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THE USE OF ECCENTRIC AND CIRCULAR ORBITS IN THE DESIGN OF A MARS NETWORK CONSTELLATION

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The Use of Eccentric and Circular Orbits in the Design of a Mars Network Constellation

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Abstract

This study examined different constellation configurations to determine their suitability for the Mars Network. Some variations on the baseline case of four circular orbits were initially studied. Eccentric orbits were then used to determine their effects on several figures of merit that were selected as representative of the design goals. It was eventually found that the use of eccentric orbits in combination with circular orbits can improve aspects of some navigation and communication figures of merit. The ability to use orbits at different inclinations helps smooth coverage over the middle and upper latitudes for these figures of merit. This configuration has more variability than one consisting of circular orbits, so the occurrence of unfavorable arrangements also results in degradation of some figures of merit. Some cases were used which improved different figures of merit, so a solution could be chosen depending on specified requirements.

Introduction

The current baseline for the Mars Network constellation consists of six circular orbits at 800 km altitude, with two at 172° inclination and four evenly spaced at 111° inclination. This configuration was obtained after examining a series of tradeoffs between navigation and communication figures of merit. These tradeoffs confirm that, in general, increasing the altitude of the circular orbits aids navigation, but degrades communication. Eccentric orbits do not remain at one altitude, however, and may present an additional solution to the design of the constellation. One of the reasons the four highly inclined orbits in the baseline have the same inclination and altitude was to cause their right ascension of ascending node to rotate at the same rate. If this rate is different for each orbit, the relative orientation of the orbits will change with time and eventually result in an unfavorable configuration. Using four identical eccentric orbits at different right ascensions of ascending node would also avoid this possibility and is one configuration worth considering. Unfortunately, the high inclination necessary to obtain coverage for the poles with four orbits at the same inclination results in low coverage for the middle latitudes and high coverage of the poles. It would be desirable to keep some of these orbits at a high inclination in order to provide some coverage to the poles and move others to a lower inclination so as to aid in coverage at the middle latitudes. It is still necessary, though, to avoid undesirable configurations by keeping precession rate (the rate of change of right ascension of ascending node) the same. This may be achieved, within limits, by varying the periapse, apoapse and inclination to obtain the same values for the following quantity:

$$d\Omega/dt = -1.5nJ_2(R_m/a)^2(\cos i)(1-e^2)^{-2}$$

$d\Omega/dt$	= rate of change of the right ascension of ascending node in deg/day
n	= mean motion in deg/day
R_m	= radius of Mars
a	= semi-major axis of orbit
i	= inclination of orbit
e	= eccentricity

In this way, a combination of circular and eccentric orbits at different altitudes and inclinations could be chosen that would continue to repeat the same relative configurations. For eccentric orbits, the perigee also drifts, so the relative position of each perigee becomes important in addition to the phasing. This paper presents the results of analyzing a variety of different configurations with circular orbits, eccentric orbits and combinations of the two types. Some effects of changing the relative phasing and the positions of each perigee are also discussed.

Method

For this preliminary analysis of different orbit configurations, "vapors" (a software program developed by L. Romans at JPL) was used to model the orbits and calculate the figures of merit. It used a J2 perturbation model to integrate the orbits.

Many different figures of merit were originally considered as a way to aid in selecting different constellation configurations. The primary navigation figures of merit that were used were the average time to obtain a fix with one meter accuracy and the maximum time to obtain a fix to one meter accuracy. The primary communication figures of merit were the maximum wait time between sightings of a satellite, and the average of optimistic and pessimistic values for Mbits/sol/Watt. The optimistic values assumed that the data rate could be varied to achieve any desired rate, and the pessimistic values used a single data rate. In practice, the optimistic values were typically twice the value of the pessimistic values. Other figures of merit used in some cases were the percent of cases that could obtain a fix to 10 meters in 10 minutes, the percent of cases that could obtain a fix to one meter in one hour, and the percent of time that a satellite was in view.

To produce the navigation figures of merit, a series of runs were made at random times. For each run, accuracies were calculated at each point on the ground until all points had reached a desired accuracy. The results from all the runs were used to calculate the figures of merit. A sample of 200 random times taken from over 1000 Earth days was finally judged as sufficient to capture all of the possible combinations for the analyzed cases. In order to obtain the communication figures of merit, a run over a period of 10 days was typically used. The grid of stations on the ground was spaced 5° in latitude and 20° in longitude, because features tended to be uniform over longitude when enough simulations over time were run. With this grid, the results could then be viewed as a contour plot.

For the case of two circular orbits and two eccentric orbits (in addition to the two equatorial orbits), either the inclination or periapse altitude was held constant and the other parameter was varied (with the apoapse adjusted to cause the precession rate to agree with the circular orbits) for a series of cases. The results could then be averaged over longitude and the effect of changing inclination for a given periapse altitude, or the reverse, could be visualized. This could then aid in choosing the optimum solutions.

Results

Initially, various runs were made that were simply variations on the baseline case, using different phasing, altitudes and relative orientations. The effect of using elliptical orbits on the figures of merit was then studied. Finally, the method of using combinations of circular and elliptical orbits was used and refined to search for a solution.

Figures of merit were determined for the baseline case with the orbital parameters given in Table 1 and used for comparison with subsequent cases.

Table 1: Orbital parameters for baseline case

Satellite #	alt_p (km)	alt_a (km)	i (deg)	Ω (deg)	ω (deg)	M (deg)
1	800	800	172	0	0	0
2	800	800	172	180	0	0
3	800	800	111	0	0	0
4	800	800	111	90	90	0
5	800	800	111	180	0	180
6	800	800	111	270	270	0

Although a few cases were examined which involved altering the two equatorial orbits, it was decided to leave them in their original configuration and focus on the four highly inclined orbits for most of the cases. First, the phasings of the four highly inclined orbits were varied in order to see the effects of this parameter. Most changes degraded both navigation and communication performance, but one case did present some possible advantages. In this case, the mean anomaly for satellite 5 was changed from 180 to 0. For navigation, this decreased the average time for a fix at the equator at the expense of slightly longer average and maximum times for a fix at the upper latitudes. It also decreased maximum wait time from approximately 245 mins to 205 mins. One improvement occurred near the equator where the data rate was increased by 50 Mbits/sol/Watt. Several cases were also checked with the four circular orbits at higher altitudes, and, as expected, the communication performance dropped off significantly as navigation parameters improved.

The use of four identical eccentric orbits in different orientations was then examined. The case shown in Table 2 resulted in somewhat degraded navigation parameters and similar communication parameters when compared to the baseline. In general, these types of configurations had the same problems at the middle latitudes as the baseline case.

Table 2: Sample case with 4 elliptical orbits (Case 2)

Satellite #	alt_p (km)	alt_a (km)	i (deg)	Ω (deg)	ω (deg)	M (deg)
1	800	800	172	0	0	0
2	800	800	172	180	0	0
3	600	1000	111	0	0	0
4	600	1000	111	90	90	180
5	600	1000	111	180	0	180
6	600	1000	111	270	270	270

Several cases with combinations of circular and eccentric orbits showed promise, so a simple method to visualize the results of a wide number of cases was used. As stated, the dependence on longitude in the contour plots was typically minimal, so the values over longitude could be averaged without loss of information. As a quick way to examine the design space, the results over different latitudes could be plotted versus either different periapse altitudes or inclinations of the eccentric orbits. Figure 1 is one sample of these plots and helps show why a 115° inclination was chosen for the eccentric orbits. The plot shows undesirable values for the middle latitudes when an inclination less than 115° was used. A higher inclination was not chosen because of effects on the other figures of merit.

