CALIBRATION OF GYROS WITH TEMPERATURE DEPENDENT SCALE FACTORS[⊥]

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ABSTRACT

The general problem of gyro calibration can be stated as the estimation of the scale factors, misalignments and drift-rate biases of the gyro using the on-orbit sensor measurements. These gyro parameters have been traditionally treated as temperature-independent in the operational flight dynamics ground systems at NASA Goddard Space Flight Center (GSFC), a scenario which has been successfully applied in the gyro calibration of a large number of missions. A significant departure from this is the Microwave Anisotropy Probe (MAP) mission where, due to the high thermal variations expected during the mission phase, it is necessary to model the scale factors as functions of temperature.

This paper addresses the issue of gyro calibration for the MAP gyro model using a manufacturer-supplied model of the variation of scale factors with temperature. The problem is formulated as a least squares problem and solved using the Levenberg-Marquardt algorithm in the MATLAB© library function 'NLSQ'. The algorithm was tested on simulated data with Gaussian noise for the quaternions as well as the gyro rates and was found to consistently converge close to the true values. Significant improvement in accuracy was noticed due to the estimation of the temperature-dependent scale factors as against constant scale factors.

INTRODUCTION

The general problem of gyro calibration can be stated as the estimation of the scale factors, misalignments and the biases of the gyro using the on-orbit sensor measurements

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(ref.1). Usually, the scale factors of the gyros are deemed to be constants which will be estimated a priori and only small, possibly slowly varying corrections to these manufacturer-supplied values will be estimated during the mission (ref. 2-5). But in some missions such as the Microwave Anisotropy Probe (MAP) (ref. 6), due to the high thermal variations expected during the mission, the scale factors are themselves functions of temperature. In most of these cases, the functional form of the variation of the gyro scale factors will be known a priori from the manufacturer and certain associated parameters for their actual values on-orbit are to be estimated as part of the gyro calibration exercise.

The existing gyro calibration utilities used at the NASA Goddard Space Flight Center (GSFC) assume that the scale factors are constants. Therefore the objective of the work reported in this document is to formulate this temperature-dependent gyro calibration and provide a methodology and a tool to solve it.

PROBLEM FORMULATION

The problem of temperature-dependent gyro calibration is formulated in this section. With a view to applying this methodology to the MAP gyro calibration scenario, the gyro model conforming to MAP (ref.7) was used here. The 3x1 vector of gyro rates is given by:

$$\omega(t) = \boldsymbol{M}\begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} - \boldsymbol{b} \tag{1}$$

where

$$S_i = a_{io} + a_{i1}v + a_{i2}v^2 + a_{i3}v^3, \quad \text{i=1,2,3} \tag{2}$$

are the temperature dependent scale factors, v is the voltage, N_i are the gyro telemetry counts, $\boldsymbol{b}$ is the 3x1 bias vector to be estimated, and $\boldsymbol{M}$ is the 3x3 misalignment matrix. The scale factors are actually functions of temperature but are provided by the manufacturer in terms of gyro thermistor voltage variations.

The gyro calibration problem now reduces to estimating the 28x1 state vector $\boldsymbol{X}$

$$\boldsymbol{X} = [\{a_{ij}\},\{M_{kl}\}, \boldsymbol{b}^T, \boldsymbol{q}_0^T]^T \tag{3}$$

where $\{a_{ij}\}$ are the 12 coefficients of the scale factors defined in Equation (2), $\{M_{kl}\}$ are the 9 elements of $\boldsymbol{M}$, and $\boldsymbol{q}_0$ is the 4x1 epoch inertial-to-body quaternion . It is convenient to define the 3x4 matrix $\boldsymbol{A}$ whose elements are $\{a_{ij}\}$.

It is possible to reduce the number of parameters in the state vector by 3 by redefining the alignment matrix to incorporate the linear part of the scale factor corrections by normalization. We are considering applying this reduction in a future version of the algorithm.

