A FULLY COUPLED MULTI-RIGID-BODY FUEL SLOSH DYNAMICS MODEL APPLIED TO THE TRIANA STACK [1]

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ABSTRACT

A somewhat general multibody model is presented that accounts for energy dissipation associated with fuel slosh and which unifies some of the existing more specialized representations. This model is used to predict the nutation growth time constant for the Triana Spacecraft, or Stack, consisting of the Triana Observatory mated with the Gyroscopic Upper Stage or GUS (includes the solid rocket motor, SRM, booster) . At the nominal spin rate of 60 rpm and with 145 kg of hydrazine propellant on board, a time constant of 116 s is predicted for worst case sloshing of a spherical slug model compared to 1,681 s (nominal), 1,043 s (worst case) for sloshing of a three degree of freedom pendulum model.

INTRODUCTION

It is a common practice to resort to simple pendulum-like models as a means of capturing the effect of internal fluid motion on the attitude behaviour of spacecraft. Although approximate, these models do provide physical insight and do permit time history simulation of interactions that include energy dissipation. A common approach is to represent the fluid, often the fuel or the propellant, as a spherical, rigid, dissipative slug centered at the vehicle center of mass [1,2] . In Reference 3, the analysis is extended to accommodate filled ellipsoidal tanks.

A number of the slosh models are variations of the simple pendulum. Reference 4, for example, allows a single rigid body spacecraft rotation and a single pendulum rotation, that is two degrees of freedom (DOF). A parallel development is followed in Reference 5 but, in this instance, only a part of the propellant mass is assigned to the pendulum with the balance remaining fixed in the vehicle. In addition, a torsional stiffness and viscous rate-dependent torque are added at the pivot to allow for a tank with an elastomeric diaphram propellant management device (PMD). Another model like that of Reference 4 is that given in Reference 6, except now translational motions of the vehicle are included. Reference 7 has effectively 3 DOF, one a rigid body spacecraft rotation and two lateral pendulum rotations. While no equations are given, Reference 8 appears to account for coupled vehicle translation, rotation and 2 DOF for each of four pendulums. There is no fixed component of propellant mass and no stiffness at the pivot, but a damping ratio of 0.01 is imposed. A symbolic dynamics model builder is employed in Reference 9 to capture 3-axis attitude behaviour coupled with what appears to be a 1 DOF pendulum for each of four tanks. A portion of the propellant is fixed and stiffness and viscous effects are present at the pivot. Reference 10 alludes to use of a different symbolic code generator to provide the coupled vehicle, pendulum dynamics. What is different for this case is that pendulum parameter estimates are derived using an independent Navier-Stokes fluid model (in this case for an ellipsoidal tank).

[1] Presented @ *2001 Flight Mechanics Symposium*, NASA Goddard Space Flight Center, Greenbelt, MD June 19-21, 2001. (This work was carried out under the NASA GSFC Triana contract 2113-012.)

Presented in this paper is a general momentum-based multibodied dynamics model applicable to a wide range of slosh model configurations including all of those discussed above. The model can be applied directly, for simulation purposes, or it can be used to derive more simplified and,or linearized models. This is demonstrated by presenting equations, in velocity format, that result from applying the general equations to the case of a single spherical slug slosh model. Both the vehicle and the slosh body orientation kinematics are solved for using Euler parameters.

