A TWO-WHEEL OBSERVING MODE FOR THE MAP SPACECRAFT

Scott R. Starin and James R. O'Donnell, Jr., Ph.D.
NASA/GSFC, Flight Dynamics Analysis Branch, Greenbelt, MD 20771 USA

Abstract

The Microwave Anisotropy Probe (MAP) is a follow-on to the Differential Microwave Radiometer (DMR) instrument on the Cosmic Background Explorer (COBE). Due to the MAP project's limited mass, power, and budget, a traditional reliability concept including fully redundant components was not feasible. The MAP design employs selective hardware redundancy, along with backup software modes and algorithms, to improve the odds of mission success. This paper describes the effort to develop a backup control mode, known as ObservingII, that will allow the MAP science mission to continue in the event of a failure of one of its three reaction wheel assemblies. This backup science mode requires a change from MAP's nominal zero-momentum control system to a momentum-bias system. In this system, existing thruster-based control modes are used to establish a momentum bias about the sun line sufficient to spin the spacecraft up to the desired scan rate. Natural spacecraft dynamics exhibits spin and nutation similar to the nominal MAP science mode with different relative rotation rates, so the two reaction wheels are used to establish and maintain the desired nutation angle from the sun line. Detailed descriptions of the ObservingII control algorithm and simulation results will be presented, along with the operational considerations of performing the rest of MAP's necessary functions with only two wheels.

Introduction

The MAP Observatory will nominally use three reaction wheels for most of its attitude control requirements. See references 1 and 2 for further information on MAP. In the event of a failure of one of MAP's three reaction wheel assemblies (RWAs), it is not possible to achieve three-axis control using the remaining two wheels. Because of this, two of the attitude control algorithms implemented on the MAP spacecraft will no longer be usable. The two are Inertial Mode, used for slewing to and holding inertial attitudes, and Observing Mode, which implements the nominal dual-spin science mode. As a result of a Red Team Review (Sept. 12-13, 2000), the pre-launch development of a strategy for completing the mission in the case of a wheel failure became an imperative. The bulk of this paper will discuss the design for a two-wheel science mode for the MAP spacecraft. However, in addition to this backup science mode, there are a number of other changes that need to be made to the MAP onboard flight software in order for it to be able to fulfill even a degraded science mission. In this section, the philosophy used in designing these changes is shown, followed by a discussion of the changes and additions themselves.

Wheel Failure Design Philosophy

In order to be able to deliver and implement a backup two-wheel control design in a timely fashion, a design philosophy was first adopted.[2] The elements of this philosophy are as follows:

- Wherever possible, existing control algorithms already implemented and tested would be used as is, or with as few changes as possible. ⇒ Reduces development and testing time.
- Where completely new algorithms are needed, such as in the new science mode, they would be implemented in a manner consistent with the current flight software design. This design makes extensive use of tables of the parameters needed for proper configuration. ⇒ Allows for flexibility on-orbit for configuration and tuning of the control algorithms.
- New and changed algorithms would be prioritized by when they are necessary, and the development and testing schedule set up to reflect this. ⇒ Most effectively uses available resources to maximize the chances of mission success in the event of a wheel failure.

Required Mission Functions and Implementation Plan

The following functions are required in order to be able to carry out the MAP mission. After the description of each function, the way in which this function is implemented is shown. The selected implementation was based on the philosophy discussed above.

- Safehold at Low System Momentum: The existing MAP Safehold/CSS controller, named for its use of coarse sun sensor signals for attitude and derived rate information, works as is with only two wheels.
- Two-Wheel Science Mode (ObservingII): This mode will be fully discussed below. It is important to note here that the two-wheel science mode works by first establishing a 20–25 newton·meter·second (Nms) momentum bias about the Sun line. The controller then increases the nutation angle to the desired value to approximate the dual spin of the nominal science mode controller.
- Establishing and Removing Momentum Bias: The existing MAP Delta H Mode, a thruster-based mode nominally designed for dumping system momentum, can be used to establish or remove the science mode momentum bias. The only necessary changes are to parameters in an existing flight software table.
- Thruster Operations for Orbit Maneuvers and Maintenance: In order to fulfill its mission, MAP must get to the L_2 libration point, which means that it must be able to perform a number of orbit maneuvers during phasing loops about the Earth (on the final phasing loop, MAP performs a lunar swingby which provides the final "push" to L_2). Once at L_2, orbit maintenance maneuvers must also be performed, nominally four times a year. The existing MAP Delta V Mode performs this function in both the nominal and two-wheel case.

- Thruster-Based Inertial Mode: With the loss of Inertial Mode, a wheel-based mode that allows MAP to be slewed to any inertial attitude, an alternative way is needed to get the spacecraft in the right attitude for thruster operations. With two relatively small changes to the existing Delta V controller, it will be possible to use it for this function, as a Thruster-Based Inertial Mode.
- Safehold with a Momentum Bias: The two-wheel Safehold/CSS also works with a momentum bias as long as the bias is close to its nominal orientation about the Sun line. **Figure 1** shows an example of the performance of this mode.
- Momentum Bias Adjustment: A big difference between the nominal and two-wheel science modes is that the nominal mode performs its dual spin about the Sun line, while the two-wheel mode performs its dual spin about a momentum bias, nominally applied about the Sun line. This momentum bias is fixed in inertial space, however, and will move relative to the Sun line approximately 1°/day. A new thruster control mode is needed to perform the small daily adjustments to the momentum bias that will be necessary in flight.