The kinematic equation relating the attitude to the gyro rates is (ref. 8)

$$\dot{q} = \tfrac{1}{2}\boldsymbol{\Omega q} \tag{4}$$

where $\boldsymbol{q}$ is the attitude quaternion and $\boldsymbol{\Omega}$ is given by

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{5}$$

Here $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)$ is the spacecraft angular velocity measured by the gyros with components along the body axes.

X is estimated by minimizing the error between the attitude quaternions computed from sensor measurements and the attitude obtained by integrating the kinematic equation above. The closed form solution to the equation is given by (ref. 8):

$$\boldsymbol{q}(t_{i+1}) = \left[\cos\left(\frac{\omega \Delta t}{2}\right)\mathbf{I} + \frac{1}{\omega}\sin\left(\frac{\omega \Delta t}{2}\right)\boldsymbol{\Omega}_i \right]\boldsymbol{q}(t_i) \tag{6}$$

where

$$\omega = \left[\omega_x^2(t_i) + \omega_y^2(t_i) + \omega_z^2(t_i)\right]^{1/2}$$

$$\Delta t = t_{i+1} - t_i$$

$$\boldsymbol{\Omega}_i = \boldsymbol{\Omega}(\omega(t_i))$$

$\boldsymbol{q}(t_i)$ is the attitude quaternion at time t_i, and $\boldsymbol{q}(t_{i+1})$ is the propagated attitude quaternion at time t_{i+1}. This closed form solution is used in the present formulation to get the propagated attitude quaternions $\boldsymbol{q}_p(t)$ for evaluating the objective function matrix for minimization using the Levenberg-Marquardt algorithm. The quaternion residuals $f(X)$ are now defined by

$$f_i(X) = \boldsymbol{q}_p(t_i) - \boldsymbol{q}_t(t_i) \tag{7}$$

where $\boldsymbol{q}_t(t)$ is the true quaternion at time t obtained using the attitude sensor measurements. The problem will then be to find the state vector that minimizes the cost function

$$L(X) = \Sigma_i\,(f_i(X))^2 \tag{8}$$

with $(f_i(X))^2$ defined as the sum of the squares of the individual components of $f_i(X)$. As will be seen below, the above cost function formulation works well. Note that the residuals defined in Equation (7) are different from the traditional attitude residuals represented by three small rotations about the true attitude (see for example, ref. 3). This traditional approach to the current problem is being investigated. Also, by suitably introducing a weight matrix in the cost function, non-uniform weighting can be allowed. This is not done in the current formulation and will be attempted in the future version.

SOLUTION METHODOLOGY

The Levenberg-Marquardt Method

The general nonlinear least squares problem can be stated as:

Find X such that L(X), defined in equation (8), is a minimum.

The Levenberg-Marquardt (L-M) (ref. 9) method uses a trust-region approach to shrink the step sizes to minimize the cost function at each iteration and the state update is given by

$$X_{new} = X_{old} + \text{Inv}\,(J^T J + D).J^T F$$

where D is the diagonal matrix given by

$$D = \lambda.\ \text{Diag}\,(J^T J\,),$$

λ is a multiplication factor, J is the matrix of first partial derivatives $\partial F/\partial X$, and F is the residual vector .

The general procedure for this method is to set λ to 1.0 for the first iteration. If the first attempt at an iteration reduces the cost function then λ is reduced for the next iteration by a factor of 10. If the first step increases the cost function, then λ is increased by a factor of 5 until the cost function is reduced. The final value of λ for the iteration is used for the next iteration. Rather than compute the sum of squares of $f_i(X)$, 'NLSQ' requires the user-defined function to compute the matrix-valued function

$$F(X) = [f_1(X)\, f_2(X)\, f_3(X)\, \ldots\, f_m(X)]^T$$

Here each $fi(X)$ could be a vector in which case the objective function will be matrix-valued.

The MATLAB Function 'NLSQ'

The MATLAB function 'NLSQ' is a general function to solve the non-linear least squares problem defined in the previous section. The default algorithm used in this function is that of L-M algorithm. The calling parameters for this function are

[X,OPTIONS,F,J] = NLSQ('FUN',X0,OPTIONS,'GRADFUN',P1,P2,..)