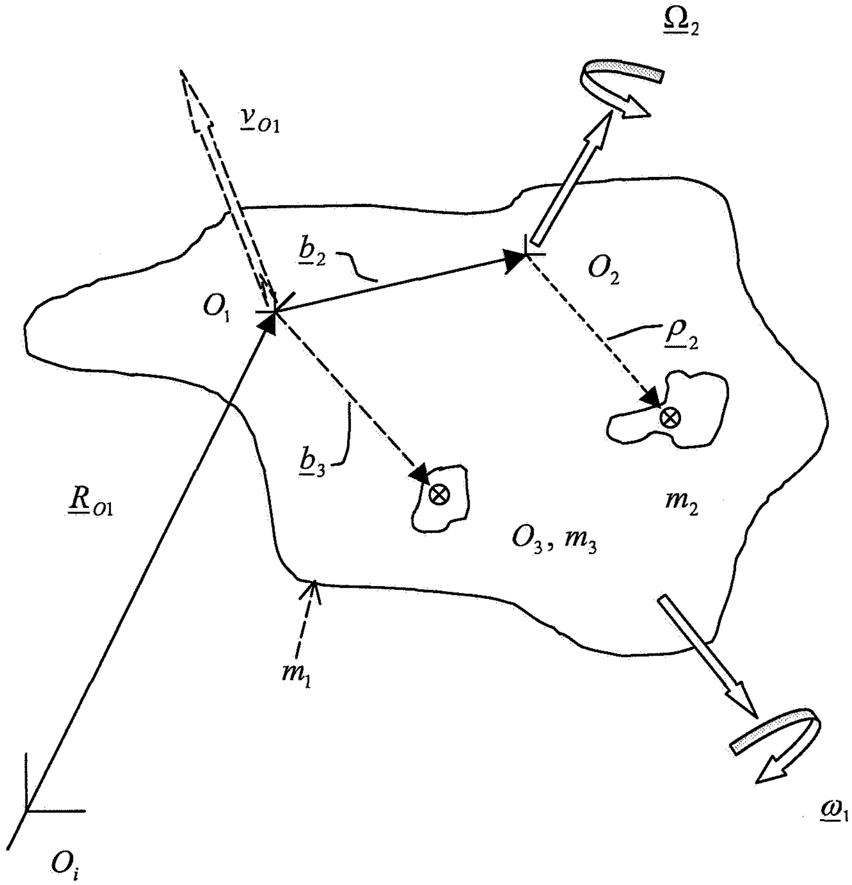

Figure 1 Position, velocity vectors of generic 3-body configuration. Reference point O_1, fixed in core body of mass m_1 and located by position vector $\underline{R}_{O1}$ relative to inertial reference at O_i, translates with absolute velocity $\underline{v}_{O1}$. Inertial angular velocity of core body is $\underline{\omega}_1$. Slosh mass m_2 has angular velocity $\underline{\Omega}_2$ relative to the core. The reference fixed to m_2 at O_2 is positioned with respect to O_1 by $\underline{b}_2$. Offset of the center of mass of body 2 from O_2 is $\underline{\rho}_2$ which, for a pendulum, becomes length L_2. Body 3 of mass m_3 represents any fixed, nonsloshing, portion of propellant mass. Offset vector of the center of mass of m_3 (O_3) from O_1 is $\underline{b}_3$.

These models are used to predict nutation time constant for the Triana spacecraft, an important design parameter for the Nutation Control System in use during orbit reboost. Effectiveness of the model is enhanced by adopting pendulum parameters extrapolated from test data already in existence for spherical tanks with a diaphram type PMD. Time constants produced by pendulum models are compared with those found for a maximum energy dissipating spherical slug slosh model. Sensitivity to parameters such as spin rate, stiffness and viscosity at the pivot are also examined.

SYSTEM DYNAMICS, KINEMATICS

Dynamic equations for both the general model and for the spherical slug model follow.

Momentum Rate Equations for the General 3-Body Model

The Newton, Euler formulation,as developed in Reference 11 for articulated interconnected rigid bodies, is used here to generate governing motion equations for a rigid 3-body model, one body representing the vehicle or spacecraft and two bodies for the propellant (one an articulated slosh mass and one fixed to the vehicle). Only a minor extension is required to the two body problem, contained in reference 11, to accommodate a third nonarticulating mass. Consequently, details of the derivation are not presented. Figure 1 depicts the generic configuration used here in order to have a model applicable to a wide range of configurations. The fully coupled motions consist of 3 DOF translation and 3 DOF rotation of rigid core body 1 with respect to inertial together with the 3 DOF rotation of body 2 relative to the core body. The momentum rate equations for the system and for body 2 about its attach point take on the following relatively straightforward appearance:

$$\dot{\underline{p}} = \underline{f}^{ext}; \qquad \text{(1a) system translation}$$

$$\dot{\underline{h}} + (\underline{v}_{O1} \otimes \underline{p}) = \underline{\Gamma}^{ext}; \qquad \text{(1b) system rotation}$$

$$\dot{\underline{h}}_2 + [\underline{v}_{O1} + (\underline{\omega}_1 \otimes \underline{b}_2)] \otimes \underline{p}_2 = \underline{\Gamma}_2^{ext} + \underline{\Gamma}_{21}; \qquad \text{(1c) slosh mass rotation}$$

where,

$$\underline{p} = m\underline{v}_{O1} - (\underline{c} \otimes \underline{\omega}_1) - (\underline{c}_2 \otimes \underline{\Omega}_2); \qquad \text{(1d) linear momentum of system}$$

$$\underline{h} = (\underline{c} \otimes \underline{v}_{O1}) + \underline{\underline{J}} \circ \underline{\omega}_1 + \underline{\underline{J}}_{12} \circ \underline{\Omega}_2; \qquad \text{(1e) angular momentum of system}$$

$$\underline{h}_2 = (\underline{c}_2 \otimes \underline{v}_{O1}) + \underline{\underline{J}}_{21} \circ \underline{\omega}_1 + \underline{\underline{J}}_2 \circ \underline{\Omega}_2; \qquad \text{(1f) angular momentum of slosh mass}$$

with,

$\underline{f}^{ext}$ external force on all bodies

$\underline{\Gamma}^{ext}$ external torques about body 1 reference point, $\underline{\Gamma}_2$ external torques about body 2 attach point

