INTEGRATION OF LIBRATION POINT ORBIT DYNAMICS INTO A UNIVERSAL 3-D AUTONOMOUS FORMATION FLYING ALGORITHM

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ABSTRACT

The autonomous formation flying control algorithm developed by the Goddard Space Flight Center (GSFC) for the New Millennium Program (NMP) Earth Observing-1 (EO-1) mission is investigated for applicability to libration point orbit formations. In the EO-1 formation-flying algorithm, control is accomplished via linearization about a reference transfer orbit with a state transition matrix (STM) computed from state inputs. The effect of libration point orbit dynamics on this algorithm architecture is explored via computation of STMs using the flight proven code, a monodromy matrix developed from a N-body model of a libration orbit, and a standard STM developed from the gravitational and coriolis effects as measured at the libration point. A comparison of formation flying Delta-Vs calculated from these methods is made to a standard linear quadratic regulator (LQR) method. The universal 3-D approach is optimal in the sense that it can be accommodated as an open-loop or closed-loop control using only state information.

Introduction

The Guidance, Navigation, and Control Center (GNCC) at GSFC is currently demonstrating an enhanced autonomous formation flying (EFF) system. In future missions, autonomous systems are expected for not only low Earth orbit formations, but libration orbits as well. In anticipation of such missions, the GSFC GNCC is investigating options for closed-loop autonomous navigation and maneuver control of satellite formations that is based on libration orbit dynamics.

As a stepping stone to next generation Earth sciences missions and technologies, the Folta-Quinn [Folta-98] algorithm was selected to be flown as a new technology onboard the NMP Earth Observing-1 spacecraft. A major accomplishment of the NMP EO-1 mission is successful completion of numerous paired scene observations with Landsat-7 to validate advanced mapping technology. To enable the paired scene process, the EO-1 spacecraft must fly in formation with Landsat-7, maintaining the same groundtrack as Landsat-7 within 3-km as shown in Figure 1. A minimum along-track separation of nominally one-minute is necessary to achieve this goal.

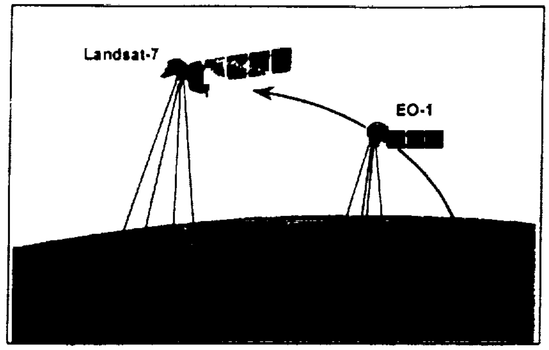

Figure 1. EO-1 Follows Landsat-7 By One Minute

BACKGROUND

Formation Flying Mechanics and Definitions

Formation flying involves position maintenance of multiple spacecraft relative to measured separation errors. It therefore demands use of an active control scheme to maintain the relative positions of the spacecraft. Ultimately, this will be performed autonomously onboard the spacecraft, in a manner similar to that which is already demonstrated by GSFC for the EO-1 mission.

Autonomous orbit control of a single spacecraft requires that a known control regime be established by the ground that is consistent with mission parameters. This data must then be provided to the spacecraft. When orbital perturbations carry the spacecraft close to any of the established boundaries, the spacecraft reacts (via maneuver) to maintain itself within its control box. Once a control box is provided to the spacecraft, no further ground interaction is required. Enhanced formation flying takes the next step up the technological ladder by permitting the spacecraft themselves to establish where their own control boxes should be. This requires cooperation between all the members of the formation, and therefore a depth of communication between all the individual satellites that is not practical (or in some cases even possible) from the ground. This may occur through cooperative "agreement" by controllers of all the spacecraft in the formation or by maintaining a relative position from a designated 'lead', or by some hybrid of these two methods.

The analysis presented here is based on combining the work of the EO-1 EFF program, recent advances in the modeling of the libration orbit dynamics in terms of dynamical systems, and standard control methods. This work begins to extend the approach to realistic libration orbit control cases by developing the linear system required for the control framework via linearization of the non-linear dynamics of circular restricted three body (CRTB) motion.