This function starts at the state vector X_0 and finds a minimum to the sum of squares of the functions described in *FUN*. *FUN* is usually an M-file which returns a vector of objective functions: F=*FUN*(X). *FUN* should return $F(X)$ and not the sum-of-squares since the cost function is computed implicitly in the algorithm.

OPTIONS is a vector of optional parameters to be defined. GRADFUN is an optional function which returns the partial derivatives of the functions at a given X. If *GRADFUN* is not supplied, numerical derivatives are computed within the function.

APPLICATION TO SIMULATED DATA

General Simulation Procedure

- Assign true values for the misalignment matrix, M, the scale factor coefficients matrix, A, the bias vector, b, and the epoch quaternion q_0.
- Assign start and end times t_i and t_f.
- Assign m, the number of time tags.
- Assign a temperature/voltage profile for the simulation time interval $[t_i,t_f]$.
- Assume an angular velocity profile for the spacecraft in $[t_i,t_f]$.
- Calculate the scale factor vector $S = [A]\ V$, where A is the 3x4 matrix of scale factor coefficients, $V = [\ 1\ v\ \ v^2\ \ v^3]^T$, v being the voltage as a known function of time t.
- Calculate the telemetry counts after adding the bias using $[S]^{-1}\ M^{-1}\ (N(t)+b)$ where $[S]$ is the diagonal matrix of the scale factor components of S.
- Starting from q_0 , get the quaternion history using the recursive relation given in Equation (6) above, normalize it and save it as "true" attitudes q_t(t).

Specific Simulation Scenario

The simulated data spanned 1 hour 15 minutes and consisted of inertial pointing periods interspersed with three slews of rate 0.1 deg/sec rotating +/-45 deg about the x, y axes and +/-90 deg about z axis. The slews lasted 1 hour totally with a gap of 7 minutes between slews. The temperature variation in the gyros in the early launch phase was taken to be between 5-10 degrees which corresponds to about 1 to 2 volts variation in voltage as per discussion with the MAP Attitude Control System (ACS) engineers. Based on this, a sinusoidal voltage variation was assigned with an amplitude of 2 volts and period of 1 hour, i. e.,

$v = 2\sin 2\pi t/P$

where t is the time and P, the period, is 1 hour.

The following set of true gyro parameters and epoch quaternion, denoted by the subscript "t", were used:

$$A_t = \begin{bmatrix} 0.4945513578201 & .00036598584787 & .00001662910926 & -.00000087266463 \\ 0.4945513578201 & .00036598584787 & .00001662910926 & -.00000087266463 \\ 0.4945513578201 & .00036598584787 & .00001662910926 & -.00000087266463 \end{bmatrix} \mathrm{X}\, 10^{-5}$$

$b_t = [0.5, 0.5, 0.5]^T$ deg/hr

$$M_t = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$q_0 = [0.5\ 0.5\ 0.5\ 0.5]^T$$

The quaternion measurements were corrupted with noise equivalent to 40 arc seconds root-sum-squared (RSS) in Euler angles to correspond to the MAP star tracker accuracy and the gyro measurements were corrupted with Gaussian noise of 0.016 deg/hr (1-σ).

Estimation Steps

The core of the estimation procedure is the evaluation of the matrix of residuals $\boldsymbol{F(X)}$:

- Offset $\boldsymbol{M}, \boldsymbol{A}$, $\boldsymbol{b}$ and $\boldsymbol{q_0}$ from the true values to get the initial state vector $\boldsymbol{X}_0$
- Calculate the gyro rates using $\boldsymbol{S}=\boldsymbol{A}\,\boldsymbol{V}$, $\omega = \boldsymbol{M\,S\,N} - \boldsymbol{b}$ where $\boldsymbol{N}$ is the vector of telemetry counts
- Starting from $\boldsymbol{q_0}$, use Equation (6) above to get the propagated attitude quaternions, $\boldsymbol{q_p}(t)$, at the sampling intervals and normalize it at every time instant.
- Compute the residual vectors via Equation (7) at times $t_1, \ldots\ t_m$
- Update the 28x1 state vector $\boldsymbol{X}$ using the L-M algorithm via the MATLAB function 'NLSQ'.