$\underline{\Gamma}_{21}$ torque exerted on body 2 by body 1 about O_2 (e.g. $-k_{pend}\,\theta_{pend}^{\;i} - c_{pend}\,\dot{\theta}_{pend}^{\;i}$)

c_{pend}, k_{pend} viscous coefficient and and torsional stiffness at body 2 attach point

$\theta_{pend}^{\;i}$ angular displacement about the ith pendulum axis; i = 1, 2, or 3 and, mass properties,

$m \;=\; m_1 + m_2 + m_3;$ system mass

$c_j \;=\; \int_{mj} \underline{r}_j \, dm;$ first mass moment of body j about its reference point

$\underline{r}_j$; location of elemental mass in body j from its reference point

$\underline{c} \;=\; \underline{c}_1 + \underline{c}_2 + \underline{c}_3;$ first mass moment of system

$\underline{\underline{J}}$; system moment of inertia about body 1 reference point

$\underline{\underline{J}}_2$; moment of inertia of slosh mass about its local reference point

$\underline{\underline{J}}_{12} \;=\; \underline{\underline{J}}_2 + \underline{c}_2 \circ \underline{b}_2 \underline{\underline{1}} - \underline{c}_2\, \underline{b}_2$; coupling moment of inertia matrix (symmetric in matrix form).

Also,

$\underline{(\;)}$ refers to a vector; $\underline{\underline{(\;)}}$ refers to a dyadic; $\underline{\underline{1}}$ is the unit dyadic ; $\otimes$ stands for vector cross product;

$\circ$ is a dot product; and dot( ) represents time derivative with respect to inertial.

With these equations as a base, and by adopting more restrictive assumptions, one can generate simpler models, as may be desirable in controls design. For example, if translation is not of interest then $\underline{p} = m\underline{v}_{O1} = \underline{0}$. Reference points are made to coincide with the core body reference if $\underline{b}_j = \underline{0}$ and setting $\underline{c}_j = \underline{0}$ places body reference point at its center of mass. Note, while the differing DOF are ultimately coupled, the first order momentum rates above are uncoupled and amenable to numerical integration as is. Of course, this is done while simultaneously solving the algebraic velocity, momentum relations together with the kinematics discussed later.

Velocity Rate Equations for 2-Body Model with Spherical Slug

A common slosh model appearing in the literature is that in which the entire mass of fluid is replaced by a single rigid spherical slug with mass uniformly distributed throughout the tank. The slug exerts a viscous torque on the rigid body vehicle proportional to its angular velocity relative to the vehicle. The general model equations are applied to this case using a core body reference attached to system center of mass. Substituting momentum relations from equations (1d), (1e), (1f) into equations (1a), (1b), (1c) gives, after a certain amount of vector algebra, the equations in velocity form:

$$m\dot{\underline{v}}_{CM} \;=\; \underline{f}^{ext}; \qquad \text{(2a) system translation}$$

$$(\underline{\underline{I}} - \underline{\underline{I}}_{SS}) \circ \dot{\underline{\omega}}_1 \;=\; \underline{\Gamma}^{ext} - \underline{\omega}_1 \otimes \underline{\underline{I}} \circ \underline{\omega}_1 + c_{SS}\, \underline{\Omega}_{SS}; \qquad \text{(2b) system rotation}$$

$$\underline{\underline{I}}_{SS} \circ \dot{\underline{\Omega}}_{SS} \;=\; -\underline{\underline{I}}_{SS} \circ \dot{\underline{\omega}}_1 - \underline{\omega}_1 \otimes \underline{\underline{I}}_{SS} \circ \underline{\Omega}_{SS} - c_{SS}\, \underline{\Omega}_{SS}; \qquad \text{(2c) slug rotation}$$

with,

$\underline{v}_{CM}$ velocity at mass center;

$\underline{\underline{I}}$ system moment of inertia about system center of mass;

$\underline{\underline{I}}_{SS}$ moment of inertia of spherical slug about its center of mass (same about all axes, all axes principal)

m_{SS} mass of spherical slug;

$\underline{b}_{SS}$ position vector to mass center of spherical slug from system center of mass;

c_{SS} constant viscous coupling coefficient giving rise to interbody torques, $\pm c_{SS}\, \underline{\Omega}_{SS}$, in equations (2b), (2c);

$\underline{\Omega}_{SS}$ angular velocity of spherical slug with respect to core body;

$\dot{\underline{\Omega}}_{SS}$ time rate of change in angular velocity of spherical slug with respect to core body;

and, $\underline{\underline{I}} \;=\; \underline{\underline{J}}_1 + \underline{\underline{I}}_{SS} + m_{SS}(\underline{b}_{SS}{}^2 - \underline{b}_{SS}\underline{b}_{SS})\,;$ (3)

with,

$\underline{\underline{J}}_1$ moment of inertia of core about system center of mass.

