Hypersonic Boundary Layer Instability over a Corner

P. Balakumar and Hongwu Zhao
Aerospace Engineering Department
Old Dominion University, Norfolk, VA 23529

Abstract

A boundary-layer transition study over a compression corner was conducted under a hypersonic flow condition. Due to the discontinuities in boundary layer flow, the full Navier-Stokes equations were solved to simulate the development of disturbance in the boundary layer. A linear stability analysis and PSE method were used to get the initial disturbance for parallel and non-parallel flow respectively. A 2-D code was developed to solve the full Navier-stokes by using WENO(weighted essentially non-oscillating) scheme. The given numerical results show the evolution of the linear disturbance for the most amplified disturbance in supersonic and hypersonic flow over a compression ramp. The nonlinear computations also determined the minimal amplitudes necessary to cause transition at a designed location.

Introduction

It is well known that boundary layer transition has a dramatic influence on aerodynamic behavior of hypersonic aircraft. Both static and dynamic stability of such vehicles depend on the spatial distribution and location of the laminar to turbulent transition. The prediction of laminar to turbulent transition in hypersonic boundary layers is a critical part of the aerodynamic design and control of hypersonic vehicles. In general, the transition is a result of nonlinear response of the laminar boundary layers to forcing disturbance. It follows five stages: 1) Receptivity. 2) Linear instability. 3) Nonlinear stability and saturation. 4) Secondary instability. 5) Breakdown to turbulence. The receptivity of boundary layers to disturbance is the process of converting environmental disturbance into instability waves in the boundary layers. The receptivity mechanism provides important initial conditions in terms of amplitude frequency and phase for the instability wave in the boundary layers. In quiet environments, the initial amplitudes of these unstable waves are small compared to any characteristic velocity and length scale in the flow. Goldstein[1] theoretically explained using asymptotic methods how the Tollmien-Schlicting waves are generated near a leading edge of a flat plate by the long wavelength acoustic disturbance and also showed the development of these waves in the boundary layer at their initial stage. In last few decades, linear stability theory[2] and PSE[3] methods had been used extensively to analyze the transition process in incompressible and compressible flat-plate and axis-symmetric boundary layers. The transition onset point

can be predicted using the N-factor method. However, linear theory is applicable only to some specific transition problems, and even then it describes just the first stage of transition, that is, the slow growth of the primary instability. Subsequent stages are due to nonlinear interactions. Especially for boundary layer flows with discontinuities, the linear stability theory and the PSE method cease to be valid. Due to the existence of interaction between shock and boundary layer, the evolution of disturbance in the boundary layer will be nonlinear. So a full Navier-Stokes equation should be solved to find the evolution of disturbance.

In this study, we will investigate the instability and the transition onset point in the supersonic and hypersonic flows over a compression corner as shown in Fig.1 by solving full Navier-Stoks equations. A fifth-order WENO scheme[4,5] had been used to accurately compute the physical shock interaction with boundary layer and also the development of instability waves in the boundary. From these numerical results, the minimal amplitudes necessary to cause transition at a designated location can be determined. This investigation will also improve our understanding of the effects of corner shocks which exist in most of the hypersonic and supersonic vehicles on the transition onset point.

Formulations and numerical methods

The governing equations are the unsteady full two-dimensional compressible Navier-Stokes equation. It can be written for computation in conservation form as following:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = \frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y}$$

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e_t \end{bmatrix} \qquad E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (\rho e_t + p)u \end{bmatrix} \qquad E_v = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ u\tau_{xx} + v\tau_{xy} - q_x \end{bmatrix}$$

$$F = \begin{bmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ (\rho e_t + p)v \end{bmatrix} \qquad F_v = \begin{bmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ u\tau_{yx} + v\tau_{yy} - q_y \end{bmatrix}$$

The gas is assumed to be thermally and calorically perfect. The viscousity and heat conductivity coefficients are calculated using Sutherland's law together with a constant Prandtl number p_r. The variables ρ, p, T, and velocity are non-dimensionalized by their corresponding reference variables ρ_∞, p_∞, T_∞ and $\sqrt{RT_\infty}$ respectively. The

reference value for length is computed by $\sqrt{\nu x_0/U_\infty}$, where x_0 is the location of the leading edge of the corner.