After examining these plots, several cases were chosen as having relatively desirable characteristics. For this analysis, the average time to obtain a fix to one meter was used as the main characteristic for comparing different constellations. In the baseline, this figure of merit has some particularly bad values in the middle latitudes between 30° and 40° latitude. Therefore, one objective was to improve the values in this area. The cases were then run individually and the one shown in Table 3 was selected as having possibly the best figures of merit when the different trades were considered.

Table 3: 700 km Periapse Case (Case 3)

Satellite #	alt_p (km)	alt_a (km)	i (deg)	Ω (deg)	ω (deg)	M (deg)
1	800	800	172	0	0	0
2	800	800	172	180	0	0
3	800	800	111	0	0	0
4	700	1330.1	115	90	90	0
5	800	800	111	270	0	180
6	700	1330.1	115	180	270	0

The navigation parameter of average time to obtain a fix generally improved for case 3, while the maximum time to obtain a fix showed some degradation. Figures 2 and 3 show the results for the average time to reach a fix to one meter for the baseline and case 3. Case 3 shows approximately a 0.1 hour improvement in the equatorial region and a 0.2 to 0.4 hour improvement in the middle latitudes. This comes at the expense of an increase of about 0.1 to 0.2 hours at the upper latitudes and at the poles. These results were expected because the eccentric orbits had a lower inclination, which should improve the values for the lower latitudes and degrade the values at the upper latitudes. The results for the maximum time to obtain a fix are shown in Figures 4 and 5. The equatorial values are approximately the same, but the values at all other regions have increased by 0.2 to 0.5 hours. Examining the graphs closely also reveals that the degraded region in the middle latitudes appears to have improved near the equatorial region and to have worsened near the interface to the higher latitudes. The increased times may occur because the eccentric orbits cause the constellation to have a greater range of configurations. Those increased times could typically be lowered for the upper latitudes by increasing the periapse altitude, but this tended to degrade communications.

The communication parameter of average Mbits/sol/Watt had mixed results for case 3 and the maximum wait time was improved at the expense of the times at the upper latitudes. The average Mbits/sol/Watt for each case are shown in Figures 6 and 7. They show that case 3 has an improved performance at the equator of about 30 Mbits/sol/Watt, about the same result in the middle latitudes and has about 60 Mbits/sol/Watt less data at the upper latitudes. Evaluating the maximum wait time shows that case 3 reduces the maximum value from 245 minutes to 205 minutes. Looking at Figures 8 and 9 shows that this improved performance of about 0.3 hours at the middle latitudes comes at the expense of 0.2 to 0.5 hours at some regions in the upper latitudes.

Case 4, shown in Table 4, was found to improve some figures of merit for the upper latitudes, but it did not have the desirable characteristic of a periapse lower than the baseline altitude. In general, it gave results similar to case 3, but it did improve the maximum time to obtain a fix at the upper latitudes. The navigation parameters at the poles improved, but the equatorial fixes took about 0.2 hours longer. The maximum wait time was decreased to 203 minutes, and the amount of data returned was decreased by about 50 Mbits/sol/Watt in the equatorial region.

Table 4: 900 km Periapse Case (Case 4)

Satellite #	alt_p (km)	alt_a (km)	i (deg)	Ω (deg)	ω (deg)	M (deg)
1	800	800	172	0	0	0
2	800	800	172	180	0	0
3	800	800	111	0	0	0
4	900	1107.15	115	90	90	0
5	800	800	111	180	0	180
6	900	1107.15	115	270	270	0

Tables 6 and 7 attempt to summarize the differences between the figures of merit for each case. As an interesting comparison, a case using a periapse of 400 km, given in Table 5, is included.

Table 5: 400 km Periapse Case (Case 5)

Satellite #	alt_p (km)	alt_a (km)	i (deg)	Ω (deg)	ω (deg)	M (deg)
1	800	800	172	0	0	0
2	800	800	172	180	0	0
3	800	800	111	0	0	0
4	400	1716.1	115	90	90	0
5	800	800	111	180	0	180
6	400	1716.1	115	270	270	0

The region used as the "mid-lat's" in the tables varied between figures of merit in order to make it easier to select values that conveyed the general range of the results. For the average time to obtain a fix, the mid-lat's ranged from approximately 25° to 45°, and for the maximum time to obtain a fix the boundaries were 15° to 45°, The range for data rate was from 15° to 50°, and for maximum wait time, from 20° to 40°. In general, the tables show an improvement in performance at the middle or lower latitudes and a degradation in performance at the upper latitudes when elliptical orbits are used. Case 3 was selected as one of the best cases using eccentric orbits in part because of it's increase in performance at the middle latitudes for average time to obtain a fix and it's improvement of communication parameters at the equator.

Table 6: Summary of Navigation Figure of Merits for Each Case

	Average Time for Fix to 1 m (hrs)			Maximum Time for Fix to 1 m (hrs)		
Case	equatorial	mid-lat's	upper-lat's	equatorial	mid-lat's	upper-lat's
1	1.0	1.8 - 2.1	1.0 - 1.1	1.8 - 2.0	3.6 - 4.0	2.0
2	1.0	1.8 - 2.0	1.0 - 1.1	1.9 - 2.0	3.6 - 5.0	2.0 - 3.0
3	0.9	1.5 - 1.7	1.1 - 1.3	1.9	3.4 - 4.5	2.2 - 2.6
4	0.9 - 1.0	1.5 - 1.8	1.1 - 1.2	1.9	3.3 - 4.4	2.0 - 2.2
5	0.9 - 1.0	1.2 - 1.9	1.1 - 1.2	1.9	3.5 - 5.1	2.1 - 3.2

Table 7: Summary of Communication Figure of Merits for Each Case

Case	Data Rate (Mbits/sol/Watt)			Maximum Wait Time (hrs)		
	equatorial	mid-lat's	upper-lat's	equatorial	mid-lat's	upper-lat's
1	475	200 - 260	230 - 400	0.8 - 0.9	2.0 - 3.0	1.5 - 1.8
2	510	200 - 290	180 - 380	0.8 - 0.9	2.0 - 3.1	1.7 - 2.0
3	520	200 - 260	180 - 340	0.8 - 0.9	2.6	2.0 - 2.2
4	470	200 - 240	180 - 330	0.8 - 0.9	2.0 - 2.8	2.0
5	470	200 - 280	160 - 340	0.8 - 0.9	2.0 - 3.4	2.0

Figures 10 through 15 show a summary of the differences between the baseline case and case 3. In each plot, the absolute value of latitude was used and the values for the cases were averaged across longitudes. The values for the baseline were then subtracted from those for case 3 to produce a rough picture of the differences. Figure 10 confirms the improvement in the average time to obtain a fix at the lower and middle latitudes. The remaining average results in Figure 10 and the maximum values in Figure 11 show a general increase. The highest and lowest values for maximum time to obtain a fix in Figure 11 come from the fact that the undesirable region in the middle latitudes has been shifted a little further from the equator in case 3. The data rate at the equator is greater for case 3 as shown in Figure 12, but, again, the values at the upper latitudes have been reduced. It is interesting to note that the maximum time to obtain a fix was decreased for case 3, but this results in larger times at all the latitudes greater than were the maximum wait time occurs. The results from two other figures of merit that were examined for the baseline and case 3 are shown in Figures 14 and 15. Figure 14 reveals that case 3 improves the percent of time that a satellite is in view for almost all of the latitude range. The percent of cases at each station able to obtain a fix to 10 m in 10 minutes is also greater in most places for case 3 as shown in Figure 15.

