

A NONLINEAR SPACECRAFT ATTITUDE CONTROLLER AND OBSERVER WITH AN UNKNOWN CONSTANT GYRO BIAS AND GYRO NOISE

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ABSTRACT

A nonlinear control scheme for attitude control of a spacecraft is combined with a nonlinear gyro bias observer for the case of constant gyro bias, in the presence of gyro noise. The observer bias estimates converge exponentially to a mean square bound determined by the standard deviation of the gyro noise. The resulting coupled, closed loop dynamics are proven to be globally stable, with asymptotic tracking which is also mean square bounded. A simulation of the proposed observer-controller design is given for a rigid spacecraft tracking a specified, time-varying attitude sequence to illustrate the theoretical claims.

INTRODUCTION

Combined observer-controller designs for the attitude control of rigid flight vehicles are a subject of active research^{1,2}. Successful design of such architectures is complicated by the fact that there is, in general, no separation principle for nonlinear systems. In contrast to linear systems, “certainty equivalence” substitution of the states from even an exponentially converging observer into a nominally stabilizing state feedback control law does not necessarily guarantee stable closed-loop operation for the coupled systems^{3,4}.

However, in reference 2, a separation principle is found to exist for the problem of forcing the attitude of a rigid vehicle to asymptotically track a (time-varying) reference attitude using feedback from sensors with persistent nonzero bias errors. A persistency of excitation argument demonstrated that the bias estimates provided by the observer are exponentially convergent to the true bias values. Reference 2 also proves that the certainty equivalence use of the observer bias estimates in the nonlinear feedback control algorithm proposed in reference 5 resulted in a stable closed-loop operation, with asymptotically perfect tracking.

Here we extend the analysis of reference 2 to include noise in the gyro reading. The Converse Lyapunov Theorem demonstrates that in this case the bias estimates provided by the observer converge exponentially to a mean square bound proportional to the variance of the noise. We then consider the certainty equivalence use of these observer estimates in the nonlinear feedback control algorithm and show that, as in reference 2, the perturbation introduced by this strategy into the closed-loop dynamics can be represented as a bounded function of the vehicle states multiplying the observer transients and the noise. We demonstrate that the stability properties of the controller are, in fact, maintained in the face of the perturbations, with asymptotic tracking to a mean square bound.

The paper is organized as follows. Section II contains definitions of the terms used in the controller and observer. In Section III the nonlinear observer for the constant gyro bias with added noise is developed and the convergence is proven. Section IV presents the nonlinear controller design and the proof of stability of the closed loop system and the convergence of the tracking errors. Section V presents simulation results, followed by conclusions in Section VI.

DEFINITIONS

The attitude of a spacecraft can be represented by a four component quaternion, consisting of a rotation angle and unit rotation vector, known as the Euler axis

$$\mathbf{q} = \begin{bmatrix} \mathbf{e} \sin \frac{\phi}{2} \\ \cos \frac{\phi}{2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\varepsilon} \\ \eta \end{bmatrix}$$

where ϕ is the rotation angle, \mathbf{e} is the Euler axis, $\boldsymbol{\varepsilon}$ and η are the vector and scalar portions of the quaternion, respectively. Note that $\|\mathbf{q}\| = 1$ by definition. The quaternion represents the rotation from an inertial coordinate system to the spacecraft body coordinate system. A rotation matrix can be computed from the quaternion as⁶

$$\mathbf{R}(\mathbf{q}) = (\eta^2 - \boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon})\mathbf{I} + 2\boldsymbol{\varepsilon} \boldsymbol{\varepsilon}^T - 2\eta\mathbf{S}(\boldsymbol{\varepsilon})$$

where $\mathbf{S}(\boldsymbol{\varepsilon})$ is a cross product matrix formed from the vector $\boldsymbol{\varepsilon}$.

$$\mathbf{S}(\boldsymbol{\varepsilon}) = \begin{bmatrix} 0 & -\varepsilon_z & \varepsilon_y \\ \varepsilon_z & 0 & -\varepsilon_x \\ -\varepsilon_y & \varepsilon_x & 0 \end{bmatrix}$$

A desired target attitude is represented by the quaternion, $\mathbf{q}_d^T = [\boldsymbol{\varepsilon}_d^T, \eta_d]$. The attitude error used in the controller is defined as a rotation from the desired body frame to the actual body frame and is computed according to⁷

$$\tilde{\mathbf{q}}_c = \begin{bmatrix} \tilde{\boldsymbol{\varepsilon}}_c \\ \tilde{\eta}_c \end{bmatrix} = \mathbf{q} \otimes \mathbf{q}_d^{-1} = \begin{bmatrix} \eta_d \mathbf{I} - \mathbf{S}(\boldsymbol{\varepsilon}_d) & -\boldsymbol{\varepsilon}_d \\ \boldsymbol{\varepsilon}_d^T & \eta_d \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \eta \end{bmatrix}$$