In order to simulate the propagation of disturbance in the boundary layer by direct numerical simulation, an initial small disturbance should be added to the uniform steady flow at the leading edge of the corner. This initial disturbance is produced by linear stability theory or PSE method in the boundary layer. It was superimposed on the steady uniform flow according to the following rule.

$$\tilde{u}(x,y,t) = \hat{u}(y)e^{i\alpha x - i\omega t} + \hat{u}^*(y)e^{-i\alpha x + i\omega t}$$

Where, α is the wave number in the streamwise direction, ω is the disturbance frequency.

In order to discern the small disturbance from the mean flow, a high order accurate scheme is necessary. In this study a fifth order WENO scheme was used for spatial discrete and three stage TVD Runge-Kutta for time iterations. The mean flow is obtained at first by steady computation, then by adding the initial disturbance profile to the mean flow at the leading edge and begin marching along the time step, we can simulate the propagation of initial disturbance along the streamwise direction in the boundary layer.

Results and discussions

1 Steady Solution

In order to simulate the disturbance propagation in the boundary layer, a steady result for hypersonic flow going through the compression corner should be obtained first. The grid for computation is shown as Fig.1. Here a 1001×301 grid is used. This grid is generated by potential and stream function method and is stretched close to the wall.
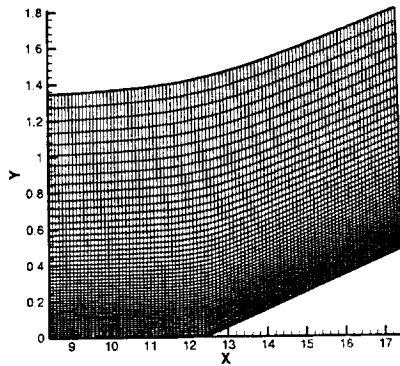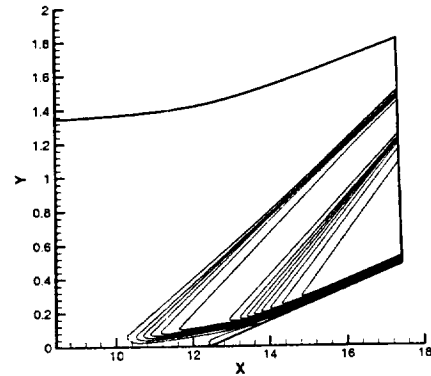

Fig. 1 The computation grid
Fig. 2 The contour of velocity in y direction

The flow is initially undisturbed, that is, only a boundary layer flow over the corner is present. This undisturbed flow field is obtained by numerically solving the full two-dimensional compressible Navier-stoks equation. The results of the mean flow are illustrated by Fig.2-6. The velocity contour is plotted as in Fig. 2. It can be seen that across the corner, a shock is produced. The density contour and Mach contour are given in Fig. 3 and Fig.4 respectively. The pressure distribution along the wall of corner is plotted as Fig. 5. The stream lines close to the corner vertex point are shown in Fig.6. It can be seen that a flow separation bubble is produced across the corner. The pressure is increased in the separation bubble to form an adverse pressure gradient so that a reverse flow in the separation bubble is produced. It is the interaction between the shock and vortex, which produce the non-linear influence on disturbance propagation downstream. The location of the separation and reattachment has a big influence on the transition of corner flow from laminar flow to turbulence.
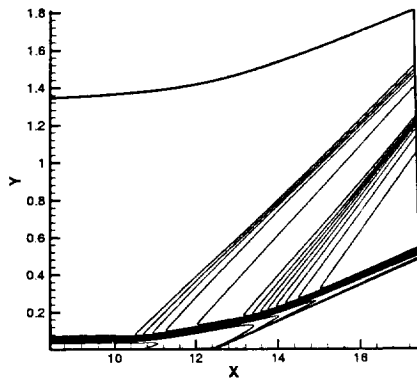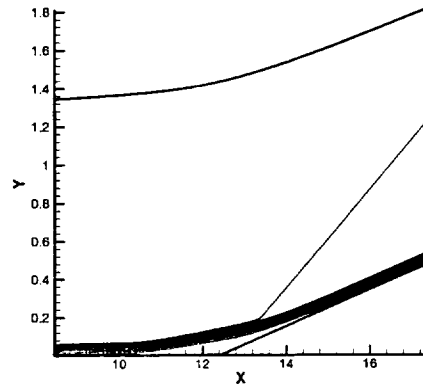
Fig.3 The Contour of Density