This procedure is repeated till the convergence criterion of the L-M method is achieved. In order to scale the independent variables and make their range of variation uniform across the state vector, the components of the bias vector and the coefficients of the scale factors are multiplied by 10^{-6} before using them in the cost function evaluation. It was found that without this, the problem becomes ill-conditioned and the procedure terminates.

DISCUSSION OF RESULTS

Numerous estimation runs were made for widely ranging values of the a priori state vector X_0. Convergence close to the true value was obtained consistently. The following is a typical result:

$$\boldsymbol{b} = [\ 0.51543 \quad 0.5052 \quad 0.4954\ ] \text{deg/hr}$$

$$\boldsymbol{M} = \begin{bmatrix} -1.00000000000000 & 0.00000000131127 & -0.00000000043832 \\ 0.00000000160429 & -1.00000000000000 & -0.00000000916929 \\ 0.00000000030366 & -0.00000001133398 & -1.00000000000000 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 0.44509621842136 & 0.00032942235090 & 0.00001491460493 & -0.00000080505695 \\ 0.44509603800549 & 0.00032951201204 & 0.00001499181614 & -0.00000080575904 \\ 0.44509511127285 & 0.00033038158483 & 0.00001508609330 & -0.00000097751931 \end{bmatrix} 10^{-5}$$

$$\boldsymbol{q_0} = [\ 0.500001\ 0.500008\ 0.49994\ 0.49995]^T$$

The biases agree to an accuracy of .015 deg/hr and the products of misalignment and scale factor matrices match to an accuracy of 10^{-10}, which corresponds to 0.017% error. Figure 1 gives the plot of the errors between estimated and true Euler angles and Figure 2 gives the deviations plot of the components of the products of the M and S matrices defined in equation (1) as a function of time (i.e., as temperature varies).

It was found that the solutions were independent of the initial guesses. Some of the initial guesses tried were: 0.9 for all diagonal elements of $\boldsymbol{M}$, 0.1 for all diagonal elements of $\boldsymbol{M}$, zero for all components of $\boldsymbol{b}$, 2 deg/hr for all components of $\boldsymbol{b}$, zero for all scale factor coefficients and 1.0 for all scale factor coefficients and different combinations thereof. Results differing only in the 12^{th} or 13^{th} significant figure were obtained with these initial guesses, which is essentially due to the limitation of the machine precision. Runs were also made not including measurements during maneuver times and the resulting accuracies did not vary significantly.

Further, estimation was carried out with only constant scale factors in the state vector and setting all other coefficients to zero as in the conventional gyro calibration methods. As seen in Figure 3, which gives the errors in the Euler angles for this case, as compared to those of the temperature-dependent gyro calibration case above, the improvement in accuracy is significant due to the enhanced state vector. Note that the asterisks in this figure represent the same data as in Figure 1, but on a different scale.