As written here, the translation is relative to inertial, whereas the system rotation is with respect to axes fixed in the core body. Since the system center of mass was chosen as main reference and the reference for the slug is at its center of mass, the system rotational motion is uncoupled from translation. The rotation equations, however, remain coupled and are in agreement with those presented elsewhere [1,2]. They are valid for non-principal axes, as well, in which case off-diagonal elements appear in the system moment of inertia matrix.

By way of observation, note that the system rotation equation (2b) can be put in an even simpler form if only principal axes are used:

$$\underline{\underline{I}}_{EQ} \circ \dot{\underline{\omega}}_1 \;=\; \underline{\Gamma}^{ext} - \underline{\omega}_1 \otimes \underline{\underline{I}}_{EQ} \circ \underline{\omega}_1 + c_{SS}\,\underline{\Omega}_{SS}\,; \qquad (4a)$$

where,

$$\underline{\underline{I}}_{EQ} \;=\; \underline{\underline{I}} - \underline{\underline{I}}_{SS}\,. \quad \text{(an 'equivalent' system moment of inertia)} \qquad (4b)$$

Rotation Kinematics

The dynamics equations presented depend implicitly on body attitude. Here Euler parameters are employed to track attiude for both vehicle and slosh body. The Euler parameter rate equations, which depend on angular velocity, are integrated in step with the dynamics. This allows one to continuously update the transformations between differing coordinate frames. More information is needed in the case of the pendulum model which feeds back position and rate dependent torques. In this case, Euler angles and Euler angle rates are also calculated for the pendulum mass. For such an application one might also consider using a space-fixed rotation sequence, particularly if large amplitude oscillations are anticipated that might affect the joint torque parameters.

Software Implementation

Independent computer software solutions, are developed in a MATLAB environment, for both the general model and for the spherical slug model. Numerical integration of the equations described above provides time histories of response. Of particular interest is angle of nutation and its rate of growth as characterized by some time constant. Since, for cases considered here, nutation is expected to closely approximate exponential growth, the time period chosen is time required for amplitude to increase by a factor e = 2.7183. A linear curve fit to the logarithm of the nutation response is used to extract this constant.

APPLICATION TO THE TRIANA SPACECRAFT

Background

The Triana spacecraft shown schematically in Figure 2 is intended to be launched from the Shuttle Orbiter payload bay by the Italian Research Interim Stage (IRIS) cradle-spin-table-launcher assembly. The Spacecraft Axis System (SAS) of coordinates, fixed in the spacecraft, has an origin at the intersection of the IRIS/spacecraft separation plane (SEP) and the centerline of the STAR-48 SRM. The Observatory Axis System (OAS) has its origin at the intersection of the GUS/ Observatory separation plane and the centerline of the STAR-48 SRM. Triana is to be ejected at a nominal spin of 60 rpm about the 'z' axis as indicated in Figure 2. Nominally, on ejection, mass totals 2989 kg with principal moments of inertia about the center of mass of 1843.5195 $kg.m^2$ about the transverse ('x', 'y') axes and 575.4170 $kg.m^2$ about the spin direction.