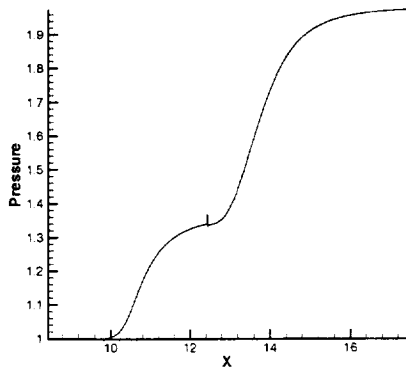Fig.4 The Contour of Mach Number
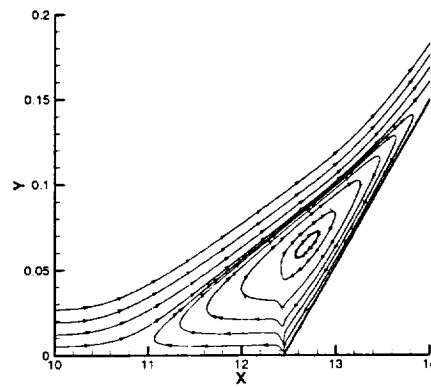
Fig. 5 The pressure distribution along the wall

Fig. 6 Streamlines over the corner

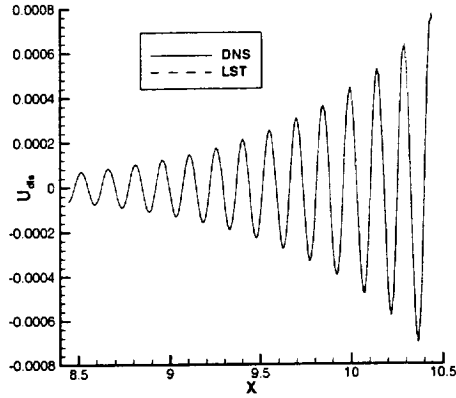## 2 Compare parallel result with that of linear stability theory
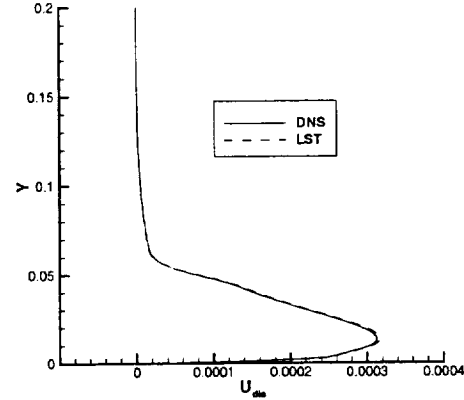

Fig. 7 Disturbance along x direction

Fig. 8 Disturbance profile at x=9.0

Because the linear stability theory is a well developed method for flat plate transition problem, it can be used to check the code validation. A direct numerical simulation is first conducted for the parallel flow going through the flat plate and then we compare its results with that of linear stability theory. Fig.7 and Fig.8 show the comparison between the results of these two methods. It can be seen that the results of direct numerical simulation fit well with the results of linear stability theory.

