as the sum of the absolute value of all control effort. The total ΔV is calculated for simulations of each of the 15 initial conditions, with all noise turned off. The RMS is then calculated over the set of 15 total ΔV 's with no noise, and found to be 2.2634 m/s. The maneuver interval in this case is one minute.

Next, the process noise is turned on and the simulation is run again for each of the 15 points. In addition to determining the total ΔV , the steady-state tracking error is calculated as well. Steady-state tracking error is measured by defining Δx , which is a statistical determination of how far the satellite is from its reference orbit (*during the second half of the simulation.*)

Let

$$\Delta x^j = RSS(\Delta x_n^j), \quad (16)$$

where

$$\Delta x_n^j = \sqrt{(r_n - r_n^{ref})^2 + (i_n - i_n^{ref})^2 + (c_n - c_n^{ref})^2}. \quad (17)$$

r_n , i_n , and c_n are the radial, in-track, and cross-track positions of each satellite at some time n ; and r_n^{ref} , i_n^{ref} , and c_n^{ref} are the radial, in-track, and cross-track reference positions for each satellite at some time n . Figure 2 shows Δx_n plotted against time for the one-satellite scenario at different maneuver intervals.

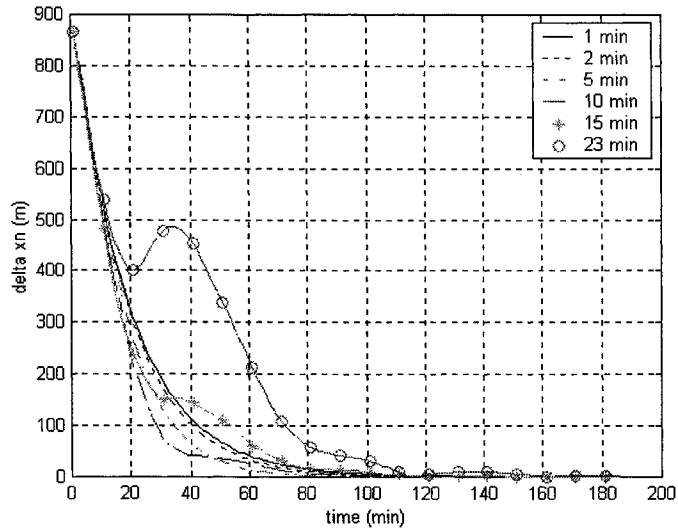


Figure 2: Position tracking error (Δx_n) versus time

The RMS is calculated over the set of 15 ΔV 's and 15 Δx 's for each maneuver interval. Table 2 and Figure 3 summarize the results of the simulations using various maneuver intervals.

Table 2: Data corresponding to Figure 3

Maneuver interval	1 min	2 min	5 min	10 min	15 min	23 min
RMS ΔV (m/s)	8.4514	5.8260	3.3272	2.1793	1.6827	1.3920
RMS Δx (m)	58.3374	74.2114	143.2634	317.4274	646.8495	1058.5662

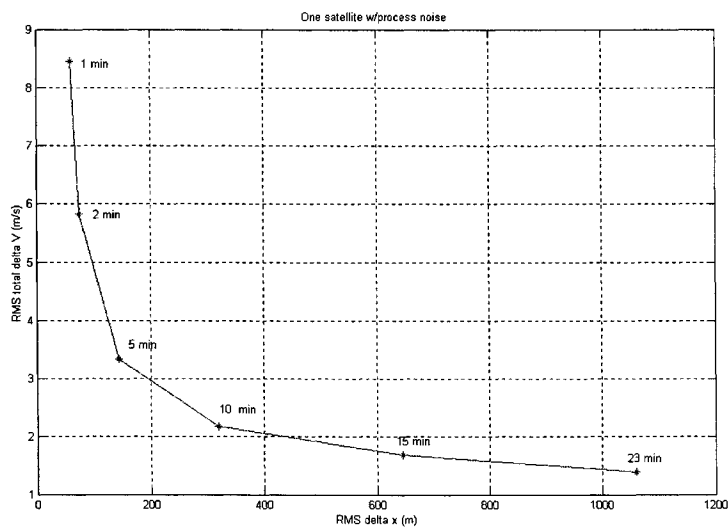


Figure 3: RMS of total ΔV versus the RMS of Δx for one satellite with process noise