Conclusion

Overall, the combination of eccentric and circular orbits was able to decrease the high values in the middle latitudes for the average time to obtain a fix, but not the maximum time to obtain a fix. The selected case had little change in the average amount of returned data and improved the maximum wait time at the expense of the upper latitude wait times. In general, it was found that some middle latitude figures of merit could be improved, but only with some penalty to the upper latitudes. Other cases were found that would improve the maximum time to obtain a fix as well as the other navigation parameters at the expense of the data rate. Depending on the priority of these parameters, some of these other cases could be more suitable. One beneficial result of using eccentric orbits would be a lower propellant requirement. For this reason, they might also be able to serve as a suitable backup when it is desirable to economize on propellant. Future research might focus on examining configurations with just one or three eccentric orbits, the effects of using a very low periapse or inclination, the use of polar satellites and consideration of sun-synchronous satellites for daylight effects. For the simulation itself a more detailed model might be considered along with other figures of merit, the optimum grid size and the number of random times necessary for each run.

Acknowledgments

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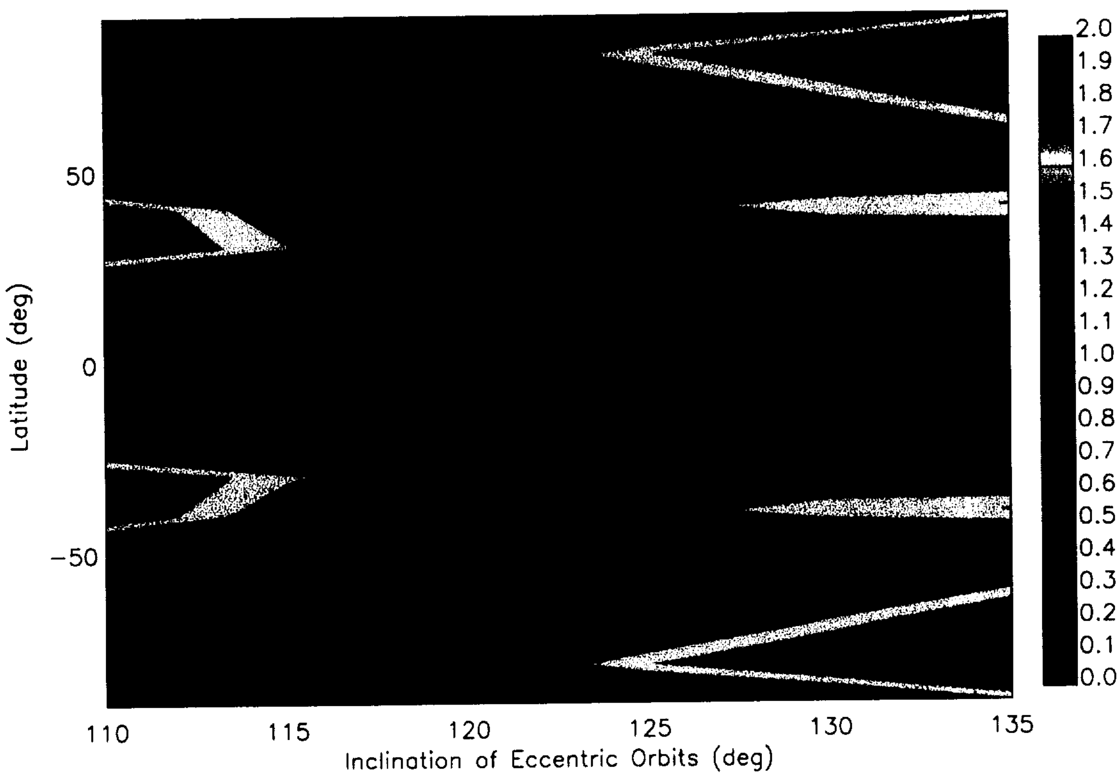