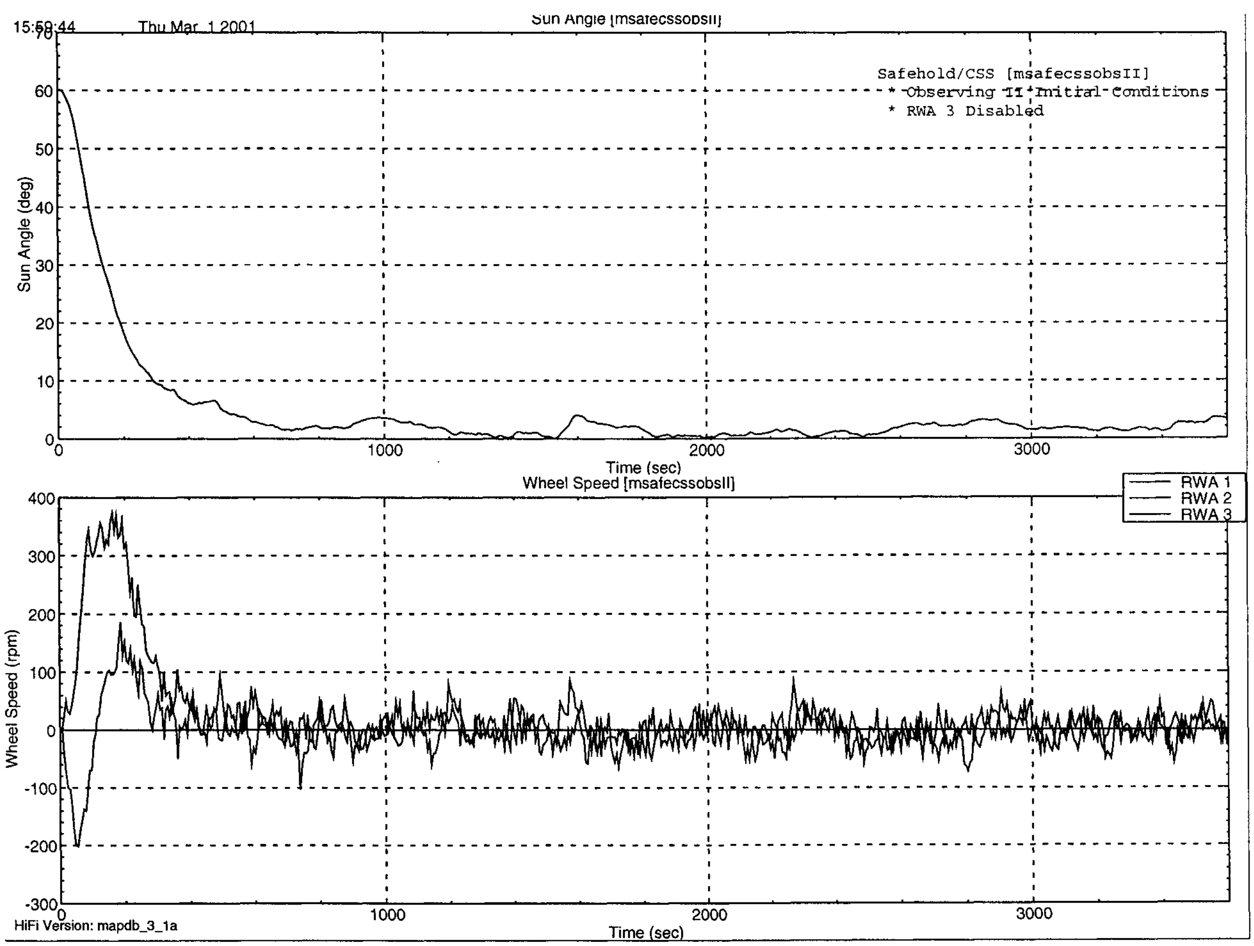

Figure 1: Safehold/CSS with Two Wheels and 20 Nms Sun line Momentum Bias

Upon reviewing the necessary functions needed for MAP to fulfill its mission, one thing becomes clear that both drives and simplifies the development schedule. Of the functions that are not supported by existing algorithms on the spacecraft, only the Thruster-Based Inertial Mode function is necessary in order for MAP to be able to reach the second Lagrange point (L_2). The two-wheel science mode and momentum bias adjustment functions are not needed until MAP reaches L_2, at least 90 days after launch. In order for the spacecraft to be able to get there, it is necessary to have a means to get the spacecraft into the correct attitude for its critical phasing loop thruster operations so that it can achieve the correct orbit for its lunar swingby. Because of this, the development of the backup algorithms and software is being done in two phases. Phase 1 includes what is needed to get to L_2 and will be completed before launch. It will be tested and available to be uploaded as a patch to the onboard software in the event of a wheel failure.

In the remainder of this paper, the three new or changed algorithms that are being designed for the MAP two-wheel contingency will be discussed. Fairly brief descriptions of the thruster-based inertial mode and momentum bias adjustment algorithms and implementations will be given, followed by extensive discussion of the two-wheel science mode.

Thruster-Based Inertial Mode: "Zero ΔV" Delta V

One of the design goals for the two-wheel contingency algorithms was to use existing control algorithms already implemented and tested on the spacecraft as much as possible, with as few changes as possible. Because of the nominal ACS flight software design, it was found that it was easier to implement a thruster-based inertial control mode through small changes to the existing Delta V Mode, rather than the existing Inertial Mode. The existing Delta V Mode was already set up to hold an inertial attitude during a burn; only two changes were necessary to allow for its use in the two-wheel contingency case for slewing to and holding inertial attitudes:

- The current Delta V Mode is meant for thruster operations, not simply holding an attitude, and automatically exits when a commanded burn is completed. In order for the mode to be used as a thruster-based inertial mode independent of an orbit maneuver, an option to allow for a "Zero ΔV" Delta V was necessary. If commanded in this fashion, the mode would only exit when it timed out (controlled by a flight software table value) or upon command into another control mode (or into a "conventional" Delta V orbit maneuver). A preliminary implementation of the "Zero ΔV" Delta V Mode was tested and showed acceptable performance.
- The existing Delta V Mode was not meant to execute large inertial slews; attempting to do so would result in gyro rate saturation because the algorithm does not include any rate-limiting action. In the two-wheel contingency case, an attitude limiter is added to the Delta V Mode controller to effectively limit its slew rate and allow it to be used for inertial hold and slews of arbitrary length.