As one can see, with process noise on, the amount of control or total ΔV needed is higher (8.4514 versus 2.2634 m/s). One way to reduce total ΔV is to reduce the number of maneuvers performed. The trade off for reducing ΔV using this method is that the Δx increases. In other words, reducing the number of maneuvers decreases the amount of control needed, but the displacement error due to the noise increases.

Three-Satellite Simulation

Next, I tested the controller design in a three-satellite simulation. For this study, I used the “best” a and b determined above for the state weighting matrix.

$$W_j = \begin{bmatrix} 0.02 & & & & & \\ & 0.02 & & & & \\ & & 0.03 & & & \\ & & & 39000 & & \\ & & & & 39000 & \\ & & & & & 36000 \end{bmatrix} \quad (18-a,b)$$

$$W = \begin{bmatrix} W_j & & \\ & W_j & \\ & & W_j \end{bmatrix}$$

The reference orbit is the “circular horizontal plane” formation proposed by Alfried, Schaub, and Gim⁸. The horizontal plane is formed by the in-track and cross-track basis vectors. The radial projection is half the length of the in-track projection. This ellipse is then inclined out-of-plane so the projection on the horizontal plane is circular. Finally, the satellites are arranged such that their projections on the horizontal plane are always spaced 120 degrees apart. The formation appears to be a rotating equilateral triangle in the circular horizontal plane with the hub at the center of the triangle. For initial conditions, all satellites are started at the hub with zero velocity:

$$X_0^j = [0 \ 0 \ 0 \ 0 \ 0 \ 0]. \quad (19)$$

One difference between this study and the previous studies is in the calculation of the discrete cost weighting matrices, W_d and V_d . Using the MATLAB symbolic toolbox, the state transition matrix can be expressed symbolically. With this, the exact definite integrals shown in Equations 6a, 6b, and 7 can be calculated exactly rather than approximated by Equations 8a, 8b, and 8c.

With noise turned off, the simulation is run at the different maneuver intervals. In addition to Δx , another determination of tracking error is calculated which I will call μ_x . μ_x is similar to Δx except that instead of taking the time-wise RSS of the Δx_n vector, the mean is taken. This is still done for the second half of the simulation, after the initial convergence. Table 3 and the Figure 4 illustrate the results.

Table 3: Data corresponding to Figure 4

Satellite # 1

Maneuver Interval	1 min	2 min	5 min	8 min	9 min	10 min	15 min	23 min
ΔV (m/s)	9.007	8.179	5.660	4.654	4.452	4.310	3.881	3.454
μ_x (m)	10.224	1.676	1.463	0.979	0.883	0.907	3.851	17.108

Satellite # 2

Maneuver Interval	1 min	2 min	5 min	8 min	9 min	10 min	15 min	23 min
ΔV (m/s)	14.121	12.63	8.609	6.732	6.363	6.052	5.044	4.650
μ_x (m)	6.713	2.071	1.662	1.606	1.646	1.902	7.688	33.768

Satellite # 3

Maneuver Interval	1 min	2 min	5 min	8 min	9 min	10 min	15 min	23 min
ΔV (m/s)	10.807	9.370	7.027	6.066	5.863	5.696	5.142	4.694
μ_x (m)	5.857	2.505	0.471	0.795	1.087	1.693	9.442	35.049