Figure 1: Average time to get a fix to 1 meter for a periapse altitude of 700 km

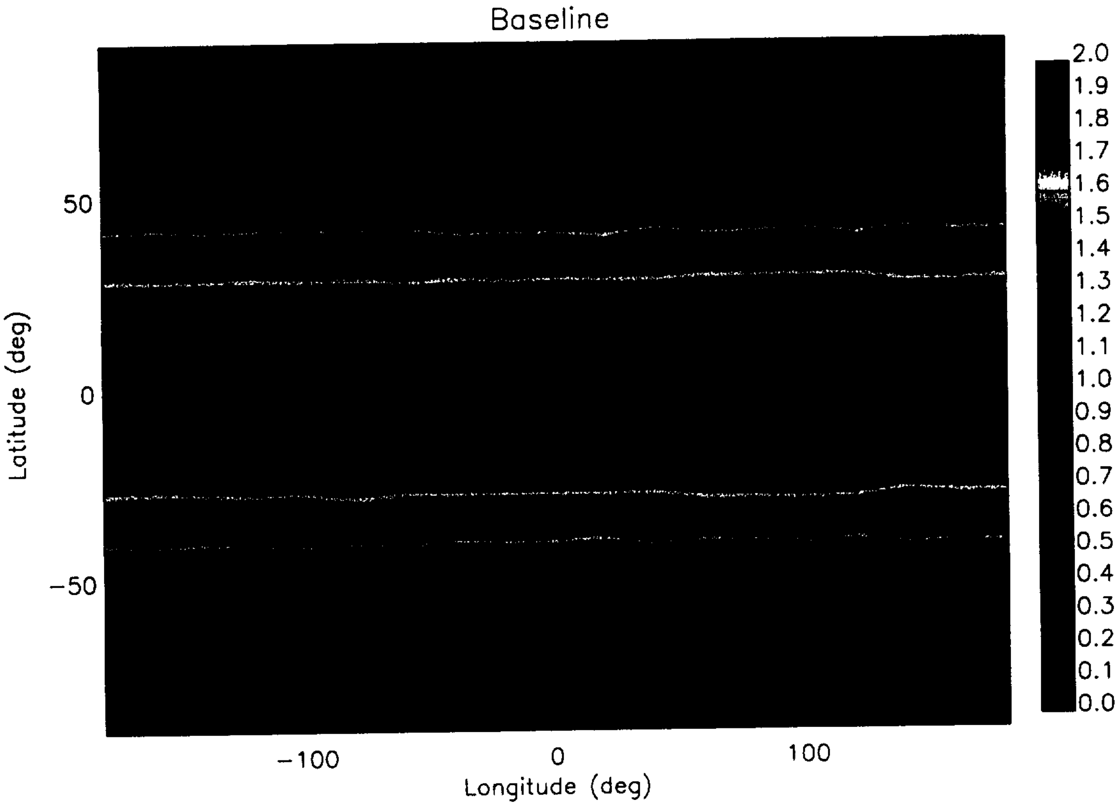

Figure 2: Average time in hours to achieve 1 meter fix for the baseline case

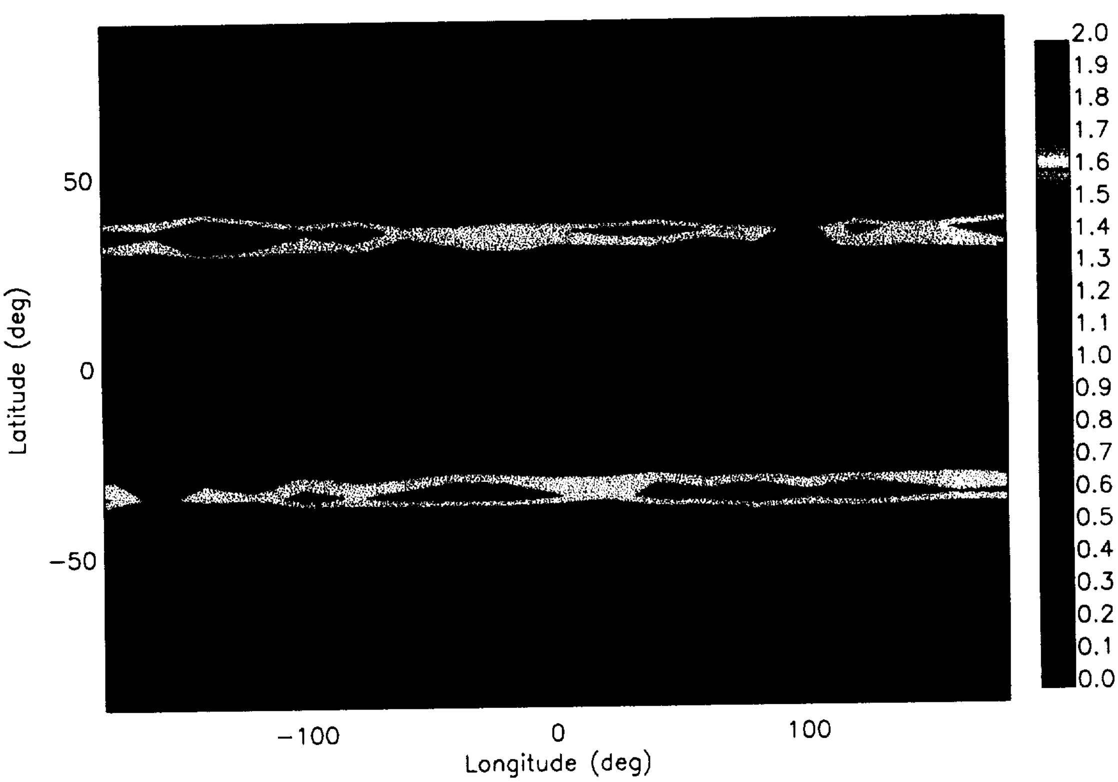

Figure 3: Average time in hours to achieve 1 meter fix for case 3

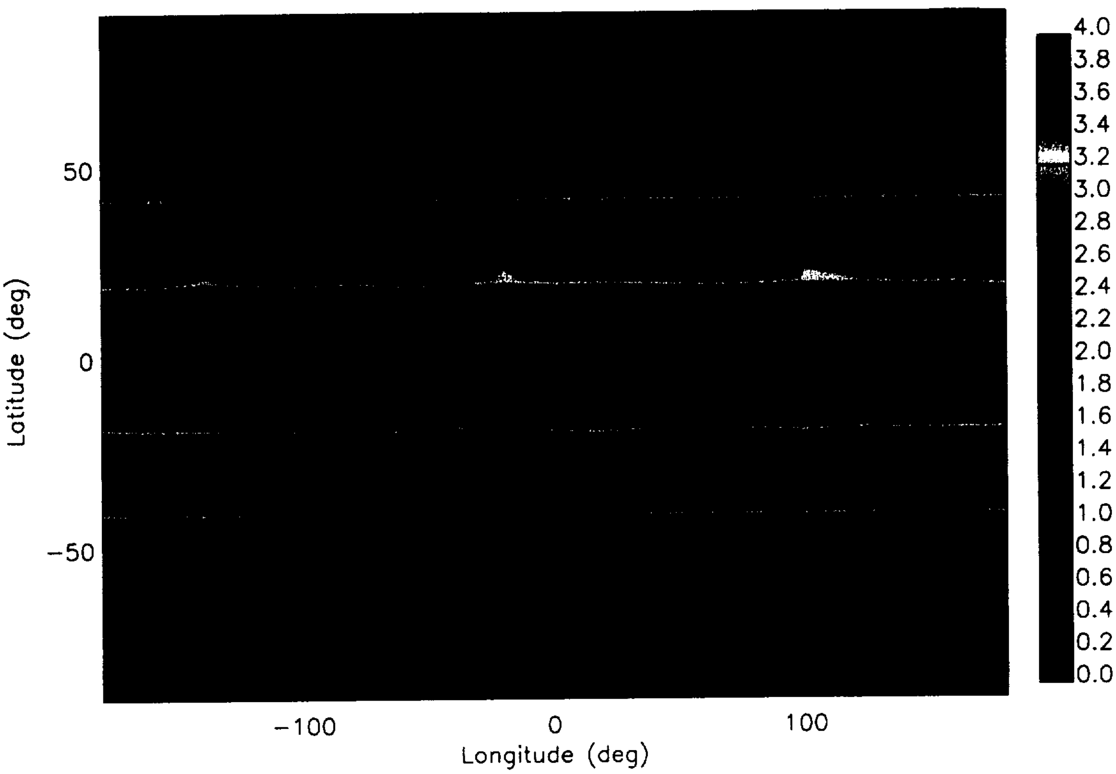

