STATE-DEPENDENT PSEUDO-LINEAR FILTER FOR SPACECRAFT ATTITUDE AND RATE ESTIMATION

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ABSTRACT

This paper presents the development and performance of a special algorithm for estimating the attitude and angular rate of a spacecraft. The algorithm is a pseudo-linear Kalman filter, which is an ordinary linear Kalman filter that operates on a linear model whose matrices are current state estimate dependent. The nonlinear rotational dynamics equation of the spacecraft is presented in the state space as a state-dependent linear system. Two types of measurements are considered. One type is a measurement of the quaternion of rotation, which is obtained from a newly introduced star tracker based apparatus. The other type of measurement is that of vectors, which permits the use of a variety of vector measuring sensors like sun sensors and magnetometers. While quaternion measurements are related linearly to the state vector, vector measurements constitute a nonlinear function of the state vector. Therefore, in this paper, a state-dependent linear measurement equation is developed for the vector measurement case. The state-dependent pseudo linear filter is applied to simulated spacecraft rotations and adequate estimates of the spacecraft attitude and rate are obtained for the case of quaternion measurements as well as of vector measurements.

INTRODUCTION

Precise angular-rate is required for spacecraft (SC) attitude determination, and a coarse rate is needed for attitude control damping. Classically, angular-rate information is obtained from gyros; however, these days, there is a tendency to build smaller, lighter and cheaper SC where rate accuracy can be compromised. Therefore, the inclination now is to do away with gyros and use other means to determine the SC angular-rate. In fact other means are needed even in gyro-equipped satellites when the angular-rate is out of range of the SC gyros.

There are several ways to obtain the angular-rate in a gyro-less SC. When the attitude is known, one can differentiate the attitude in whatever parameters it is given and use the kinematics equation that connects the derivative of the attitude with the satellite angular-rate in order to compute the latter (ref. 1, 2). However, the differentiation of the attitude introduces a considerable noise component in the computed angular-rate vector. To overcome this noise, one can use an active filter, like a Kalman filter (KF) (ref. 3, 4).

All these methods use the derivative of either the attitude parameters or of the measured directions, which normally determine the attitude parameters. Another approach is that of using the attitude parameters, or the measured directions themselves, as measurements in a KF. The dynamics model of that KF also includes the SC angular dynamics equation, which is a nonlinear first order vector differential equation. In this case, the kinematics equation that connects the attitude parameters, or the directions, with their derivatives are included in the dynamics model used by the filter thereby the need for differentiation is eliminated (ref. 5, 6). The KF dynamics model also includes the SC angular dynamics equation, which is a nonlinear first order vector differential equation.

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New sensor packages that uses star trackers and yield the SC attitude in terms of the attitude quaternion recently became available[1]. Therefore, it became possible to use the quaternion supplied by such sensors as measurements. This approach was indeed used in a recent work (ref. 7) where the nonlinear dynamics equation was converted to a *state-dependent pseudo-linear equation* but the measurement equation was linear. The pseudo-linear dynamics model together with he linear quaternion measurement model enabled the use of PSEudo LInear KAlman (PSELIKA) filter.

While the use of quaternion measurement yielded very satisfactory results, the algorithm was limited to one type of instrumentation package, consequently, the popular vector measuring devises like sun sensors, magnetometers, horizon sensors and others could not benefit from the PSELIKA estimator for estimating attitude and angular rate.

In the present work we add a development, which enables the use of the elementary vector observations as filter measurements rather than the quaternions derived from them. On one hand, this approach allows the use of a wide range of instruments to track the SC attitude, but on the other hand, it introduces nonlinear measurement equations. However, as shown in this paper, these measurement equations can be transformed into a state-dependent pseudo-linear equation thereby permitting the use of the PSELIKA filter.

In the next section we briefly describe the development of the filter dynamics equation and, in particular, the development of the state-dependent pseudo-linear SC angular equation. Then, in the following section we present the development of the pseudo-linear measurement equation associated with vector measurements. In the section that follows this one, we present the PSELIKA filter. Next, we present computer runs for vector measurements that, as mentioned, involve pseudo-linear dynamics as well as pseudo-linear measurement equations, and compare them with the results obtained when using quaternion measurements, which involve pseudo-linear dynamics but truly linear measurement equations. Finally, in the last section we present the conclusions derived from this work.