CRTB Control

Problems of single spacecraft control of circular restricted three-body (CRTB) motion have been previously investigated using state-space equations to characterize the linearized equations of motion ([Hoffman-93], [Wie-98]). State-space analysis methods in control theory provide a useful framework for defining goals and the optimal control of satellites designed to fly about a reference orbit. We advance formation flying control using state-space by incorporating control algorithms into a proven algorithm. The mathematical foundation for the explanation of CRTB motion will be briefly addressed, but only to the degree required to understand the results of the simulations presented.

A State Space Model

As with the current literature about this subject we use three-dimensional Cartesian coordinates denoted by capital letters to describe a system with an origin at the barycenter of the primary bodies and small letters will be used for an origin at a libration point. Figure 2 illustrates the coordinate system and sample orbit that will be used here. Here *m* is an infinitely small mass in the gravitational field of the primary bodies M_1 and M_2.

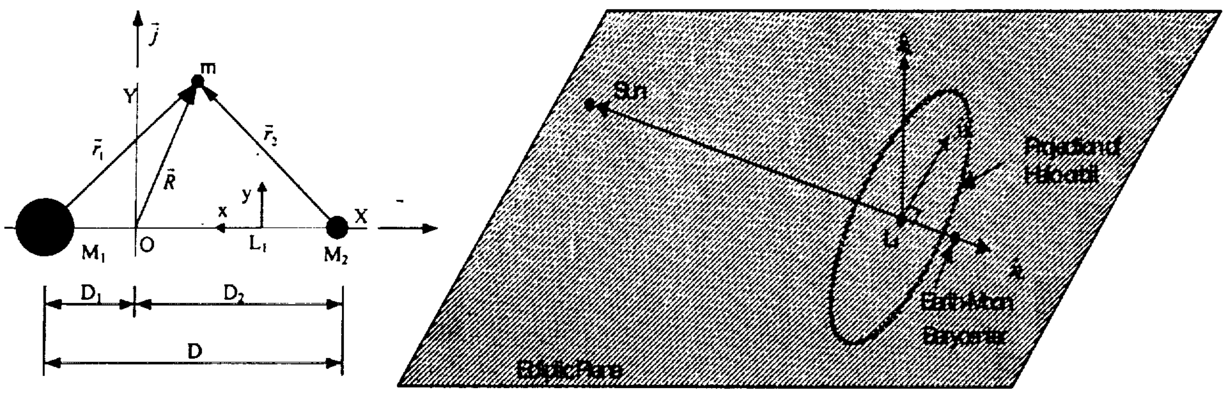

Fig. 2: Coordinate system and sample libration orbit

The linearized equation of motion for m close to the libration point is:

$$\ddot{x} - 2n\dot{y} = U_{XX}x\,,\quad \ddot{y} + 2n\dot{x} = U_{YY}y\,,\ \text{and}\quad \ddot{z} = U_{ZZ}z$$

where $U_{XX} = \dfrac{\partial^2 U}{\partial X^2}$, $U_{YY} = \dfrac{\partial^2 U}{\partial Y^2}$ and $U_{ZZ} = \dfrac{\partial^2 U}{\partial Z^2}$. U_{XX}, U_{YY} and U_{ZZ} are calculated at the respective libration point to get the respective equation of motion and are constants. As a result of the linearization, *x* and *y* are coupled whereas *z* is now completely independent and is a simple harmonic. These equations can also be written in state-space form as below where $\mathbf{x}^j$ represents the

$$\dot{\mathbf{x}}^j = A^j\mathbf{x}^j \qquad \text{where} \qquad \mathbf{x}^j = \begin{bmatrix} x^j \\ y^j \\ z^j \\ \dot{x}^j \\ \dot{y}^j \\ \dot{z}^j \end{bmatrix},\quad A^j = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ U_{XX} & 0 & 0 & 0 & 2n & 0 \\ 0 & U_{YY} & 0 & -2n & 0 & 0 \\ 0 & 0 & U_{ZZ} & 0 & 0 & 0 \end{bmatrix}$$

state of the *j*th spacecraft in the formation. This is one of the forms that will be used in the following sections.