Similarly, in the observer, the attitude error is defined as the rotation from the estimated body frame to the actual body frame as

$$\tilde{\mathbf{q}}_o = \begin{bmatrix} \tilde{\boldsymbol{\varepsilon}}_o \\ \tilde{\eta}_o \end{bmatrix} = \mathbf{q} \otimes \hat{\mathbf{q}}^{-1} = \begin{bmatrix} \hat{\eta} \mathbf{I} - \mathbf{S}(\hat{\boldsymbol{\varepsilon}}) & -\hat{\boldsymbol{\varepsilon}} \\ \hat{\boldsymbol{\varepsilon}}^T & \hat{\eta} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \eta \end{bmatrix} \quad (1)$$

where $\hat{\mathbf{q}}$ represents the attitude state of the observer. Note that $\tilde{\boldsymbol{\varepsilon}}_c = 0$, $\tilde{\eta}_c = \pm 1$ indicates that the spacecraft is aligned with the desired attitude and, similarly, $\tilde{\boldsymbol{\varepsilon}}_o = 0$, $\tilde{\eta}_o = \pm 1$ indicates that the attitude estimate is aligned with the actual attitude.

The kinematics equation for the quaternion is given as

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{\boldsymbol{\varepsilon}} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \eta \mathbf{I} + \mathbf{S}(\boldsymbol{\varepsilon}) \\ -\boldsymbol{\varepsilon}^T \end{bmatrix} \boldsymbol{\omega} = \frac{1}{2} \mathbf{Q}(\mathbf{q}) \boldsymbol{\omega}$$

where $\boldsymbol{\omega}$ is the spacecraft angular velocity. The angular velocity is typically measured by a gyro, which can be corrupted with both systematic and random errors. In the case of gyro bias and random noise, the gyro reading, $\boldsymbol{\omega}_g$, can be written as

$$\boldsymbol{\omega}_g = \boldsymbol{\omega} + \mathbf{b} + \mathbf{v}(t)$$

where $\boldsymbol{\omega}$ is the true angular velocity, \mathbf{b} is the gyro bias (which in this work is treated without noise), and $\mathbf{v}(t)$ is an added noise. An estimate of the angular velocity is given as $\hat{\boldsymbol{\omega}} = \boldsymbol{\omega}_g - \hat{\mathbf{b}}$. The bias error is defined as the difference between the true and estimated bias

$$\tilde{\mathbf{b}} = \mathbf{b} - \hat{\mathbf{b}} \quad (2)$$

Finally, a measure of the discrepancy between the actual and desired angular velocity in the controller is computed as⁷

$$\tilde{\boldsymbol{\omega}}_c = \boldsymbol{\omega} - \mathbf{R}(\tilde{\mathbf{q}}_c)\boldsymbol{\omega}_d \quad (3)$$

which is defined such that $\tilde{\mathbf{q}}_c = \frac{1}{2}\mathbf{Q}(\tilde{\mathbf{q}}_c)\tilde{\boldsymbol{\omega}}_c$.

NONLINEAR OBSERVER FOR CONSTANT GYRO BIAS

Following the development of reference 1, a state observer for the bias can be defined as

$$\dot{\hat{\mathbf{q}}} = \frac{1}{2}\hat{\mathbf{Q}}(\hat{\mathbf{q}})\mathbf{R}^T(\tilde{\mathbf{q}}_o)(\boldsymbol{\omega}_g - \hat{\mathbf{b}} + k\tilde{\boldsymbol{\epsilon}}_o \operatorname{sgn}(\tilde{\eta}_o)) \quad (4)$$

$$\dot{\hat{\mathbf{b}}} = -\frac{1}{2}\tilde{\boldsymbol{\epsilon}}_o \operatorname{sgn}(\tilde{\eta}_o) \quad (5)$$

The gain, k , is chosen as a positive constant. The $\mathbf{R}^T(\tilde{\mathbf{q}}_o)$ resolves the angular velocity terms in the observer frame.

Computing the derivatives of $\tilde{\mathbf{q}}_o$ in (1) and $\tilde{\mathbf{b}}$ in (2) results in the following differential error equations

$$\dot{\tilde{\mathbf{q}}}_o = \begin{bmatrix} \dot{\tilde{\boldsymbol{\epsilon}}}_o \\ \dot{\tilde{\eta}}_o \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \tilde{\eta}_o \mathbf{I} + \mathbf{S}(\tilde{\boldsymbol{\epsilon}}_o) \\ -\tilde{\boldsymbol{\epsilon}}_o^T \end{bmatrix} \begin{bmatrix} -\tilde{\mathbf{b}} - \mathbf{v}(t) - k\tilde{\boldsymbol{\epsilon}}_o \operatorname{sgn}(\tilde{\eta}_o) \\ \tilde{\boldsymbol{\epsilon}}_o \end{bmatrix} \quad (6)$$

$$\dot{\tilde{\mathbf{b}}} = \frac{1}{2}\tilde{\boldsymbol{\epsilon}}_o \operatorname{sgn}(\tilde{\eta}_o) \quad (7)$$