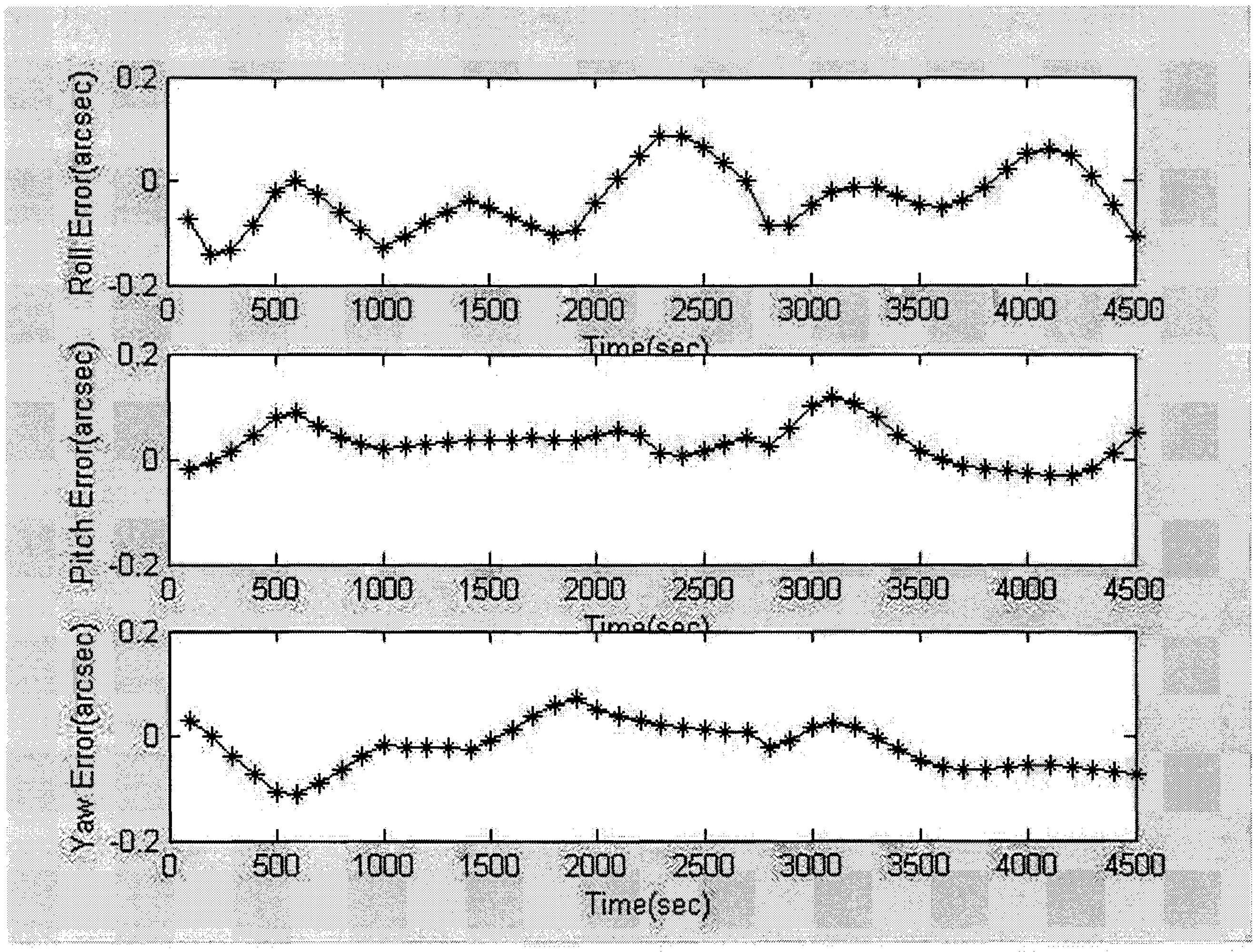

Fig 1. Euler Angle Error Plot After Gyro Calibration

