Flux Compression Magnetic Nozzle

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Abstract

In pulsed fusion propulsion schemes in which the fusion energy creates a radially expanding plasma, a magnetic nozzle is required to redirect the radially diverging flow of the expanding fusion plasma into a rearward axial flow, thereby producing a forward axial impulse to the vehicle. In a highly electrically conducting plasma, the presence of a magnetic field B in the plasma creates a pressure $B^2/2\mu$ in the plasma, the magnetic pressure. A gradient in the magnetic pressure can be used to decelerate the plasma traveling in the direction of increasing magnetic field, or to accelerate a plasma from rest in the direction of decreasing magnetic pressure. In principle, ignoring dissipative processes, it is possible to design magnetic configurations to produce an "elastic" deflection of a plasma beam. In particular, it is conceivable that, by an appropriate arrangement of a set of coils, a good approximation to a parabolic "magnetic mirror" may be formed, such that a beam of charged particles emanating from the focal point of the parabolic mirror would be reflected by the mirror to travel axially away from the mirror. The degree to which this may be accomplished depends on the degree of control one has over the flux surface of the magnetic field, which changes as a result of its interaction with a moving plasma.

The simplest way to accomplish this is <u>to allow the plasma to compress the magnetic flux to within a very thin boundary layer conforming to an envelop of a set of current carrying coils</u> as illustrated in Figure 1.1. In this case, by arranging a set of coils on a parabolic surface of revolution, a parabolic magnetic mirror is likely to be generated. Of course, the 3-D flow of an extended body of plasma is considerably more complicated than a beam of plasma, deviations from perfect parabolic reflections are to be expected. Such deviations may be characterized by an average angular deviation, $\delta\theta_D$, of the net momentum of the deflected plasma from the axial direction. The fusion reactions create an explosion at the focal point of the hypothetical parabolic mirror. Due to its high electrical conductivity and high velocity, the expanding fusion plasma is an MHD flow with a high magnetic Reynolds number, on the order of 10^7. The flow compresses the magnetic flux against the coils serving as a wall to the magnetic flux. The magnetic flux is trapped between the fast moving plasma and the coils because of the currents induced in the coils and the plasma. Currents are induced in the plasma due to the motion of the plasma against the magnetic field, while currents are induced in the coils to oppose the change of the total magnetic flux linked to the coils. The flux compression gives rise to a sharp increase in the intensity of the magnetic field and thus in the magnetic pressure acting on the plasma. The plasma is deflected and swept out by this magnetic pressure through the rear axially. The magnetic pressure reacts on the coils to produce a thrust for propulsion. In this paper we discuss the feasibility of such a parabolic magnetic flux compression nozzle and the overall energy constraints on such a nozzle, leading to the efficiency and other performance parameters of such a nozzle.

From an energetic viewpoint, energy is transferred from the plasma motion to the magnetic field during the compression of the magnetic flux. In a perfectly elastic deflection, the field energy is completely returned to the plasma's kinetic energy. Deviation from a perfect elastic deflection arises from three main sources: (1) Non-isentropic processes in the plasma leading to an increase in entropy and heat production (Q_{plasma}) in the plasma, (2) Ohmic heating in the coils (Q_{nozzle}), and (3) the intentional extraction of an amount (E_{rc}) of the magnetic field energy by the electrical circuit connected to the coils for the purpose of recharging the system for the next pulse. Thus the net amount of charged particle energy available for the nozzle action to produce thrust is,

$$E_{cp,\,nozzle} = E_{cp} - Q_{plasma} - Q_{nozzle} - E_{rc} = (1-\varepsilon_c)\varepsilon_k\, E_{cp} \tag{1.1a}$$

where,

$$\varepsilon_k = 1 - \frac{Q_{plasma} + Q_{nozzle}}{E_{cp}}, \tag{1.1b}$$

and ε_c is the fraction of available plasma kinetic energy siphoned off by the circuit for regenerative purposes. The MHD processes are highly isentropic giving values of ε_k closed to unity, typically $\varepsilon_k \sim 0.85$. The resulting increase in entropy resides in the exhaust stream.

It must be realized that, unlike optical reflections, the deflection of a beam of plasma in such a magnetic geometry does not remain in the plane of motion of the plasma. The plasma particles will also experience a $\mathbf{B} \times \nabla B$ drift in the azimuthal direction. Thus a ring of plasma expanding from the mirror's focal point will be reflected axially and rotated azimuthally. This rotation is advantageous in that it provides shear stabilization against Rayleigh-Taylor instabilities and also enhances the axisymmetry of the flow. The reaction to this rotation produces a torque on the coils, which may be used to rotate the coils and/or the whole vehicle (for external thermal cycling or to produce artificial gravity internally via centripetal acceleration). Alternatively the torque on the coils could be nullified by reversing the magnetic field from pulse to pulse.