Momentum Bias Adjustment

As mentioned above, the two-wheel science mode performs a dual spin about a momentum bias nominally applied about the Sun line. Because this momentum bias remains fixed in inertial space, the Sun line moves relative to the bias approximately 1°/day. Therefore, in order to keep the science mode spin about the Sun line, the momentum bias must be adjusted daily. Because MAP nominally only has one 37-minute ground contact per day, this momentum bias thruster operation must not be overly complicated.

In order to implement this, an algorithm has been designed that uses the instantaneous estimate of spacecraft attitude to determine when the body z-axis lies in the plane of the ecliptic, the plane in which the momentum bias should be applied or adjusted. See **Figure 2** for axis definitions. With a momentum bias of 20 Nms, this occurs once every 90 seconds. In order to move the direction of the momentum bias 1° inertially, when the body z-axis is in the ecliptic plane the momentum can be adjusted in either the body z-axis or the body xy-plane. Because there is more momentum in the body z-axis, it takes more Δ momentum in that axis to move the direction 1° than in xy. So, xy-adjustments can be made for five or six days, each of which slightly increase the magnitude of the momentum bias while adjusting its direction, followed by one day with a z-axis adjustment to return the momentum bias magnitude to its nominal value. The size of the momentum bias adjustment burns, which will be determined by comparing an average value of body momentum and the Sun line direction, are less than one second of burn per thruster for one or two thrusters. Fuel usage is on the order of a total of 30 seconds of thruster firing over 90 days, which is a fairly insignificant portion of the fuel budget.

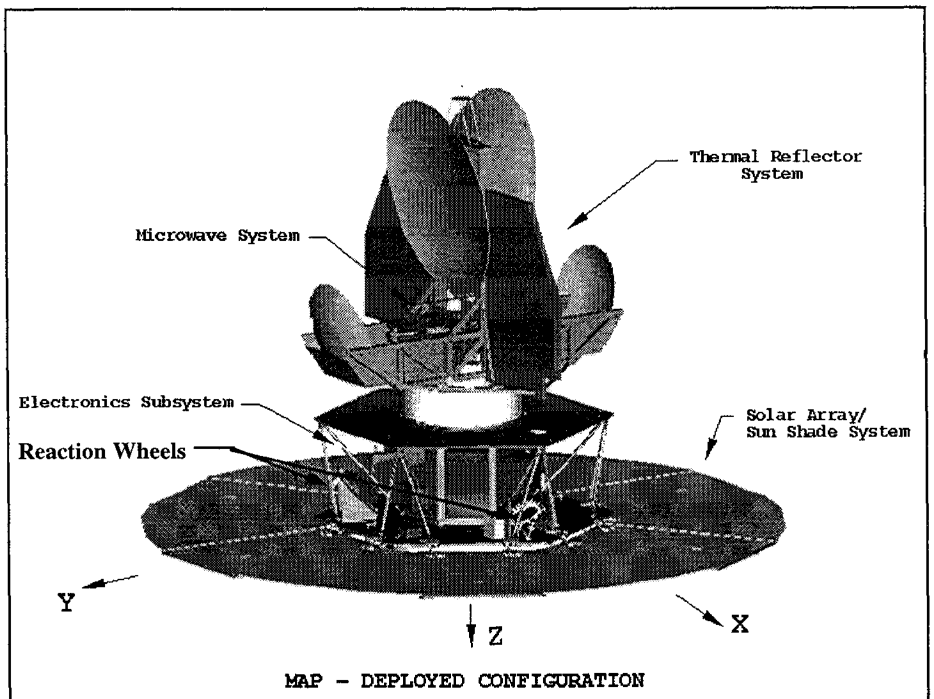

Figure 2: Configuration of MAP spacecraft with solar arrays deployed.

MAP Two-Wheel Science Mode

The MAP Observatory is equipped with three reaction wheels, which are arranged symmetrically about the spacecraft's z-axis. Wheel #1 lies in the +x/–z quarter-plane of the spacecraft body reference frame, and its unit torque vector is $n_1 = [-0.866\ 0\ -0.5]^T$. Wheels #2 and #3 have the same 30° cant with respect to the z-axis, with unit vectors $n_2 = [0.433\ -0.75\ -0.5]^T$ and $n_3 = [0.433\ 0.75\ -0.5]^T$, respectively. The nominal-mission Observing Mode uses the three reaction wheels to establish commanded Euler angle rates.[1] A 3-1-3 Euler angle rotation is used, with the angles named ϕ, θ and ψ, respectively. The rates to be commanded are:

$$\dot{\phi} = 1\ rph = 0.1°/s \tag{1}$$

$$\dot{\theta} = 0°/s; \quad with\ \theta = 22.5° \tag{2}$$

$$\dot{\psi} = 0.464\ rpm = 2.8°/s \tag{3}$$

The angle θ, *i.e.* the angle between the Sun vector and the unit vector along the geometrical z-axis, is maintained at a constant value of 22.5°. The rate in ψ represents a spin about the z-axis, and the rate in ϕ is therefore a precession of the z-axis.