3 Compare non-parallel result with that of PSE method

In spite of the qualitative success of assumption of the parallel flow, the parallel stability theory does not explain some important phenomena, and the experiments have shown systematic difference with the theory. Apart from predicting a minimum critical Reynolds number that is lower than that given by the parallel stability theory, evidence from experiments shows that the growth rate of the disturbance is not only a function of the coordinate normal to the wall, but is also different for different streamwise location. So a non-parallel stability result is preferred to predict the transition in boundary layer. In this study, a comparison between the results of non-parallel numerical simulation and PSE method had been sought for a supersonic flow to go through a flat plate. Fig. 9 shows the comparison of maximal disturbance value of density and velocity in streamwise direction, which are obtained by PSE and non-parallel DNS method respectively. It can be seen from these two figures that the results of these two methods are very close to each other. This gives a code validity verification for full Navier-Stokes computation to take account the influence of boundary layer increasing on disturbance propagation.
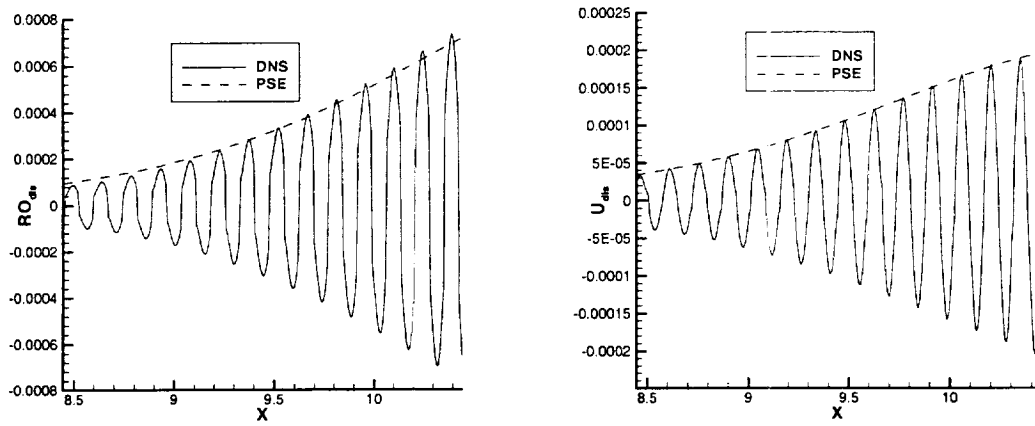
Fig. 9 Comparison of maximal disturbance value between DNS and PSE

4 Instability simulating in the boundary layer of the corner

After a validation check of the code by linear stability theory and PSE method, next we can conduct a direct numerical simulation of the boundary layer instability for hypersonic flow over a compression corner. The motivation of this work is to determine the flow condition in front of the inlet to the propulsion system in the X-31 flight vehicle in the flight conditions without any tripping devices and also to determine what is the efficient way to cause the transition before the inlet. The configuration of the corner and computation grid is illustrated as Fig.1. The deflection angle of the corner is 5°. The incoming flow Mach number is 5.373. The initial disturbance is introduced at the leading edge of the corner by PSE method. Fig.10 shows the initial velocity disturbance profiles in x direction and y direction receptively. It can be seen that the disturbance will disappear outside the boundary layer.
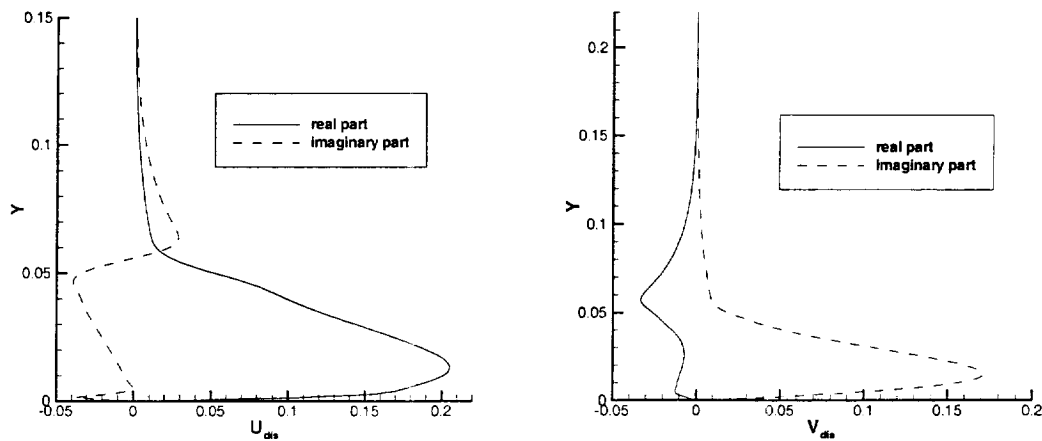