THE STATE-DEPENDENT DYNAMICS EQUATION

The angular dynamics of a constant mass SC is given in the following equation (ref. 8, p. 523)

$$I\dot{\omega} + \dot{h} + \omega \times (I\omega + h) = T \tag{1}$$

where I is the SC inertia matrix, ω is the angular velocity vector, h is the angular momentum of the momentum wheels, and T is the external torque acting on the SC. Since the inertia matrix I is invertible we may write this equation as

$$\dot{\omega} = I^{-1}[(I\omega + h)\times]\omega + I^{-1}(T - \dot{h}) \tag{2}$$

where $[a\times]$ denotes the cross product matrix of the general vector a . Defining

$$F'(\omega) = I^{-1}[(I\omega + h)\times] \tag{3.a}$$

and

$$u(t) = I^{-1}(T - \dot{h}) \tag{3.b}$$

Eq. (2) can be written as

$$\dot{\omega} = F'(\omega)\omega + u(t) \tag{3.c}$$

As was noted by Cloutier, D'Souza, and Mracek (refs. 9, 10) the decomposition of $I\omega \times \omega$ into $[I\omega\times]\omega$ is not unique. In fact there are eight possible ways to decompose the vector (ref. 11), consequently there are eight primary

dynamics matrices which express the angular dynamics of an SC; that is, we have $F_i^{'}(\omega)$ $i = 1, 2, ..., 8$ for which

$$F_1^{'}(\omega)\omega = F_2^{'}(\omega)\omega = ... = F_8^{'}(\omega)\omega \text{ but}$$

$$F_1^{'}(\omega) \neq F_2^{'}(\omega) \neq .... \neq F_8^{'}(\omega) \tag{4}$$

Although there are only eight primary representations, one can generate infinite *secondary* dynamics matrices as linear combinations of the primary matrices by forming (refs. 9, 10)

$$E_i^{'}(\omega) = \sum_{j=1}^{8} \alpha_{i,j} F_j^{'}(\omega) \quad \text{where} \quad \sum_{j=1}^{8} \alpha_{i,j} = 1 \quad i = 1, 2, ..., \to \infty \tag{5}$$

This can be easily proven when noting that

$$E_i^{'}(\omega)\omega = \sum_{j=1}^{8} \alpha_{i,j} F_j^{'}(\omega)\omega = \sum_{j=1}^{8} \alpha_{i,j} f(\omega) = f(\omega) \sum_{j=1}^{8} \alpha_{i,j} = f(\omega) \tag{6}$$

where $f(\omega) = I^{-1}[(I\omega + h)\times]\omega$. We conclude then that the representation of the SC dynamics equation in the form

$$\dot{x} = F_\omega(x)x + u_\omega(t) \tag{7}$$

is not unique, and that there are exactly eight different ways to express the nonlinear SC dynamics equation by basic state-dependent linear equations, and infinite secondary such equations.

The attitude is best described by the quaternion of rotation. The quaternion dynamics equation is (ref. 8, p. 512)

$$\dot{q} = \tfrac{1}{2}Q\omega \tag{8}$$

where

$$Q = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \tag{9}$$

Grouping Eqs. (2) and (8) yields

$$\begin{bmatrix} \dot{\omega} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} I^{-1}[(I\omega + h)\times] & 0 \\ \tfrac{1}{2}Q & 0 \end{bmatrix} \begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} I^{-1}(T - \dot{h}) \\ 0 \end{bmatrix} \tag{10}$$

In order to use the last equation as a dynamics equation in a KF, we need to add to it white noise to account for modeling inaccuracies. This results in the following dynamics equation.

$$\begin{bmatrix} \dot{\omega} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} I^{-1}[(I\omega + h)\times] & 0 \\ \tfrac{1}{2}Q & 0 \end{bmatrix} \begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} I^{-1}(T - \dot{h}) \\ 0 \end{bmatrix} + \begin{bmatrix} w_\omega \\ w_q \end{bmatrix} \tag{11}$$

where w_ω accounts for the inaccuracies in the modeling of the SC angular dynamics and w_q accounts for modeling errors of the quaternion dynamics.

THE STATE-DEPENDENT MEASURMENT EQUATION

As mentioned earlier, a measurement system has been introduced recently[1], which yields the attitude directly in a form of a quaternion of rotation. In this case the measurement equation is linear; namely,

$$\mathbf{q}_m = \begin{bmatrix} 0_{4\times 3} & I_4 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \mathbf{v}_q \tag{12}$$

where $\mathbf{q}_m$ is the measured quaternion and $\mathbf{v}_q$ is the noise associated with this "measurement". The actual measurements are vector measurements obtained from star trackers out of which the "measured" quaternion is computed. Traditionally though, it is the vector measurements which are used directly to estimate the attitude. In this case the link between the measurements and the attitude expressed by the quaternion of rotation is a nonlinear one. However, this nonlinear relationship can too be expressed as a state-dependent pseudo-linear function. This is shown next. Let $\mathbf{r}$ denote the measured vector expressed in the reference coordinate system and let $\mathbf{b}$ denote the measurement of this vector as obtained in body coordinates. The relationship between the two vectors is expressed by