The above error equations are rewritten as

$$\begin{bmatrix} \dot{\tilde{\mathbf{q}}}_o \\ \dot{\tilde{\mathbf{b}}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\tilde{\mathbf{Q}}(\tilde{\mathbf{q}}_o)(-\tilde{\mathbf{b}} - k\tilde{\boldsymbol{\epsilon}}_o \operatorname{sgn}(\tilde{\eta}_o)) \\ \frac{1}{2}\tilde{\boldsymbol{\epsilon}}_o \operatorname{sgn}(\tilde{\eta}_o) \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}\tilde{\mathbf{Q}}(\tilde{\mathbf{q}}_o)\mathbf{v}(t) \\ 0 \end{bmatrix} \quad (8)$$

The system in (8) is divided into the nominal system of reference 2 plus a perturbation

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}) + \mathbf{D}(t)$$

where $\mathbf{x}^T = \begin{bmatrix} \tilde{\boldsymbol{\epsilon}}_o^T & \tilde{\mathbf{b}}^T \end{bmatrix}$ and $\mathbf{D}(t)^T = \begin{bmatrix} -\frac{1}{2}[\tilde{\eta}_o \mathbf{I} + \mathbf{S}(\tilde{\boldsymbol{\epsilon}}_o)]\mathbf{v}(t) & 0 \end{bmatrix}$. Through a persistency of excitation argument, the nominal system $\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x})$ is proven to be exponentially stable². Therefore, according to the Converse Lyapunov Theorem³, a Lyapunov function and positive, finite constants c_1 , c_2 , c_3 , and c_4 exist and satisfy the following

$$c_1\|\mathbf{x}\|^2 \leq V_o \leq c_2\|\mathbf{x}\|^2$$

$$\begin{aligned}\dot{V}_o &\leq -c_3 \|\mathbf{x}\|^2 \\ \left\| \frac{\partial V_o}{\partial \mathbf{x}} \right\| &\leq c_4 \|\mathbf{x}\|\end{aligned}$$

The perturbed Lyapunov function then satisfies

$$\dot{V}_o \leq -c_3 \|\mathbf{x}\|^2 + \left\| \frac{\partial V_o}{\partial \mathbf{x}} \right\| \|D\| \quad (9)$$

Since $\tilde{Q}^T \tilde{Q} = I$, $\|D\| = \frac{1}{2} \|\mathbf{v}\|$ and (9) becomes

$$\dot{V}_o \leq -c_3 \|\mathbf{x}\|^2 + \frac{c_4}{2} \|\mathbf{x}\| \|\mathbf{v}\| \quad (10)$$

If $\mathbf{v}(t)$ is uniformly bounded, the system is globally stable. The state $\mathbf{x}(t)$ converges exponentially to a ball determined by the bound on $\mathbf{v}(t)$, and then remains within that ball³.

Consider the case that the noise $\mathbf{v}(t)$ is a bounded, zero mean, wide sense stationary (WSS) process with a mean square value of $\sigma^2 I$. Applying Young's inequality⁴ to (10) results in

$$\dot{V}_o \leq -\frac{c_3}{2} \|\mathbf{x}\|^2 + \frac{c_4}{2c_3} \|\mathbf{v}\|^2 \quad (11)$$

The time average of (11) is computed as

$$\frac{1}{T} \int_0^T \|\mathbf{x}\|^2 d\tau \leq \frac{c_4}{c_3^2} \frac{1}{T} \int_0^T \|\mathbf{v}\|^2 d\tau + \frac{2}{c_3 T} [V_o(0) - V_o(T)]$$

Taking the limit as $T \rightarrow \infty$

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \|\mathbf{x}\|^2 d\tau \leq \lim_{T \rightarrow \infty} \frac{c_4}{c_3^2} \frac{1}{T} \int_0^T \|\mathbf{v}\|^2 d\tau = \frac{c_4}{c_3^2} 3\sigma^2$$

The root mean square (RMS) bound is then given as

$$\|\tilde{\mathbf{b}}\|_{\text{RMS}} \leq \|\mathbf{x}\|_{\text{RMS}} \leq \sqrt{3c_4} \frac{\sigma}{c_3}$$