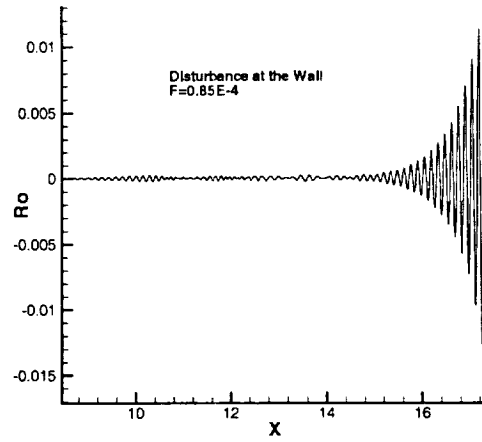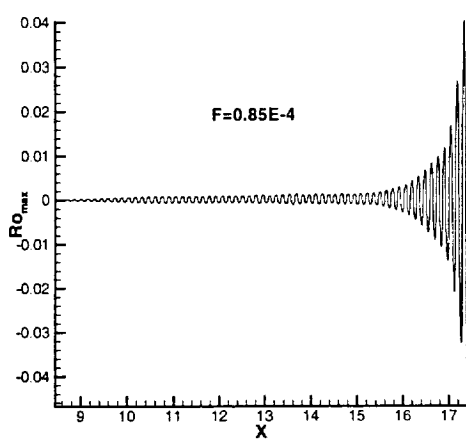Fig.10 Velocity disturbance profiles at the leading edge

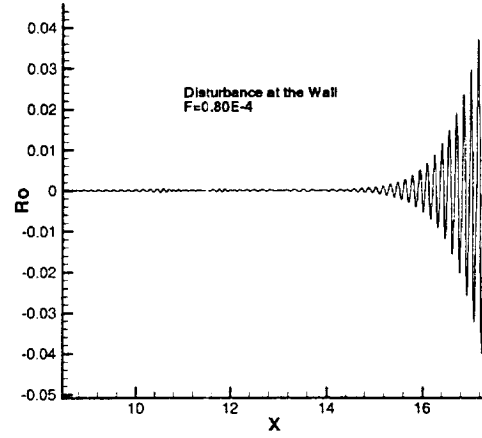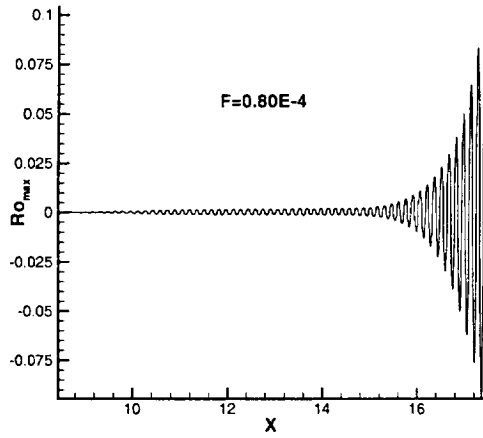Fig. 11 The density disturbance with F=0.85×10^{-4}, Amp=0.0001