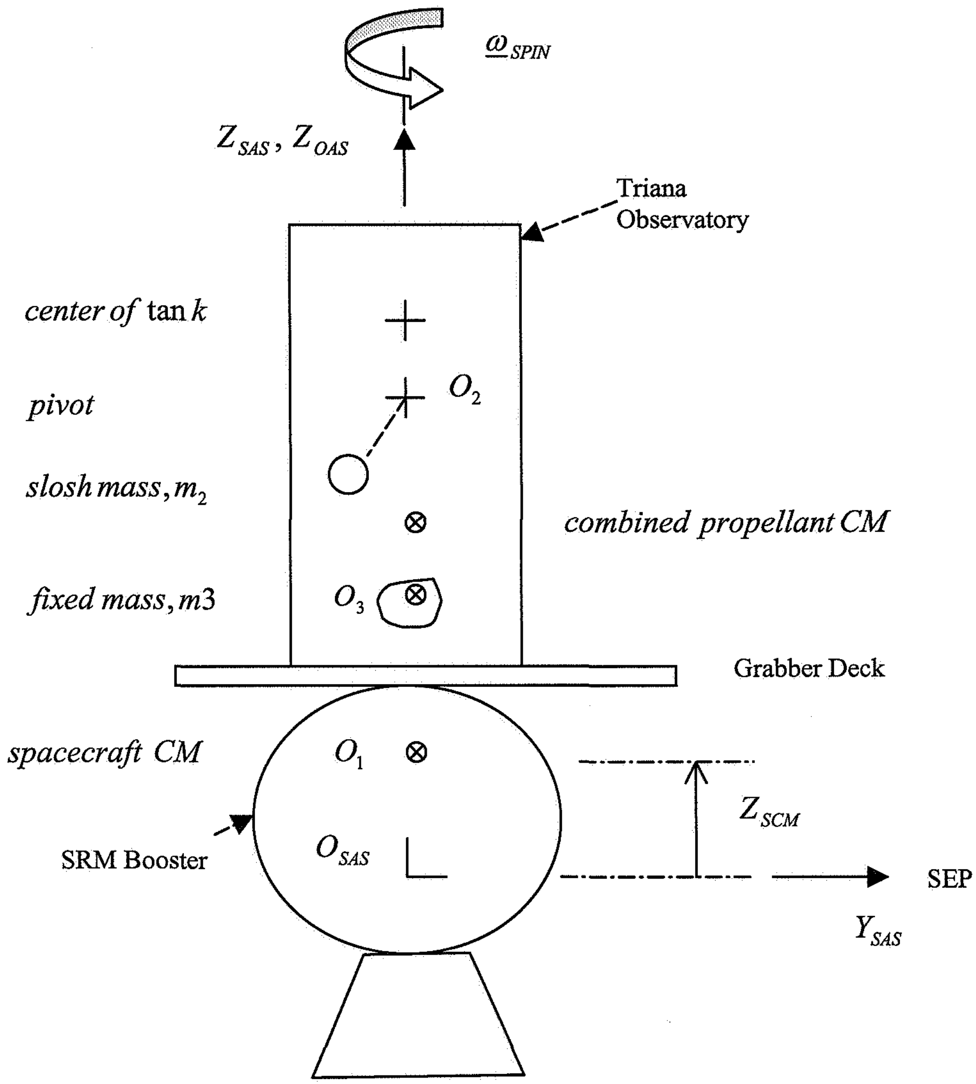

Figure 2 Schematic of Triana spacecraft showing key elements of the 2-body pendulum slosh model, coordinate system alignment and axis of spin.

Triana Observatory Propellant and Tank

Hydrazine thrusters provide active attitude control of the TRIANA Observatory. The liquid hydrazine is housed in a single 28 inch diameter pressurized spherical tank centered on the 'z' axis parallel to the direction of spin. What is referred to as an elastomeric diaphram (membrane, or bladder) Propellant Management Device (PMD), with its plane normal to 'z', is added to assist with expulsion of the propellant. This membrane is welded in place at the mid-section of the tank thus separating it into two distinct compartments; one containing the hydrazine and one containing a pressurized gas. Additional details are given in Reference 5.

A Pendulum Slosh Model for Triana

A complete transient dynamic analysis is complex and requires a rigid body rotational model for the tank coupled with the partial differential equations and associated boundary conditions of the fluid. Hence, it is common to resort to equivalent mechanical analogs, such as the pendulum, in an attempt to capture first order fluid force, torque interactions. For a spherical tank with diaphram, lateral slosh associated with surface waves is assumed to be the predominant influence. There is some measured data available for tanks of this type (Reference 12). Existing test data has been extrapolated to a number of different spacecraft as, for example, in References 9, 13 and 14 and, more specifically here, for the Triana Observatory in Reference 5. It is the work of the later reference that is made use of here. The same characteristics at the pivot are used since they depend on thickness of the diaphram (0.0625 inches) and not on fill level of the tank and they include a torsional stiffness of 54.74 Nm/Rad and a viscous rate coefficient of 4.569 Nm/(RAD/s). A number of other parameters depend on fill level, which is determined here, for known propellant mass of 145 kg contained in a spherical segment, to be 0.689 (as a fraction of diameter). Least squares fit relationships, based on earlier test data, are given in Reference 5 for pendulum mass, pendulum length (L2) and distance of the pivot from the center of the tank (h1), as well as a linear two-point fit for an equivalent rigid body inertia (Io) of the fixed mass component about its mass center. They are evaluated here for the above fill level giving:

$L_2 = 0.1126$ m; $m_2 = 49.44$ kg; $m_3 = 95.56$ kg; Io $= 1.0281\ kg.m^2$

$h_1 = 0.06854$ m (below center of tank);

$h_0 = 0.29847$ m; (leaves original center of mass location for propellant unchanged at 0.25846 m from tank center).

Since the pendulum model here has 3 DOF, the rigid body Io about the 'z' axis is divided between pendulum and fixed mass in proportion to their mass (the fixed mass still has principal transverse moments of inertia of Io). Note, on implementation into the general model the core body reference was positioned at the system center of mass nominally 0.55613 m from SEP. Also, the center of the tank is 1.47617 m from SEP, thus completing the information needed to specify the pendulum model as used here.