$$\mathbf{b} = D\mathbf{r} + \mathbf{v} \tag{13}$$

where D is the direction cosine matrix that transforms vectors from the reference to the body coordinates, and $\mathbf{v}$ denotes the zero mean white noise associated with this measurement. It is well known that D is the following function of the quaternion elements (ref. 8, p. 414)

$$D = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \tag{14}$$

when the last expression is substituted into Eq. (13) we obtain

$$\mathbf{b} = \begin{bmatrix} (q_1^2 - q_2^2 - q_3^2 + q_4^2)r_1 + 2(q_1q_2 + q_3q_4)r_2 + 2(q_1q_3 - q_2q_4)r_3 \\ 2(q_1q_2 - q_3q_4)r_1 + (-q_1^2 + q_2^2 - q_3^2 + q_4^2)r_2 + 2(q_2q_3 + q_1q_4)r_3 \\ 2(q_1q_3 + q_2q_4)r_1 + 2(q_2q_3 - q_1q_4)r_2 + (-q_1^2 - q_2^2 + q_3^2 + q_4^2)r_3 \end{bmatrix} + \mathbf{v} \tag{15}$$

An expansion and rearrangement of the terms in the last equation yields

$$\mathbf{b} = \begin{bmatrix} q_1^2 r_1 + q_1q_2r_2 + q_1q_3r_3 - q_2^2 r_1 + q_1q_2r_2 - q_2q_4r_3 - q_3^2 r_1 + q_3q_4r_2 + q_1q_3r_3 + q_4^2 r_1 + q_3q_4r_2 - q_2q_4r_3 \\ q_1q_2r_1 - q_1^2 r_2 + q_1q_4r_3 + q_1q_2r_1 + q_2^2 r_2 + q_2q_3r_3 - q_3q_4r_1 - q_3^2 r_2 + q_2q_3r_3 - q_3q_4r_1 + q_4^2 r_2 + q_1q_4r_3 \\ q_1q_3r_1 - q_1q_4r_2 - q_1^2 r_3 + q_2q_4r_1 + q_2q_3r_2 - q_2^2 r_3 + q_1q_3r_1 + q_2q_3r_2 + q_3^2 r_3 + q_2q_4r_1 - q_1q_4r_2 + q_4^2 r_3 \end{bmatrix} + \mathbf{v}$$

$$\ldots \ (16)$$

which can be written as

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[1] Van Bezooijen, R.W.H., "AST Capabilities," Lockheed Martin Advanced Technology Center, Palo Alto, CA 95304-1191. (Slide presentation).

$$\mathbf{b} = \begin{bmatrix} q_1r_1 + q_2r_2 + q_3r_3 & -q_2r_1 + q_1r_2 - q_4r_3 & -q_3r_1 + q_4r_2 + q_1r_3 & q_4r_1 + q_3r_2 - q_2r_3 \\ q_2r_1 - q_1r_2 + q_4r_3 & q_1r_1 + q_2r_2 + q_3r_3 & -q_4r_1 - q_3r_2 + q_2r_3 & -q_3r_1 + q_4r_2 + q_1r_3 \\ q_3r_1 - q_4r_2 - q_1r_3 & q_4r_1 + q_3r_2 - q_2r_3 & q_1r_1 + q_2r_2 + q_3r_3 & q_2r_1 - q_1r_2 + q_4r_3 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \mathbf{v}$$

$$\ldots (17)$$

Let

$$H(\mathbf{r},\mathbf{q}) = \begin{bmatrix} q_1r_1 + q_2r_2 + q_3r_3 & -q_2r_1 + q_1r_2 - q_4r_3 & -q_3r_1 + q_4r_2 + q_1r_3 & q_4r_1 + q_3r_2 - q_2r_3 \\ q_2r_1 - q_1r_2 + q_4r_3 & q_1r_1 + q_2r_2 + q_3r_3 & -q_4r_1 - q_3r_2 + q_2r_3 & -q_3r_1 + q_4r_2 + q_1r_3 \\ q_3r_1 - q_4r_2 - q_1r_3 & q_4r_1 + q_3r_2 - q_2r_3 & q_1r_1 + q_2r_2 + q_3r_3 & q_2r_1 - q_1r_2 + q_4r_3 \end{bmatrix}$$

$$\ldots (18)$$

then Eq. (17) can be written as

$$\mathbf{b} = H(\mathbf{r},\mathbf{q})\mathbf{q} + \mathbf{v} \tag{19}$$

which can be written as

$$\mathbf{b} = \begin{bmatrix} 0 & H(\mathbf{r},\mathbf{q}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \mathbf{v} \tag{20}$$

We have thus succeeded in expressing the vector measurement equation as pseudo-linear equation.