Fig. 12 The density disturbance with F=0.80×10^{-4}, Amp=0.0001

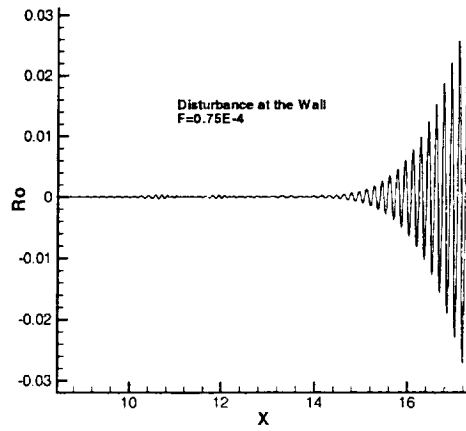

Fig. 13 The density disturbance with F=0.75×10^{-4}, Amp=0.0001

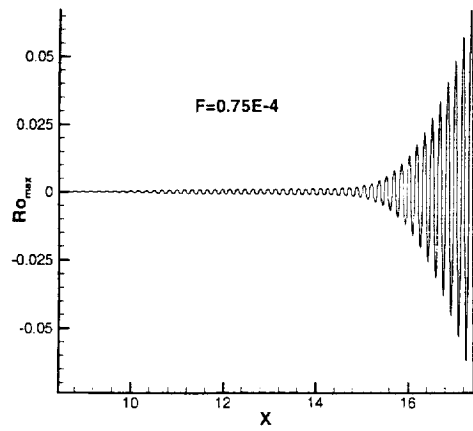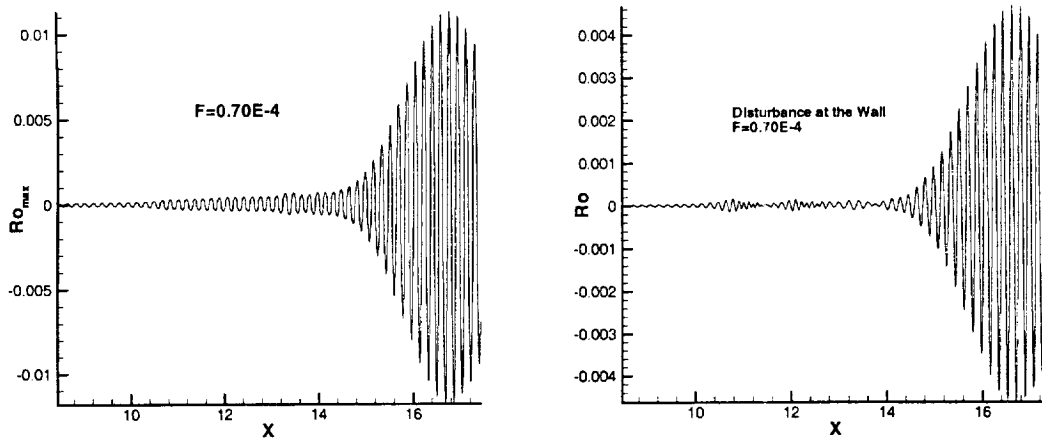Fig. 14 The density disturbance with $F=0.85\times10^{-4}$, Amp=0.0001