Figure 4: Maximum time in hours to achieve 1 meter fix for baseline case

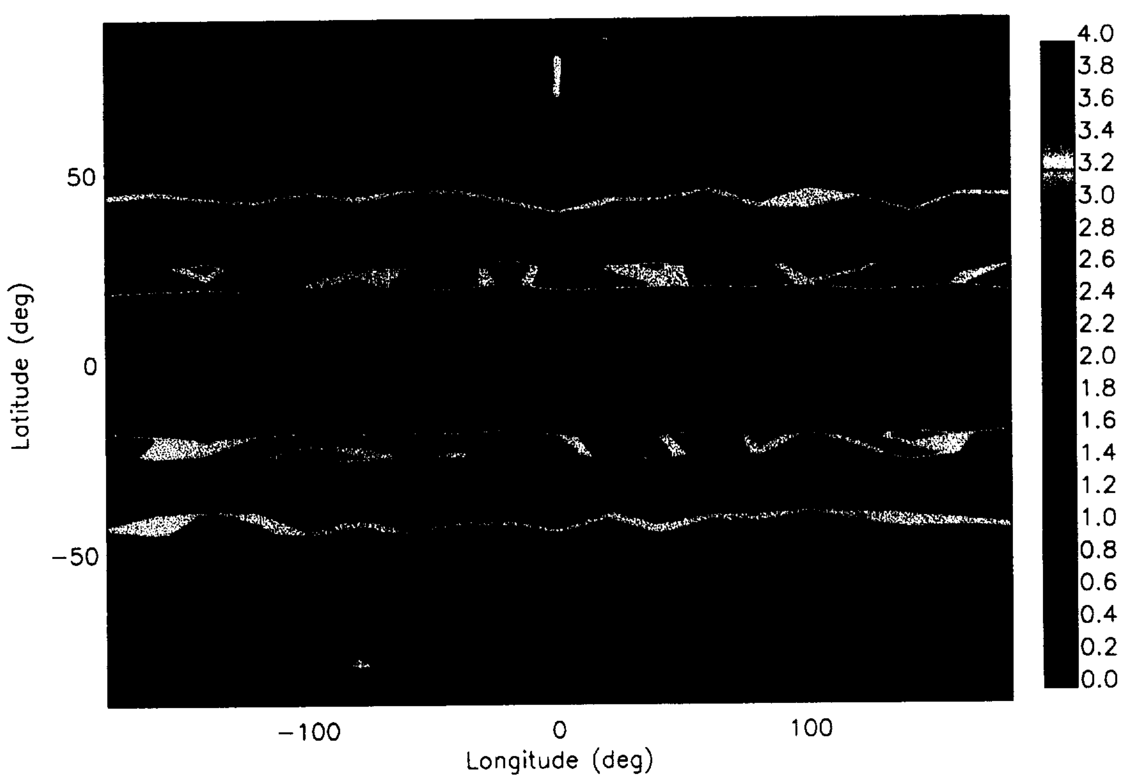

Figure 5: Maximum time in hours for a 1 meter fix for case 3

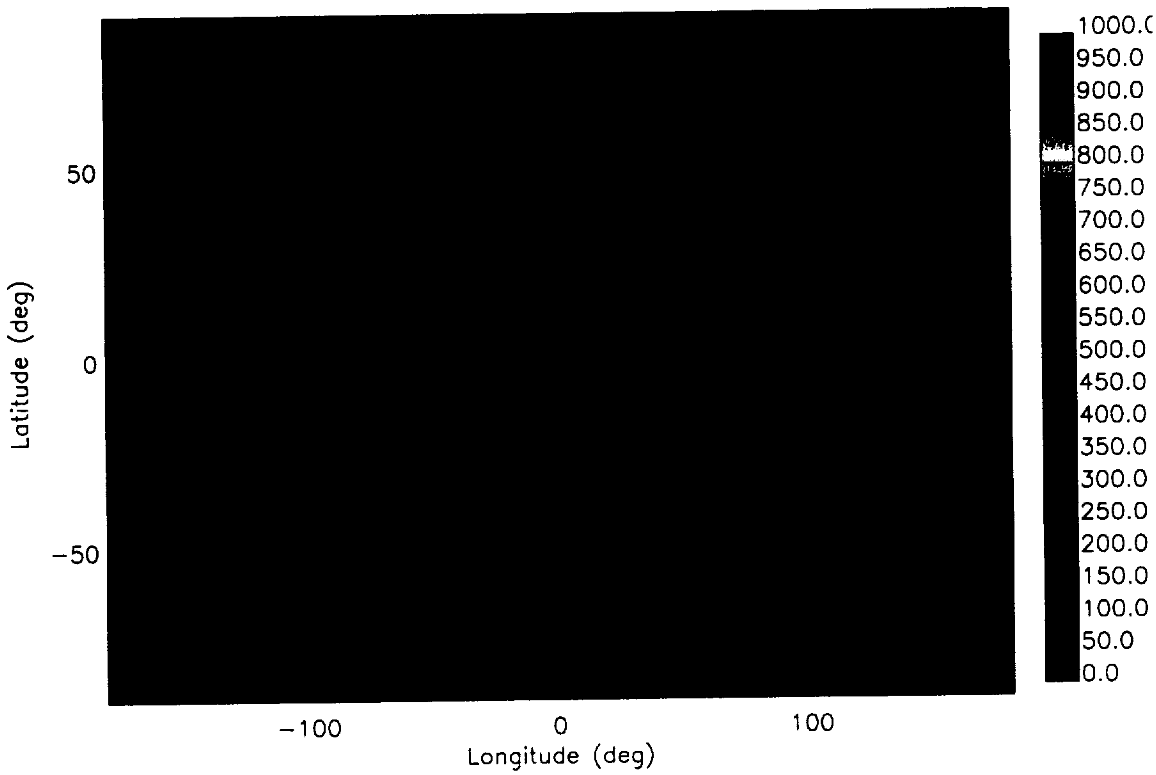

Figure 6: Average Mbits/sol/Watt for baseline case

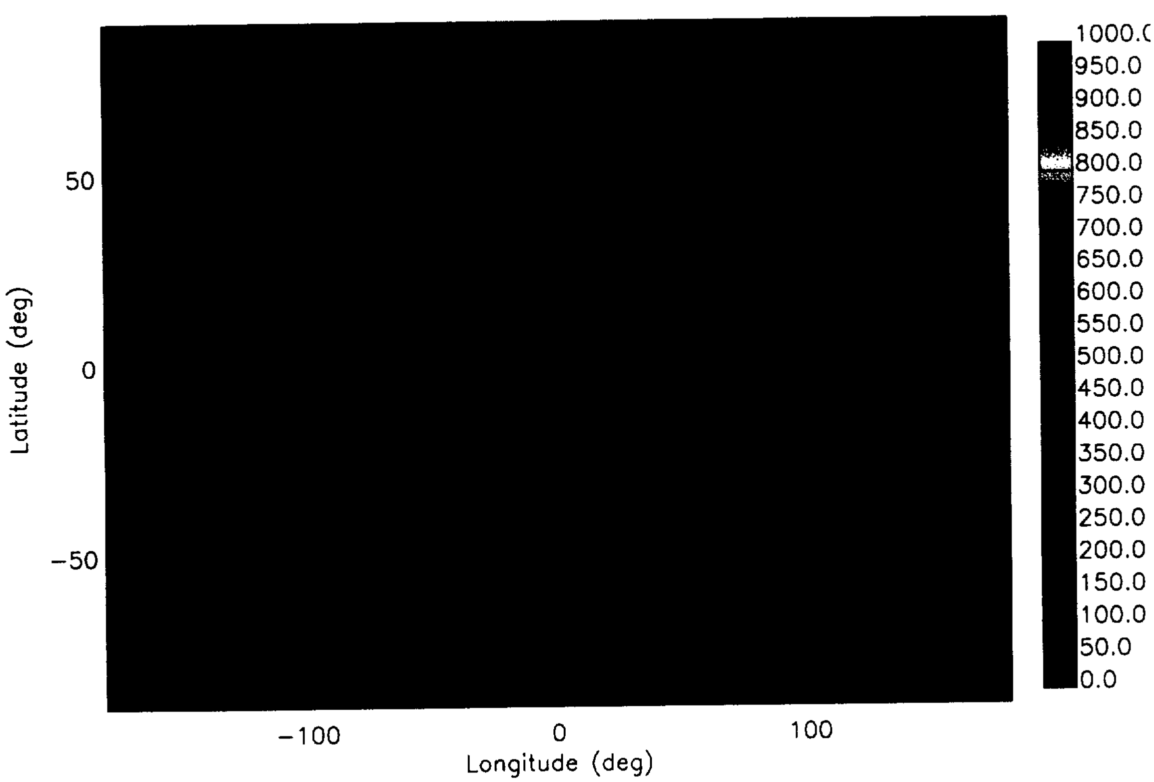