THE PDEUDO-LINEAR KALMAN FILTER

The pseudo linear KF (PSELIKA) is an ordinary linear Kalman filter where the state-dependent coefficients are functions of the best available state estimate. Let

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} \tag{21.a}$$

$$F(\boldsymbol{\omega}) = \begin{bmatrix} I^{-1}[(I\boldsymbol{\omega} + \mathbf{h})\times] & 0 \\ \frac{1}{2}Q & 0 \end{bmatrix} \tag{21.b}$$

$$\mathbf{u}(t) = \begin{bmatrix} I^{-1}(\mathbf{T} - \dot{\mathbf{h}}) \\ 0 \end{bmatrix} \tag{21.c}$$

$$\mathbf{w}(t) = \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_q \end{bmatrix} \tag{21.d}$$

then Eq. (11) can be written as

$$\dot{\mathbf{x}} = F(\boldsymbol{\omega})\mathbf{x} + \mathbf{u}(t) + \mathbf{w}(t) \tag{22}$$

From Eq. (12) we obtain the following corresponding *quaternion* measurement equation

$$\mathbf{q}_{m,k+1} = H^q_{k+1} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \mathbf{v}_{q,k+1} \tag{23.a}$$

where

$$H^q_{k+1} = \begin{bmatrix} 0_{4\times3} & I_4 \end{bmatrix} \tag{23.b}$$

and from Eq. (20) we obtain the following corresponding *vector* measurement equation

$$\mathbf{b}_{m,k+1} = H^b_{k+1}(\mathbf{r},\mathbf{q}) \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \mathbf{v}_{b,k+1} \tag{23.c}$$

where

$$H^b_{k+1} = \begin{bmatrix} 0 & H(\mathbf{r},\mathbf{q}) \end{bmatrix} \tag{23.d}$$

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Filter initialization:

Compute:

$$Q_k = E\{w(t_k)w(t_k)^T\} \tag{24.a}$$

$$R_{q,k+1} = E\{v_{q,k+1}v_{q,k+1}^T\} \tag{24.b}$$

$$R_{b,k+1} = E\{v_{b,k+1}v_{b,k+1}^T\} \tag{24.c}$$

and choose an approximate value for the initial estimate of the state vector. In the absence of such initial estimate, choose:

$$x_0^T = \begin{bmatrix} \omega_0^T & q_0^T \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24.d}$$

Time propagation:

Solve simultaneously the differential equations

$$\dot{\hat{x}} = F(\hat{x})\hat{x} + u(t) \tag{25.a}$$

$$\dot{P} = F(\hat{x})P + PF^T(\hat{x}) + Q \tag{25.b}$$

between t_k and t_{k+1} using the definition $\hat{x}(t_k) = \hat{x}_{k/k}$, $P(t_k) = P_{k/k}$, $\hat{x}_{k+1/k} = \hat{x}(t_{k+1})$, and $P_{k+1/k} = P(t_{k+1})$. (The subscript q/p denotes the estimate at time t_q based on the measurements up to time t_p).

Measurement update:

For <u>quaternion</u> *measurements:*

$$K_{k+1} = P_{k+1/k}H_{k+1}^{qT}[H_{k+1}^q P_{k+1/k}H_{k+1}^{qT} + R_{q,k+1}]^{-1} \tag{26.a}$$

$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}[q_{m,k+1} - H_{k+1}^q \hat{x}_{k+1/k}] \tag{26.b}$$

$$P_{k+1/k+1} = [I - K_{k+1}H_{k+1}^q]P_{k+1/k}[I - K_{k+1}H_{k+1}^q]^T + K_{k+1}R_{q,k+1}K_{k+1}^T \tag{26.c}$$

For <u>vector</u> *measurements:*

$$K_{k+1} = P_{k+1/k}H_{k+1}^{bT}(r_{k+1},\hat{q}_{k+1/k})[H_{k+1}^b(r_{k+1},\hat{q}_{k+1/k})P_{k+1/k}H_{k+1}^{bT}(r_{k+1},\hat{q}_{k+1/k}) + R_{b,k+1}]^{-1} \tag{27.a}$$