In this study, several computational cases have been undertaken for different disturbance frequency and initial disturbance amplitude to show the influence of the frequency and amplitude on disturbance propagation in the boundary layer of the corner. Fig.11-14 shows the maximal density disturbance amplitude growth and the density disturbance growth at the wall with the same initial disturbance amplitude and different dimensionless disturbance frequency. In the cases when the frequency is between 0.85×10^{-4} and 0.75×10^{-4}, the disturbance is unstable and keeps growing downstream in the boundary layer. But in the separation bubble region, the disturbance is saturated and grows again after the separation region. In the case of $F=0.70\times10^{-4}$, the disturbance is stable, but it grows in the separation bubble region and begin decaying at small distance after the separation bubble region.
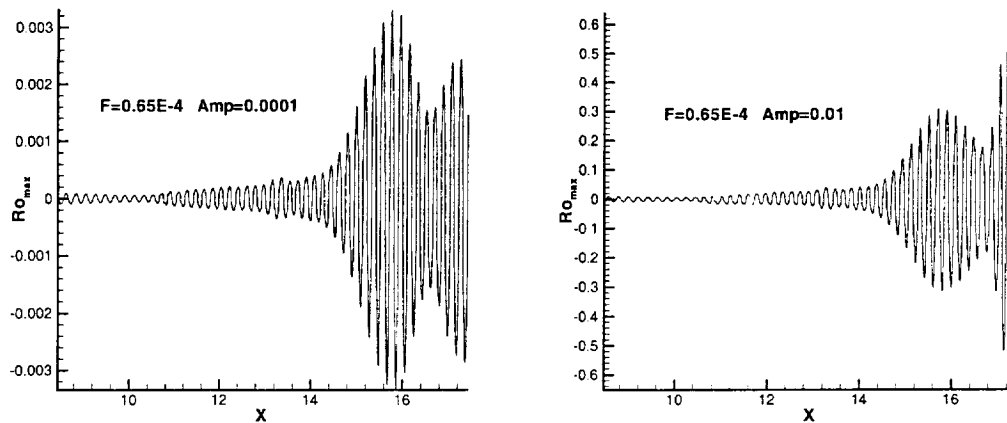

Fig.15 Maximal density disturbance with the same frequency
and different initial amplitude

Fig.15 shows the disturbance propagation in the boundary layer with the same disturbance frequency F=0.65×10^{-4} but with different initial disturbance amplitude. It can be seen from these figures that the disturbance decay first before it reaches the separation bubble region, but it becomes unstable in the separation bubble region. After a small saturation at some distance after the separation region, the disturbance grows again. With much bigger initial disturbance amplitude, the disturbance amplitude will reach to big enough at some specific downstream location to produce a nonlinear saturation. A secondary instability will begin to take effect. So by choosing the appropriate initial disturbance amplitude, we can make transition develop at some specific location.

Concluding remarks

The boundary layer instability over a two-dimensional compression corner under the hypersonic flow condition is studied by solving the full Navier-Stokes equation in this study. The computational validation is checked by comparing parallel and non-parallel results for flow over a flat plate with that of the linear stability theory and PSE method respectively. By the direct numerical simulating, the disturbance propagating procedure in the boundary layer flow with discontinuity could be understood in detail. According to the results, it can be seen that the separation bubble has a significant influence on the disturbance propagation in the boundary layer of the compression corner. In this region, disturbance becomes unstable when the disturbance frequency is below 0.70×10^{-4}. The minimal disturbance amplitude at the leading edge to cause transition at some specific location downstream can be determined by the direct numerical simulation. So the direct numerical simulation for full two-dimensional compressible Navier-Stokes shows a basic picture for the disturbance propagation in the boundary layer with flow discontinuity. But more computational cases are needed to study the influence of different disturbance frequency and initial disturbance amplitude on transition location. A three-dimensional computation is needed as well in order to know more details about the nonlinear interaction between different modes and secondary instability.

Reference

[1]Goldstein M. E. and Hultgren S. L., "Boundary-layer receptivity to long wave free-stream disturbances", Ann. Rew. Fluid Mech., 1989, Vol.21, pp. 137-166.

[2]Mack M. L., "Boundary-layer stability theory", Jet Propulsion Laboratory, California Institute of Technology, 900-227 Rev. A, 1969.

[3]Herbet T., "Parabolized stability equations", Ann. Rew. Fluid Mech., 1997, Vol.29, pp. 245-283.

[4]Atkins H. L., "High-order ENO methods for the unsteady compressible Navier-Stokes equations", AIAA paper 91-1557, 1991.

[5]Shu C. W., "Essentially Non-Oscillatory and Weighted Essentially Non-Oscillatory schemes for hyperbolic conservation laws", NASA/CR-97-206253, 1997.