NONLINEAR CONTROLLER DESIGN

The complete attitude dynamics for a rigid spacecraft are given as

$$\begin{aligned}H\dot{\boldsymbol{\omega}} - S(H\boldsymbol{\omega})\boldsymbol{\omega} &= \mathbf{u} \\ \dot{\mathbf{q}} &= \frac{1}{2} Q(\mathbf{q})\boldsymbol{\omega}\end{aligned}$$

where H is a constant, symmetric inertia matrix and \mathbf{u} is the applied external torque, for example, from attached rocket thrusters. The goal of the controller is for the actual, measured attitude $\mathbf{q}(t)$ to asymptotically track a

(generally) time-varying desired attitude $\mathbf{q}_d(t)$ and angular velocity $\boldsymbol{\omega}_d(t)$, related for consistency by $\dot{\mathbf{q}}_d = \frac{1}{2}\mathbf{Q}(\mathbf{q}_d)\boldsymbol{\omega}_d$. It is assumed that $\boldsymbol{\omega}_d(t)$ is bounded and differentiable with $\dot{\boldsymbol{\omega}}_d(t)$ also bounded.

The passivity based controller of reference 5 utilizes the composite error metric

$$\mathbf{s} = \tilde{\boldsymbol{\omega}}_c + \lambda \tilde{\boldsymbol{\epsilon}}_c = \boldsymbol{\omega} - \boldsymbol{\omega}_r \quad (12)$$

where from (3), $\boldsymbol{\omega}_r = \mathbf{R}(\tilde{\mathbf{q}}_c)\boldsymbol{\omega}_d - \lambda \tilde{\boldsymbol{\epsilon}}_c$, $\lambda > 0$. Taking the derivative of (12) and multiplying by H results in

$$\mathbf{H}\dot{\mathbf{s}} = \mathbf{H}\dot{\boldsymbol{\omega}} - \mathbf{H}\dot{\boldsymbol{\omega}}_r = \mathbf{u} + \mathbf{S}(\mathbf{H}\boldsymbol{\omega})\boldsymbol{\omega} - \mathbf{H}\boldsymbol{\alpha}_r \quad (13)$$

where

$$\boldsymbol{\alpha}_r = \dot{\boldsymbol{\omega}}_r = \mathbf{R}(\tilde{\mathbf{q}}_c)\dot{\boldsymbol{\omega}}_d - \mathbf{S}(\tilde{\boldsymbol{\omega}}_c)\mathbf{R}(\tilde{\mathbf{q}}_c)\boldsymbol{\omega}_d - \lambda \mathbf{Q}_1(\tilde{\mathbf{q}}_c)\tilde{\boldsymbol{\omega}}_c$$

and $\mathbf{Q}_1(\tilde{\mathbf{q}}_c) = \tilde{\eta}_c \mathbf{I} + \mathbf{S}(\tilde{\boldsymbol{\epsilon}}_c)$ as defined above. With these definitions, the control law

$$\mathbf{u} = -\mathbf{K}_D \mathbf{s} + \mathbf{H}\boldsymbol{\alpha}_r - \mathbf{S}(\mathbf{H}\boldsymbol{\omega})\boldsymbol{\omega} \quad (14)$$

for any symmetric, positive definite \mathbf{K}_D results in closed-loop dynamics

$$\mathbf{H}\dot{\mathbf{s}} - \mathbf{S}(\mathbf{H}\boldsymbol{\omega})\mathbf{s} + \mathbf{K}_D \mathbf{s} = 0$$

As shown in reference 5, these dynamics, together with the definition of the composite error, produces the desired stability and tracking properties.

In the current application, the control law (14) cannot be implemented because exact measurements of the angular velocity $\boldsymbol{\omega}$ are not available. Instead a certainty equivalence approach is employed using the estimates $\hat{\boldsymbol{\omega}}$ from above, resulting in

$$\mathbf{u} = -\mathbf{K}_D \hat{\mathbf{s}} + \mathbf{H}\hat{\boldsymbol{\alpha}}_r - \mathbf{S}(\mathbf{H}\hat{\boldsymbol{\omega}})\hat{\boldsymbol{\omega}}_r \quad (15)$$

where $\hat{\mathbf{s}} = \hat{\boldsymbol{\omega}} - \boldsymbol{\omega}_r$, $\hat{\boldsymbol{\omega}}_c = \hat{\boldsymbol{\omega}} - \mathbf{R}(\tilde{\mathbf{q}}_c)\boldsymbol{\omega}_d$, and

$$\hat{\boldsymbol{\alpha}}_r = \mathbf{R}(\tilde{\mathbf{q}}_c)\dot{\boldsymbol{\omega}}_d + \mathbf{S}(\mathbf{R}(\tilde{\mathbf{q}}_c)\boldsymbol{\omega}_d)\hat{\boldsymbol{\omega}}_c - \lambda \mathbf{Q}_1(\tilde{\mathbf{q}}_c)\hat{\boldsymbol{\omega}}_c.$$