$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}[b_{m,k+1} - H_{k+1}^b(r_{k+1},\hat{q}_{k+1/k})\hat{x}_{k+1/k}] \tag{27.b}$$

$$P_{k+1/k+1} = [I - K_{k+1}H_{k+1}^b(r_{k+1},\hat{q}_{k+1/k})]P_{k+1/k}[I - K_{k+1}H_{k+1}^b(r_{k+1},\hat{q}_{k+1/k})]^T + K_{k+1}R_{b,k+1}K_{k+1}^T \tag{27.c}$$

SIMULATION RESULTS

In order to test the PSELIKA filter a simulation program was constructed. The simulation included a given profile of the wheel momentum, $h(t)$, that generated the desired true angular velocity vector. Based on that data the corresponding true SC attitude was computed in terms of the true quaternion. White measurement noise was added to the true quaternion and the resultant noisy quaternion served as the measured signal for the case where the PSELIKA filter used measured quaternions to estimate the SC attitude and angular rate. The noise vector was zero mean and white. Its standard deviation was 0.00001 for each component. Fig. 1 presents the true and estimated angular velocity

components. The estimation error of the angular velocity for this case is presented in Fig. 2. Since the measured quantity was the quaternion itself, the attitude errors were on the level of the noise. They were, of course, very small.
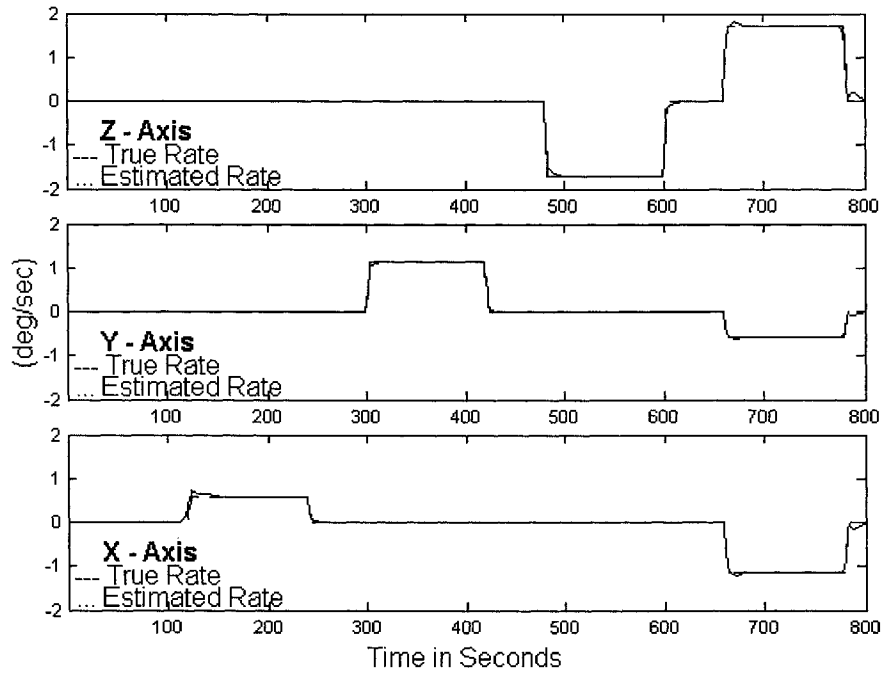

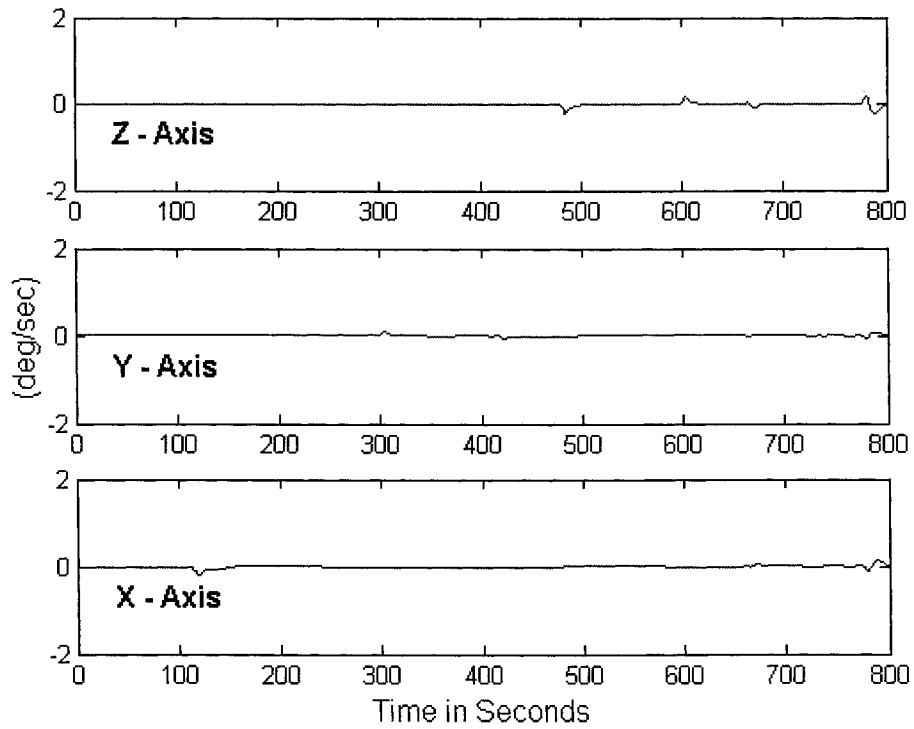Fig. 1: True and Estimated SC Angular Rate Using Quaternion Measurements.