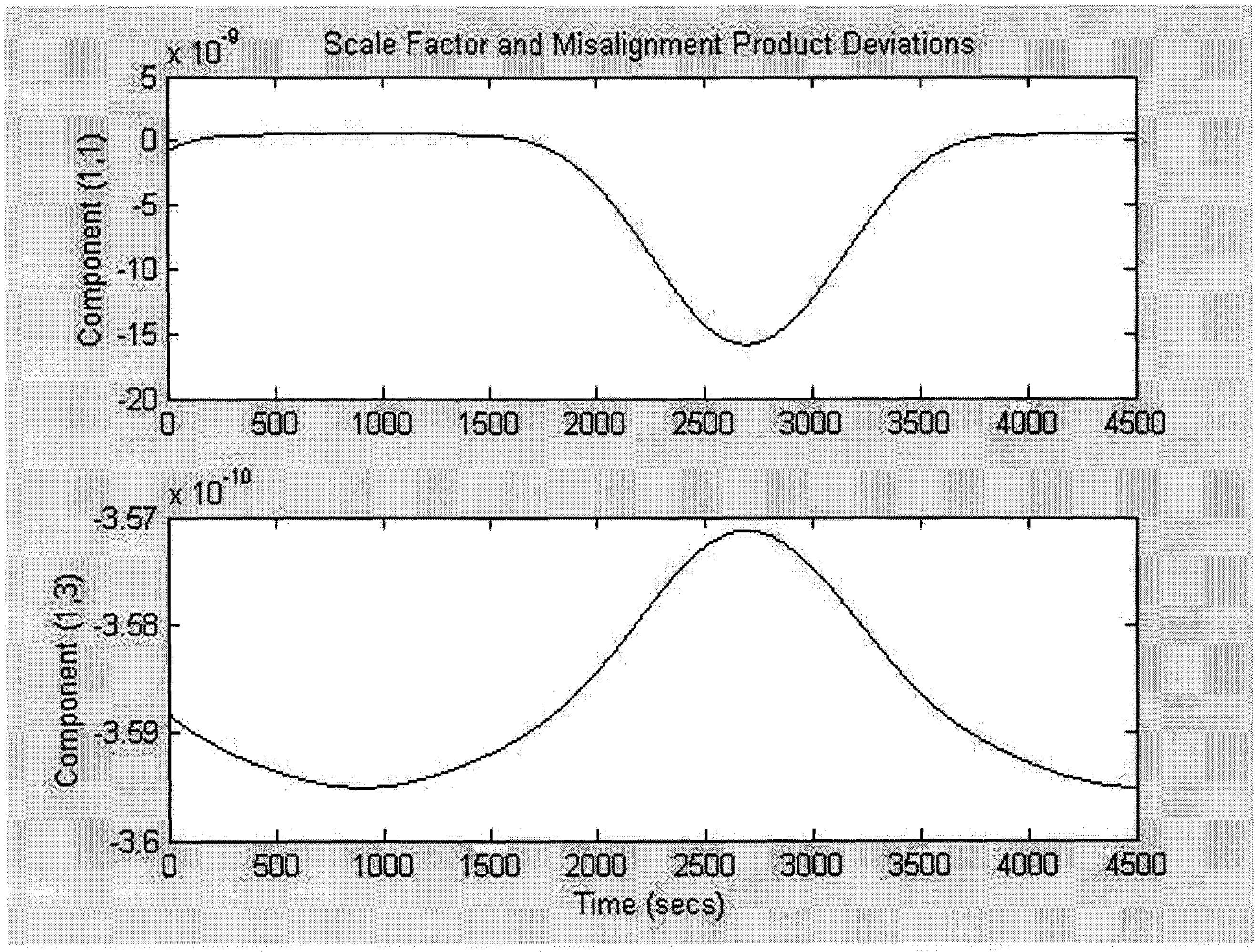

Fig. 2. Deviations Plot of the Components of the Products of Misalignment and Scale Factors