Figure 7: Average Mbits/sol/Watt for case 3

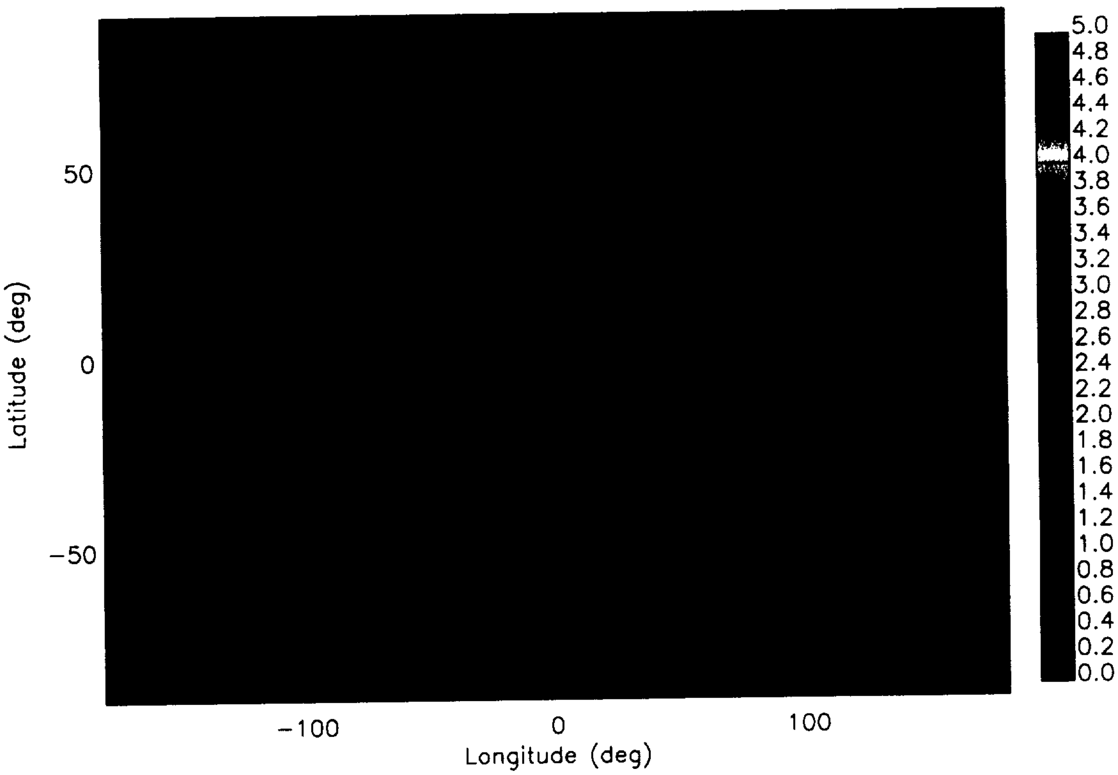

Figure 8: Maximum wait time in hours for baseline case

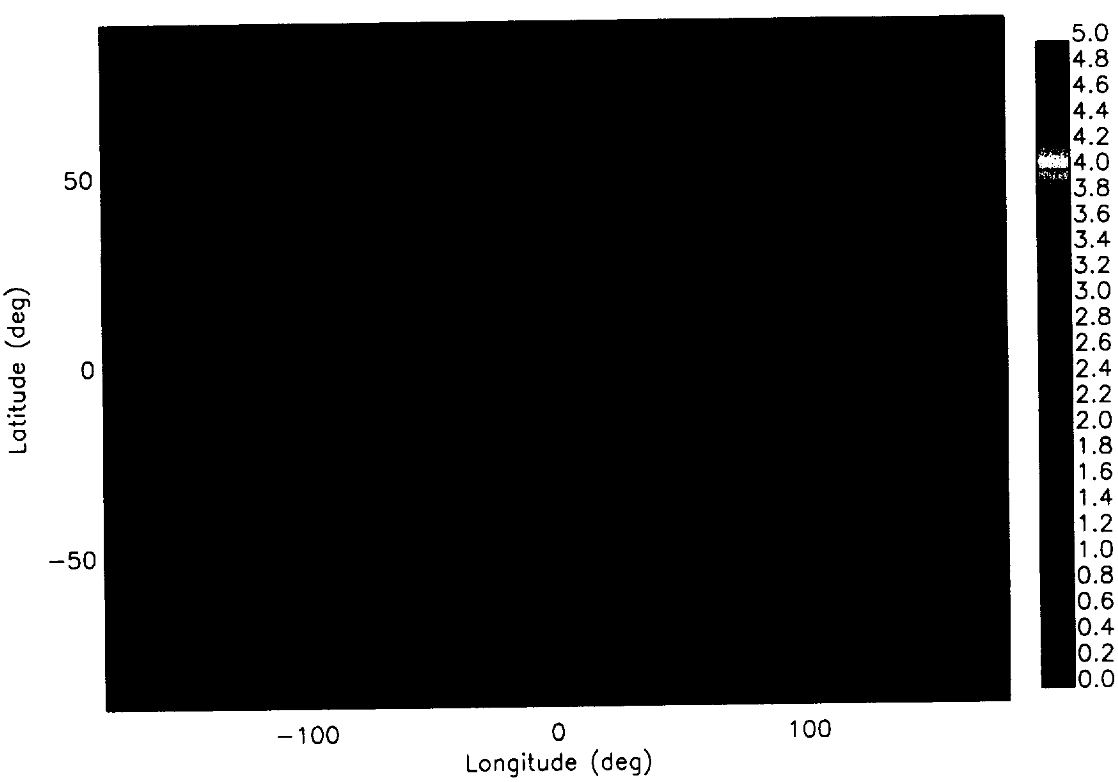

Figure 9: Maximum wait time in hours for case 3

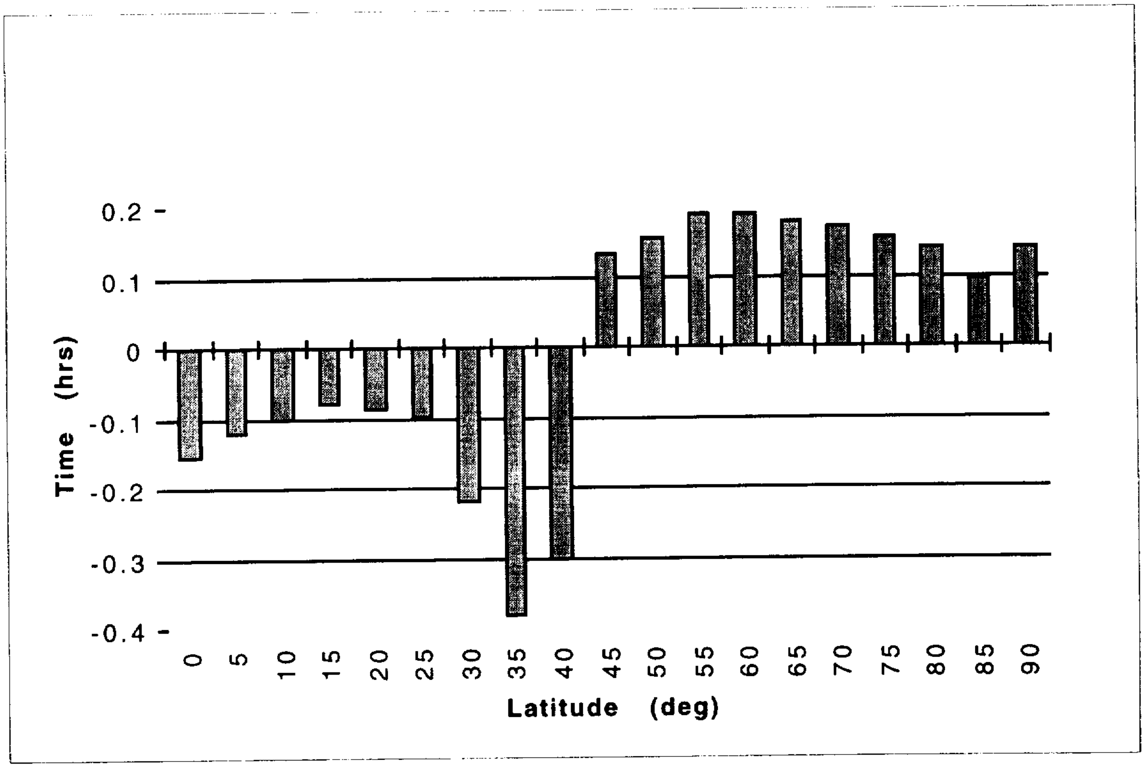

Figure 10: Difference between average times to obtain a fix (Case 3 - Baseline)