Maximum Dissipation Spherical Slug Model

Analysis has been done to estimate maximum energy dissipation for a spherical slug slosh model with the slug at the system center of mass of a spinning axisymmetric rigid body[2]. It is possible to analytically solve for angular rates of the slug in terms of the steady rates of the nutating rigid body. Since applied torque depends on those rates and on viscous coefficient, the energy dissipation can be expressed as a function of the viscous torque coefficient. The value of this coefficient which renders the dissipation rate stationary is, as established by Flatley,

$$c_{SS}{}^{F} = (I_{SS}\, I_{SPIN}\, \omega_{SPIN}) \,/\, I_T \,;$$

where,

ω_{SPIN} is spin rate;

I_{SPIN}, I_T refers to system moments of inertia about spin and transverse axes, respectively.

Also, equating this dissipation rate to that contained in the approximate Energy Sink model gives a minimum time constant:

$$\tau_{MIN}{}^{F} = 2 I_T{}^2 \,/\, [I_{SS}\, \omega_{SPIN}\, (I_T - I_{SPIN})]$$

For the Triana tank $I_{SS} = (2/5)\, m_{SS}\, r^2 = 7.3342\ kg.m^2$.

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[2] Flatley, T., Notes to Tobin re-'An Energy Dissipation Model', NASA GSFC 11/15/93 as attached to NASA GSFC (Code 712) document from: Houghton, Martin B. to: Ward, D. entitled: "FUSE Nutation Time Constant," December 15, 1993.

Table 1 shows viscous cofficient and minimum time constant, calculated using nominal Triana mass properties, for low, nominal and high end Triana spacecraft spin rates. These coefficients represent the maximum dissipation theoretically achievable for such a model.

Table 1
Viscous coefficient and related nutation time constant for maximum spherical slug dissipation

ω_{SPIN}, rpm	c_{SS}^{F}, NMs	τ_{MIN}^{F}, s
52	12.4658	134.21
60	14.3836	116.32
68	16.3014	102.63

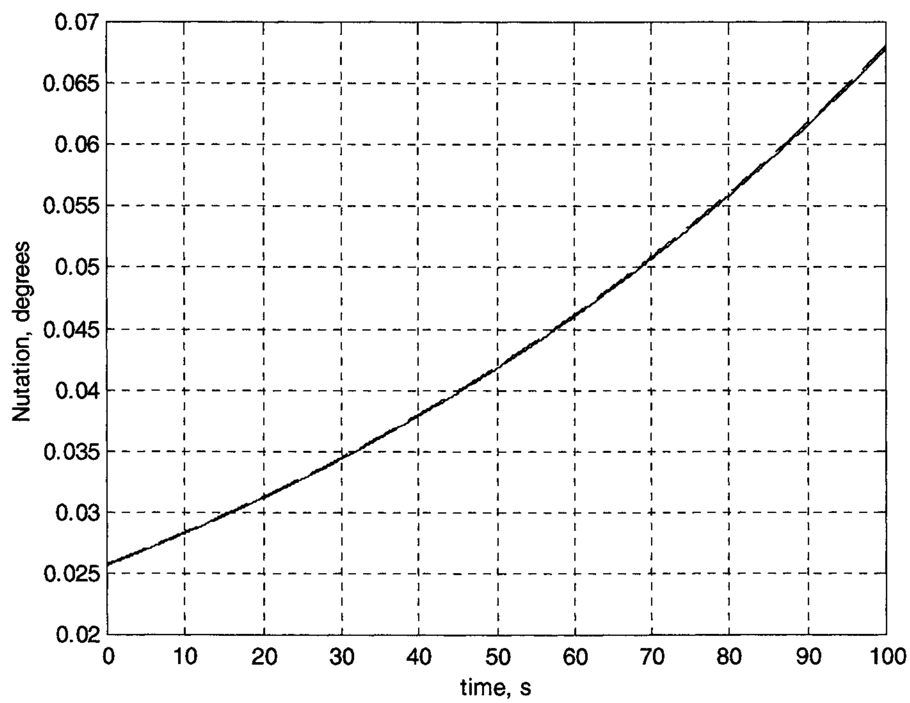

Figure 3 Spherical slug slosh: general model versus specialized 2-body model at 68 rpm.

Results for Spherical Slug Slosh Model

Results presented here are based on nominal parameter values unless otherwise noted.

Figure 3 compares nutation growth found using a 2-body spherical slug model, similar to that currently in use in the literature, with that predicted using the general model. Agreement is seen to be reasonable.

The general model is applied next to the FUSE spacecraft reported on by Houghton in the memo of footnote 2, where again, the agreement appears reasonable when using a spherical slug slosh model (Table 2).