Substituting (15) into (13), along with (12), and noting that $\tilde{\mathbf{s}} = \mathbf{s} - \hat{\mathbf{s}} = -\tilde{\mathbf{b}} - \mathbf{v}(t)$, $\tilde{\boldsymbol{\alpha}}_r = [-\mathbf{S}(\mathbf{R}(\tilde{\mathbf{q}}_c)\boldsymbol{\omega}_d) + \lambda \mathbf{Q}_1(\tilde{\mathbf{q}}_c)](\tilde{\mathbf{b}} + \mathbf{v}(t))$, and $\tilde{\boldsymbol{\omega}}_c - \hat{\boldsymbol{\omega}}_c = \boldsymbol{\omega} - \hat{\boldsymbol{\omega}} = -\tilde{\mathbf{b}} - \mathbf{v}(t)$ produces the closed-loop dynamics

$$\mathbf{H}\dot{\tilde{\mathbf{s}}} - \mathbf{S}(\mathbf{H}\boldsymbol{\omega})\tilde{\mathbf{s}} + \mathbf{K}_D \tilde{\mathbf{s}} = [\mathbf{S}(\boldsymbol{\omega}_r)\mathbf{H} + \mathbf{H}\mathbf{S}(\mathbf{R}(\tilde{\mathbf{q}}_c)\boldsymbol{\omega}_d) - \lambda \mathbf{H}\mathbf{Q}_1(\tilde{\mathbf{q}}_c) - \mathbf{K}_D](\tilde{\mathbf{b}} + \mathbf{v}(t)) \quad (16)$$

The terms on the right hand side of (16) can be rewritten as $\Delta(\tilde{\mathbf{q}}_c, \boldsymbol{\omega}_d)(\tilde{\mathbf{b}} + \mathbf{v}(t))$. Since $\|\tilde{\mathbf{q}}_c\| = 1$ by definition and $\|\boldsymbol{\omega}_d\| < \infty$ by assumption

$$\gamma \equiv \sup_{t \geq t_0} \sup_{\|\tilde{\mathbf{q}}_c\|=1} \|\Delta(\tilde{\mathbf{q}}_c, \boldsymbol{\omega}_d(t))\| < \infty$$

Using the Lyapunov function $V_c = \frac{1}{2} \mathbf{s}^T \mathbf{H} \mathbf{s}$, the derivative of V_c along closed-loop trajectories of (15) satisfies the inequality

$$\dot{V}_c = -\mathbf{s}^T \mathbf{K}_D \mathbf{s} + \mathbf{s}^T \mathbf{\Delta} (\tilde{\mathbf{b}} + \mathbf{v}) \leq -k_D \|\mathbf{s}\|^2 + \gamma \|\mathbf{s}\| (\|\tilde{\mathbf{b}}\| + \|\mathbf{v}\|)$$

where k_D is the smallest eigenvalue of \mathbf{K}_D . Using Young's inequality⁴ on the last term above, \dot{V}_c is rewritten as

$$\dot{V}_c \leq -\frac{k_D}{2} \|\mathbf{s}\|^2 + \frac{\gamma}{2k_D} (\|\tilde{\mathbf{b}}\|^2 + \|\mathbf{v}\|^2) \quad (17)$$

Thus, from the definition of V_c , and recalling from the observer analysis that $\|\tilde{\mathbf{b}}\|$ and $\|\mathbf{v}\|$ are bounded, \mathbf{s} is also seen to be uniformly bounded. Similarly $\dot{\mathbf{s}}$ is uniformly bounded, since all the terms in (16) are bounded. Integrating (17)

$$\int_0^T \|\mathbf{s}\|^2 dt \leq \frac{\gamma}{k_D^2} (\int_0^T \|\tilde{\mathbf{b}}\|^2 dt + \int_0^T \|\mathbf{v}\|^2 dt) + \frac{2}{k_D} (V_c(0) - V_c(T))$$

Substituting $\|\mathbf{s}\|^2 = \|\tilde{\boldsymbol{\omega}}_c\|^2 + 2\lambda \tilde{\boldsymbol{\omega}}_c^T \tilde{\boldsymbol{\epsilon}}_c + \lambda^2 \|\tilde{\boldsymbol{\epsilon}}_c\|^2$ from (12)

$$\int_0^T \|\tilde{\boldsymbol{\epsilon}}_c\|^2 dt \leq \frac{\gamma}{k_D^2 \lambda^2} (\int_0^T \|\tilde{\mathbf{b}}\|^2 dt + \int_0^T \|\mathbf{v}\|^2 dt) + \frac{2}{k_D \lambda^2} (V_c(0) - V_c(T)) - \frac{2}{\lambda} \int_0^T \tilde{\boldsymbol{\omega}}_c^T \tilde{\boldsymbol{\epsilon}}_c dt \quad (18)$$