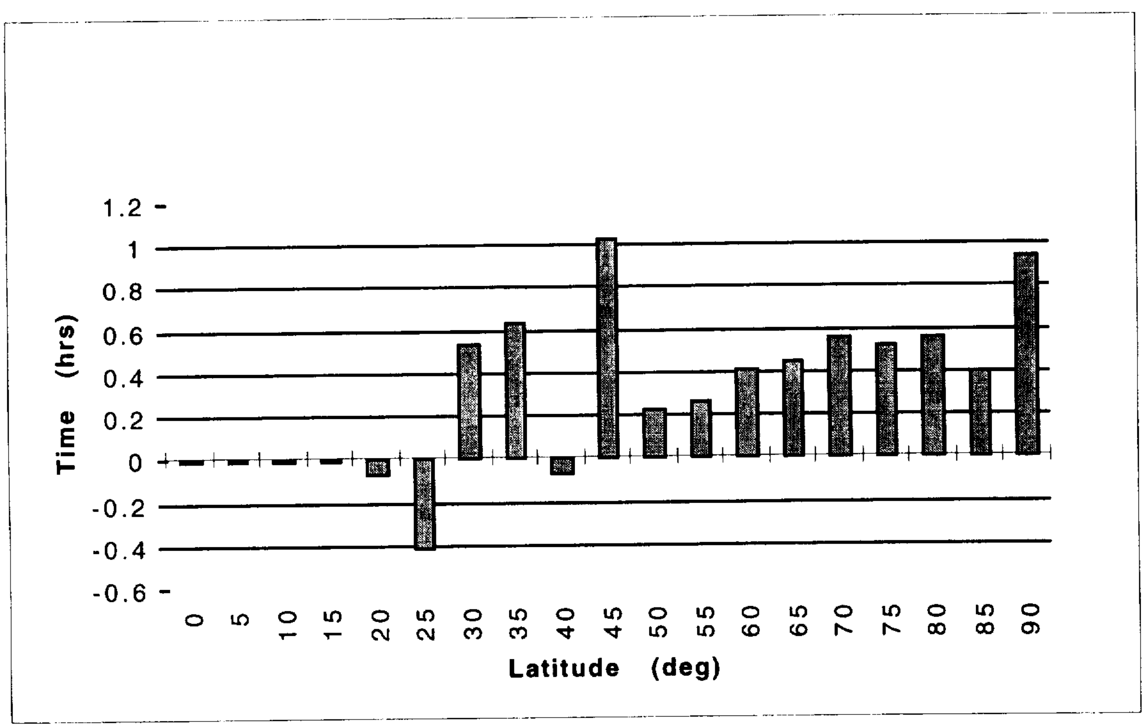

Figure 11: Difference between maximum times to obtain a fix (Case 3 - Baseline)

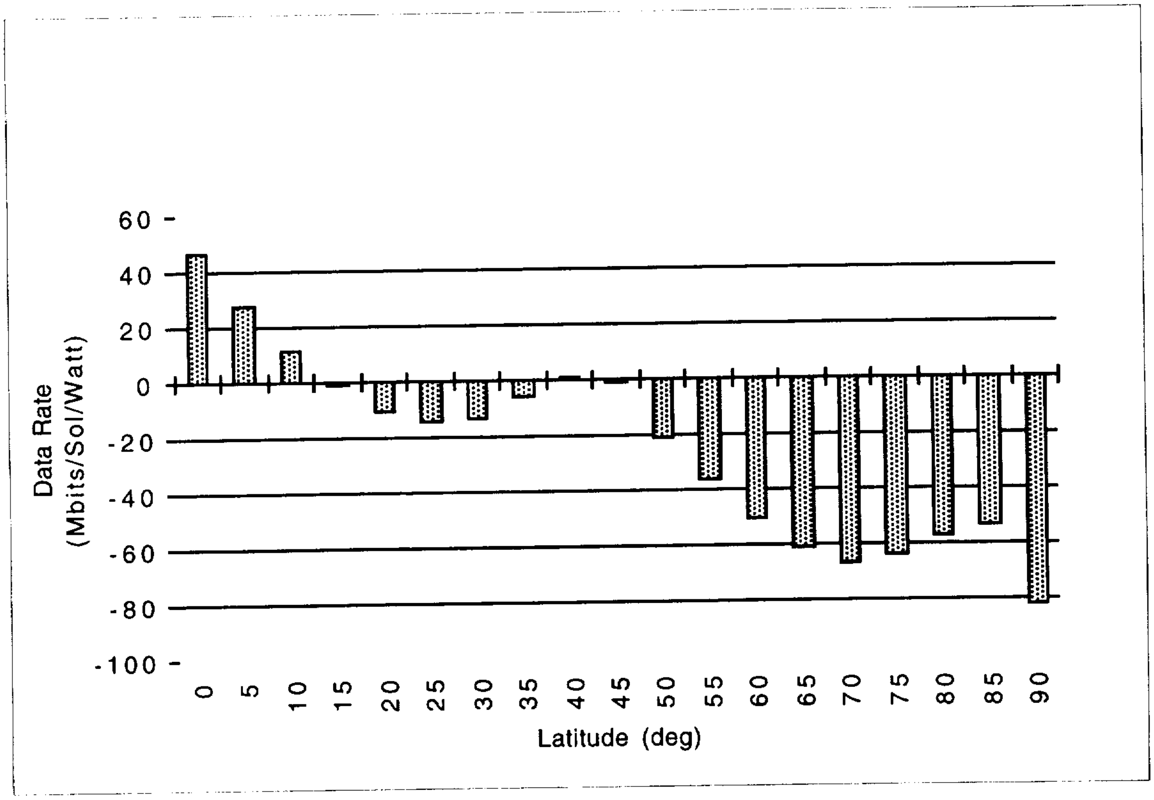

Figure 12: Difference in data rate (Case 3 - Baseline)