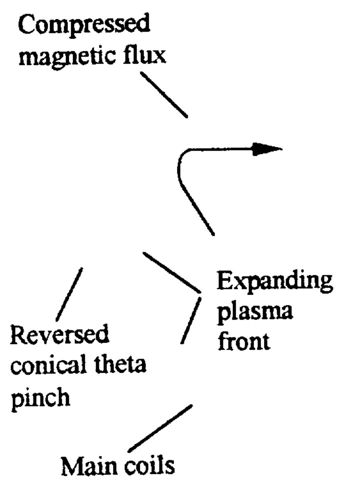

Figure 1.1. Paraboloic Magnetic Nozzle.

The compressed magnetic flux forms a boundary layer with the magnetic field lines running more or less parallel to the parabolic envelop of the coils, except near the vertex of the parabola. The solenoidal nature of the magnetic fields produced by the coils invariably leave a "hole" at the vertex of the magnetic nozzle. To patch up this hole, a reversed conical theta pinch is used as shown in Figure 1.1.

The coils are protected from the moderated intermediate neutrons (10 keV - 100 keV) and other radiation by a shielding jacket consisting of a graphite core and cooling channels lined with zirconium alloys and cladded on the exterior with SiC. Though SiC has superior chemical properties to graphite, it is difficult to manufacture SiC in thick pieces. It is also much heavier, more brittle, harder to machine than graphite. SiC is less stable to oxidizing corrosion than zirconium alloys/ceramics such as ZrB_2. The composite structure with graphite core, Zr alloys as wall lining in contact with the coolant, and SiC on the exterior for refractory protection appears the best use of these materials for this application. Due to the high temperature operation and to avoid oxidation corrosion, considerations are given to the use of a high-temperature, non-oxidizing coolant such as helium or molten lithium hydride (LiH). Lithium hydride has the added advantage of being a good neutron moderator and absorber. All the heat generated from the shielding and absorption of neutrons is piped to the radiator panels to be radiated out into space. The feasibility of the above scheme to control the flux surface of the compressed magnetic field then depends on the possibility that this shielding jacket can be made sufficiently thin. This in turn depends on the moderated neutrons having sufficiently low energy.

Following Zel'dovich and Raizer,[1] the average macroscopic radial velocity and mass density of the expanding plasma may be taken to be,

$$v = \left(\frac{2E_{cp,nozzle}}{m_p}\right)^{1/2} = \left(\frac{2(1-\varepsilon_c)\,\varepsilon_k\,E_{cp}}{m_p}\right)^{1/2}, \qquad (1.2)$$

$$\rho = \frac{m_p}{4\pi R^3/3}$$

where m_p is the total mass of the expanding plasma and R is the outer radius of the expanding sphere. In what follows, we assume an imperfect "parabolic nozzle" with a mean axial angular deviation of $\delta\theta_D$, having an orifice subtending an angle $2\theta_a$ at the focal point. The axial component of the momentum of the spherically expanding plasma intercepted by the nozzle is given by,

$$\begin{aligned} p_{z,\exp} &= \int_0^{2\pi}\int_{\theta_a}^{\pi}\int_{r=0}^{R_0} \rho_0 v \cos\theta \; r^2 \sin\theta \, dr\, d\theta\, d\phi \\ &= \frac{1}{4} m_p v \sin^2\theta_a \end{aligned} \qquad (1.3)$$

The axial momentum of the reflected plasma with a mean angular deviation of $\delta\theta_D$ from the axis is given by,

[1] Ya. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*, Vol. II, Chap. VIII, Section 3, p. 571 - 573, Academic Press, New York, 1967.

$$p_{z,ref} = \int_0^{2\pi} \int_{\theta_a}^{\pi} \int_{r=0}^{R_0} \rho_0 v r^2 \sin\theta \, dr \, d\theta \, d\phi = m_p v \cos^2 \frac{\theta_a}{2} \cos \delta\theta_D \tag{1.4}$$

Thus, the net axial forward impulse imparted to the magnetic nozzle is the sum of the above two momenta,

$$p_z = \int_0^{2\pi} \int_{\theta_a}^{\pi} \int_{r=0}^{R_0} \rho_0 v r^2 \sin\theta \, dr \, d\theta \, d\phi = m_p v \left(\cos \delta\theta_D + \sin^2 \frac{\theta_a}{2} \right) \cos^2 \frac{\theta_a}{2} \tag{1.5}$$

It is convenient to measure the efficiency of the nozzle as the ratio of the momentum of the deflected plasma to the momentum if all the available charged particle energy of the plasma is converted into axial momentum, thus,

$$\eta_j = \frac{p_z}{m_p \sqrt{\dfrac{2E_{cp}}{m_p}}} = \sqrt{\varepsilon_k (1-\varepsilon_c)} \left(\cos \delta\theta_D + \sin^2 \frac{\theta_a}{2} \right) \cos^2 \frac{\theta_a}{2} \tag{1.6}$$

For a nozzle with an orifice half angle $\theta_a = 5\pi/12$, the theoretical maximum nozzle efficiency is 86.3%. With $\varepsilon_c = 0.042$, $\varepsilon_k = 0.85$, and a mean deviation of 15° from a perfect axial reflection, the nozzle efficiency drops to 78.9%.