Table 2
Comparison of Nutation Time Constants for the FUSE Spacecraft using a Spherical Slug Slosh Model (60 rpm)

	Spin Axis Moment of Inertia, kgm^2	Transverse Axis Moment of Inertia, kgm^2	Time Constant, s from Houghton	Time Constant, s from General Model
prior to booster firing	885	4602	2126	2164
post burnout	546	2096	1058	1065
post separation	500	1450	826	830

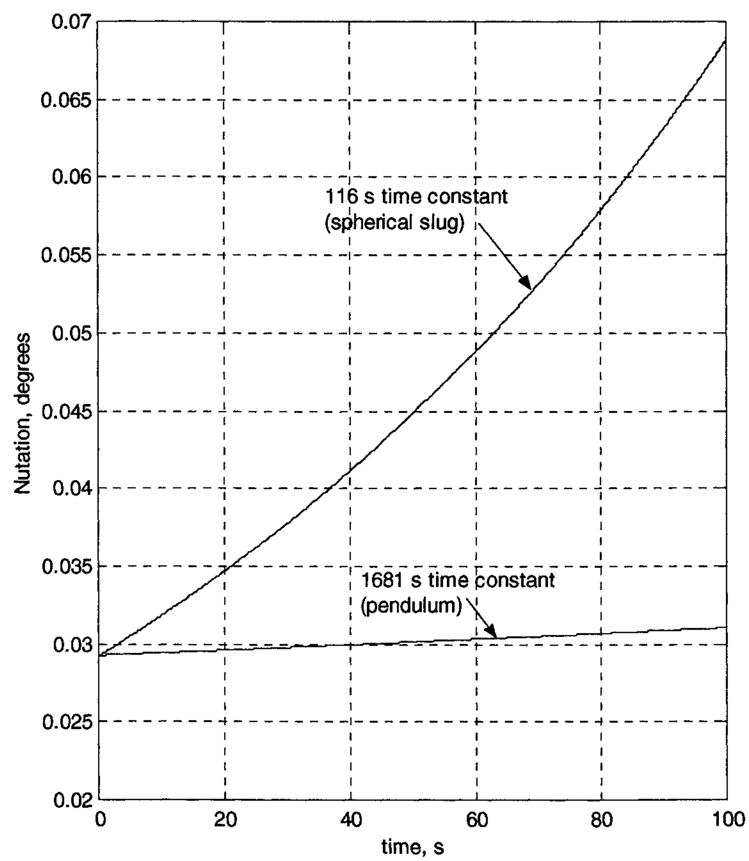

Figure 4 Nutation growth predicted using the general model with the tuned Triana 3 DOF pendulum versus the 2-body spherical slug with maximum dissipation at 60 rpm.

Results for Pendulum Slosh Model

Figure 4 compares nutation growth for the worst case spherical slug slosh model for the propellant with that predicted using a pendulum slosh model in the general program with parameters tuned to the Triana spacecraft. The growth rate is significantly higher for the spherical slug case as reflected in the time constant which is more than an order of magnitude below that for the pendulum model.