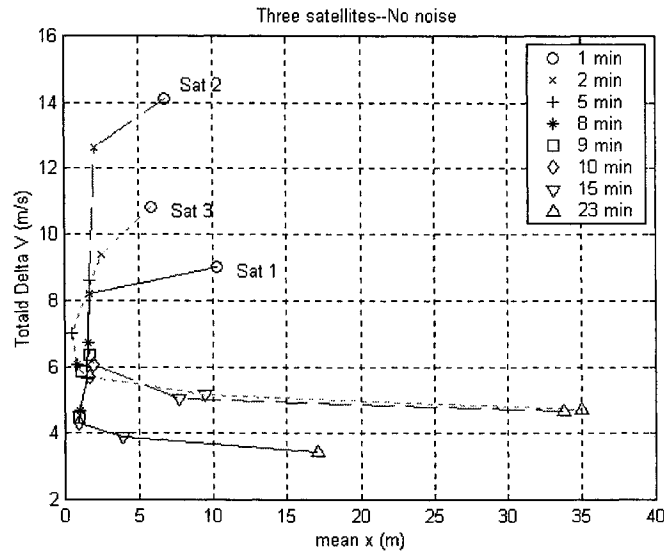


Figure 4: Total ΔV versus μ_x for three-satellite formation with no noise

As expected, with fewer maneuvers, less total ΔV is required. However, μ_x actually is at a minimum around the 8-10 minute maneuver intervals. This differs from the one satellite case where the position tracking error was smaller with more maneuvers.

Figure 5 shows the closed-loop poles in the polar plane (z-plane) as they vary with increasing maneuver intervals. From Phillips and Nagle⁹, the poles are of the form

$$z = e^{sT}, \quad (20)$$

where z is the discrete closed-loop pole, s is the continuous closed-loop pole, and T is the maneuver interval.

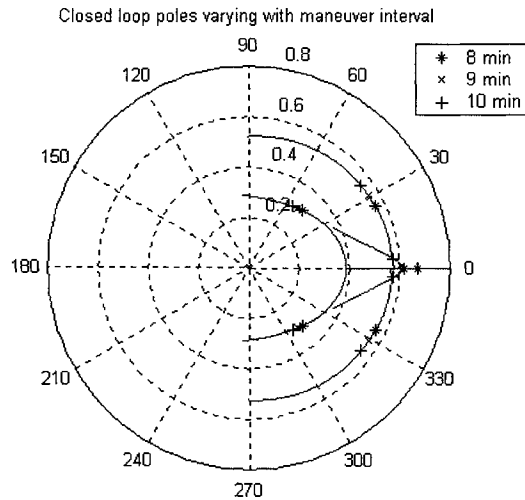


Figure 5: Closed-loop poles varying with maneuver interval (one satellite)

Poles within the unit circle are stable. Specifically looking at the set of poles on the far right in Figure 5, at the 8 minute maneuver interval they are still on the real axis; at the 9 minute maneuver interval they have just left the real axis; and at the 10 minute maneuver interval they have further diverged. Converting this set of poles back to the s-plane, the settling time for a second order system is approximated by Nise¹⁰ as

$$T_s = \frac{-8}{(s1 + s2)}, \quad (21)$$

where $s1$ and $s2$ are the corresponding s-plane poles at a given maneuver interval. A second order system assumption is valid because these poles are the most dominant. Figure 6 shows the relationship between settling time and maneuver interval.

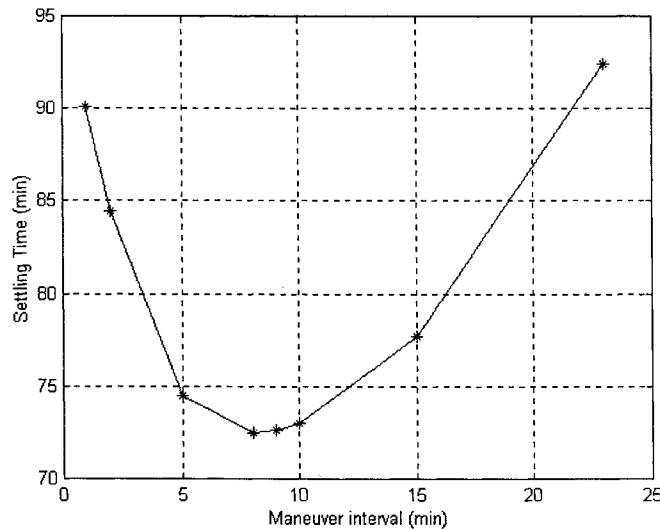


Figure 6: Settling time versus maneuver interval

Note that around the 8-10 minute maneuver interval, a minimum settling time occurs. This helps to explain the minimum μ_x around the same maneuver interval range.