Only the component of plasma motion normal to the magnetic field is available for compressing the magnetic flux. A spherical coordinate system (r, θ, ϕ) is used with the polar axis pointing rearward. The parabolic reflecting surface is described by the equation $r(1-\cos\theta) = 2a_1$ where a_1 is the distance from the focus to the vertex of the parabola. This is also a flux surface, thus the magnetic field lines are parallel to the surface. Ignoring the small $\mathbf{B} \times \nabla B$ drift, the normal component of the plasma velocity at a point (r, θ, ϕ) is given by,

$$v_n = v \sin\frac{\theta}{2} = \left(\frac{2(1-\varepsilon_c)\varepsilon_k E_{cp}}{m_p} \right)^{1/2} \sin\frac{\theta}{2}. \tag{1.7}$$

To deflect the plasma, the magnetic pressure in the compressed layer of flux must be sufficient to absorb the recoil momentum flux of the plasma. The recoil momentum flux is twice the momentum flux normal to the flux surface since the normal component of the velocity reverses its direction in the elastic deflection. Thus,

$$\frac{B^2}{2\mu} = \rho (2v_n)^2 = \frac{m_p}{4\pi r^3/3} 4v^2 \sin^2 \frac{\theta}{2} = \frac{6(1-\varepsilon_c)E_{cp}}{\pi r^3} \sin^2 \frac{\theta}{2} \tag{1.8}$$

We would like to have this layer as thick as possible to allow for the thickness of the shielding jacket around the coils. Assuming that the field energy is wholly derived from the plasma, the maximum thickness of the layer is determined by the amount of plasma kinetic energy available for compressing the magnetic flux. Consider an annular ring formed by revolving an annular elemental area spanned by the segments from (r_1, θ, ϕ) to (r_2, θ, ϕ) and from $(r_1, \theta+d\theta, \phi)$ to $(r_2, \theta+d\theta, \phi)$ about the axis of the parabola where r_1 and r_2 are the inner and outer radial coordinates of the flux surface at the polar angle θ. It is reasonable to assume that the magnetic field energy in this ring is derived from the conical shell of plasma between the polar angles θ and $\theta+d\theta$. The plasma kinetic energy that is available for compressing the magnetic flux is the part associated with the component of the plasma velocity normal to the magnetic field. With $B^2/2\mu$ being the field energy density of the field, the condition that the energy in the compressed flux cannot exceed the plasma energy available for compression is,

$$\int_{r=r_1}^{r_2} \int_{\phi=0}^{2\pi} \frac{B^2}{2\mu} r^2 \sin\theta \, d\theta \, d\phi \, dr \le \int_{r=0}^{R_0} \int_{\phi=0}^{2\pi} \frac{1}{2} \rho_0 v_n^2 r^2 \sin\theta \; d\theta \; d\phi \; dr \tag{1.9}$$

In the above we have ignored the initial seeding flux produced by the nozzle, as this is small compared to the field energy created by the flux compression. Upon simplification, the above relationship yields a condition for the maximum ratio of the radial coordinates as,

$$\ln\left(\frac{r_2}{r_1}\right) \le \frac{1}{24}, \text{ or, } \frac{r_2}{r_1} \le 1.043\,. \tag{1.10}$$

It is interesting to note that the maximum amount of kinetic energy of the plasma available for flux compression is,

$$E_{available} = \int_{\phi=0}^{2\pi} \int_{\theta=\theta_a}^{\pi} \int_{r=0}^{R} \frac{1}{2}\rho v^2 \sin^2\frac{\theta}{2}\, r^2 \sin\theta \; dr\, d\theta\; d\phi\,. = \frac{1}{2}\left(1-\varepsilon_c\right)\varepsilon_k\, E_{cp}\left(1-\sin^4\frac{\theta_a}{2}\right) \tag{1.11}$$

showing that the energy available for flux compression is less than half the total amount of plasma kinetic energy. On the other hand, up to this amount of energy may be stored in the magnetic field of the magnetic nozzle or, equivalently, in the coils of the magnetic nozzle. Most of the magnetic energy stored in the coils is returned to the plasma with a small fraction (typically 2 ~ 3 %) of it is dissipated as heat in the cryogenically cooled coils.