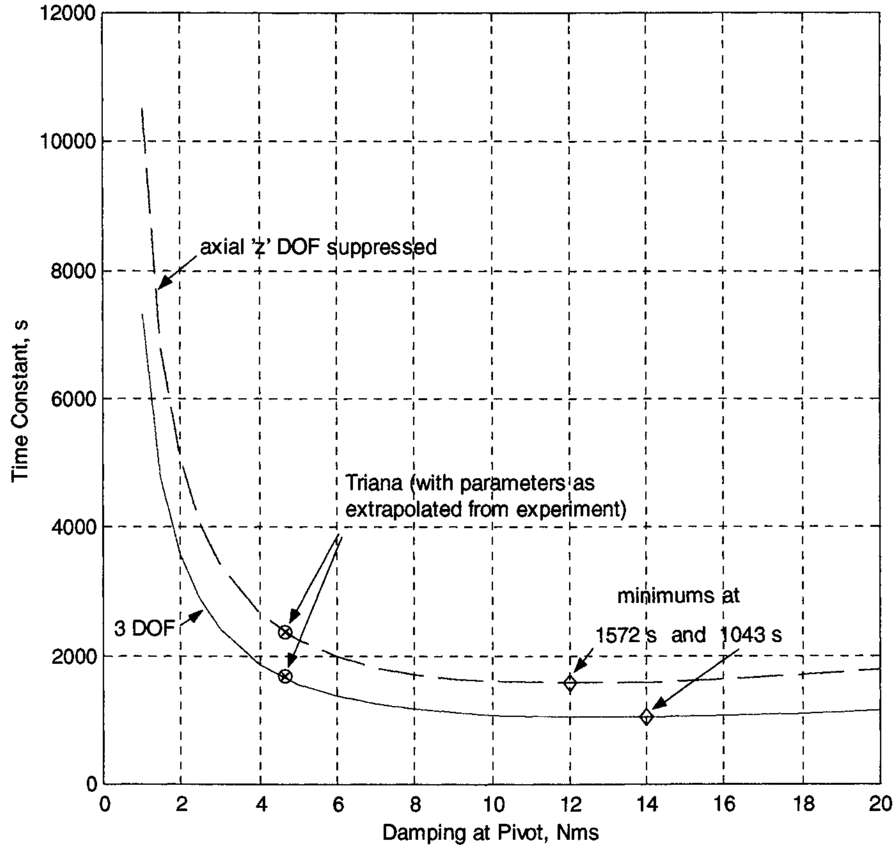

Figure 5 Sensitivity to viscous damping coefficient of pendulum slosh model for Triana spacecraft (60 rpm).

The effect of viscous damping at the pendulum pivot is illustrated in Figure 5. The nominal extrapolated value of 4.569 Nms yields a 1681s time constant. This curve allows one to gauge the effect changes in viscosity will have and it also gives a minimum, or worst case, of 1043 s. Suppressing the pendulum rotation (swirl) about its axis resulted in somewhat less dissipation. This effect was confirmed by cases run with close to zero moment of inertia about this axis for which the time constant appeared to converge to these same levels. Note, stiffness about this axis is nominally set to zero.

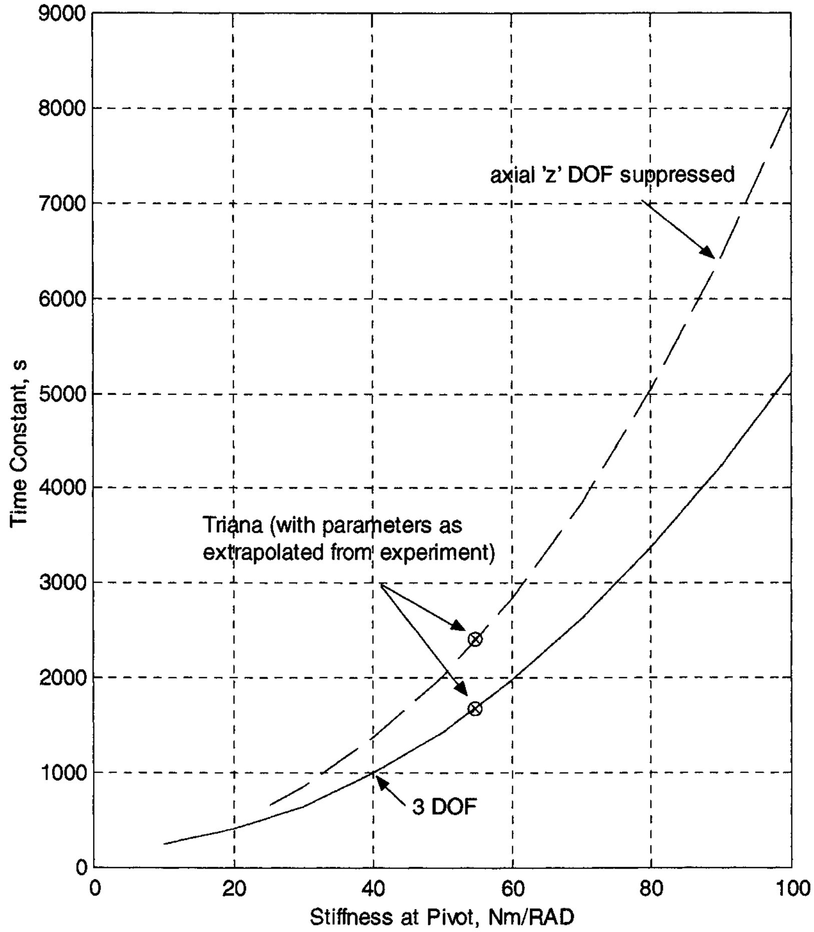

Figure 6 Sensitivity to stiffness at pendulum pivot for Triana spacecraft (60 rpm).

For the Triana pendulum the pivot and mass are nominally aligned along the axis of spin. Figure 6 points to increased dissipation with reduced stiffness. The 3 DOF case gives a more conservative time constant estimate compared to when the 'z' DOF is suppressed.

CONCLUDING COMMENT

A general 3-body slosh model is presented and validated for the case in which the propellant is represented solely as a spherical slug. The model can readily be extended to include any number of additional tanks as well as additional slosh modes if desired. It is applied to the Triana spacecraft using a pendulum slosh model as well. Sensitivity to some of the important parameters is demonstrated. The estimates should prove useful in gauging a worst case energy dissipation scenario. An independent 2-body maximum dissipation spherical slug model is used as well to provide a worst case nutation time constant prediction.

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