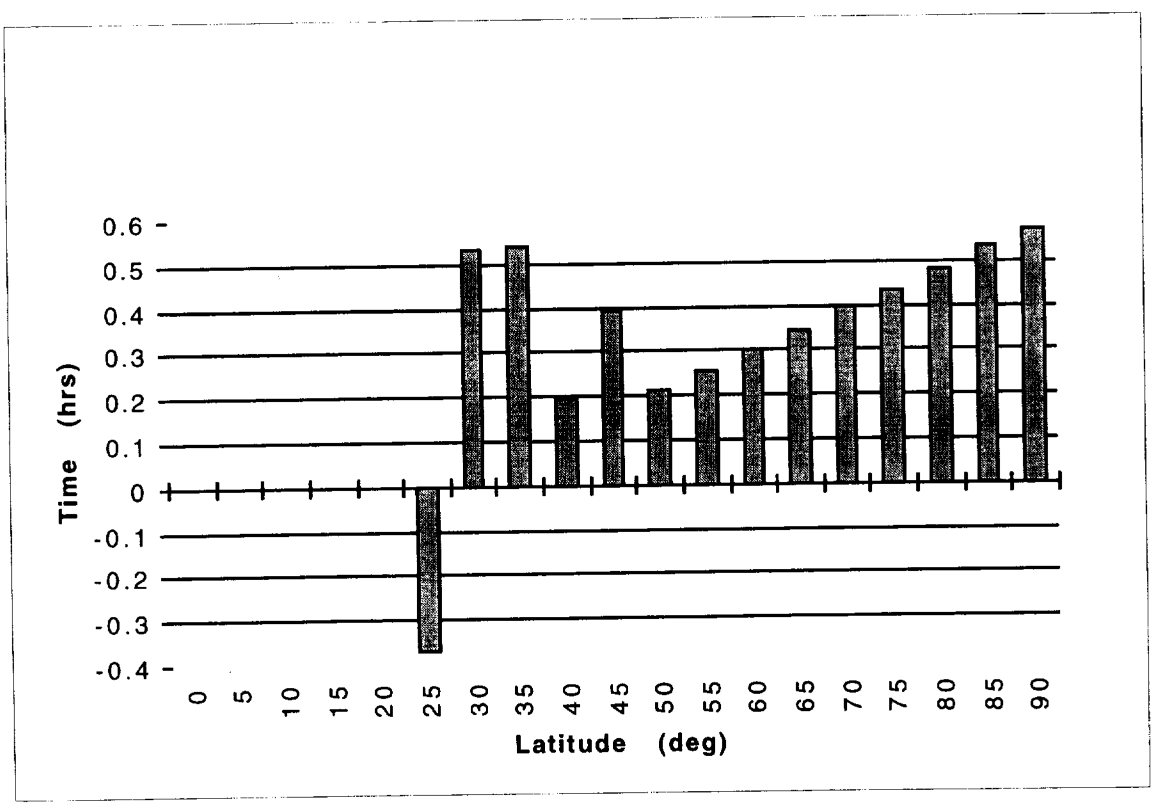

Figure 13: Difference in maximum wait time (Case 3 - Baseline)

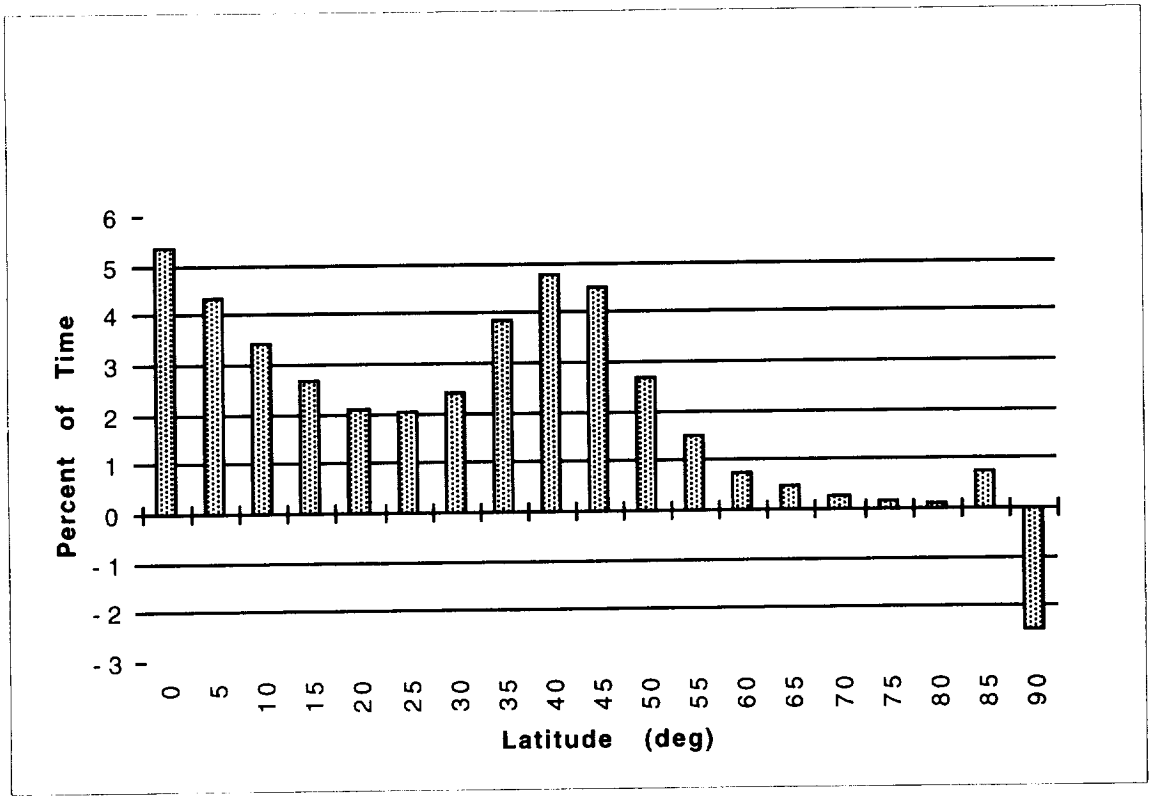

Figure 14: Difference in percent of time a satellite is in view (Case 3 - Baseline)

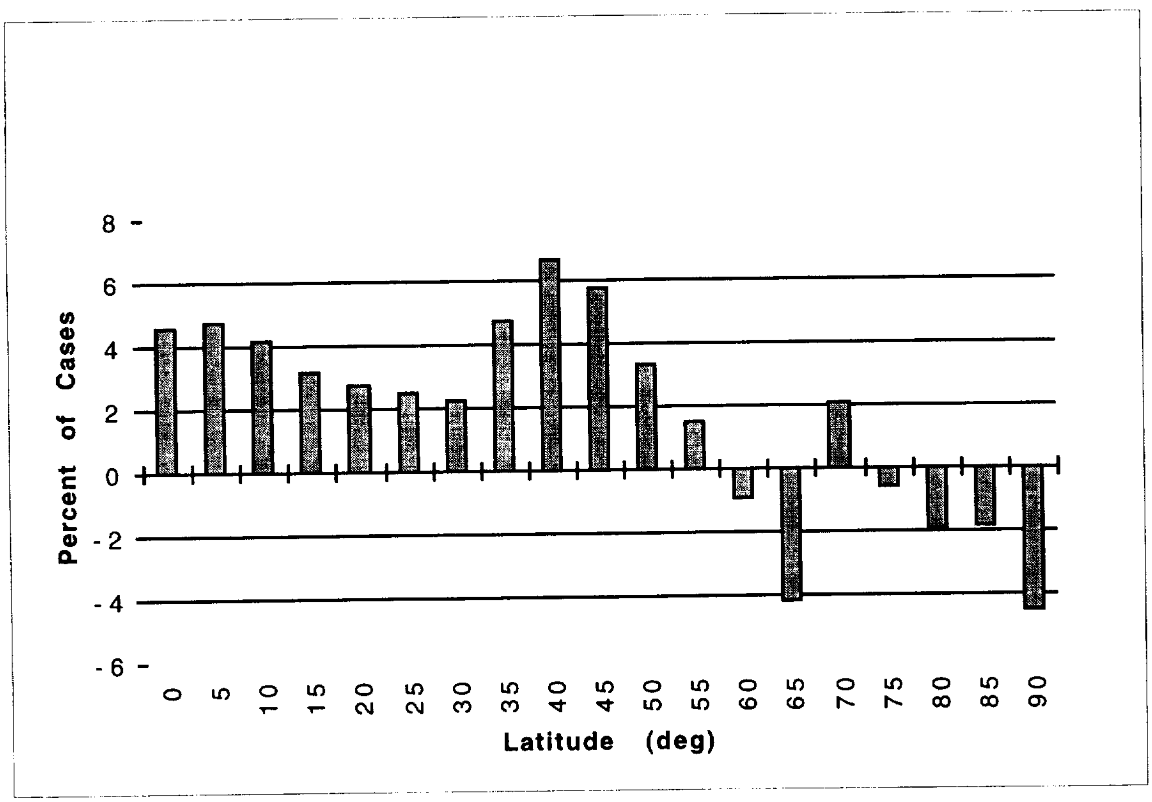

Figure 15: Difference in % of cases to obtain a fix to 10 m in 10 mins (Case 3 - Baseline)