Since the precession rate is so much slower than the body spin rate, the body momentum is nearly parallel to the z-axis, and this momentum is precessed in the inertial space by the wheels. In other words, the spacecraft momentum associated with commanded rates is stored by the wheels, and nominal science operations are kept at very low system momentum. These rates are established by continually generating a small delta-quaternion, Δq, and then commanding the wheels to enact that Δq. Because the system momentum is small, tiny corrections in the calculated Δq values are sufficient to adjust for the rotation of the Sun vector in inertial space. This allows the precession of the z-axis to remain symmetric about the Sun line. Thus, the combination of three rates – spin, precession, and Sun vector rotation – sweeps the instrument boresights over the entire sky in a period as short as six months.

Because of the large difference between various rates, this nominal science mode may be said to operate under the condition of being able to store momentum of a specified magnitude, but of an essentially arbitrary direction with respect to the body frame. With the loss of the use of a reaction wheel, however, the ability to store momentum in any arbitrary direction is lost. The two remaining wheels are limited to storing momentum in directions parallel to the plane spanned by their unit torque vectors:

$$\begin{aligned} \vec{H}_{RWA} &= \frac{d}{dt}\vec{N} = \frac{d}{dt}\left(\underline{n}_\alpha J_\alpha \omega_\alpha + \underline{n}_\beta J_\beta \omega_\beta\right) \\ &= \underline{n}_\alpha J_\alpha\left(\frac{d}{dt}\omega_\alpha\right) + \underline{n}_\beta J_\beta\left(\frac{d}{dt}\omega_\beta\right) \end{aligned} \tag{4}$$

Here and throughout the paper, H denotes momentum; N, torque; $\underline{n}$, any characteristic unit vector (here denoting the wheel spin axes); J, the moment of inertia of a wheel about its spin axis; ω, angular velocity. The subscripts α and β refer to the two operable wheels after the failure of the third. If wheel #1 had failed, α would be 2 and β would be 3.

Since the wheels are arranged in a pyramid about and canted 30° to the z-axis, the nominal method of having the momentum nearly parallel to the z-axis becomes unfeasible. In the zero-momentum scheme of attitude control, two wheels may be shown capable of enacting an algorithm to reach any given attitude[3,4]. Unfortunately such an algorithm requires large slews for even very small attitude corrections and is therefore unacceptable for the purposes of smooth rate establishment. As for smooth feedback algorithms, Byrnes and Isidori[5] showed that "a rigid satellite with (one or) two independent actuators cannot be locally asymptotically stabilized using continuously differentiable static or dynamic state feedback." And though Morin *et al* [6] have shown that a time-varying or discontinuous feedback controller may stabilize the attitude using only two inputs, their design requires the use of external torques in the form of jets or other propulsion-based attitude control. Since every indication is that control of the full-state attitude of a rigid spacecraft is not possible using two internal torque inputs, a design approach that is completely different from the nominal design is required.

Dynamical Considerations

The basic concept for an alternate control approach using two wheels was presented by O'Donnell *et al.*[2] In the event of a wheel failure, the Safehold control mode is capable of directing the spacecraft z-axis, and therefore the solar arrays, toward the Sun and holding an instrument-safe, power-positive condition until new software can be uploaded.

The newly installed software would spin the spacecraft up to a momentum of approximately 20–25 Nms (2.3–2.9°/sec) about the spacecraft z-axis using thrusters. This value provides rates of about the same magnitude as the nominal Observing mode. Once this momentum vector is established, the backup algorithm would increase the Sun angle, θ, to the desired value of 22.5° and maintain that angle as well as possible. Since thrusters may be used to periodically realign the momentum vector with the Sun vector, such maintenance should require little effort since the Sun angle would then coincide with the nutation angle of the z-axis about the momentum vector.

The natural dynamics of the rigid body allows this approach because of the mass properties of the MAP observatory. The current estimates for end-of-life (EOL) moments of inertia about the x- and y-axes are nearly equal (I_{xx} = 572 kg·m^2 and I_{yy} = 580 kg·m^2), and the moment of inertia about the z-axis is smaller (I_{zz} = 496 kg·m^2), so that the body is prolate. EOL values are used because most of the MAP fuel budget is used when travelling out to its target orbit at L_2; the beginning-of-life (BOL) ratio is $\sigma = 0.84$.

This near symmetry greatly reduces any effects a non-zero value of the xy-product of inertia might have, so long as the z-axis is in close alignment with a principal axis. As established by launch vehicle requirements, the geometrical z-axis is within 0.25° of the nearest principal axis. Misalignments of this magnitude have proved negligible in the development of this backup mode. Since they have little bearing on the remainder of the paper, the differences between the geometrical axes and the principal axes will no longer be mentioned. The axis frame "x-y-z" will refer to both axis frames simultaneously.

If we define a transverse moment of inertia, $I_T \equiv \sqrt{I_{xx} \cdot I_{yy}} = 576$ kg·m^2, then the ratio of the z- and transverse moments of inertia is $\sigma = 0.86$. From rigid-body dynamics, the ratio between the inertial nutation rate(ϕ-rate; analogous to precession of nominal mode) and body nutation rate (ψ-rate; spin with respect to momentum vector) is:

$$\frac{\dot{\phi}}{\dot{\psi}} = \frac{1-\sigma}{\sigma} \cos\theta. \tag{5}$$

For a nominal Sun angle of $\theta = 22.5°$, this ratio is approximately 0.15 at EOL, or 0.18 at BOL. For a system momentum of 20 Nms, the inertial nutation rate would be about 0.33 revolutions per minute, and the body nutation rate would be about 3 revolutions per hour. See **Figure 3** below for a comparison of nominal and back-up scan patterns.