GSFC Controller Application

The GSFC Formation Flying Algorithm is adaptable to generic formation flying problems and permits full closed-loop three axis orbital maneuver autonomy onboard any spacecraft. This algorithm, being demonstrated on the Earth Observer-1 formation flying mission, solves the position maintenance problem by combining the boundary value problem, initial and target states, and Battin's 'C*' matrix formulation to construct a state transition matrix. In this example, the goal of the algorithm is for a spacecraft to perform maneuvers which cause it to move along a specific transfer orbit. The transfer orbit is established by determining a path which will carry the spacecraft from some initial state, $(\mathbf{r}_0, \mathbf{v}_0)$, at a given time, t_0, to a target state, $(\mathbf{r}_t, \mathbf{v}_t)$, at a later time, t_t. The target state found will place the spacecraft in a location relative to the control spacecraft so as to maintain the desired formation. Back propagating the target state to find the initial state the spacecraft would need at time t_0 to achieve the target state at time t_t without executing a maneuver gives rise to the desired state, ($\mathbf{r}_d$, $\mathbf{v}_d$) at time t_0. The initial state can now be differenced from the desired state to find:

$$\begin{pmatrix} \delta \mathbf{r} \\ \delta \mathbf{v} \end{pmatrix} = \begin{pmatrix} \mathbf{r}_0 - \mathbf{r}_d \\ \mathbf{v}_0 - \mathbf{v}_d \end{pmatrix} \tag{3.24}$$

The original application of the FQ algorithm used a state transition matrix calculated using universal variables and the F&G series in a two–body formulation. For the application here, we derive the state transition matrix using the matrix exponential as shown above. That state transition matrix is then partitioned as follows:

$$\Phi(t_0,t_1) \equiv \left[\frac{\Phi_1(t_0,t_1),\Phi_2(t_0,t_1)}{\Phi_3(t_0,t_1),\Phi_4(t_0,t_1)}\right] = \left[\frac{\tilde{R}^*(t_0),R^*(t_0)}{\tilde{V}^*(t_0),V^*(t_0)}\right] = \left[\frac{V^T(t_1),-R^T(t_1)}{-\tilde{V}^T(t_1),V^T(t_1)}\right] = \Phi(t_1,t_0)^{-1}$$

Where the starred quantities are based upon the position/velocity partitions of $\Phi(t_0,t_1)$, and unstarred quantities are based on a $\Phi(t_1,t_0)$, which Battin calls the guidance matrix and navigation matrix respectively. If a reversible Keplerian path is assumed between the two states, one should expect the forward projection of the state from t_0 to t_1 to be related to the backward projection of the state from t_1 to t_0. From these sub-matrices, a C* matrix is computed as follows:

$$\mathrm{C}^*(t_0) = \mathrm{V}^*(t_0)\left[\mathrm{R}^*(t_0)\right]^{-1}$$

The expression for the impulsive maneuver applied herein follows immediately:

$$\Delta \mathrm{V} = \left[\mathrm{C}^*(t_0)\right]\delta \mathrm{r}_0 - \delta \mathrm{v}_0$$

<u>Mondromy Matrix</u>

The computation process of the stable and unstable manifolds for a libration point orbit is associated with particular orbit design parameters and is accomplished numerically in a straightforward manner. The procedure is based on the availability of the monodromy matrix (the variational or state transition matrix after one period of motion) associated with the libration point orbit. As with any discrete mapping of a fixed point, the characteristics of the local geometry of the phase space can be determined from the eigenvalues and eigenvectors of the monodromy matrix. These are characteristics not only of the fixed point, but of the libration point orbit. The local approximation of the stable and unstable manifolds involves calculating the eigenvectors of the monodromy matrix that are associated with the stable and unstable eigenvalues. Using this information , this approximation can be propagated to any point along the halo orbit using the state transition matrix.

<u>Summary</u>

The intend is to develop STMs via mathematically proven systems developed by GSFC for libration orbits and use them in an augmented EO-1 formation flying algorithm for realistic control of formations in libration orbits. The use of the Mondromy matrix, a STM, contains more dynamical information of the libration orbit than does the pseudo-gravitational matrix of the libration point itself. These changes from a Two-Body to a Circular Restricted Three Body approach provides a flight proven formation flying system that is suitable to libration orbit missions.

References

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