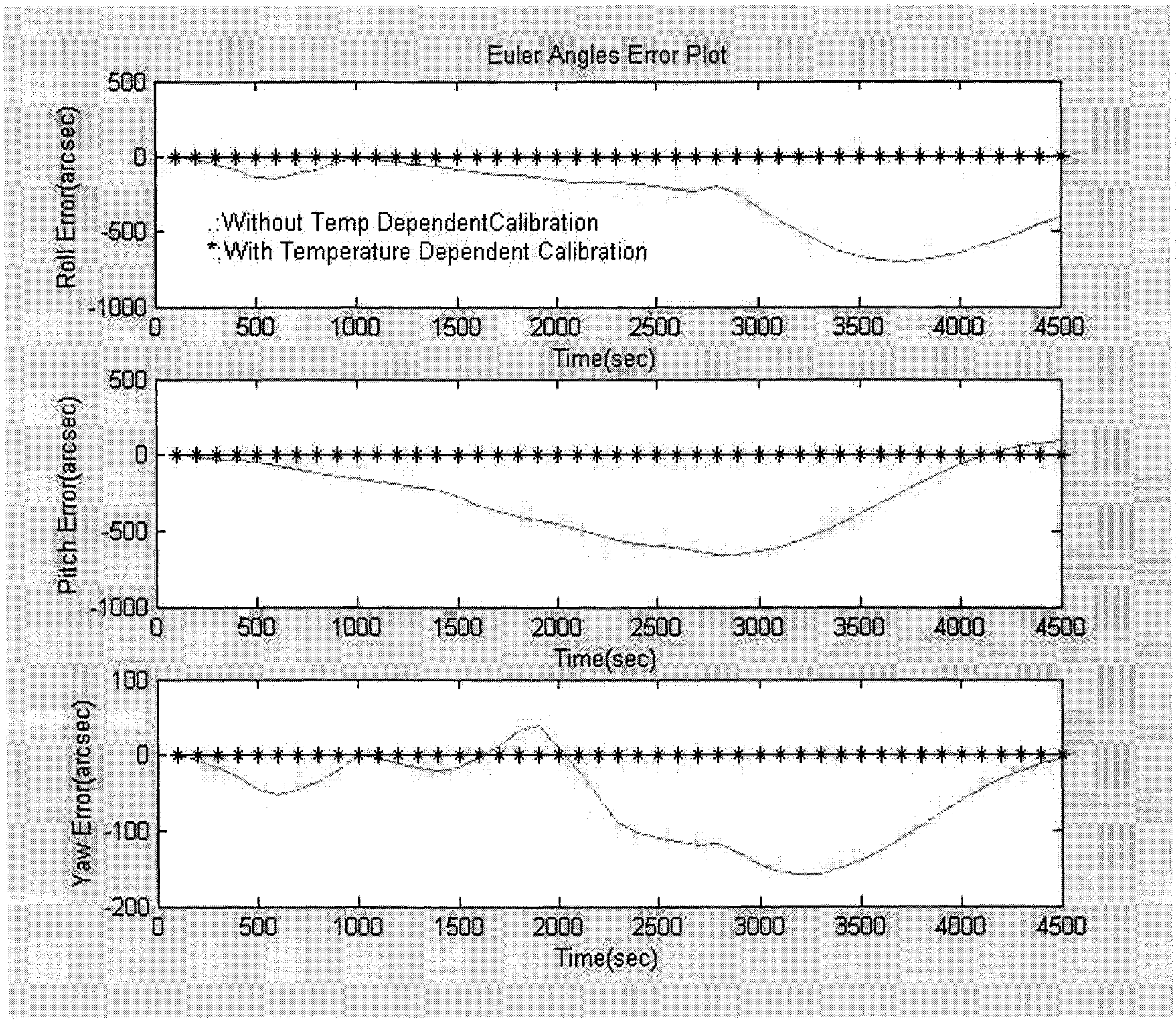

Fig 3. Euler Angle Errors Improvement Due to Temperature-Dependent Gyro Calibration

CONCLUSIONS

A gyro calibration problem with temperature dependent scale factors was formulated. The Levenberg-Marquardt algorithm in the MATLAB function NLSQ was used to solve the resulting least squares problem. Encouraging results using simulated data, conforming to the MAP mission, show the feasibility of applying it for recovering the spacecraft gyro scale factors. The advantage of this approach is that no partial derivatives of the cost function with respect to the state vector are needed. This helps to augment or remove components from the state vector very easily. Also, since the entire state vector is estimated using a single span of data, no operator intervention is needed.

SYMBOLS

$\boldsymbol{\omega}$	Vector of gyro rates
$\boldsymbol{M}$	Misalignment matrix
$\boldsymbol{S}$	Temperature-dependent scale factors
$\boldsymbol{N}$	Gyro telemetry counts
$\boldsymbol{b}$	Bias vector
$\boldsymbol{X}$	State vector
a_{ij}	Scale factor coefficients
$\boldsymbol{q}_0$	Epoch quaternion
$\boldsymbol{\Omega}$	Matrix of gyro rate components
$f(X)$	Vector of residuals
$\boldsymbol{q}_t$	True quaternion
$\boldsymbol{q}_p$	Propagated quaternion
$\delta\boldsymbol{q}$	Quaternion error
Δt	Time interval

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