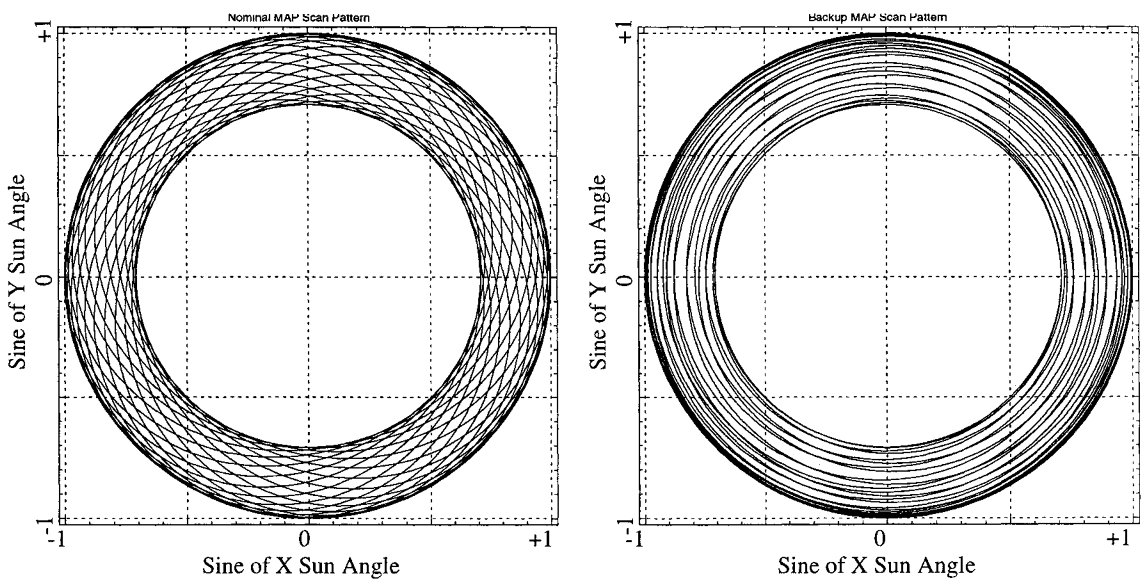

Figure 3: Comparison of scan patterns in nominal and two-wheel science modes.

Were there no internal sink for energy dissipation on the Observatory, whatever Sun angle was established by the control law would remain constant except for the gradual rotation of the Sun vector in inertial space. However, the fuel tank on MAP, with its elastomeric diaphragm, provides just such an energy sink in the form of fuel slosh. Therefore, the control law must be able to respond to and to counteract any increase in θ.

The inequality of the x- and y-moments of inertia, though small, is not negligible. This asymmetry results in a sinusoidal variation of the body and inertial nutation rates and the Sun angle. Specifically, the Sun angle varies in a peak-to-peak range of 1.5°. Intuitively, the variation is a result of the constant system momentum nutating through varying moments of inertia in the rotating spacecraft frame. To remove this wobble requires controlling the magnitude or direction of the system momentum in the spacecraft frame independent of that frame. Again, two obliquely mounted wheels are incapable of actuating control torques in arbitrary directions.

Because the original requirement of 22.5°±0.25° cannot be met using only two wheels, the goals for this backup control law have been changed to match the priorities dictated by the science goals. Specifically, the heating of the instrument by sunlight is to be avoided above all other concerns, including the completeness of the current scan pattern. To avoid heating of the instrument by diffracted sunlight, the Sun angle, θ, will not be allowed to exceed 22.75° during normal establishment of, and operations during, ObservingII. If 22.75° is exceeded, the algorithm should reduce θ as quickly as possible. To avoid sunlight falling directly on the instrument, θ will never be allowed to exceed 25° under any circumstances. In the event that an anomaly places the Sun angle outside of 25°, the current Safehold mode (which operates using two wheels, as noted above) will take over attitude control.

Beyond the protection of the instrument, the largest, most constant Sun angle feasible is to be maintained by the control algorithm. The attitude determination should not suffer, since a complex scan pattern may still be processed as long as the attitude data is good. This requirement is currently the limiting factor for the rate selection; the star trackers provide less accurate attitude determination as the transverse rates increase. The star tracker accuracy would have to be balanced with the increased sky coverage granted by higher rates. Lastly, there is some indication that, due to possible limitations on contact times, the algorithm should be able to establish backup operations as quickly as can be done without risking the other objectives.

Algorithm Design

The control algorithm design process was more exploratory, rather than strictly analytical. This approach was driven by factors such as nonlinear dynamics, underactuated control, a reduced set of state variables to be controlled (only θ needs to be controlled), and sparse literature from which to start. Due to limited software development resources, the algorithm was to fit into the same space in the software, with the same inputs and outputs, as the nominal science mode. The main effect of this restriction on the ObservingII algorithm was that, since Sun angle was not already input into the nominal Observing mode, the backup ObservingII mode had to rely on the measurement of body and wheel rates to establish the angle between the z-axis and the momentum vector.

The first conceptual step in the design process was to consider controllability. The wheels, which are fixed in the body frame (x-y-z), can have no effect on system momentum. They can only provide momentum to the body by temporarily storing the opposite momentum, thus maintaining an inertially constant system momentum vector. This storage must be temporary because wheel drag will eventually bring each wheel to rest. It soon became clear that the movement of the system momentum vector through the body frame was vital to controllability; if the momentum were stationary in the body frame, any momentum stored by the wheels would necessarily be released along exactly the same direction and thus undo any actuation that had been commanded.