Noting that $\tilde{\boldsymbol{\omega}}_c^T \tilde{\boldsymbol{\epsilon}}_c = 2\tilde{\eta}_c$, (18) is then

$$\int_0^T \|\tilde{\boldsymbol{\epsilon}}_c\|^2 dt \leq \frac{\gamma}{k_D^2 \lambda^2} (\int_0^T \|\tilde{\mathbf{b}}\|^2 dt + \int_0^T \|\mathbf{v}\|^2 dt) + \frac{2}{k_D \lambda^2} (V_c(0) - V_c(T)) - \frac{4}{\lambda} [\tilde{\eta}(T) - \tilde{\eta}(0)] \quad (19)$$

Computing the time average of (19) and taking the limit as $T \rightarrow \infty$

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \|\tilde{\boldsymbol{\epsilon}}_c\|^2 dt \leq \lim_{T \rightarrow \infty} \frac{\gamma}{k_D^2 \lambda^2} \frac{1}{T} (\int_0^T \|\tilde{\mathbf{b}}\|^2 dt + \int_0^T \|\mathbf{v}\|^2 dt) \leq \frac{\gamma^2}{k_D^2 \lambda^2} \left[\frac{c_4}{c_3^2} 3\sigma^2 + 3\sigma^2 \right]$$

The RMS limit of the tracking error is then

$$\|\tilde{\boldsymbol{\epsilon}}_c\|_{\text{RMS}} \leq \frac{\sqrt{3\gamma\sigma}}{k_D \lambda} \left[\frac{c_4}{c_3^2} + 1 \right]^{\frac{1}{2}}$$

SIMULATION RESULTS

The spacecraft attitude controller/observer design is tested with a Matlab simulation. The inertia matrix is a diagonal matrix with principal moments of inertia of $[90, 100, 70]^T$ kg-m². Table I lists the initial conditions for the observer and controller, as well as the true gyro bias, true initial angular velocity, and desired angular velocity. The gains are chosen as $k=1$, $\mathbf{K}_D=k_D \mathbf{I}$ with $k_D=10$, and $\lambda=3$. The standard deviation of the gyro noise was first set to 0.57 deg/sec and then to 0.057 deg/sec.

Figures 1 and 2 show the observer bias errors with the two different standard deviations for the gyro noise, respectively. In each figure the top plot shows $\|\tilde{\mathbf{b}}(t)\|$ and the bottom plot shows the $\|\tilde{\mathbf{b}}(t)\|_{\text{RMS}}$. In Figure 1,

$\|\tilde{\mathbf{b}}(t)\|$ converges to less than 0.3 deg/sec, and in Figure 2 to less than 0.03 deg/sec. In both cases, the RMS errors are less than the standard deviation of the gyro noise.

Figures 3 and 4 show the attitude tracking error from the controller, again for the two different standard deviations of the gyro noise. In each figure the upper plot shows the angular error and the lower plot shows $\|\tilde{\mathbf{e}}_c(t)\|_{\text{RMS}}$. In Figure 3 the attitude error converges to less than 0.4 degrees and to less than 0.04 degrees in Figure 4. Note that the RMS error of the vector part of the quaternion is plotted with the standard deviation of the gyro noise (converted to rad/sec) for comparison of the magnitudes only.

Table I. Simulation Initial and True Values

Variable	Initial Value
$\hat{\mathbf{q}}$	$[0, 1, 0, 0]^T$
\mathbf{q}	$[0, 0, 1, 0]^T$
\mathbf{q}_d	$[0, 0, 0, 1]^T$
$\hat{\mathbf{b}}$	$[0, 0, 0]^T$ deg/sec
$\boldsymbol{\omega}$ - true	$[-5.7, 11.4, -22.9]^T$ deg/sec
\mathbf{b} - true	$[2.9, -2.9, 1.9]^T$ deg/sec
$\boldsymbol{\omega}_d$	$[0, 6.3, 0]^T$ deg/sec