Next, the process noise is turned on and the simulation is run five times for each of the maneuver intervals. The RMS is taken for the total ΔV and μ_x for each satellite at each maneuver interval. Table 4 and Figure 7 show the results.

Table 4: Data corresponding to Figure 7

Satellite # 1

Maneuver Interval	1 min	2 min	5 min	8 min	9 min	10 min	15 min	23 min
RMS ΔV (m/s)	13.734	10.140	6.653	5.283	5.107	4.888	4.334	3.734
RMS μ_x (m)	11.226	13.565	14.303	27.780	27.877	27.119	56.006	95.106

Satellite # 2

Maneuver Interval	1 min	2 min	5 min	8 min	9 min	10 min	15 min	23 min
RMS ΔV (m/s)	18.005	14.276	9.258	7.217	6.597	6.261	5.110	4.767
RMS μ_x (m)	9.021	11.779	17.701	27.796	27.505	36.165	44.963	111.545

Satellite # 3

Maneuver Interval	1 min	2 min	5 min	8 min	9 min	10 min	15 min	23 min
RMS ΔV (m/s)	14.857	11.784	8.217	6.950	6.637	6.341	5.446	4.875
RMS μ_x (m)	7.962	7.816	12.014	20.882	26.240	22.216	39.782	117.646

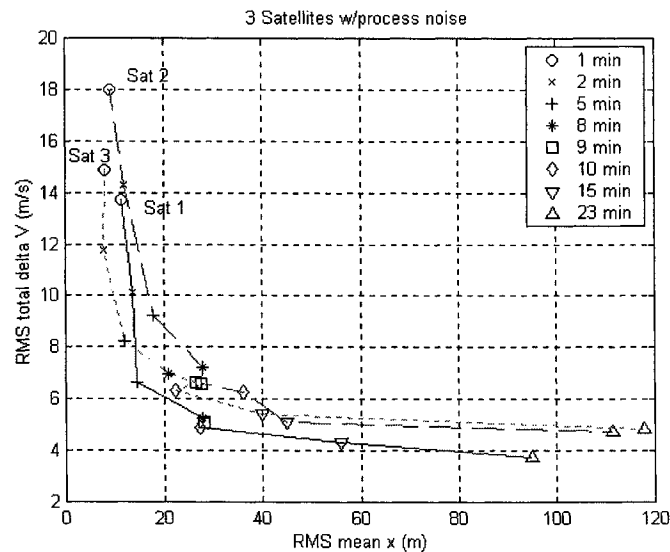


Figure 7: RMS of total ΔV versus RMS of μ_x for three-satellite formation with process noise

These results show the same trend as the one satellite case. Fewer maneuvers correspond to less total ΔV , but also lead to increases in position tracking error. A small discrepancy remains around the 8-10 minute maneuver intervals. I attribute this to only having five runs from which to take the statistics. I believe more runs would smooth these curves out.

CONCLUSION

A method for reducing formation-keeping maneuver cost has been developed. Tuning the state-weighting matrix of a single satellite yields a relationship between initial conditions and total ΔV . These results are applied to the three-satellite formation based on the desired geometry. By altering the maneuver interval, a relationship has been found between total ΔV and position tracking error. For all cases, fewer maneuvers require less ΔV . However, fewer maneuvers also tend to increase the position tracking error. Based on a closed-loop pole analysis neglecting noise, a minimum mean position tracking error is found to correspond to an 8 minute maneuver interval. This analysis is verified by the simulation. Once process noise is included, position tracking error continually increases as fewer maneuvers are performed and is inversely proportional to the ΔV needed. An 8-10 minute maneuver interval appears to be a good tradeoff between maneuver cost and position tracking error.

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