The general dynamics are governed by Euler's equation,

$$\dot{\vec{H}}_{sys} = -\vec{\omega}_b \times \vec{H}_{sys} \quad (body\ frame) \tag{6}$$

which may be expanded in the following manner:

$$\dot{\vec{H}}_b = -\dot{\vec{H}}_{RWA} - \vec{\omega}_b \times \vec{H}_b - \vec{\omega}_b \times \vec{H}_{RWA}, \tag{7}$$

$$where\ \vec{T} = \vec{T}_{command} + \vec{T}_{drag} = -\dot{\vec{H}}_{RWA}. \tag{8}$$

If the simplification is made that $I_{xx} = I_{yy}$, then the control-free motion may be parameterized using the angle between the system momentum and the spacecraft z-axis, θ'', and the azimuthal angle for the body momentum, ψ'':

$$H_x = H \sin\theta'' \cos\psi'' \tag{9a}$$

$$H_y = H \sin\theta'' \sin\psi'' \tag{9b}$$

$$H_z = H \cos\theta'' \tag{9c}$$

$$\psi'' = \dot{\psi}'' t + \psi''_{t=0}. \tag{9d}$$

Here, H is the magnitude of the system momentum and remains constant as the system momentum vector sweeps a cone-like surface in the rotating body frame; if θ is constant, then the surface is truly a cone. The angles θ'' and ψ'' are analogous to the Euler angles with respect to the Sun—specifically θ, the Sun angle; and ψ, the angle between the x-axis and the xy-projection of the Sun vector. The two sets of angles are identical when the system momentum is exactly aligned with the Sun vector.

The effect of $I_{xx} \neq I_{yy}$ is that the cone-like surface, which is circular as parameterized above, is in fact elliptical. Since no further benefit is gained by using the less familiar elliptical parametric equations, the circular equations were used in the design of the algorithm.

Note that, since the motion of the body momentum vector is nearly symmetric about the z-axis, and the wheels are also symmetric about the z-axis, a geometrical simplification may be used to make one general control law work in nearly the same way no matter which wheel fails. The failed wheel must be identified and its identity entered into the software. Then, the algorithm may redefine the body frame such that the failed wheel is located on the new positive x-axis, which may be called the x'-axis. The operable wheels are then situated symmetrically about the x'-axis. The z-axis is identical with the z'-axis, and the y-axis is rotated into the y'-axis accordingly. The –y-wheel is referred to by the subscript α, and the +y-wheel is referred to by β. While there were differences in the configuration of the algorithms for the failure of each wheel, there were no problems large enough to warrant designing an entirely different algorithm for each wheel.

The first attempt at a control law design was to establish an open-loop rule for torque input to be provided by a single wheel. The body momentum vector could be calculated by measuring the body rates and multiplying by the measured spacecraft inertia matrix. When the body momentum vector was most advantageously oriented with respect to the torque direction, the wheel would torque according to the error in the Sun angle, θ. Then, when the wheel torque would have relatively little effect, the wheel speed was restored to zero, often passively by drag torque. This method appeared to perform appropriately, but the open-loop nature of the control algorithm was undesirable. So, other solutions were sought.

Most of the design effort was directed toward the use of the Lyapunov theorem to reduce a set of error variables to zero. The effort to find a Lyapunov function of all appropriate variables was, in the end, unsuccessful. One attempt was to use the following error variables to find a Lyapunov function:

$$\begin{aligned} &\textit{Transverse error}: e_T \equiv H_b \sin\theta - H_{b,desired} \sin\theta_{desired}, \\ &Z-\textit{axis error}: \quad e_Z \equiv H_b \cos\theta - H_{b,desired} \cos\theta_{desired}. \end{aligned} \tag{10}$$

Functions of these variables were chosen that were positive definite, and the wheel speeds were included. The time derivatives of the function tried, however, contained terms that were independent of the wheel torques, and so could not be influenced by the control algorithm. Finally, the Lyapunov approach was abandoned.

Though the search for a true Lyapunov function was unsuccessful, the various attempts resulted in several simple one- and two-wheel algorithms based on the error functions defined above. A few of the algorithms acted to stabilize the closed-loop system about a desired Sun angle for initial angles of 25° or less. Some algorithms could operate outside of this range, but such capability would be useless, since the Safehold mode would take over control. The control algorithms were composed of three parts: an initial kick to escape limit cycles about the Sun vector, an error-reducing function of the measured body momentum, and an anti-runaway restriction on the speed of the wheels. Since the error-reducing functions had certain similarities, a simple expression was found which included the most promising candidates. This expression, along with the other two components of the algorithm, are listed here in order of precedence (*e.g.* first listed is always commanded):

$$\textit{If } \omega_i \geq 50\,rpm,\, i \in \{\alpha, \beta\},\, \textit{then } T_i = -K\omega_i. \qquad \text{Anti-Runaway} \tag{11}$$

$$\textit{If } \theta < 10°,\, \textit{then } T_\alpha = T_\beta = T_{escape}. \qquad \text{Sun Vector Escape} \tag{12}$$

$$T_\alpha = k_{Z\alpha} e_Z + k_{T\alpha} e_T \cos(\psi + \varphi_\alpha) \qquad \text{Error-Reduction - } \alpha \tag{13a}$$

$$T_\beta = k_{Z\beta} e_Z + k_{T\beta} e_T \cos(\psi + \varphi_\beta). \qquad \text{Error-Reduction - } \beta \tag{13b}$$

According to simulations, the best-performing of these functions has been the one defined by:

$$\{k_{Z\alpha} = 0.007 \text{ Nm/Nms},\ k_{T\beta} = 0.010 \text{ Nm/Nms},\ \varphi_\beta = 30°,\ k_{Z\beta} = k_{T\alpha} = 0,\ \varphi_\alpha = 0°\} \tag{14}$$

This control law, which may be called ObservingII-A, has been tested using high fidelity simulations in order to gain confidence in the stability of the controller. At the time this paper is being written, Monte Carlo simulations are being developed to further establish the stability of the controller. Other means of proving stability should be forthcoming, but none are complete as yet.