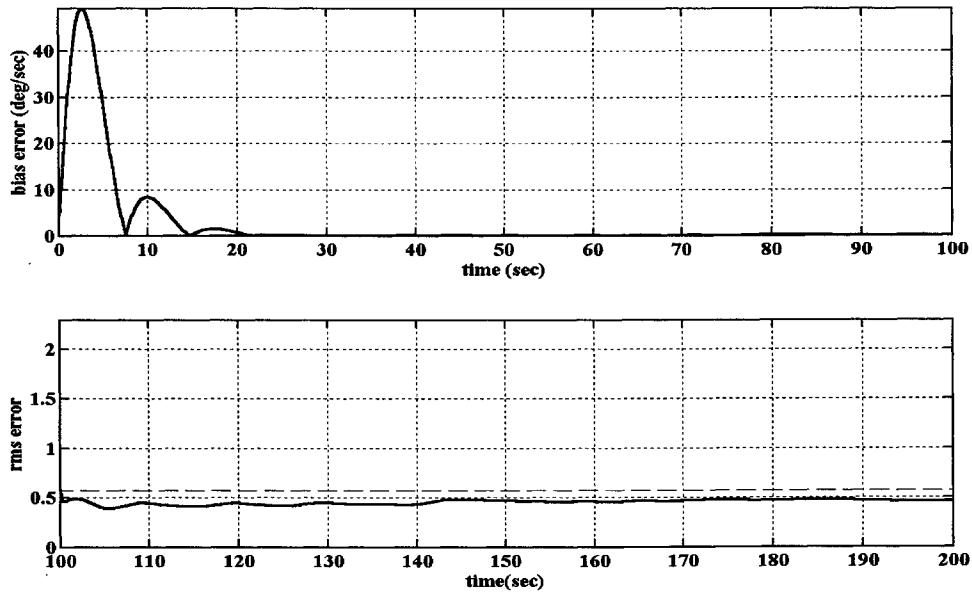


Figure 1. Bias Errors - $\|\tilde{\mathbf{b}}(t)\|$ and $\|\tilde{\mathbf{b}}(t)\|_{\text{RMS}}$ (solid lines), and $\sigma=0.57$ deg/sec (dashed line)

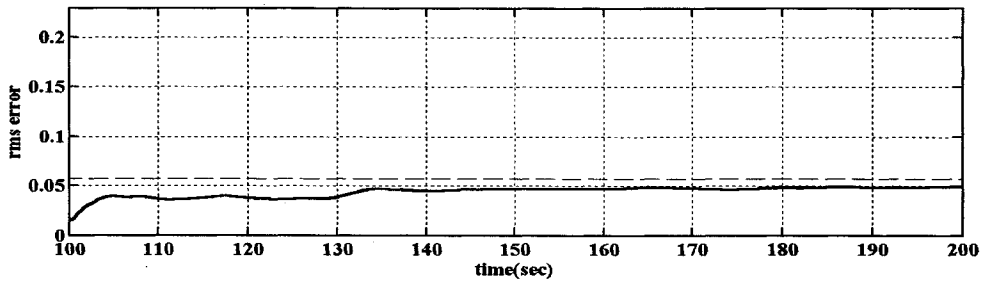
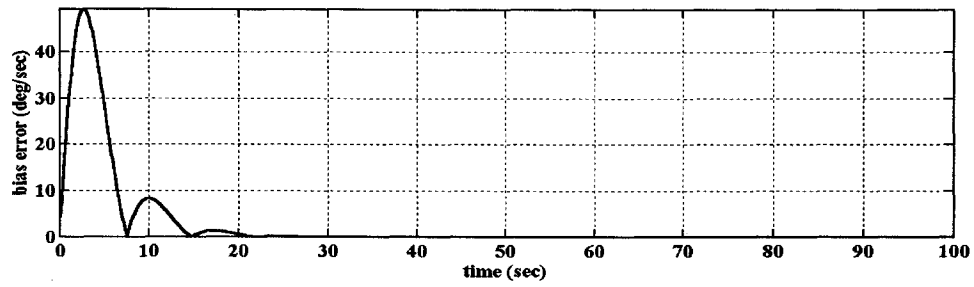


Figure 2. Bias Errors - $\|\tilde{\mathbf{b}}(t)\|$ and $\|\tilde{\mathbf{b}}(t)\|_{\text{RMS}}$ (solid lines), and $\sigma=0.057$ deg/sec (dashed line)

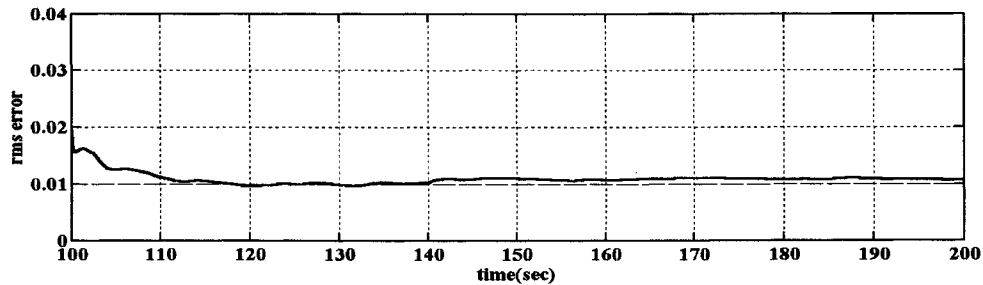
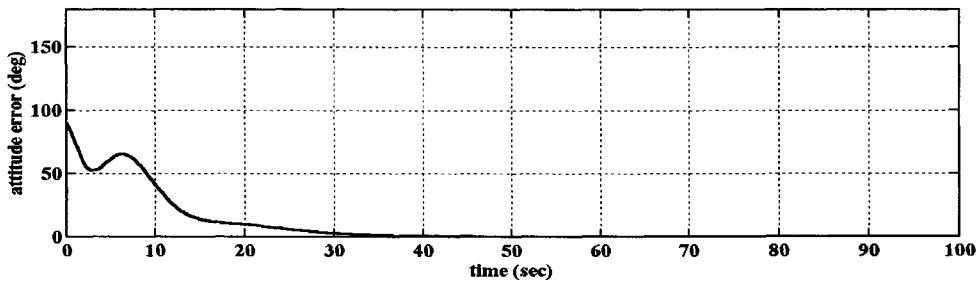