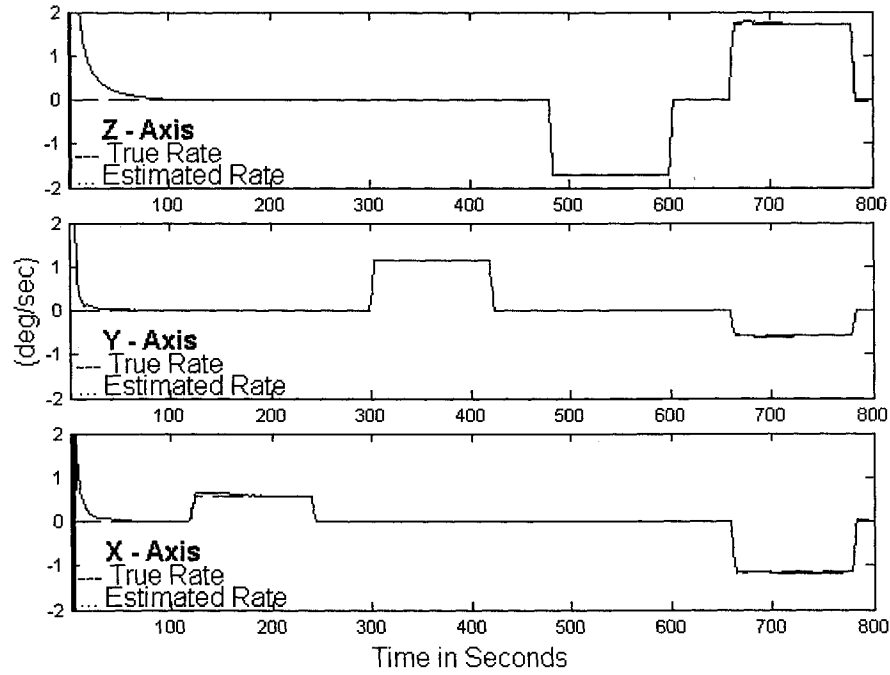Fig. 2: Rate Estimation Errors Using Quaternion Measurements.

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Fig. 3: True and Estimated SC Angular Rate Using Two Simultaneous Vector Measurements.

In order to simulate the case where the estimator used measured vectors, the following two reference vectors were assumed

$$\mathbf{r}_1^T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$
$$\mathbf{r}_2^T = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \tag{28}$$

They represented sun sensor and magnetometer measurements, respectively. The corresponding vectors in body coordinates, $\mathbf{b}_1$ and $\mathbf{b}_2$, were computed by transforming $\mathbf{r}_1$ and $\mathbf{r}_2$ by the true attitude matrix. White measurement noise was added to the true $\mathbf{b}_1$ and $\mathbf{b}_2$ vector. The noise was zero mean and its standard deviation was 0.01 per component of $\mathbf{b}_1$ and 0.005 per component of $\mathbf{b}_2$. When the two measurements were used in a sequential manner at each update time point, the filter diverged. It was speculated that although in the truly linear case performing the two sequential measurement updates is identical to performing the two updates simultaneously, in the pseudo-linear case, where the matrices involved in the updates are functions of the estimate, there could be a difference between the two possible update methods. This is particularly true when estimating attitude using vectors because two vector measurements, determine attitude whereas one vector measurement is not sufficient for that. As a result of the foregoing discussion, the two measurements were stacked together to form the following measurement equation (see Eq. 23.c)

$$\begin{bmatrix} \mathbf{b}_{1,m,k+1} \\ \mathbf{b}_{2,m,k+1} \end{bmatrix} = \begin{bmatrix} H_{1,k+1}^b[\mathbf{r}, \hat{\mathbf{q}}_{k+1}(-)] \\ H_{2,k+1}^b[\mathbf{r}, \hat{\mathbf{q}}_{k+1}(-)] \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{1,b,k+1} \\ \mathbf{v}_{2,b,k+1} \end{bmatrix} \tag{29}$$