Two-Wheel Performance

The figures presented at the end of this section show simulated operation of ObservingII-A. **Figure 4** and **Figure 5** show the case in which Wheel #1 has failed, the spacecraft has been recovered, and the thrusters have established a momentum bias of 20 Nms parallel to the Sun vector. The algorithm shows good stability and performance for initial conditions over the full range of the cone defined by $\theta \leq 25°$; in this case the Sun angle is started at 10°. When the natural motion reduces the Sun angle below the 10° Sun Escape limit, the Sun Escape torques quickly bring the wheels up to their speed limit. At the speed limit, the anti-runaway portion of the algorithm strongly counteracts any increase in wheel speed, and a back-and-forth torquing pattern begins which drives the Sun angle up again.

Starting from Sun angles less than the desired value, the algorithm establishes the desired motion in approximately 6–10 minutes, as is shown in Figure 3. Since the nominal ground contact during science operations is supposed to last 37 minutes, this time is well within the goal of operating within a contact so that results may be immediately observed, and corrections may be applied before returning to autonomous operations for the day.

The simulated performance of the controller varies slightly for the different wheel failure cases. **Figure 6** shows establishment of science operations in each failure case from the same initial conditions. To prevent the maximum Sun angle from exceeding the 22.75° limit, the target Sun angle, $\theta_{desired}$ (see Eq. 10), must be set at a slightly different, empirically determined value for each case. This fine-tuning adjusts for small body asymmetries that were neglected by generalizing the algorithm using rotational symmetry about the z-axis. There are some differences in performance that depend on the geometry of the initial conditions. For example, the case in which wheel #3 fails shows the controller taking somewhat longer to establish the science mode. This delay occurs at some initial conditions in each case, but is never more than the half-period of the inertial nutation cycle.

Though analytical modeling of fuel slosh indicates that the energy dissipation caused by slosh during this natural motion mode would be very small, it is possible that those models are in error. Such energy dissipation would cause the Sun angle to increase. **Figure 7** shows a case where the Sun angle has exceeded the allowable range for science and is close to the dangerous range ($\theta > 25°$) where direct sunlight would heat the instrument. Ideally, the Sun angle would never reach this high a value; nevertheless, the control algorithm must be and is able to stabilize the Sun angle quickly about the desired value. During operations at high Sun angle, the error reduction portion of the algorithm is generally the only active portion, as the wheel speeds do not exceed the anti-runaway limits.

Other non-ideal circumstances such as pinwheel torques or inaccurate momentum adjustments may arise. Also, changes to the drag torque model in the simulations indicate that the algorithm may be sensitive to changes in wheel drag that are not dependent on wheel speed. Because the performance of the control mode is very dependent on effects that cannot be well-understood until MAP has operated in space for some time, considerable adaptability has been built into the algorithm. By changing certain flight software table values, several types of alteration to the algorithm may be implemented.

The controller may be set on a timer so that it does not activate until a certain amount of time after a microprocessor restart. A counter may be set to start when science operations have been established, as determined by the value of θ entering and remaining within a certain range. When the counter reaches a target value, the controller will zero its commands until θ leaves the acceptable range. Another optional addition uses a quasi-Lyapunov function of the errors and torque to determine if the commanded torques are likely to be useful; non-useful torques, as determined by the sign of the function, are zeroed out. Finally, the errors may be increased over certain ranges in order to speed up the error reduction portion. These addenda to the algorithm are not necessary according to the performance of ObservingII-A in simulation. However, since the simulations are incomplete compared to the unpredictability of deep space operations, it was felt that every practical means of improving the adaptability of the completed control software should be included.

Conclusion

Due to cost and mass budget considerations, the MAP Observatory has no redundant reaction wheel to back up the three nominally operational wheels. In the event of a wheel failure, it is possible to perform all of the necessary attitude control using thrusters and the two remaining wheels. This paper has briefly described the use of thrusters in slewing for critical burns; the necessary software changes will be available at launch for immediate upload should a wheel fail before the spacecraft reaches L_2.

A two-wheel science mode has been detailed in which a thruster-generated momentum bias is used to sweep the instrument boresights across the sky in a manner similar to the nominal science mode. However, this natural-motion science mode results in a decrease in data density over the sky. The momentum bias must point at the Sun as much as is feasible, so it must be adjusted by the thrusters often for the science mode to operate safely and effectively.

Because of the intractability of the attitude control problem, the science mode algorithm has been designed to allow for the adjustment of gains multiplying functions of the angle error. In addition to these gains, other parameters may be used to adjust various failsafe modes built into the algorithm. All of these parameters may be adjusted in a flight software table which is part of the existing architecture. This design philosophy allows a measure of security against unpredictable dynamical effects.

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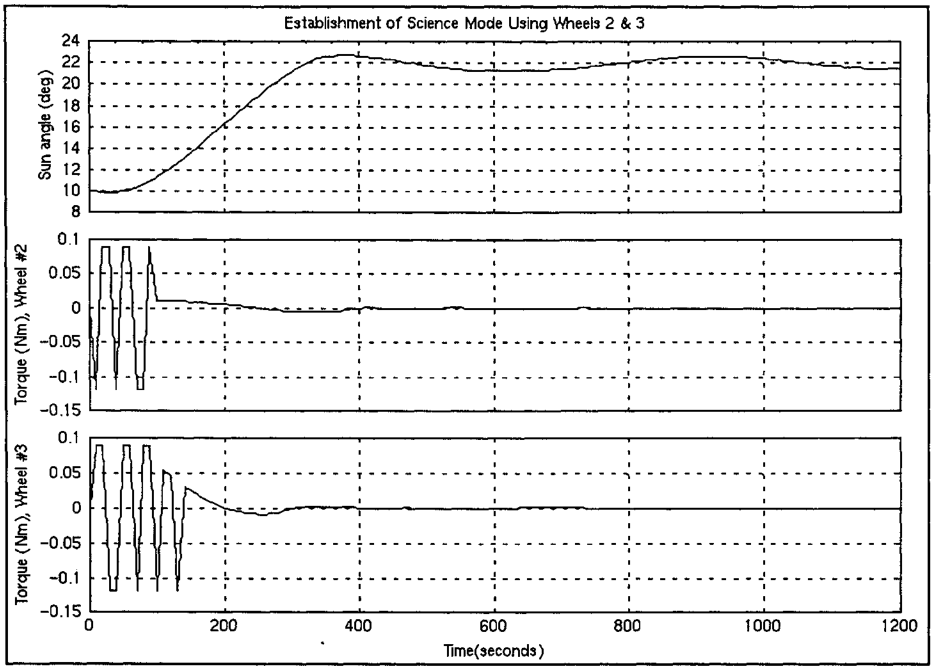