Figure 3. Attitude Tracking Error, $\|\tilde{\mathbf{e}}_c(t)\|_{\text{RMS}}$ (solid lines), and $\sigma=0.01$ rad/sec (dashed line)

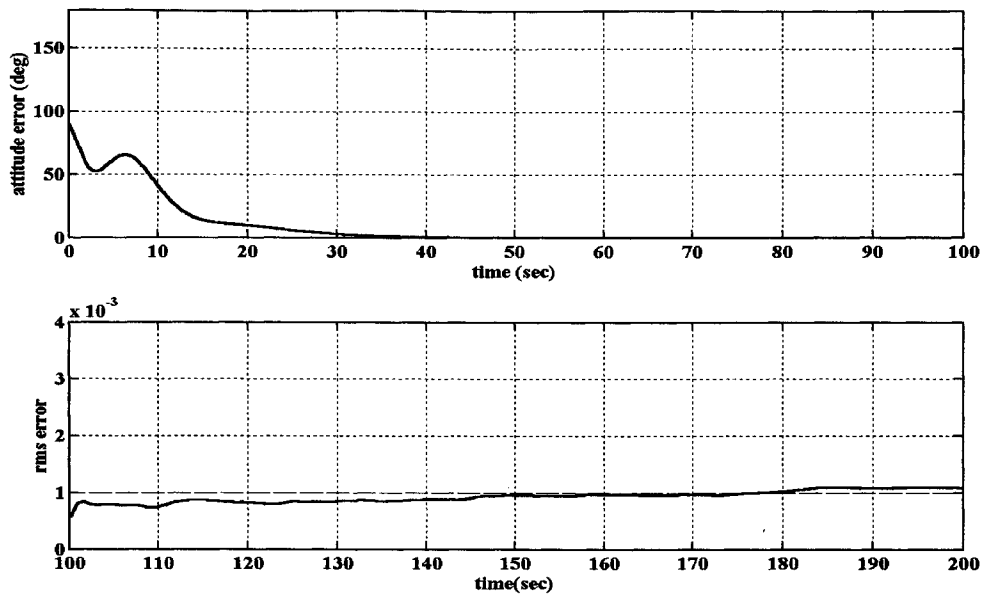


Figure 4. Attitude Tracking Error, $\|\tilde{e}_c(t)\|_{\text{RMS}}$ (solid lines), and $\sigma=0.001$ rad/sec (dashed line)

CONCLUSIONS

A nonlinear controller/observer is presented for spacecraft attitude applications, given a constant gyro bias and gyro noise. The gyro bias estimates converge exponentially to a mean square bound determined by the standard deviation of the noise, relying on a persistency of excitation argument, which proves asymptotic identification of the gyro bias in the absence of noise. The nonlinear controller is a passivity based controller. The control input requires the use of the gyro bias estimate from the nonlinear observer. The closed loop stability properties of this nonlinear controller coupled with the nonlinear observer are analyzed and the system is found to be globally stable, leading to a separation principle for the nonlinear system, with asymptotic tracking to a mean square bound.

REFERENCES

1. H. Nijmeijer and T.I. Fossen, *Lecture Notes in Control and Information Sciences, New Directions in Nonlinear Observer Design*, Springer-Verlag, 1999, pp. 135-159.
2. J. Deutschmann and R.M. Sanner, "A Coupled Nonlinear Spacecraft Attitude Controller/Observer With an Unknown Constant Gyro Bias", submitted to the IEEE Conference on Decision and Control, paper no. CDC01-REG1379, Dec. 4-7, 2001, Orlando, FL.
3. H. Khalil, *Nonlinear Systems*, Prentice-Hall, Inc., 1996.
4. M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, Inc., 1995.
5. O. Eglund and J.M. Godhavn, "Passivity-Based Adaptive Attitude Control of a Rigid Spacecraft", *IEEE Transactions on Automatic Control*, Vol. 39, No. 4, April 1994, pp. 842-846.
6. B. Wie, *Space Vehicle Dynamics and Control*, AIAA Education Series, 1998.
7. R.M. Sanner, "Adaptive Attitude Control Using Fixed and Dynamically Structured Neural Networks", AIAA Guidance, Navigation and Control Conference, San Diego, CA, July 29-31, 1996.