Indeed, using the two vector measurements together in the filter updates, the filter converged and produced satisfactory results. Figure 3 presents the true and estimated rates obtained in this case, whereas Fig. 4 presents the angular rate estimation error for this case. Finally, in Fig. 5 the attitude estimation error is presented for this case of simultaneous vector measurements.
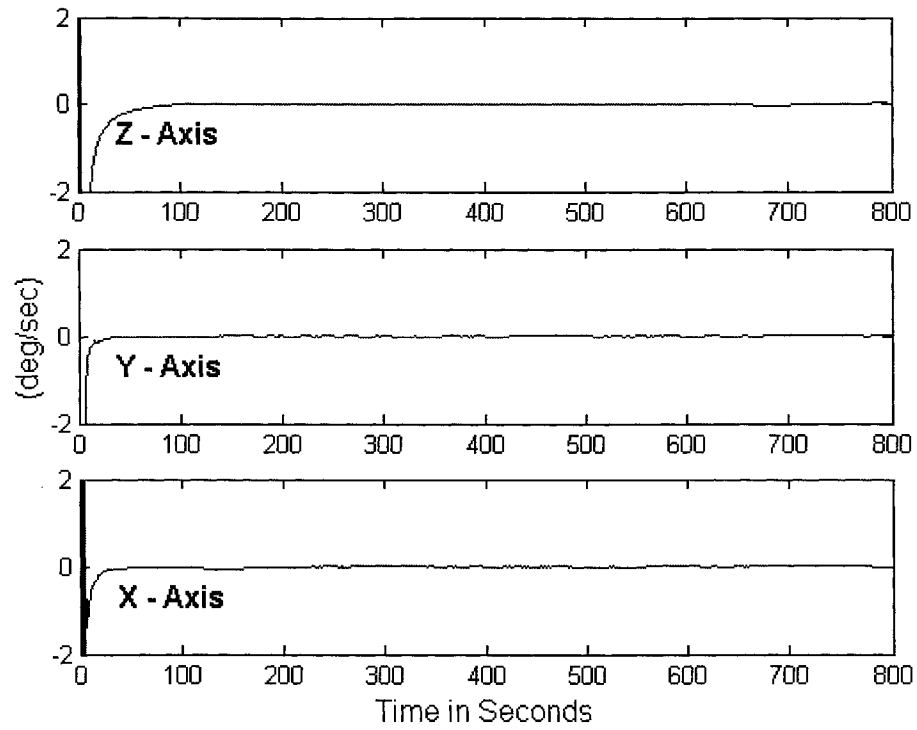
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Fig. 4: Angular Rate Estimation Errors Using Two Simultaneous Vector Measurements.
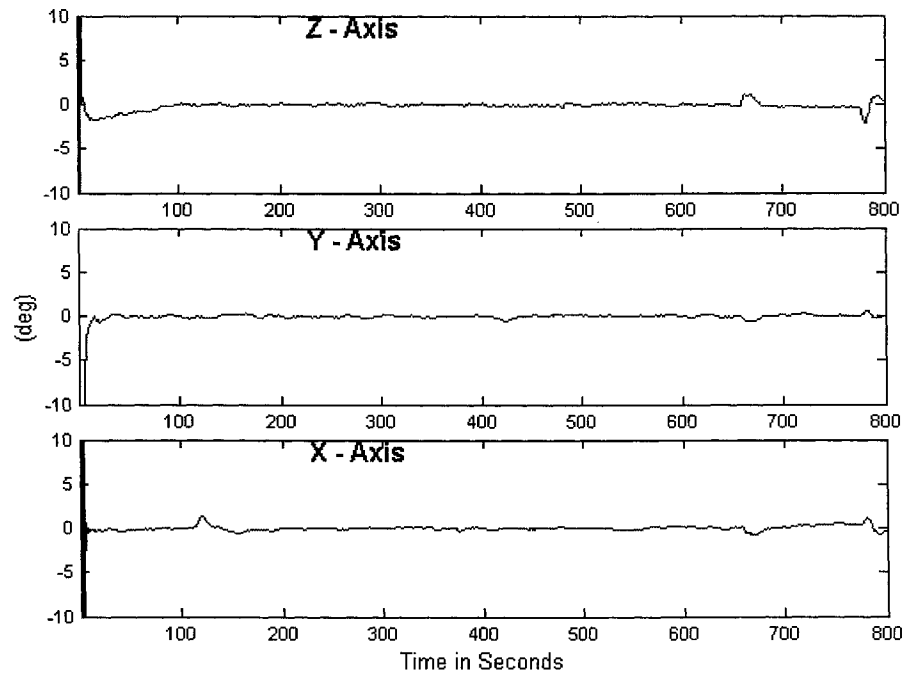

Fig. 5: Attitude Estimation Errors Using Two Simultaneous Vector Measurements.