Figure 4: Case of wheel #1 failure. The jagged torque profile shows typical commands during Sun Escape, a portion of the ObservingII algorithm which is used to avoid limit cycles at low Sun angles. After error reduction, the mode is established, and torque commands become unnecessary unless the Sun angle leaves the operating range.

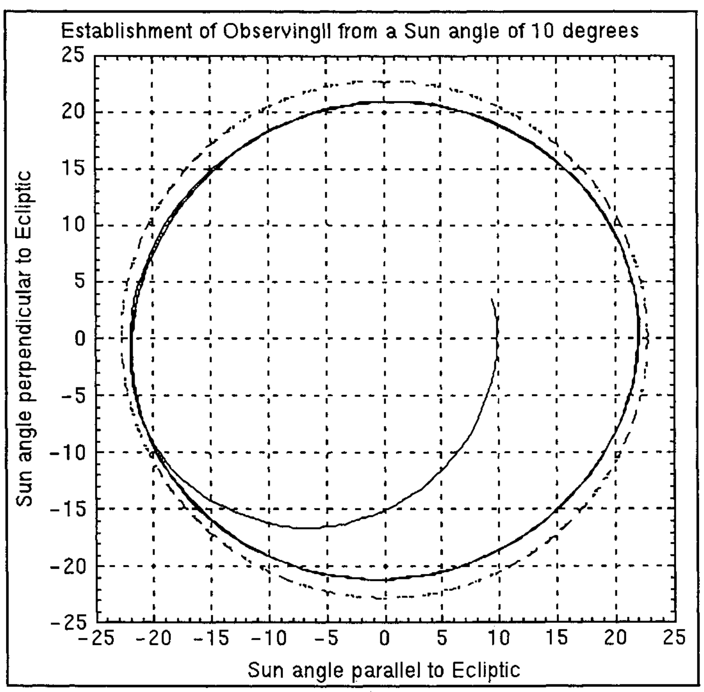

Figure 5: Case of wheel #1 failure. Establishment of mode showing the trajectory of the z-axis direction. The Sun is located at the origin; the dashed line is the 22.75° performance limit.

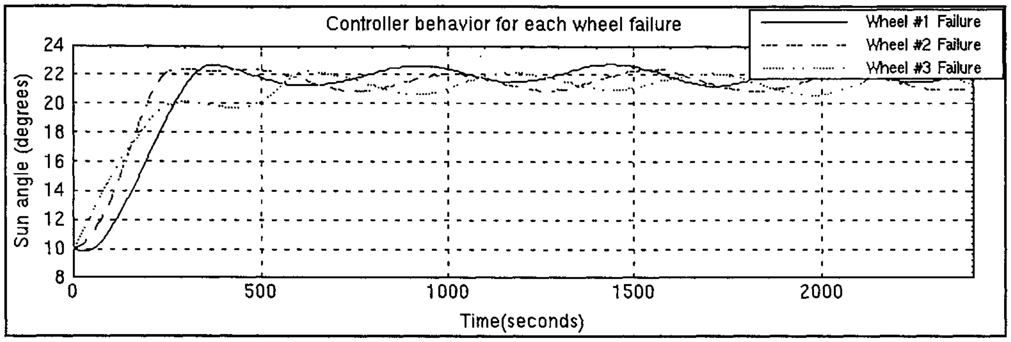

Figure 6: Establishment of ObservingII science mode in all three cases of wheel failure. Initial conditions for each case were identical.

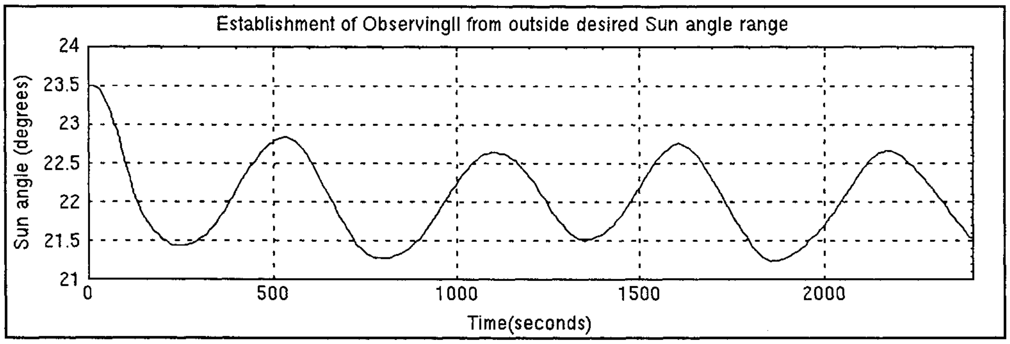

Figure 7: Because the spacecraft symmetry axis has the minimum moment of inertia, the control algorithm must be able to counteract Sun angle increases resulting from energy dissipation. From an initial Sun angle of 23.5°, science operations are restored within 10 minutes.

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