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CONCLUSIONS

In this work we presented two versions of a pseudo-linear Kalman filter for spacecraft attitude and rate estimation, which is based on the ability to express the nonlinear parts of the system model as state-dependent linear models. This formulation of the model enabled the evaluation of the state-dependent matrices in terms of the best available state estimate, and thus to express the system model as deterministic matrices multiplying random vectors.

In the first version of the filter, the measurement was that of the attitude quaternion, which is related linearly to the estimated state vector; therefore, only the nonlinear dynamics model of the spacecraft rotation had to be formulated in a state-dependent linear model. The second version treated a more general case where the measurements were vector measurements, in which case, the measurement model too was nonlinear and had to be formulated as a state dependent linear model.

Simulations were performed which showed that, as expected, the first filter whose quaternion measurements were based on star tracker measurements yielded very good rate estimates and exceptionally good attitude results. The second filter, though, was unable to produce good results when the vector measurements were performed sequentially. However, when the two vector measurements were performed simultaneously the filter performed satisfactorily. The results were, of course, less accurate than with the previous algorithm, but that was because the simulated measurements supplied by a sun sensor and magnetometers whose inherent accuracy is less than that of star trackers which supplied the quaternion measurement to the first algorithm.

REFERENCES

1. Flately, T.W., Forden, J.K., Henretty, D.A., Lightsey, E.G., and Markley, F.L., On-Board Attitude Determination and Control Algorithm or SAMPEX," *Proceedings of the Flight Mechanics/Estimation Theory Symposium*, NASA Conference Publication 3102, NASA Goddard Space Flight Center, Greenbelt, MD,1990, pp. 379-398.

2. Harman, R.R., and Bar-Itzhack, I.Y., "Angular-rate Computation Using Attitude Differentiation," Faculty of Aerospace Engineering, Technion-Israel Institute of Technology, TAE-881, Haifa 32000, Israel, Dec. 97.

3. Harman, R.R., and Bar-Itzhack, I.Y., "Pseudo-Linear and State-Dependent Riccati Equation Filters for Angular Rate Estimation," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 5, Sept.-Oct. 1999, pp. 723-725.

4. Azor, R., Bar-Itzhack, I. Y., and Harman, R. R., "Satellite Angular-rate Estimation from Vector Measurements," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 3, 1998, pp. 450-457.

5. Oshman, Y., and Markley, F. L., "Sequential Attitude and Attitude-Rate Estimation Using Integrated-Rate Parameters," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 3, 1999, pp. 385-394.

6. Dellus, F., "Estimation of Satellite Angular Velocity Using Sequential Measurements of a Single Inertial Vector," M.Sc. Thesis, Faculty of Aerospace Engineering, Technion-Israel Institute of Technology, Haifa, Israel, June 1998 (In Hebrew).

7. Azor, R., Bar-Itzhack, I.Y., Deutschmann, J.K., and Harman, R. R., "Angular-rate Estimation Using Delayed Quaternion Measurements," *Journal of Guidance, Control, and Dynamics* (accepted for publication).

8. Wertz, J. R., (Ed.), *Spacecraft Attitude Dynamics and Control*, Reidel Publishing Co., Dordrecht, Holland, 1978.

9. Cloutier, J.R., D'Souza, C.N., and Mracek, C.P., "Nonlinear Regulation and Nonlinear H_∞ Control Via the State-Dependent Riccati Equation Technique: Part 1, Theory," The First International Conference on Nonlinear Problems in Aviation and Aerospace, Daytona Beach, FL, May 1996.

10. Cloutier, J.R., D'Souza, C.N., and Mracek, C.P., "Nonlinear Regulation and Nonlinear H_∞ Control Via the State-Dependent Riccati Equation Technique: Part 2, Examples," The First International Conference on Nonlinear Problems in Aviation and Aerospace, Daytona Beach, FL, May 1996.

11. Harman, R.R., and Bar-Itzhack, I.Y., "Pseudolinear and State-Dependent Riccati Equation Filters for Angular Rate Estimation," *J. of Guidance, Control, and Dynamics*, Vol. 22, No. 5, September-October, 1999, pp. 723-725.